ETH zürich

System Modeling – HS 2020

Exercise Set 1 Discussion

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Exercise 1 – Introduction

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Polybox:

- Theory Sheets
- Exercise Notes
- My Summary (I suggest you make your own)
- Skript

➢ You will be automatically added if you are inscribed in my exercise, otherwise send me an email

Covid-19 Announcement

- Make sure that the social distances are met!
- Check your name in the list when coming in (for contact tracing)
- Max 36 People allowed – inscribed people have priority
- **Masks mandatory at all times** – no exceptions.

System Modeling - Introduction

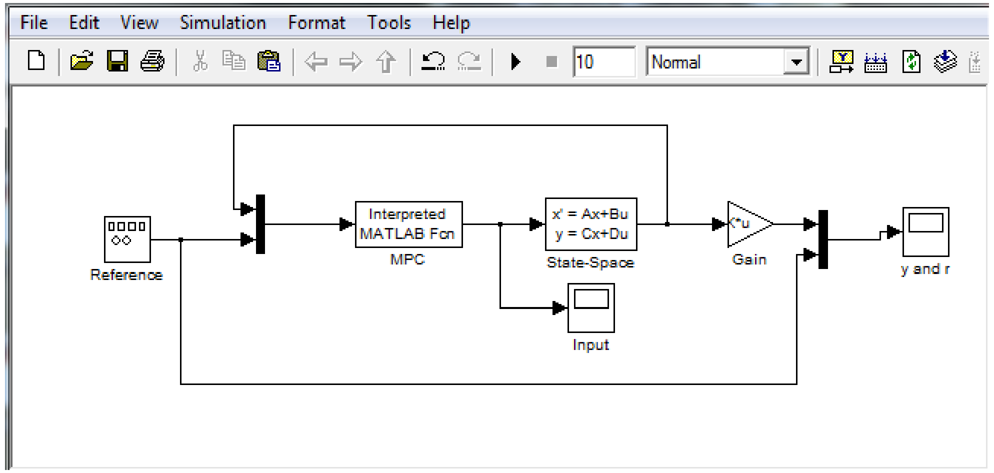
- Software: Matlab and Simulink (we work with 2018a)
- Exercises are an integral part of the lecture & exam (also Matlab/Simulink knowledge!)
- Exam style similar to last year – Multiple Choice and Boxes, Causality Diagrams
- Theory based on Theory Sheets by Leonardo Andreae, Nicolas Lanzetti & Giovanni Moscato (on polybox as well)

System Modelling Motivation

- Advanced controllers need system model: A,B,C,D
  - LQR Control:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad \Longrightarrow \quad u_{LQR} = -R^{-1}(PB)^T x(t)$$

- Simulate the real world:



Exercise 1 – Theory

A **dynamic system** is a non-static system which changes over time.

These changes can be caused by:

- input signals,
- external perturbations,
- naturally.

Example: mechanical systems are described with the general differential equation

$$m\ddot{y}(t) + d\dot{y}(t) + ky(t) = F(t) \quad \longrightarrow \quad \textbf{Parametric model} \text{ with parameters: m, d, k, F}$$

Exercise 1 – Theory

We can further distinguish three types of models:

- **Black-box models**: derived from experiments only.
- **Grey-box models:** starting from a model, one uses experiments to identify model parameters and validate the model. (e.g. multicopter)
- **White-box models:** no experiments needed, since one knows each part of the system. (e.g. pendulum with known mass & string length)

Exercise 1 – Building a Model

Two elements: Reservoir and Flows

Reservoirs: Accumulate something (energy, mass, information…)

- For every reservoir, an associated **level variable** can be defined, being a function of the **reservoir`s content**. (Often the variable in which energy is stored)

- Only systems including **at least 1 reservoir** can exhibit **dynamic** behavior.

