Two-story frame with Bouc-Wen hysteretic links as a multi-degree of freedom nonlinear response simulator

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1 Introduction

A diverse variety of engineering and dynamic systems, ranging from control applications and solid mechanics to biology and economics, feature hysteretic phenomena. This commonly encountered nonlinear behavior can be captured and described via diverse numerical models, with the Bouc-Wen representation comprising a common choice within the nonlinear dynamics and vibration engineering community. In this benchmark, the Bouc-Wen model is employed to characterize the response of the nodal connections of a two-story frame structure. The resulting shear frame with hysteretic links is proposed as a multi-degree of freedom nonlinear response simulator.

This case study can be seen as an extension to the single degree of freedom 'Hysteretic Benchmark with a Dynamic Nonlinearity' problem, which is already featured in the nonlinear benchmark catalogue [1]. Our simulator employs a similar parameterized representation of the Bouc-Wen model for each nonlinear link and builds upon it to also include strength deterioration and stiffness degradation effects in a structure with increased dimensionality. As a result, the featured parametric shear frame serves as a multi-degree of freedom nonlinear response simulator, able to model a wide range of nonlinear effects through the parametrized Bouc-Wen couplings.

The provided simulator can be utilized as a benchmark problem to validate methods and tools in structural health monitoring, model reduction, or identification applications. It has already been exploited in a reduced order modelling context to validate the performance of parametric, physics-based ROMs for nonlinear, dynamical systems in [2, 3, 4]. Compared to the existing Bouc-Wen oscillator benchmark study in [1], described in detail in [5], the proposed multi-degree of freedom simulator allows for increased complexity studies due to the higher dimensionality of the system and the potential for multi-parametric numerical examples. In addition, the proposed benchmark is provided as a framework simulator and not a single function, thus offering full flexibility for the user to modify and evaluate the shear frame based on customized needs and requirements of the underlying problem.

This document provides a detailed description of the proposed multi-degree of freedom nonlinear response simulator. Section 2 presents the assembly of the shear frame structure, including geometrical configuration and material parameters, whereas section 3 documents the mathematical formulation of the parametrized Bouc-Wen model governing the nonlinear restoring force mapping of the hysteretic links. User guidelines for proper input and accurate simulations are provided in section 4, along with a detailed overview of the capabilities of the provided software. This serves as a short overview of the user to treat the proposed benchmark as a fully adjustable case study. Finally, section 5 discusses the challenges associated with performing nonlinear system identification in the featured benchmark and proposes standardized system identification tasks related to the benchmark along with the respective provided datasets. Although the respective implementation is fully parametrized and all relevant software files are made available offering direct access to specification of the geometry, the material properties or further desired properties, this document refers to a default, standardized version of the simulator if not stated otherwise.

The code repository of the simulator is published under the Apache License, Version 2.0 on GitHub here[1], whereas the standardized datasets described in this study can be found in the Zenodo open-access repository here [2].

2 Description of shear frame

The shear frame structure proposed as a multi-degree of freedom simulator is graphically depicted in Figure 1. The three dimensional frame comprises of columns and horizontal beams, featuring identical material and cross section properties, which are modeled as finite beam elements. Additional functionality for the assembly of plates, individual springs or lumped masses is provided, based on the documentation in section 4. The geometrical and material parameters of the default version of the frame are summarized in Table 1. The nodal discretization depicted in Figure 1 is equivalent to the one employed for integration purposes, as well as for assembly of the system matrices, as no intermediate nodal points are assumed.

