Two-story frame with Bouc-Wen hysteretic links as a multi-degree of freedom nonlinear response simulator

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*This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy’s National Nuclear Security Administration under contract DE-NA0003525.

Introduction

Two story shear frame as MDOF simulator

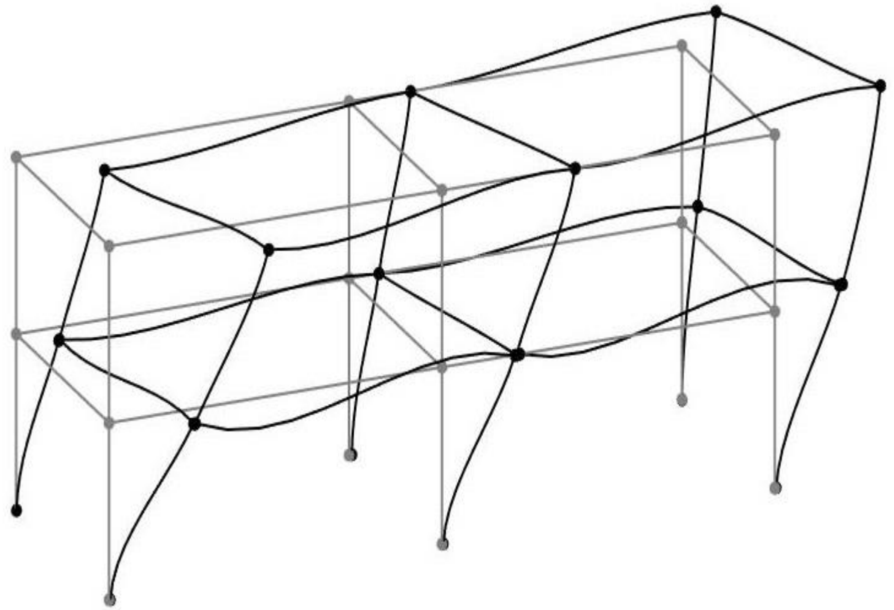
Two story shear frame with nonlinear links

Multi-degree of freedom with relatively high dimensionality

Nonlinear dynamical system
featuring ***hysteretic nonlinearities*** in nodal connections

Parametrized excitation & dependencies on structural traits

Proposed as a ***benchmark problem*** to validate methods in
structural health monitoring, model reduction, or identification applications.



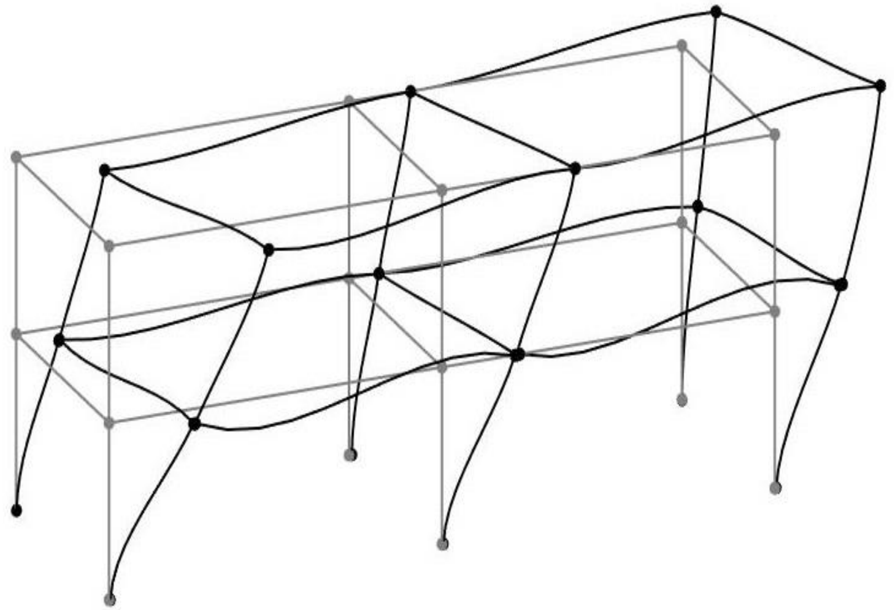
Introduction

Relation with existing SDOF benchmark

Viewed as an extension to the single degree of freedom
'Hysteretic Benchmark with a Dynamic Nonlinearity' problem

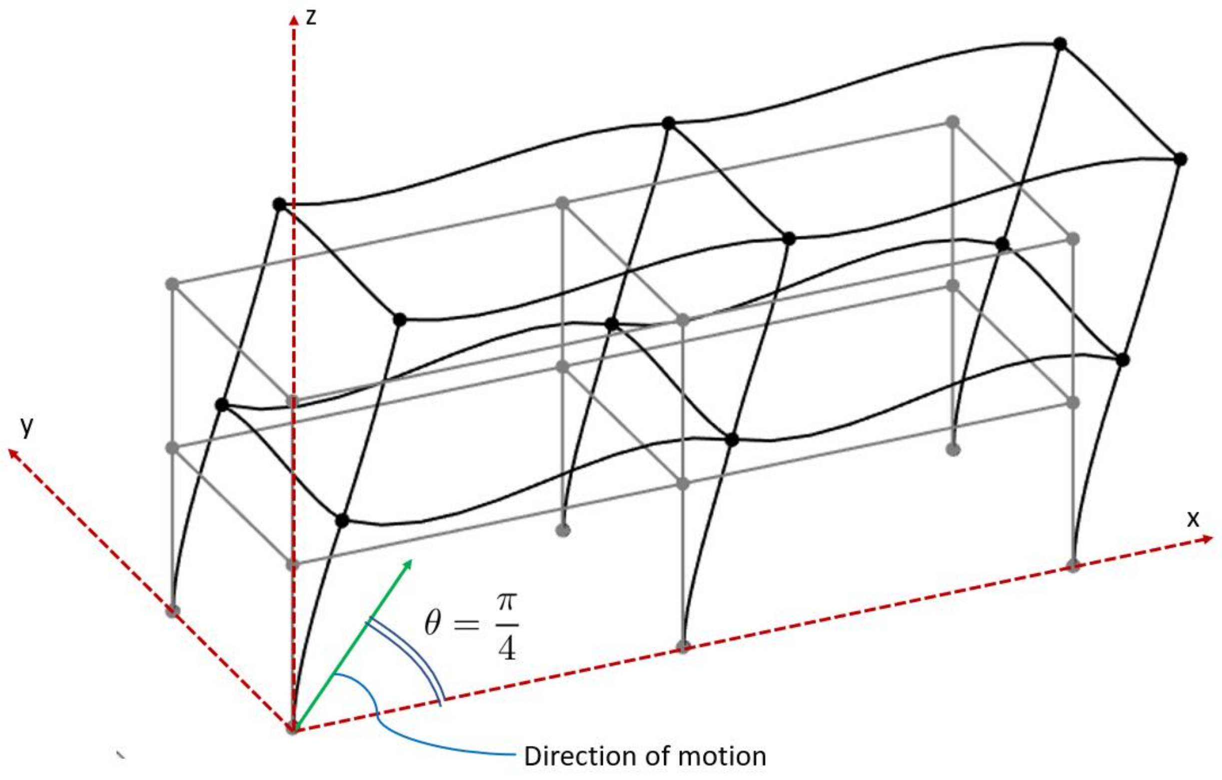
In addition, ***the MDOF frame allows for***:

- ***Increased complexity*** *studies due to the* ***dimensionality*** *of the system*
- *Potential for* ***multi-parametric numerical examples****.*
- ***Fully adjustable framework*** *providing flexibility on the user side*



Shear Frame

Configuration and properties



Description of default version

Horizontal beams and columns treated as ***three-dimensional finite beam elements***

Steel material properties & HEA cross-section

Rayleigh damping assumption

Ground-motion excitation of the xy-plane

Bottom nodes fully constrained

Additional functionality for ***plates, individual springs or lumped masses***

Nodal discretization as is (no intermediate integration points)

Shear Frame

Equation of Motion

The simulator is a nonlinear, parametric, dynamical structural system:

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{u}}(t) + \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}\right) = \mathbf{f}(t, \mathbf{p})$$

$$\mathbf{u}(t) \in \mathbb{R}^n, \mathbf{M}(\mathbf{p}) \in \mathbb{R}^{n \times n}, \mathbf{f}(t, \mathbf{p}) \in \mathbb{R}^n, \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t)\right) \in \mathbb{R}^n$$