Flows: Describe the transfer of anything between the reservoirs (heat, mass…)

Exercise 1 – Causality Diagram

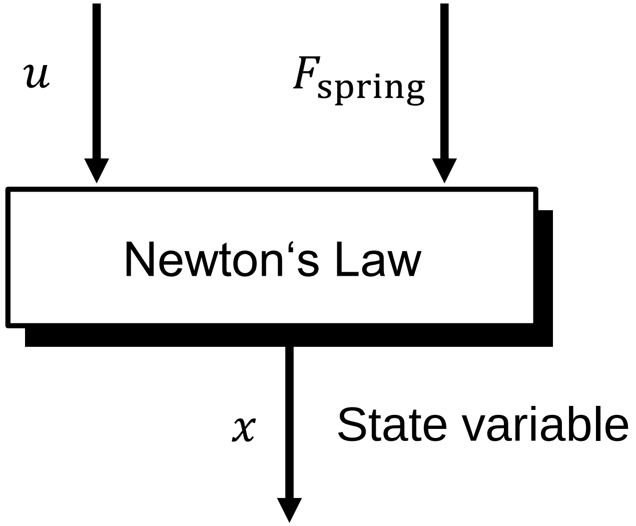
Example: Mass-Spring System

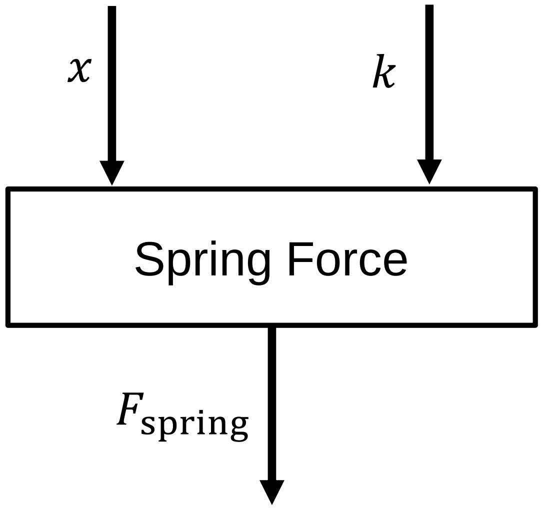
Dynamic Block

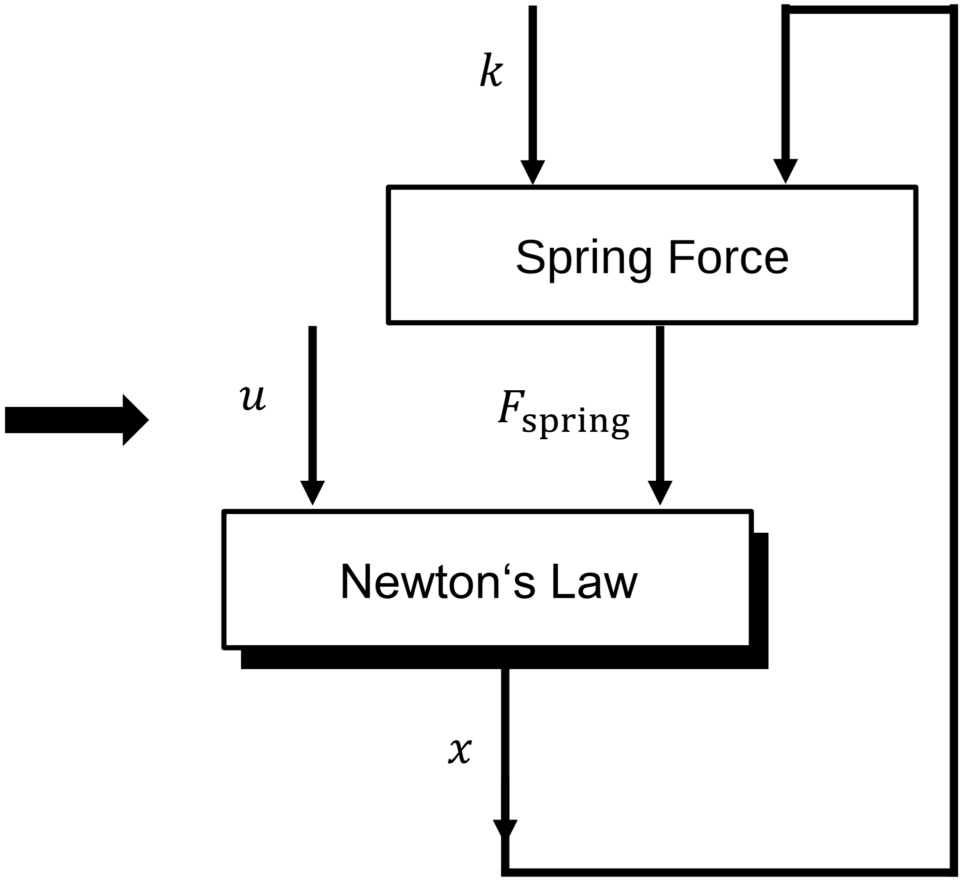
$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = u - F_{\text{spring}}$$