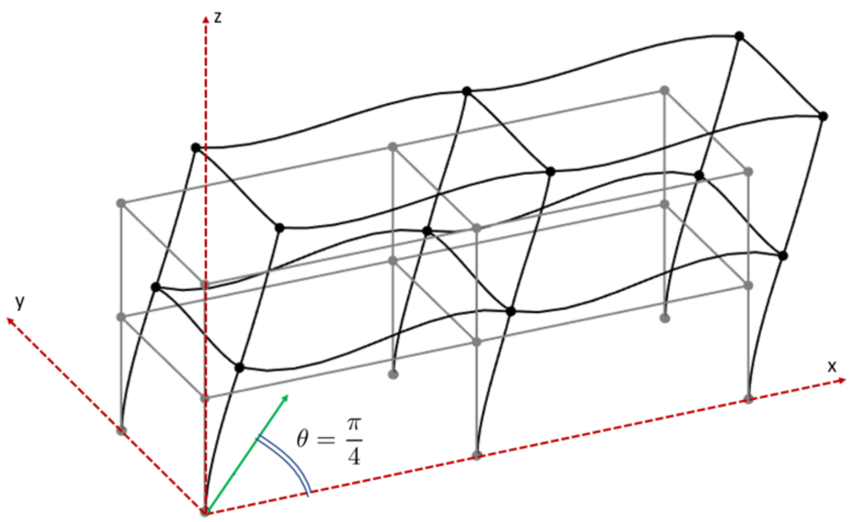


Figure 1: Configuration of the two story shear frame proposed as a simulator (default version). An example deformed state is presented in black. The green vector indicates the direction of motion of the xy-plane.

As illustrated in Figure 1, frame is defined on the basis of the depicted Cartesian system. The six bottom nodes of the frame are assumed fully constrained, representing the ground. As already explained, the geometrical configuration of the frame in Figure 1 is a representative one of the default version of the frame based on the input files provided. The xyz system presented is the reference coordinate system to define all relevant properties across this documentation. The users are offered full access to modify this setup following the guidelines of section 4. The shear frame is excited assuming a ground motion scenario, as defined via specification of a respective acceleration record. The direction of motion is defined within the xy plane

[1] https://github.com/KosVla/NonlinearBoucWenFrameBenchmark.git

[2] https://zenodo.org/record/4742248 # .YS37bd-xWUk, https://doi.org/10.5281/zenodo.4742248

(horizontal plane), under an angle θ with respect to the x-axis, as presented in Figure 1, and assumes a default value of $\pi/4$. This implies that there is no excitation along the z-axis. The user also has the possibility of defining a three dimensional angle, an accelerogram or a loading time-history as well by slight modification on the framework. Further details on defining the input excitation can be found in section 4. Based on the respective accelerogram the nodal excitation is assembled by multiplying the acceleration with the respective terms on the lumped mass matrix as a simplifying assumption.

Table 1: Geometric and mechanical properties of the frame.

Geometric configuration	Material parameters	Natural frequencies (Hz)
Frame Length(m): 7.5	Young modulus (GPa): 210	1^{st} - 3.78
Total Length(m): 2*7.5	Poisson Ratio: 0.30	2^{nd} - 4.61
Frame Width(m): 5	Density(kg/m^3): 8000	3^{rd} - 5.04
Story Height(m): 3.20	Damping: 2% modal in first two modes	4^{th} - 5.99
Cross-section properties: HEA 200		5^{th} - 7.00

The response of the two story shear frame follows the equations of motion of a nonlinear dynamical system with dependencies on l parameters, contained in vector $\mathbf{p} = [p_1, ..., p_l]^{\mathrm{T}} \in \Omega \subset \mathbb{R}^l$. The governing set of equations is:

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{x}}(t) + \mathbf{g}\left(\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{p}\right) = \mathbf{f}(t, \mathbf{p}), \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^N$ represents the displacement vector, $\mathbf{M}(\mathbf{p}) \in \mathbb{R}^{N \times N}$ denotes the (consistent) mass matrix of the system and $\mathbf{f}(t, \mathbf{p}) \in \mathbb{R}^N$ corresponds to the induced excitation. The dimension of the system is N, which physically corresponds to the total number of degrees of freedom. Nonlinear effects are encoded in the restoring force vector $\mathbf{g}\left(\mathbf{x}(t), \dot{\mathbf{x}}(t)\right) \in \mathbb{R}^R$ which represents the internal forces of the system and is further dependent, along with the external excitation, on the parameter vector $\mathbf{p}$. In the proposed multi-degree of freedom simulator the nonlinearities are caused by hysteretic links between elements, representing nonlinear behavior in nodal connections. A detailed documentation on the assembly and formulation of these links is given next.

3 Hysteretic links modeling

The nonlinear effects on the response of the shear frame presented in section 2 are induced due to the presence of hysteretic internal forces herein assumed to be concentrated on the connections (joints) of the frame. To materialize this nonlinear joint behavior virtual nodes are defined. These are illustrated in Figure 2, where the virtual nodes are annotated.

The corresponding hysteretic links materializing each connection assume no length, although the virtual nodes are depicted within a distance from the reference node in Figure 2 for demonstration purposes. In the default version of the shear frame not all connections are represented as hysteretic links. This is also presented in Figure 2. Namely, the ground nodes, the columns of the first floor and all beams are connected via the hysteretic link representation, whereas the columns of the second floor assume a typical, rigid connection. Additional versions are also provided based on the documentation in section 4. The user can flexibly modify this setup by assembling a new input file according to the guidelines in section 4 and the provided ReadMe file.