Parametric dependency on *k* parameters denoted by: $\mathbf{p} = [p_1, ..., p_k]^T \in \Omega \subset \mathbb{R}^k$

Notation:

M is the system mass matrix

f is the vector of induced excitation

u is the displacement response time history

g are the nonlinear, state-dependent internal forces

Shear Frame

Equation of Motion

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Number of ***degrees of freedom***
Dimensionality/Order of the system

Nonlinearity encoded in ***restoring forces***
Hysteretic behaviour of nodal links

Parametric dependency on *k* parameters denoted by: $\mathbf{p} = [p_1, \ldots, p_k]^T \in \Omega \subset \mathbb{R}^k$

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Hysteretic Links

Setup assumptions and visualization

Assembly of hysteretic links

Nodal connections modelled as ***zero-length links***

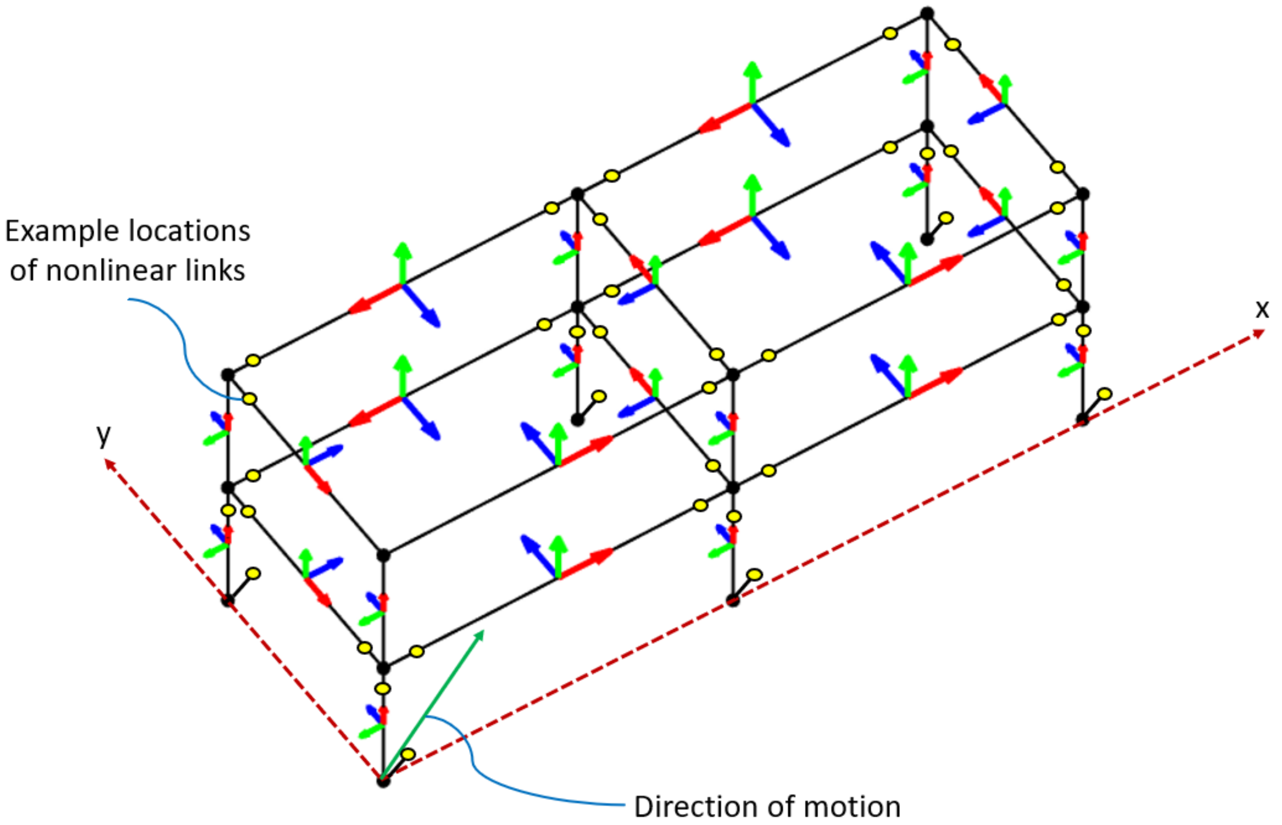
Virtual copies of nodes to enable link definition

Links located at
ground nodes, first story columns & all beams

Second story columns assume rigid connection

Alternative input assembly also available
(Links on all beam elements)

Restoring forces of links follow the
Bouc-Wen hysteretic formulation



Hysteretic Links

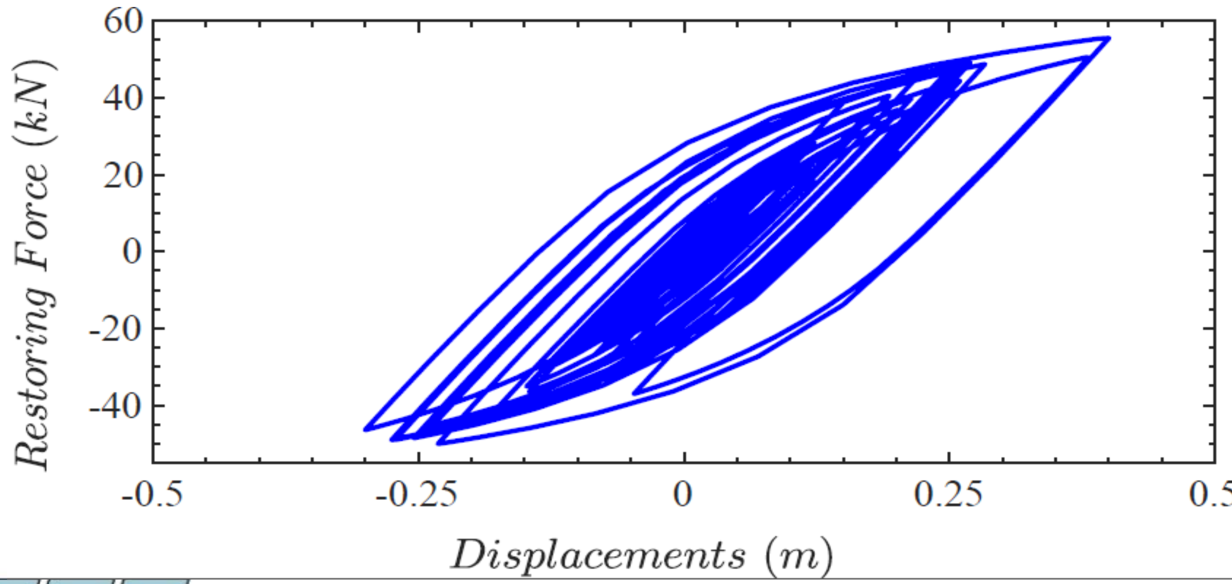
Bouc-Wen model formulation

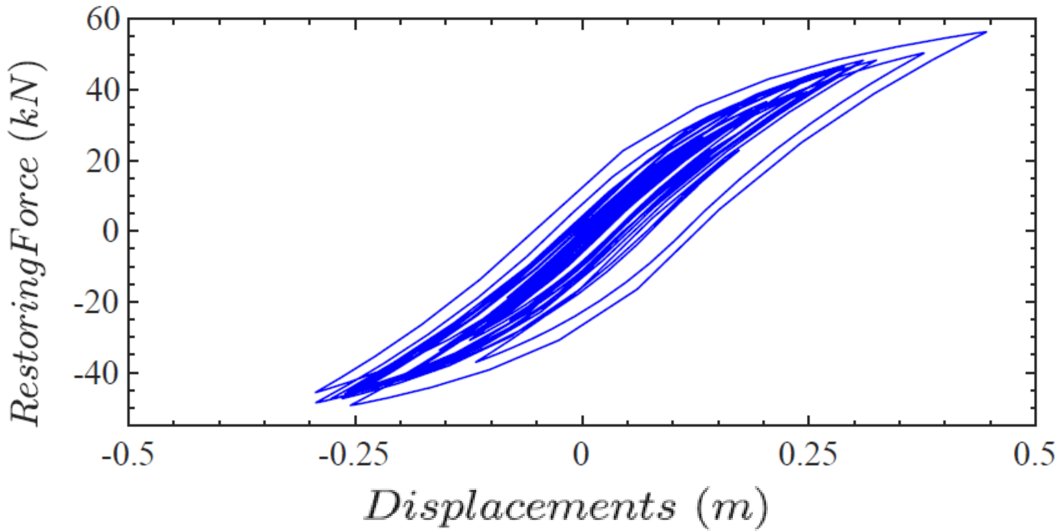
Hysteretic links response model

Total restoring force is composed of:

- ***Linear term*** => depends on the ***instantaneous*** nodal deformation
- Nonlinear, ***hysteretic term*** => ***History-dependent***

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1-\alpha) k \mathbf{z}$$





Bouc-Wen equation

Term z represents the dynamic, ***hysteretic terms***

Captures the ***memory of the system***

Obeys the first-order differential equation

$$\dot{\mathbf{z}} = A\dot{\mathbf{x}} - \beta|\dot{\mathbf{x}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{x}}|\mathbf{z}|^{w}$$

Hysteretic Links

Bouc-Wen model formulation

Hysteretic links response model

- *Total restoring force:*

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1-\alpha) k \mathbf{z}$$

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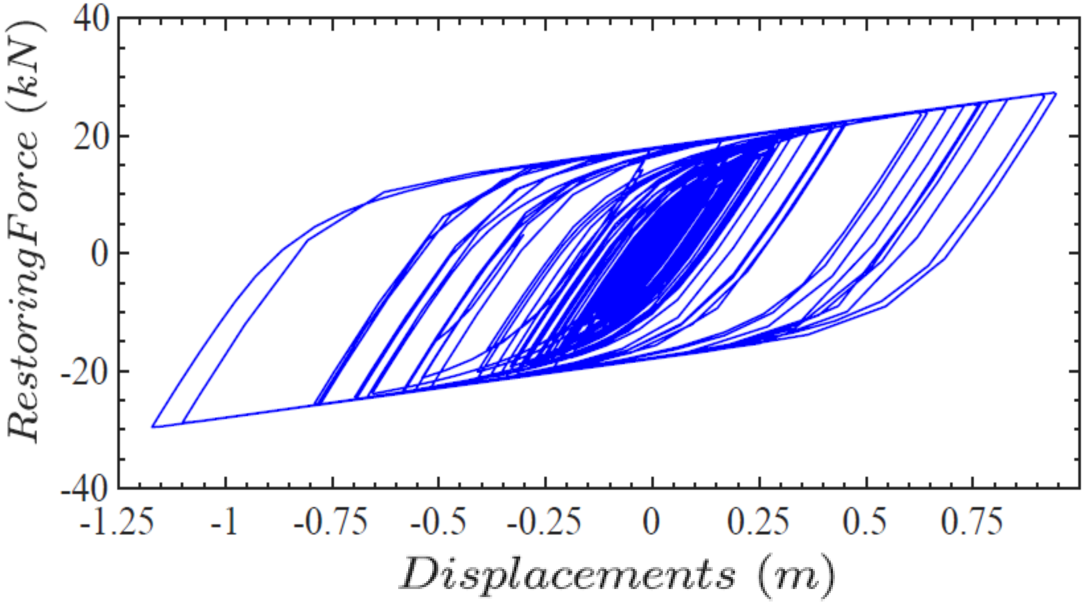
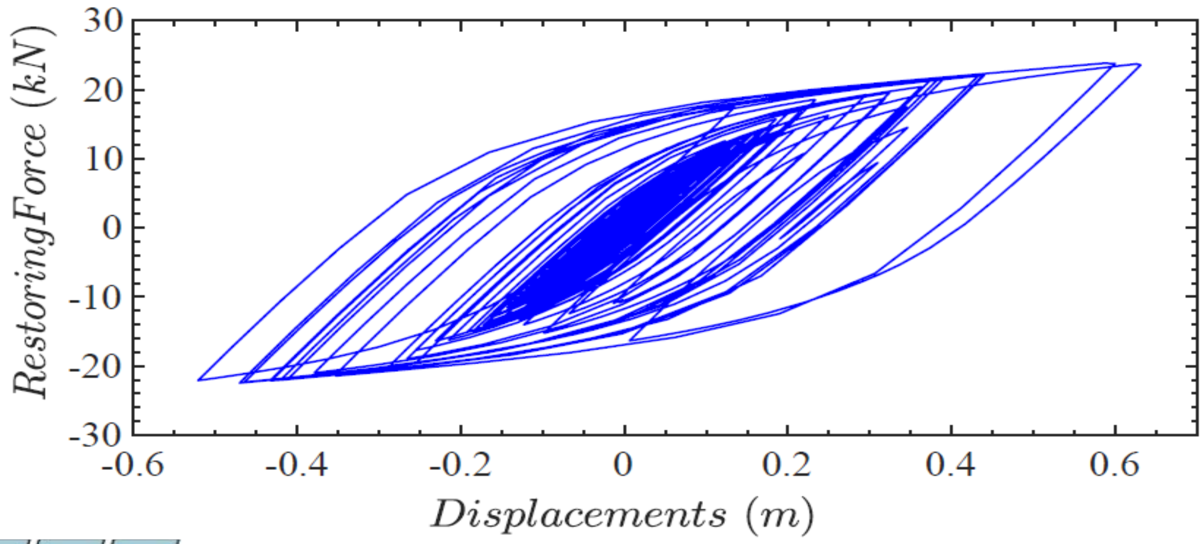




Strength Deterioration Effects

$$\nu(t) = 1.0 + \delta_{\nu}\epsilon(t)$$

Stiffness Degradation Effects

$$\eta(t) = 1.0 + \delta_{\eta}\epsilon(t)$$

Measure of the absorbed hysteretic energy: $\epsilon(t) = \int_0^t \mathbf{z}\dot{\mathbf{u}}\delta t$

Hysteretic Links

Bouc-Wen model formulation

Hysteretic links response model

- *Total restoring force:*

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1-\alpha) k \mathbf{z}$$

- *Bouc-Wen equation with degradation/deterioration effects:*

$$\dot{\mathbf{z}} = \frac{A\dot{\mathbf{u}} - \nu(t)(\beta|\dot{\mathbf{u}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{u}}|\mathbf{z}|^{w})}{\eta(t)}$$

$$\nu(t) = 1.0 + \delta_\nu \epsilon(t), \quad \eta(t) = 1.0 + \delta_\eta \epsilon(t), \quad \epsilon(t) = \int_0^t \mathbf{z}\dot{\mathbf{u}}\delta t$$

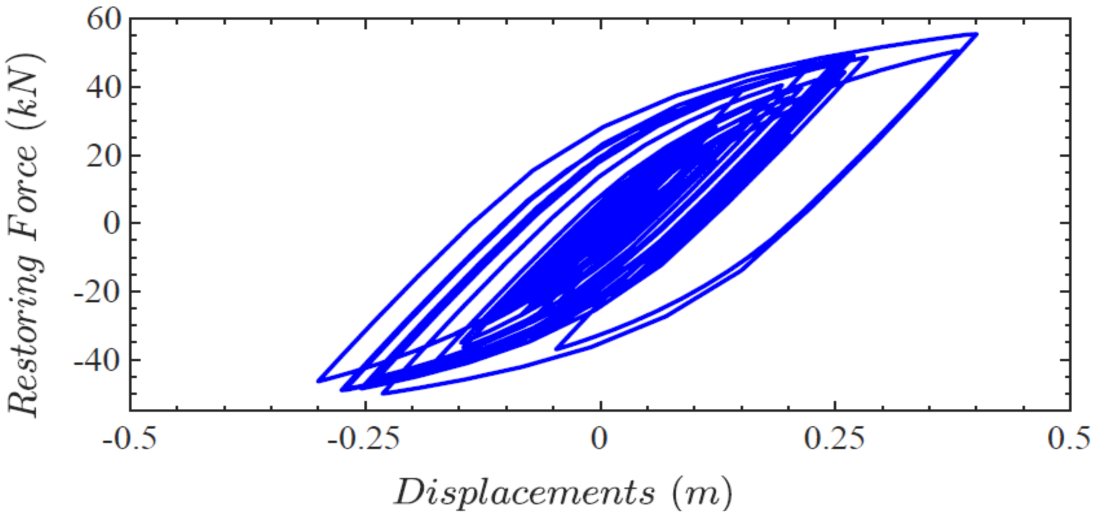
Characteristics of the Bouc-Wen links:

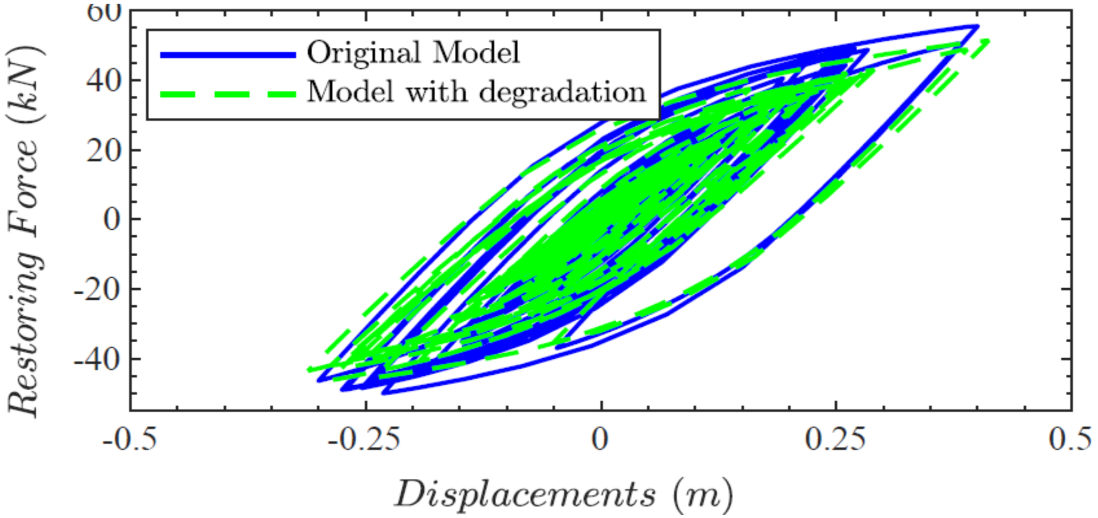
β, γ, A, w : *Control the smoothness, amplitude and shape of the hysteresis curve*

δ_ν, δ_η : *Degradation/Deterioration effects*

a, k : *Linear/Hysteretic contribution weighting*

=> Parametric dependencies of the hysteretic links





Hysteretic Links

Relation with SDOF benchmark

Hysteretic links response model

- ***MDOF total restoring force:***

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1-\alpha) k \mathbf{z}$$

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$$\dot{\mathbf{z}} = \frac{A\dot{\mathbf{u}} - \nu(t)(\beta|\dot{\mathbf{u}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{u}}|\mathbf{z}|^{w})}{\eta(t)}$$

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- ***SDOF Bouc-Wen oscillator EqoM & restoring force:***

$$m_L \ddot{v}(t) + r(v,\dot{v}) + z(v,\dot{v}) = f(t)$$

$$r(v,\dot{v}) = k_L v + c_L \dot{v}$$

- ***SDOF Bouc-Wen equation:***

$$\dot{z}(v,\dot{v}) = a\dot{v} - \beta(\gamma|\dot{v}||z|^{\nu-1}z + \delta\dot{v}|z|^{\nu})$$

Additional challenges of MDOF simulator

Increased dimensionality and interactions

Potentially ***multi-parametric context***

Framework simulator offering ***full flexibility for adaptation***

Implementations offers ***alternative framework versions***

=> ***Vectorized simulator***
with ***same Bouc-Wen model for all links***

=> ***Non-vectorized frame template***
evaluating each ***degree of freedom separately***

- ✓ Assemble ***user-defined function for Bouc-Wen*** evaluation
- ✓ Assemble ***different properties*** for each Bouc-Wen link

Hysteretic Links

Relation with SDOF benchmark

Hysteretic links response model

- *MDOF total restoring force:*

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- *SDOF Bouc-Wen equation:*

$$\dot{z}(v,\dot{v}) = a\dot{v} - \beta(\gamma|\dot{v}||z|^{\nu-1}z + \delta\dot{v}|z|^{\nu})$$

=> *Non-vectorized frame template*
evaluating each ***degree of freedom separately***

Loop over ***hysteretic links***

Assembles hysteretic link contributions
on stiffness matrix and internal forces

Loop over ***Degrees of Freedom***

- Check ***if degree is active*** => Hysteretic behaviour
- Evaluate nonlinearity => ***SDOF Bouc-Wen oscillator function***

Two story frame MDOF simulator

Inputs and Outputs

Standardized (default) version

Inputs:

- Bouc-Wen model parameters
- Excitation time history (frequency content etc.)
- Integration timestep and sampling frequency
- Initial conditions (optional)
- Damping ratios (optional)

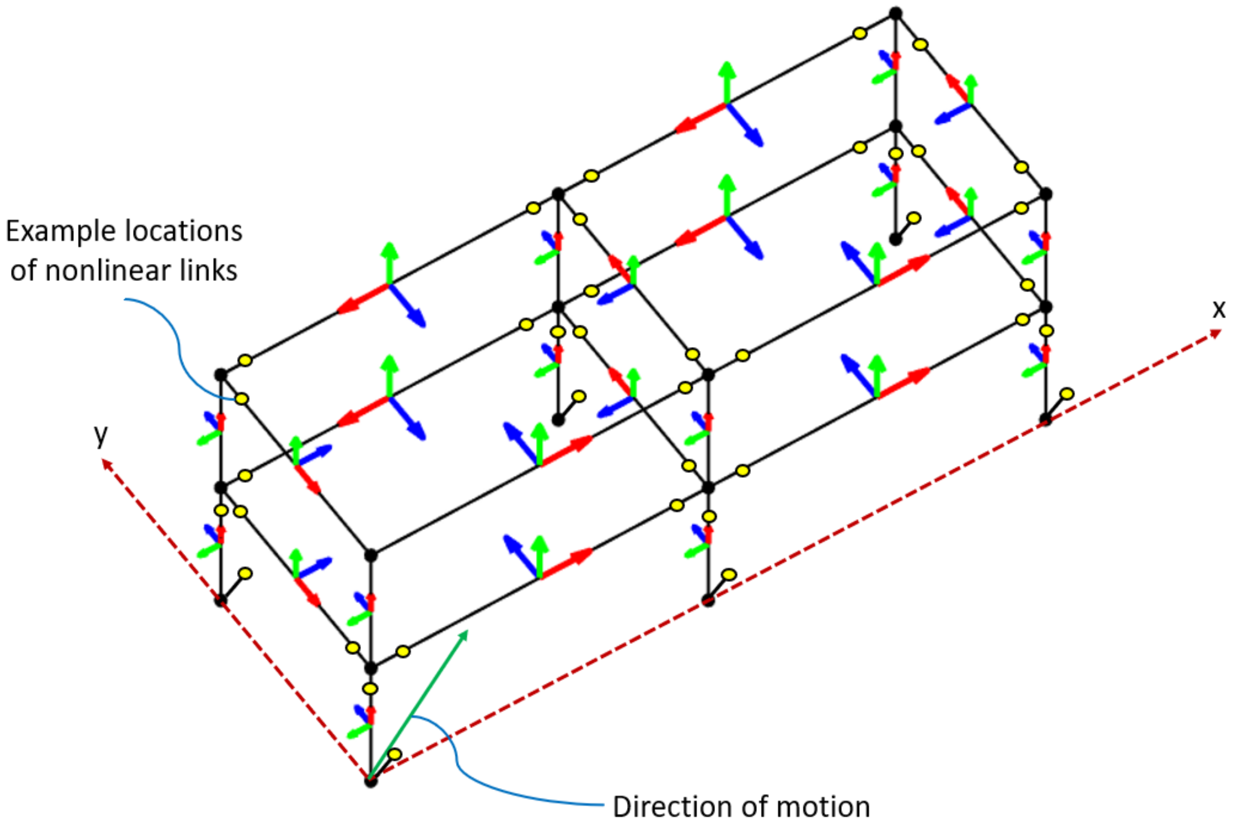
Outputs:

- Displacement time history
- Velocity and Acceleration time history
- Hysteretic curves of restoring forces (links)

Fully adjustable version

Inputs:

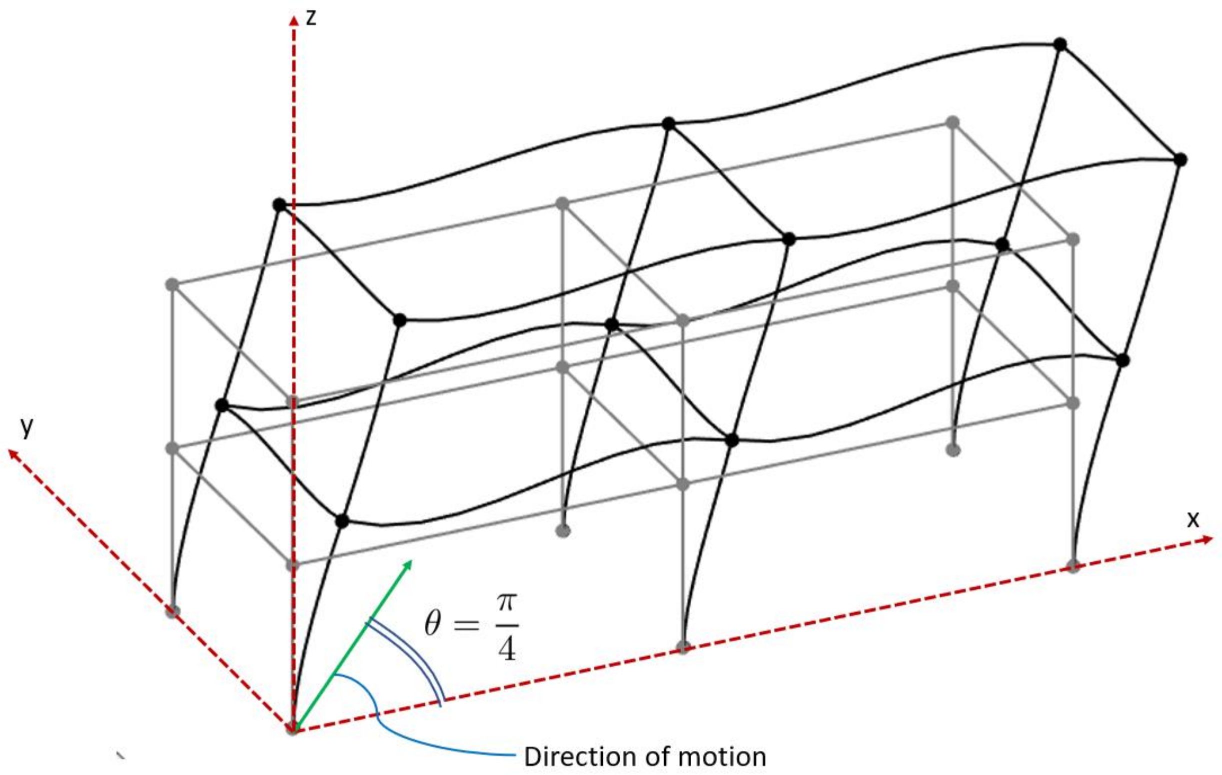
- User defined input file (geometry, materials)
- User defined Bouc-Wen function
- Different Bouc-Wen models for each link
- All inputs of standardized version



Research featuring the proposed benchmark

Parametric Reduced Order Modelling

Relevant Publication:
K. Vlachas, K. Tatsis, K. Agathos, A. R. Brink, and E. Chatzi, “A physics-based, local POD basis approach for multi-parametric reduced order models,” in International Conference on Noise and Vibration Engineering (ISMA2020)



Parametric Reduced Order Modelling framework

Derive pROM based on ***full-order simulations for training*** parametric ***configurations***

Validate pROM performance on capturing the response on ***validation parametric inputs***

Induced excitation based on ***synthetic accelerograms*** produced from stochastic model

Parametric Dependencies:

- Spectral characteristics of excitation
  => Frequency content
- Temporal characteristics of excitation:
  => Angle/ Direction of motion
- Traits of the hysteretic links
  => Shape and smoothness parameters

Research featuring the proposed benchmark

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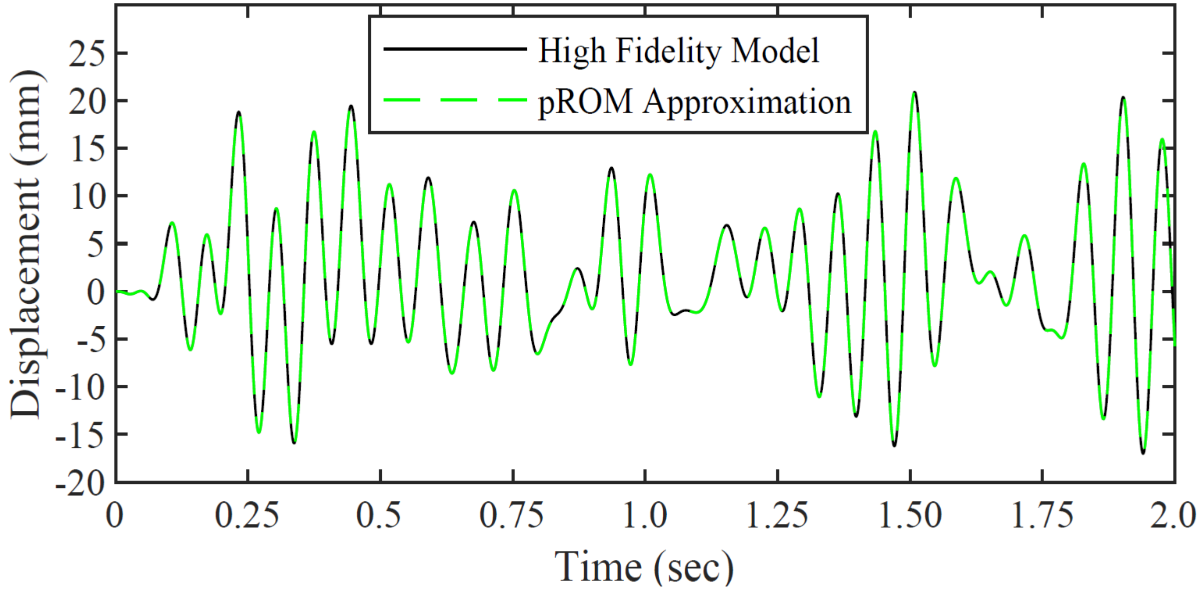
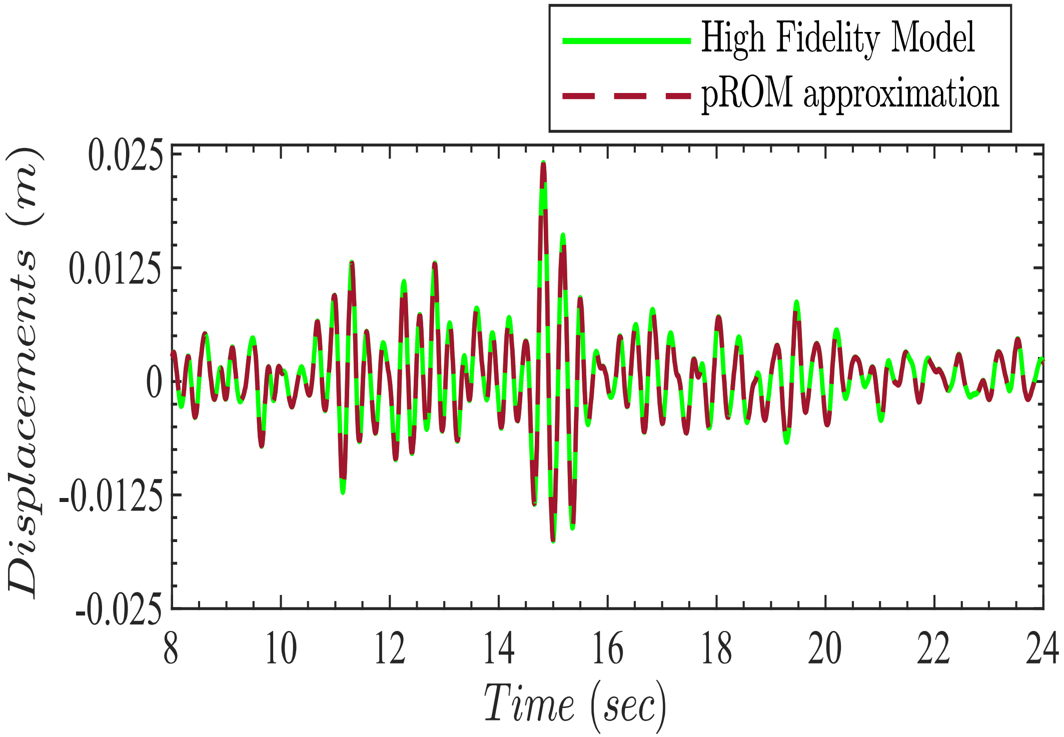
Accuracy performance of the pROM