Algebraic Block

$$F_{\text{spring}} = k \cdot x$$







Exercise 1 – Building a Model

Simple Recipe:

1) System boundaries: what are the **inputs** and **outputs**?
2) Identify relevant **reservoirs** and associated **level variables**
3) *Optional, but very useful and important*: Draw causality diagram (*Hint*: start at end)
4) Differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\text{reservoir content}) = \sum \text{inflows} - \sum \text{outflows}$$

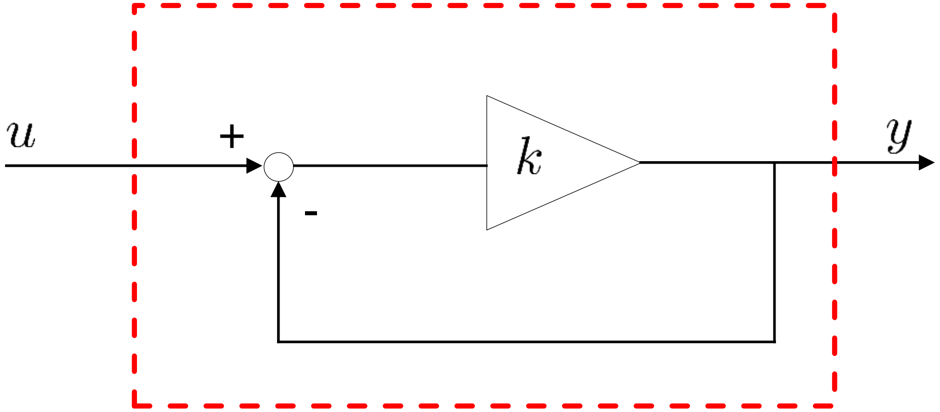
5) Algebraic relations for flows:

$$\text{flow} = f(\text{level variable}, \ldots)$$

6) Resolve implicit algebraic loops

Exercise 1 – Algebraic Loops

Example: Algebraic Loop



Algebraic loop

➔ Signal depends on itself algebraically

➔ Not suitable for SIMULINK

Exercise 1 – Problem 1

Problem 1 (System Boundaries and Causality Diagram)

The water tank has the cross sectional area F. The mass flow $\overset{*}{m}_{\text{out}}$ exits the tank through the cross sectional area E. There are two inflows: The mass flow $\overset{*}{m}_u$ is regulated through a valve, which is controlled by the input signal u; the mass flow $\overset{*}{m}_z$ is a disturbance, which is controlled by the disturbance signal z. We assume that the flows are frictionless. For the control of the water level, we can only use u, not z. The output signal of the sensor, which measures the water level h, is denoted y. The dynamics of the sensor are not negligible.