To model the nonlinear behavior of the hysteretic links our implementation is based on the Bouc-Wen formulation [6]. The Bouc-Wen model has been widely utilized to represent hysteretic phenomena [7, 8] and is considered a dominant formulation, able to represent a variety of shape curves and nonlinear or damage effects, stiffness degradation and strength deterioration. The interested reader is referred to [9, 10] for a more extensive review and discussion on this formulation.

In the two story frame simulator, the total restoring force of the hysteretic links is composed of a linear term that depends on the instantaneous nodal deformation values, and of a nonlinear, hysteretic and thus

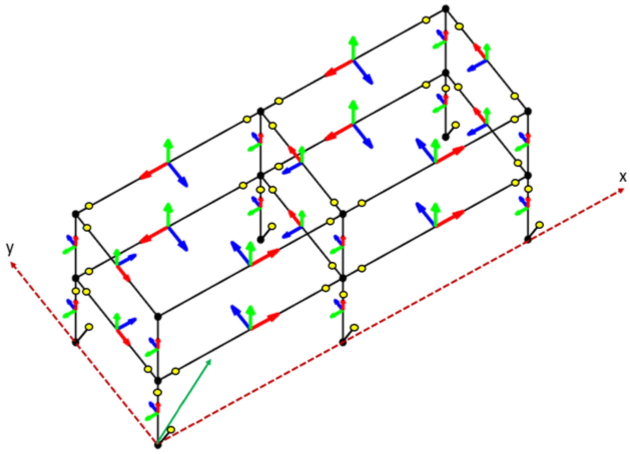


Figure 2: Graphical demonstration of the virtual nodes, employed to define the hysteretic links, visualized as yellow dots. Hysteretic links have no length, the nodes are depicted apart for visualization.

history-dependent term. The respective equation reads:

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k\mathbf{x} + (1-\alpha)k\mathbf{z} \tag{2}$$

where $\mathbf{R}$ denotes the vector of restoring forces, $\mathbf{x}$ the nodal displacements and α, k are characteristic parameters of the Bouc-Wen model. The hysteretic variable $\mathbf{z}$ represents the dynamic, hysteretic terms of the links, capturing the memory of the system, as discussed in [5]. Variable $\mathbf{z}$ controls the hysteretic forcing and obeys the first-order differential equation:

$$\dot{\mathbf{z}} = A\dot{\mathbf{x}} - \beta|\dot{\mathbf{x}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{x}}|\mathbf{z}|^{w} \tag{3}$$

where A, β, γ, w are characteristic parameters of the Bouc-Wen model and are used to determine and calibrate the shape, smoothness and amplitude of the hysteretic curve characterizing the behavior of each link. To additionally model degradation and deterioration effects, Equation (3) is modified based on the description in [11] and reads:

$$\dot{\mathbf{z}} = \frac{A\dot{\mathbf{x}} - \nu(t)(\beta|\dot{\mathbf{x}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{x}}|\mathbf{z}|^{w})}{\eta(t)} \tag{4}$$

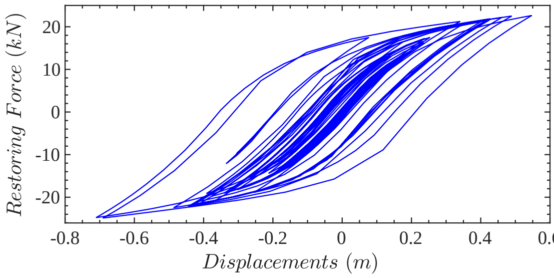
$$\nu(t) = 1.0 + \delta_\nu \epsilon(t), \quad \eta(t) = 1.0 + \delta_\eta \epsilon(t), \quad \epsilon(t) = \int_0^t \mathbf{z}\dot{\mathbf{x}}dt \tag{5}$$

where $\nu(t), \eta(t)$ are the additional terms introduced to capture strength deterioration and stiffness degradation effects respectively. Their time evolution depends on $\epsilon(t)$, which forms a measured of the absorbed hysteretic energy of the link. The parameter values for the default version of the shear frame are summarized in Table 2.