Response approximation on synthetic earthquake



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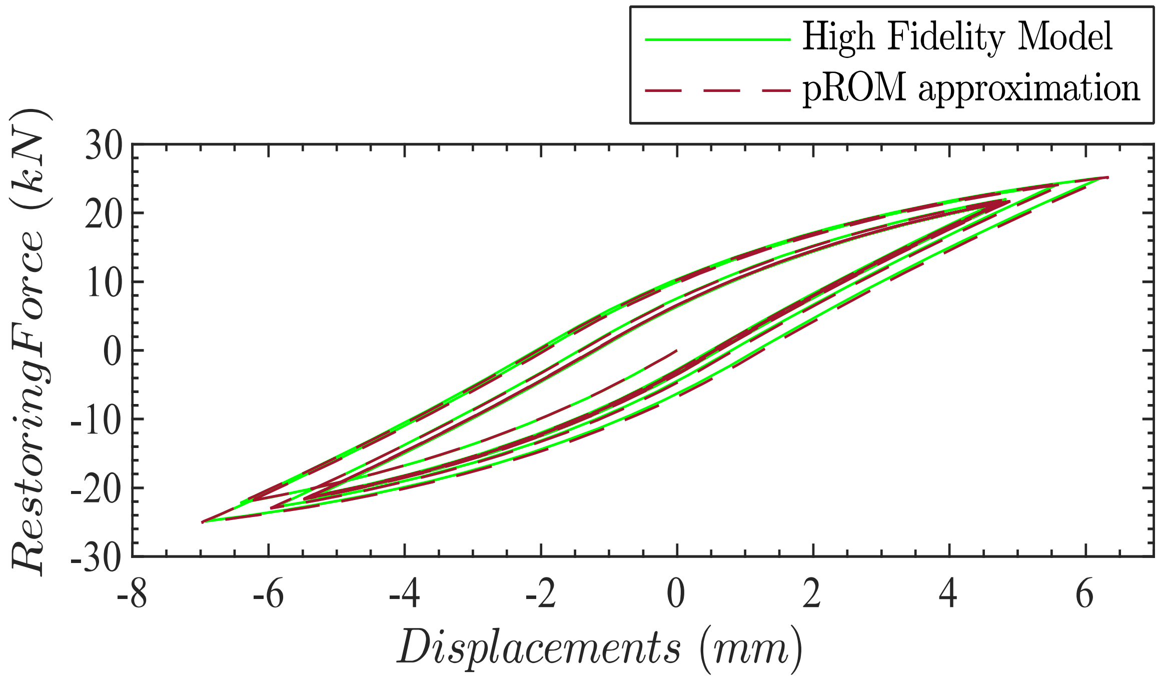

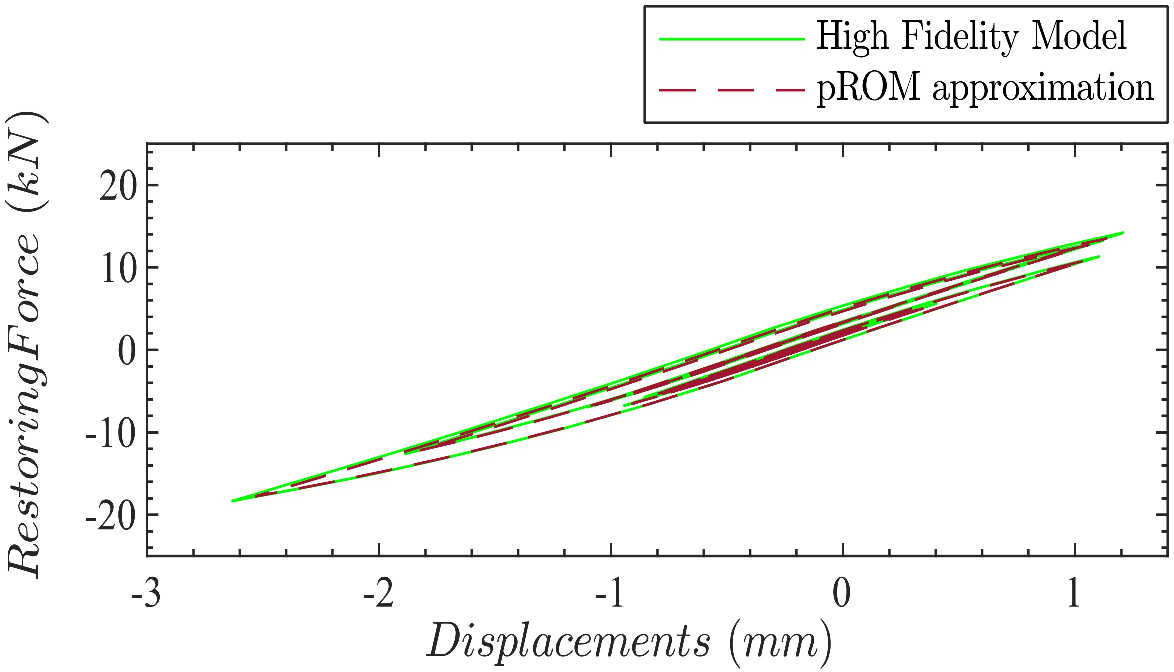

Hysteresis curve approximation

Research featuring the proposed benchmark

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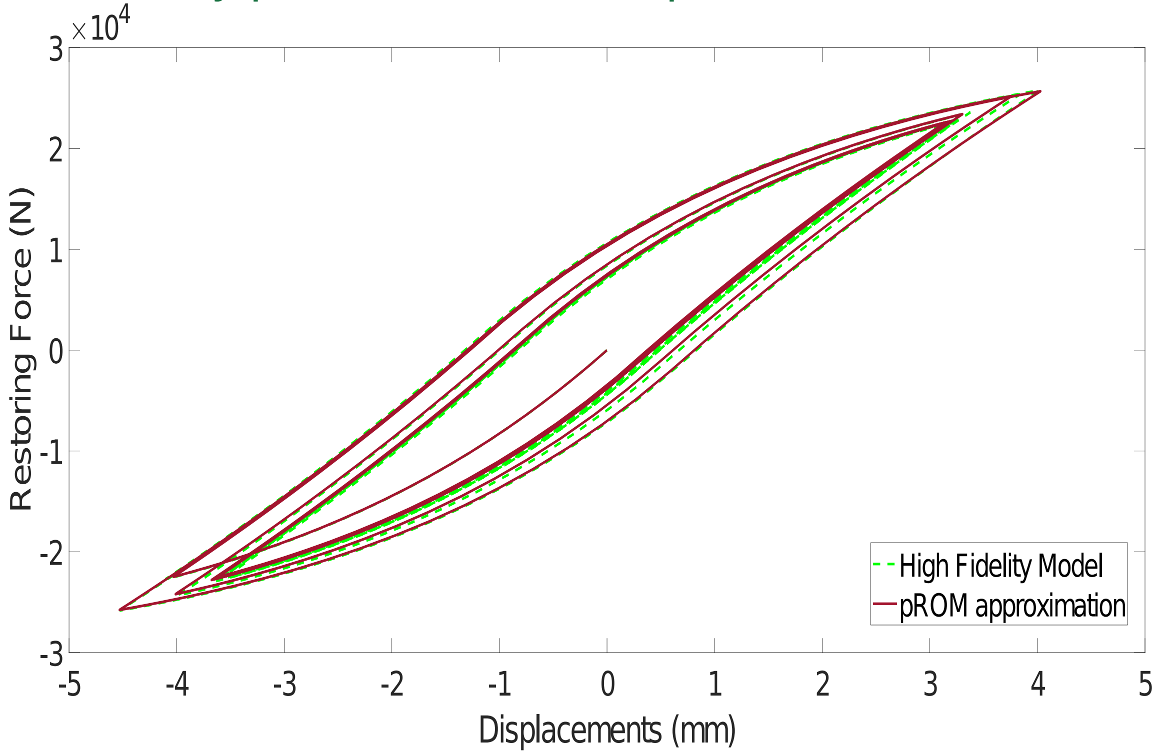