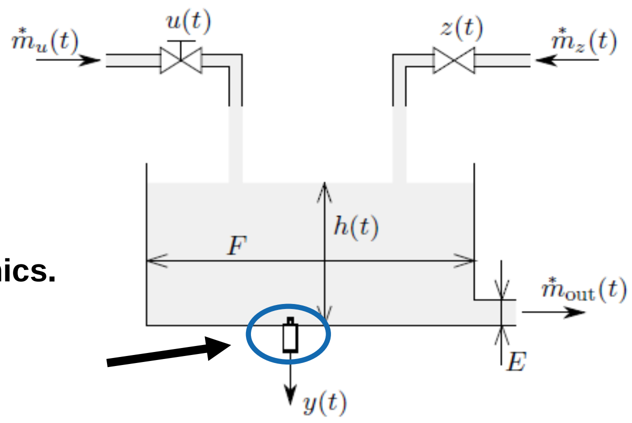


Figure 1: Sketch of the water tank.

Hint: **Sensors can have internal dynamics.**

- Infrared thermometer: algebraic block
- Mercury thermometer: Internal thermal capacity → reservoir / temperature readout will lag behind

a) What is/are the input(s) of the system? What is/are the output(s)?

Hint: **If not stated „Control Inputs", all inputs are meant**

b) Identify the relevant reservoirs and name their corresponding level variables.

c) Draw the causality diagram of the system. Highlight dynamic subsystems (containing a reservoir) with a shade. Denote all signals according to the nomenclature introduced in the description above.

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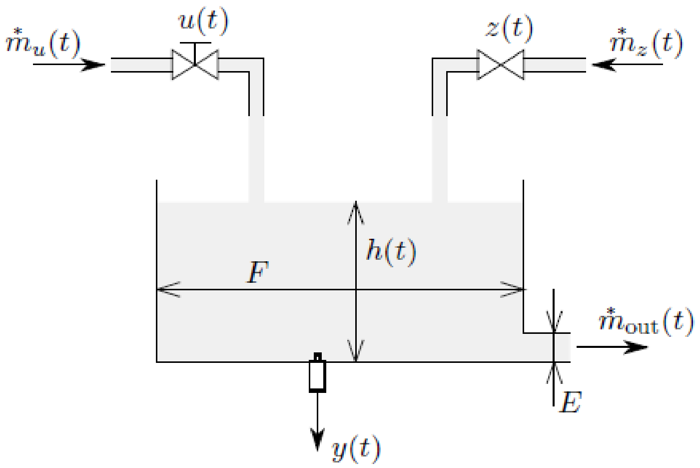

Figure 1: Sketch of the water tank.

a) What is/are the input(s) of the system? What is/are the output(s)?

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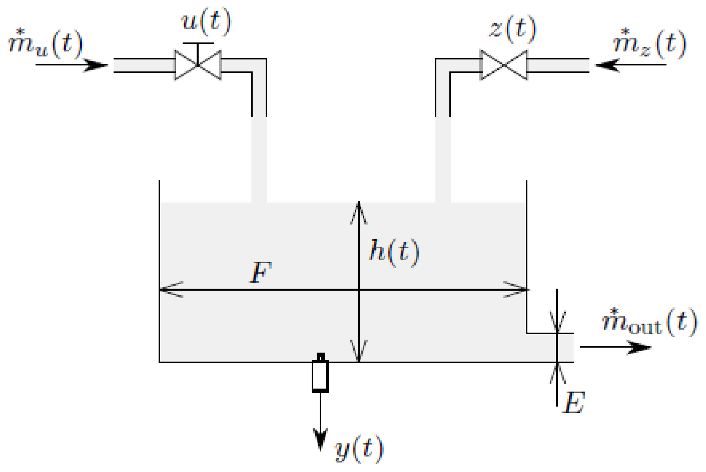

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c) Draw the causality diagram of the system. Highlight dynamic subsystems (containing a reservoir) with a shade. Denote all signals according to the nomenclature introduced in the description above.

Exercise 1 – Hints

Problem 2 (Equations)

In the next step, we want to formulate the equations of all elements in the causality diagram.

Assumptions / Specifications:

- The mass flows through the valves can be described by linear correlations:

$$\overset{*}{m}_u(t) = u(t) \cdot \overset{*}{m}_{u,\max} \qquad \text{(Control valve)}$$

$$\overset{*}{m}_z(t) = z(t) \cdot \overset{*}{m}_{z,\max} \qquad \text{(Disturbance valve)}$$

- A colleague of yours has already found out that the dynamics of the sensor can be described by a first-order system

$$\frac{\mathrm{d}}{\mathrm{d}t} y(t) = \frac{1}{\tau} \cdot \big(h(t) - y(t)\big) \qquad \text{(Sensor)}$$

  with time constant $\tau = 5$ s.