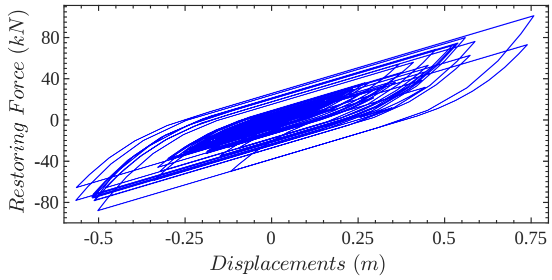
Table 2: Example physical parameters of the Bouc-Wen system.

Parameter:	α	k	A	β	γ	w	δ_ν	δ_η
Value:	0.10	2e08	1.0	3.0	2.0	1.0	0.0	0.0

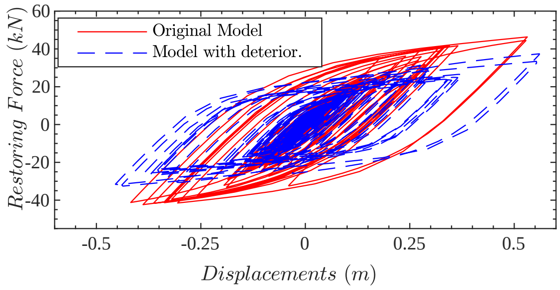
Figure 3 demonstrates the capability of the two story shear frame in representing different type of hysteretic behavior, based on the assigned values of the Bouc-Wen parameters, as specified in Table 2. Figure 3d



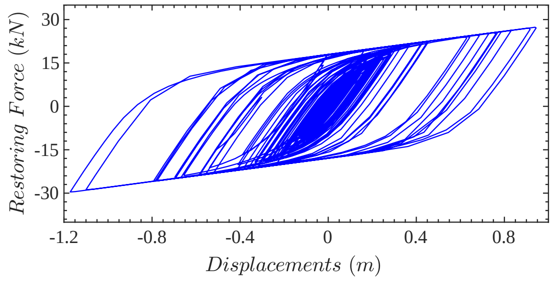
(a) Steep hysteresis curve of the shear frame link. Produced with Table 2 parameters and $\beta = 1, \gamma = 4$



(b) Medium hysteresis curve of the shear frame link. Produced with Table 2 parameters and $\beta = 2, \gamma = -2$



(c) Influence of deterioration on the hysteresis curve. Produced with parameters of Figure 3b and $\delta_\nu = 3.2$



(d) Shallow hysteresis curve of the shear frame link. Produced with Table 2 parameters and $\beta = 10, \gamma = 0.8$

Figure 3: Demonstration of model capabilities. Varying shape of hysteresis curves for the same link under different assignments of the Bouc-Wen parameters. Assumptions of Table 2 apply, if not stated otherwise.

depicts a shallow hysteresis curve, obtained by setting $\beta = 3, \gamma = 2$, whereas Figure 3a reflects the behavior of a steep hysteretic link with the following set of assigned parameters $\beta = 1, \gamma = 4$. The effect of plasticity and damage, as linked to degradation effects is depicted in Figure 3c, where the hysteretic link exhibits progressive strength deterioration and stiffness degradation effects, compared to Figure 3b.

4 User guidelines and software capabilities

The code repository of the simulator is published under the Apache License, Version 2.0 on GitHub here[3].The code repository contains a master route dedicated to the initial/current version of the simulator (*Version_0.0*), whereas updated versions with modifications will be uploaded as additional master routes, including ReadMe files to document all updated features.

The multi-degree of freedom oscillator allows for full user control in modifying the model configuration based on specific research needs. For this reason, a short overview of the critical elements defining the simulator is presented here for reference. The user is referred to the ReadMe file provided with the software implementation for further details.

The *ExampleMain** are the two main files of the repository that initiate and guide the simulation of the dynamic response of the two story shear frame under a ground motion excitation scenario. As already mentioned, the proposed benchmark has the potential of multi-parametric dependencies, pertaining to either system properties (e.g. damping coefficients), parameters of the Bouc-Wen hysteretic links, or the temporal and spectral characteristics of the excitation. Several alternative configuration files are offered for this reason, each one corresponding to a specific standardized parametric case study. These five main *Configuration** example files refer to several alternative parametric input configurations for the standardized system identification datasets described in detail in section 5.

[3]https://github.com/KosVla/NonlinearBoucWenFrameBenchmark.git

The overall implementation is summarized in the *MODEL* structure array, instantiated and initialized utilizing one of the input files provided (e.g. *InputFileLinksAll*, *InputFileLinksBeams*, *InputFileLinksFirst*, etc). It contains all system relevant parameters, matrices and output time histories. The structure and notation of the input files are described in detail next.