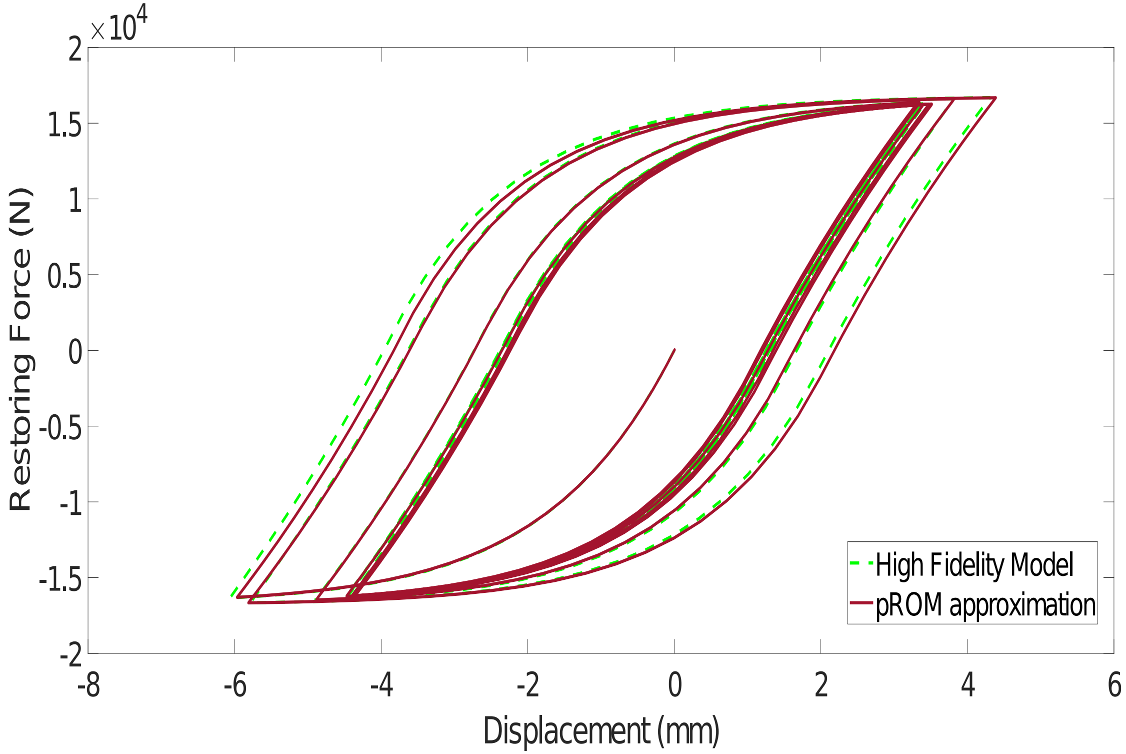

Hysteresis curve approximation

Research featuring the proposed benchmark

Parametric Reduced Order Modelling

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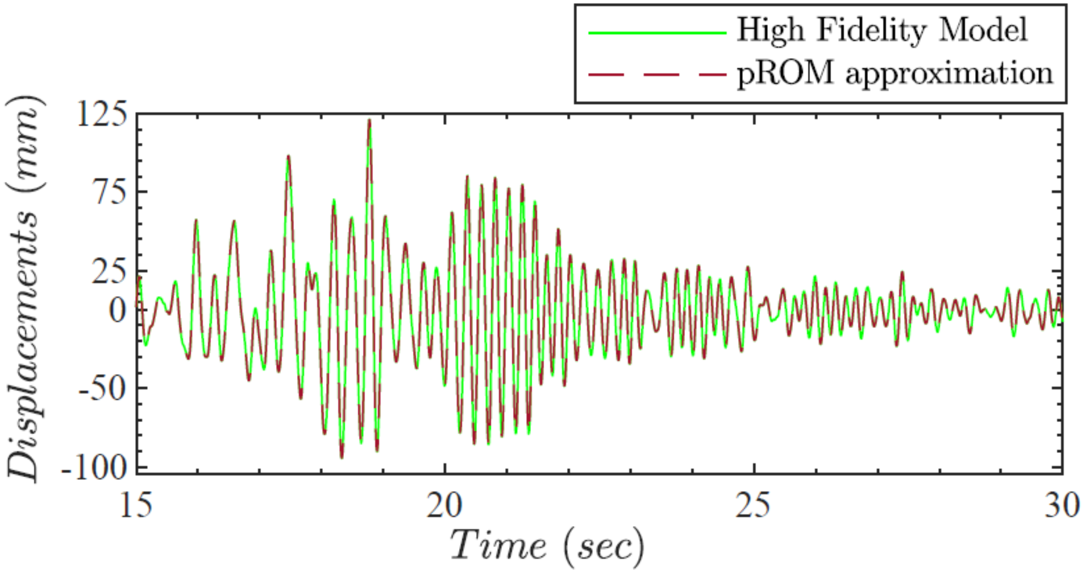
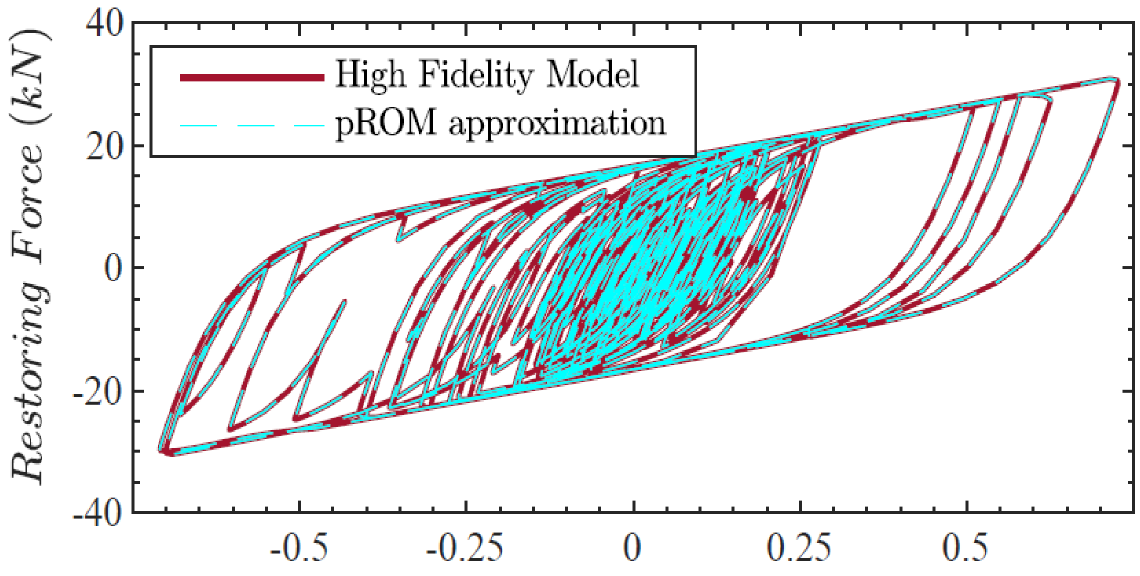
K. Vlachas, K. Tatsis, K. Agathos, A. Brink, and E. Chatzi, “A local basis approximation approach for nonlinear parametric model order reduction,” Journal of Sound and Vibration, vol. 502, p. 116055, 2021

Accuracy performance of the pROM



Response approximation on ground motion



Hysteretic curve approximation

Research featuring the proposed benchmark

Parametric Reduced Order Modelling

All relevant references featuring the two story frame

K. Vlachas, K. Tatsis, K. Agathos, A. R. Brink, and E. Chatzi, “A physics-based, local POD basis approach for multi-parametric reduced order models,” in International Conference on Noise and Vibration Engineering (ISMA2020)

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T. Simpson, N. Dervilis, and E. Chatzi, “A machine learning approach to model order reduction of nonlinear systems via autoencoder and LSTM networks,” Journal of Engineering Mechanics, (Forthcoming)

K. Vlachas, K. Tatsis, and E. Chatzi, “Earthquake-induced damage estimation in structural systems using parametric physics-based reduced-order models,” (Forthcoming)

Two story frame MDOF simulator

Structural Identification Context

Challenges expected:

- The Bouc-Wen links cause a nonlinear response characterized by hysteresis and featuring memory.
  => ***The system contains a dynamic nonlinearity***
- The behavior of the hysteretic links ***is governed by an internal, not measurable variable*** **z**
- ***Relatively high dimensionality*** and the presence of ***multiple Bouc-Wen links*** increases the complexity of any task at hand.
- The equation that governs the evolution of the hysteretic forcing is ***nonlinear with respect to parameter w***.
- On top of that, a ***finite Taylor series expansion is not possible*** due to the presence of absolute values.