4.1 Input files description

The first step of the configuration files lies in the definition of the model, which is accomplished via a dedicated input file. The input file offers the possibility to modify the geometry and material parameters of the frame, assemble nonlinear hysteretic links, based on the documentation in section 3, or include nodal spring elements, plates to model the floor surface and lumped masses. Each input file includes details in the form of comments. By inspecting the respective input files and functions the user can grasp easier the structure of the software implementation and the naming convention used throughout the code. In any case, the ReadMe file provided in the main directory also offers additional step-by-step explanations.

Several example input files are provided by default in the dedicated *InputFiles* directory. The first one, named *InputFileLinksAll* assumes that hysteretic links are located in all horizontal beams and columns of the two-story frame. This implies that one hysteretic link is located on the top of each basement column, one at the bottom and one at the top of each first story column, modelling the respective nodal connections. The one named *InputFileLinksFirst* assumes that all links depicted in Figure 2 are modeled using the Bouc-Wen hysteretic formulation of section 3. This implies that all beam connections and the column connections of the basement assume a hysteretic behavior. In addition to this file, *InputFileLinksFirst_DifferentBW* demonstrates how to deactivate the Bouc-Wen links in certain degrees of freedom and force the model to behave linearly instead. This alternative also explains how to include a different Bouc-Wen parametrization for certain hysteretic links. The last input file provided, named *InputFileLinksBeams*, assigns nonlinear links exclusively on the horizontal beam elements, leaving the column connections to behave rigidly.

On top of those input files the repository offers an automated way to define the frame assembly based on an input number of floors, and an input number of frames in each direction. This is handled by the *AutomaticMesh* function, which assembles a hysteretic link in all horizontal beams and columns following the assumptions in input file *InputFileLinksAll*, and the *AutomaticMeshColumns* function, which only assembles hysteretic links in the columns of the frame.

The files carrying out the simulation are located in the *core* directory. The function that drives the simulation is *BoucWenRun*, whereas all simulation and configuration relevant parameters and matrices, including the assembly of damping, are initialized and handled in the *GetHystereticParams* function. The framework provides the functionality of adding Rayleigh damping to the system either by specifying the damping coefficients in the input files directly or by giving the modal damping ratios and the corresponding modes. The function that handles the computation of the Rayleigh coefficients is *GetRayleighDamping*. If the user does not specify any Rayleigh damping parameters or modal ratios, the input file defines a 5% stiffness proportional damping by default. As indicated in Table 1, the provided configuration files define a 2% modal damping on the first two natural frequencies of the frame.

The hysteretic links are modeled in the *GetDofsLoop* function. For efficiency and vectorization purposes, the Bouc-Wen links are evaluated all at once, avoiding additional loops on the implementation. To achieve this, the *GetDofsLoop* function assembles a matrix that contains the Bouc-Wen parameters of each single hysteretic link in all degrees of freedom. Thus, the matrix contains the starting and end degree of freedom and the Bouc-Wen parameters of the respective degree of freedom hysteretic model, as presented in Table 1. This assembly process offers the possibility of deactivating certain degrees of freedom by modifying the input file, and specifically the *MODEL.nl_link_flags* entry. Then, the respective parameters of the Bouc-Wen models are modified accordingly to enforce a linear behavior.

Additionally, the user has the option to change the Bouc-Wen parameters for certain elements by modifying the *MODEL.nl_links_alternate*, as demonstrated in the input file *InputFileLinksFirst_DifferentBW*. For further details, the user is referred to the ReadMe file.

The Bouc-Wen model evaluation happens at *bw_Ndall*. This function takes vectorized inputs, with the parameters of all hysteretic links assembled properly, based on the notation in the *GetDofsLoop* function explained previously. The user can modify this function as desired, as long as the vectorized nature of the input and outputs is retained. A non-vectorized template is also offered, explained in detail in section 4.5.

4.2 Excitation generation

Similar to the description and implementation in [5], the example configuration files produce a xy-plane, ground motion acceleration based on a synthetic multi-sinusoidal signal. The user though can also define a three-dimensional motion, input an earthquake accelerogram directly or use a nodal loading time history. These alternatives are explained in the ReadMe documentation.

Regarding the standardized, multi-sinusoidal signal, the user is able to define the frequency components of the excitation along with a temporal parameter controlling the amplitude of the accelerogram. The external force time history is then derived as a multisinus excitation. Use of several such terms enables modeling the input as a white-noise like excitation to explore the dynamic behavior of the system across a wide range of excitation frequencies, similar to [2, 3].

The overall process follows the procedure described in the single degree of freedom hysteretic benchmark [5] for generating the time history of the excitation. For example, in the provided configuration files a multisine excitation is applied to the shear frame considering all excited frequencies in the 5 to 50 Hz band. The respective frequency resolution is $f_0 = f_s/N \approx 0.0125$ Hz, with a sampling frequency of $f_s = 150Hz$ and $N = 12000$ number of time samples. A total of 5 output periods are simulated under an input signal, whose root-mean-squared amplitude is dictated by amplitude coefficient A. Since the excitation signal represents ground motion all variables assume units of acceleration. In addition, low-pass filtering and downsampling is provided in case upsampling is needed, whereas the edge effects of the low-pass filter are addressed by adding an extra period during time integration and subsequently removing it. Based on the respective parameters in the configuration file and the remarks on the ReadMe file the user can control and define the excitation as desired. As already mentioned, the framework also accommodates user-define accelerograms, three dimensional excitation or nodal time histories input. The interested user is referred to the ReadMe for further details.

Two simplified excitation examples are also provided in the shared workspaces for a clearer visualization of the different qualities of the hysteretic curves, as depicted in Figure 3. For all cases zero initial conditions are assumed. These are also specified on the main files of the simulation and can be freely modified.

4.3 Time integration

Regarding time integration of the governing equations of motion, the software relies on the one-step-ahead approximation of velocities and displacements by employing the Newmark method [12]. To correct the initial predictions at every timestep based on the hysteretic, restoring force mapping and the dynamic equilibrium conditions, a Newton-Raphson iterative scheme is utilized. The respective tolerance and maximum iterations parameters are defined internally, in the *newmark_nlBW* function. Additionally, print-out messages inform the user for the convergence course of the integration. The integration timestep is defined in the input file, allowing the user to adjust the sampling frequency of the output based on the specified input excitation. During time integration the *assemble_nlBW* function updates the stiffness and restoring force vectors by evaluating the Bouc-Wen hysteretic models at each timestep. In addition, *apply_bc_nl* applies the boundary conditions on the updated matrices and vectors. These, along with all simulation relevant functions, are located in the *core* directory

4.4 Post-processing and interpretation of output

The degree of freedom notation for the global system follows Figure 1 and the Cartesian reference system depicted. Thus, the first degree of freedom of each node corresponds to the x-axis, the second to the y and the third to the z-axis. The rotation around the x-axis is represented by the fourth degree of freedom and so on. This numbering notation is depicted in Figures 1 and 2 as well.

As already mentioned, the output of the time integration and model evaluation step is stored inside the *MODEL* structure array. For example, the displacement time histories for every degree of freedom are stored in the field named *MODEL.U* $\in \mathbb{R}^{N*N_t}$, where each row corresponds to the time history of the respective degree of freedom and N_t denotes the simulated timesteps. Following the degree of freedom notation, to evaluate the displacement response of the n-th node along the x-axis, the user needs to plot row $(n-1)*6+1$ of the *MODEL.U* structure array. By consulting the ReadMe file provided, the user can also evaluate the output velocities and accelerations time histories or the system matrices during the last iteration.

The hysteretic curves of the nonlinear links are also retained in the structure array, in *MODEL.HistU* and *MODEL.HistR* fields. Example plotting routines are also provided to visualize the deformed configuration of the model at any step of the simulation, along with the output time histories and the hysteretic curves for each link. The ReadMe file provides a detailed explanation of all fields of the *MODEL* struct, for either pre- or post-processing purposes.

4.5 Relation to single degree of freedom benchmark

As already mentioned, the proposed multi-degree of freedom simulator employs the Bouc-Wen model for formulating the hysteretic restoring force mapping of the nonlinear links. This implies a degree of similarity or equivalence between the proposed benchmark and the single degree of freedom hysteretic benchmark [5], which is featured in the nonlinear benchmark catalogue [1]. The user is encouraged to study the respective description of the single degree of freedom oscillator and explore the equivalence between the Bouc-Wen parameters explained in section 2 and the ones utilized in [5].