$$\dot{\mathbf{z}} = \frac{A\dot{\mathbf{u}} - \nu(t)(\beta|\dot{\mathbf{u}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{u}}|\mathbf{z}|^{w})}{\eta(t)}$$

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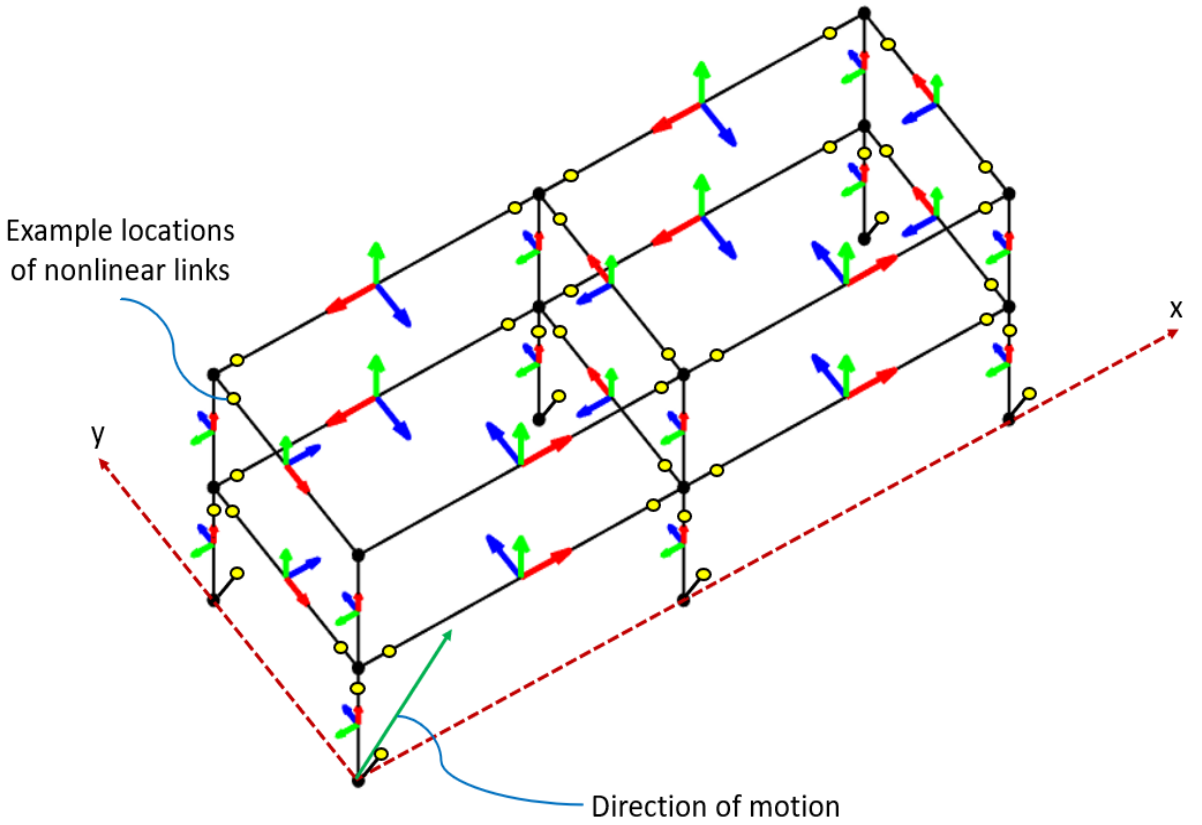
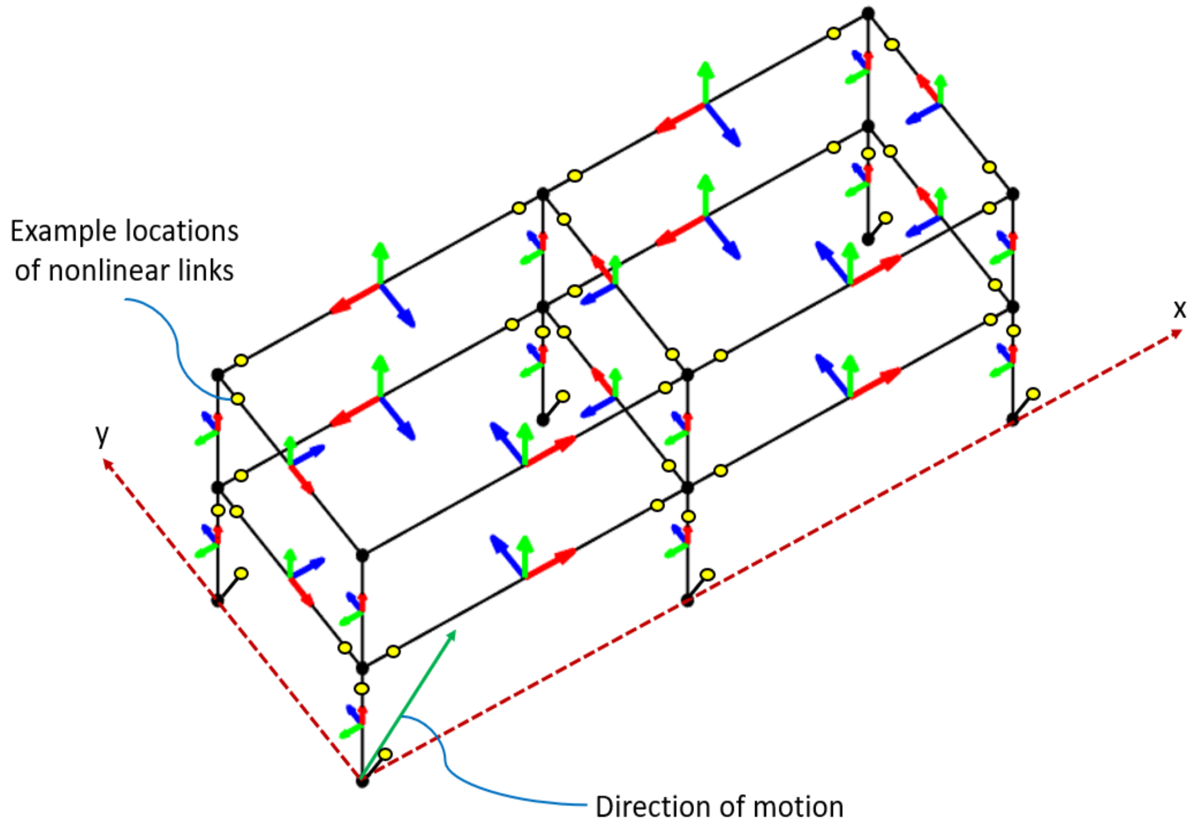


Two story frame MDOF simulator

Structural Identification Context

Potential SSYID tasks:

- **Estimation of the nominal values of parameters** a, k of the Bouc-Wen hysteretic links,
  => *Representing a form of mechanical damage induced in the healthy (linear) state*
- ***Estimation of the nominal values of parameters*** δ_ν, δ_η of the Bouc-Wen hysteretic links
  => *Representing a condition deterioration or damage growth scenario*
- ***Estimation of the nominal values of parameters*** β, γ of the Bouc-Wen hysteretic links
  => *Representing a form of model fitting or model calibration during design or operation stages of SHM*
- ***Joint input-state estimation using (limited) output measurements***



Two story frame MDOF simulator

Standardized SSYID datasets

Sets of data for reference and results comparison

- ✓ Each set contains ***test/train/validation datasets***
- ✓ ***Fixed*** Bouc-Wen ***parametrization***
- ✓ ***Random-phase multisine*** excitation (available in standardized version)
- ✓ Each dataset contains ***twenty-one signals***
- ✓ Signals designed to excite the nonlinear behavior of the system in three levels of discrete intensity:
  Strong nonlinearities / milder phenomena /even weaker effects

Two story frame MDOF simulator

Initial thoughts on SSYID approach

Relevant Publication:
K. Tatsis, T. Simpson, and E. Chatzi, “Bayesian and Genetic Methods for Model Selection of Greybox Modelling,” Workshop on Nonlinear System Identification Benchmarks ,2019

Approach overview

✓ Employ **Bayesian selection relying on physics-based models**
=> ***Bayesian filtering - The Unscented Kalman filter***

✓ Utilize **Genetic algorithms based on time series models**
=> Polynomial NARX modelling and GA optimization

Model Selection for parameter identification and similar SSYID tasks

Questions