To demonstrate the relation between the two benchmarks, the contributed software includes a configuration file and a dedicated core folder that implements a non-vectorized alternative of the main framework. The respective configuration file is *RelationwithSDOF.m*, whereas the directory is *coreBWSDOF*. This implementation offers an alternative assembly process by looping over all hysteretic links and all degrees of freedom one by one. This enables the evaluation of a single degree of freedom Bouc-Wen oscillator for each degree independently, in place of the vectorized multi degree Bouc-Wen function.

The provided alternative adds up to the flexibility of the framework, as the user can freely modify and adjust the Bouc-Wen model of each link independently as an single degree of freedom oscillator. Thus, the non-vectorized version allows the user to define different Bouc-Wen models for each link or even degree of freedom, by slightly modifying the input file. The user is referred to the ReadMe file for further details.

5 Standardized system identification datasets and challenges

The fully adjusted framework described in the previous section is offered under the Apache License, Version 2.0 on GitHub here[4].

On top of that, five standardized case studies are provided, serving as reference input-output datasets for nonlinear system identification tasks in the proposed shear frame benchmark. The respective standardized datasets can be found in the Zenodo open-access repository here [5].

The standardized case studies are summarized in Table 3. Each one is produced with different Bouc-Wen parameters, as demonstrated in Table 3. A configuration file that runs the simulation and produces the respective input-output signals is also provided on top of the results datasets for each one of the proposed case studies.

Each standardized case study contains two simulations, each one with a slightly different parametric input representing damage phenomena or model calibration. The Bouc-Wen parameters remain fixed in the test,

[4]https://github.com/KosVla/NonlinearBoucWenFrameBenchmark.git

[5]https://zenodo.org/record/4742248 # .YS37bd-xWUk, https://doi.org/10.5281/zenodo.4742248

Table 3: Bouc-Wen parametric configurations and physical representation of standardized case studies. Notation refers to Table 2. Parameter $A = 1, \delta_\nu = 0, \delta_\eta = 0$ if not stated otherwise.

Name	Parametrization	Physical Representation
Configuration 0	Linearized model : $a = 1.0, k = 6e^7, \beta = 12.6, \gamma = 5.70$	Healthy State
Configuration A	Hysteretic parameters: $a = 0.5276, k = 5.4236e^7$	Mechanical damage induced in healthy state
Configuration B	Hysteretic parameters: $\beta = 3.54, \gamma = 6.39, \delta_\eta = 0 \rightarrow 1.42, \delta_\nu = 0 \rightarrow 2.57,$ $a = 0.10, k = 6e^7$	Damage growth / Deterioration scenario
Configuration C	Hysteretic parameters: $\beta_{init} = 4.67 \rightarrow 30.87, \gamma_{init} = 1.21 \rightarrow$-40.21, $a = 0.10, k = 1e^8, A = 0.75$	Model calibration during operation
Configuration D	Hysteretic parameters: $\beta = 21.82, \gamma = 9.71, a = 0.6235 \rightarrow 0.1236$ $k = 1e8 \rightarrow 7.7234e^7$ (changes in first floor links only)	Localized damage concentrated in first floor
Configuration E	Hysteretic parameters: $\beta = 16.32, \gamma = -16.32, \delta_\eta = 0 \rightarrow 4.85,$ $\delta_\nu = 0 \rightarrow 2.31, a = 0.10, k = 5e^7$	Deteriorating links in first floor

training and validation datasets of each case study so the user can manually define the system identification task of interest and adapt the parametric dependencies of the simulator accordingly. However, as we offer the standardized datasets for output-only approaches, we also propose here a few potential problem tasks related to the multi degree of freedom simulator.

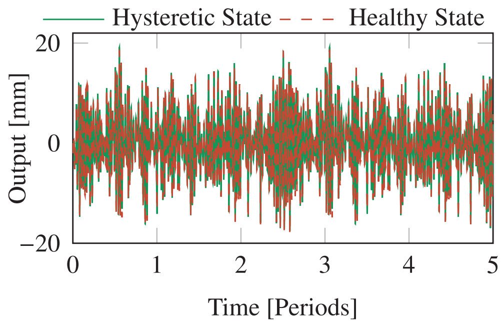
The tasks are related to system identification applications, reduced-order or surrogate modelling and machine learning based approaches and serve orientation and comparison purposes:

- Estimate parameters k and a of the hysteretic links, potentially representing a form of mechanical damage induced in the healthy state of the system due to loading conditions (Configuration A and D).
- Estimate the nominal values of the δ_η and/or δ_ν parameters of the Bouc-Wen hysteretic links, representing a deterioration or damage growth scenario (Configuration B and E).
- Estimate the change in parameters β and γ of the hysteretic links, representing a form of model fitting or calibration during design or operation stages of structural health monitoring (Configuration C).
- Perform state, and potentially jointly input estimation, with (limited) output measurements.
- Assemble output-only data-driven or physics-inspired reduced order models for efficient surrogate representations of the nonlinear, dynamical system (All Configurations).
- Perform damage detection, localization or quantification by employing machine learning based approaches on outputs measurements (All Configurations).

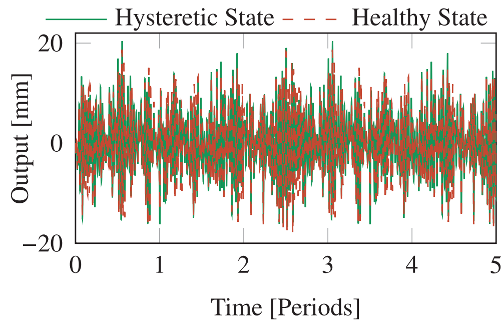
The standardized cases in Table 3 have been derived in a corresponding fashion to the proposed tasks, as for example, *Configuration A* varies the k and a parameters of the Bouc-Wen model to initiate a form of damage in the healthy state of the system. For each standardized case, the training, test and validation datasets contain twenty one input excitation signals each, produced through the random phase multisine excitation described in section 4. All datasets are noiseless.

In each dataset the twenty one signals are designed to excite the nonlinear behavior of the system in three levels of discrete intensity, namely seven signals produce a response with strong nonlinearities, seven signals lead to milder phenomena and the remaining seven induce weak nonlinearity.

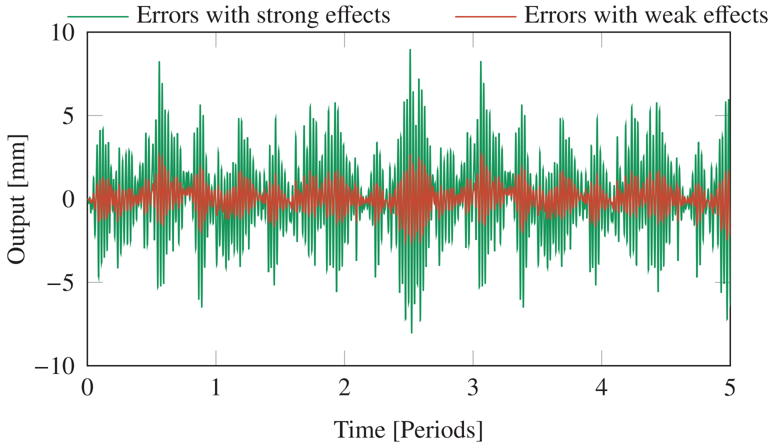
In Configuration A, for example, levels of intensity refer to the comparison against the linearized state of the model whereas in the rest Configurations the differentiation refers to an initial nonlinear configuration, as



(a) Response from input causing weak effects



(b) Response from input causing strong effects



(c) Output errors in time for the two examples

Figure 4: Displacement response output for signals with different intensities of nonlinear effects. Example outputs are based on Configuration A in Table 3.

indicated in Table 3. In Configuration B, for instance, the deteriorating model is compared against an initial nonlinear case where $\beta = 3.54, \gamma = 6.39$ but $\delta_\eta = \delta_\nu = 0$.

To obtain these states the direction of motion, the temporal (amplitude) and spectral characteristics (minimum frequency) of the excitation signal varied. An example is illustrated in Figure 4 for Configuration A, where the discrepancies from the healthy state of the model depicted in blue are caused due to the hysteretic effects of the links. Figure 4c visualizes the absolute error for the weak and strong nonlinear effects examples for better comparison.

Overall, following the observations in [5], the multi degree of freedom benchmark proposed may be associated with the following major nonlinear system identification challenges:

- The Bouc-Wen nodal links cause a nonlinear, dynamic response characterized by hysteresis and featuring memory, thus the system contains a dynamic nonlinearity.
- The behavior of the hysteretic links, and thus the restoring force mapping, is governed by an internal, not measurable variable $\mathbf{z}$.
- The dimensionality of the shear frame and the presence of multiple Bouc-Wen links increases the interactions between the components of the system and, thus, the complexity of any task at hand.
- Equation 4 that governs the evolution of the hysteretic forcing is nonlinear with respect to parameter w. Moreover, a finite Taylor series expansion is not a suited means of approximation due to the presence of absolute value.

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