- The symbols listed in Table 1 shall be used.

a) Formulate the differential equation that describes the mass balance of the water tank.

b) Derive the algebraic relations that express the flows between the reservoirs as functions of the level variables.

Exercise 1 – Hints

Problem 3 (Implementation and Simulation in Simulink)

Now, we know the causality of the system and can describe all reservoirs and flows with equations. Thus, we can start implementing the model in Simulink.

a) Create a Simulink model with the same structure as your causality diagram.
Hints:
- A new Simulink model is created through `Home` → `New` → `Simulink Model`.
- Use subsystems to represent the dynamic and algebraic blocks.
- You find the necessary blocks in the `Library browser`.
- To add additional inputs and outputs to a subsystem, open the subsystem and add `In` or `Out` blocks from the `Library browser` as necessary.

b) Implement the equations from Problem 2 in the respective subsystems. Use the parameter names listed in Table 1.
Hint: Don't forget to set the initial conditions of the integrators. This is preferably done with variables (e.g. set the initial value of the sensor output to `y_IC`).

c) Add the parameters and initial conditions (e.g. `y_IC`) to the workspace. To do this, create an `.m`-file with the following structure

```
g   = 9.81;              % [m/s^2]  gravity
rho = 1000;              % [kg/m^3] density water
```

which defines all parameters and initial conditions and, then, run the file (by clicking `Editor` → `Run` → `Run` or by pressing `F5`).

d) Simulate the water tank model for 200 s for the constant inputs $u = z = 0.5$.
Hints:
- You can use `Constant` blocks for the input signals.
- You can use a `Scope` block to visualize the simulated output signal.
- In order to see the whole simulation result, deactivate the `Limit data points to last` option of the `Scope` block.
- Configure the solver as depicted in Figure 2.

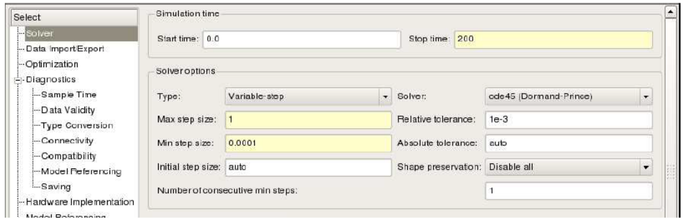

Figure 2: Solver configuration: `Simulation` → `Model Configuration Parameters` (or `Ctrl+e`).

e) In order to validate the simulation, calculate the equilibrium output of the system for the inputs $u = z = 0.5$. Does it match the simulation?

All information without warranty.

MATLAB/SIMULINK implementation:
Self-explanatory once causality diagram is known

Hint: Causality diagram top → bottom
Simulink "traditionally" left → right

Simulink Demo Checklist:

- Subsystems
- Name subsystems and blocks
- Library Browser + click&search
- In and Out Blocks
- Invert block direction (Ctrl-i / Ctrl-R)
- Constant Block
- Scope Block
- Integrator Block and Initial Conditions

Hint: Simulink can see MATLAB workspace (variables).
Use variable names for constants and create matlab script that defines the variables

Exercise 1 – Key Learnings

- Determine inputs – outputs of a dynamic system
- Reservoir-based approach:
  - Identify reservoirs
  - Identify flows
- Use mass/energy/… conservation for the reservoirs
- Formulate equations of algebraic blocks,
  e.g. Incompressible Bernoulli equation
- **Draw causality diagram**
- Implement in Simulink

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}(\text{reservoir content}) = \sum \text{inflows} - \sum \text{outflows}}$$

Bonus Exercise in Theory Sheet

1.5 Example

In recent years, the market of food trucks has seen a tremendous increase. Willing to get into the market, you decide to found the company SpaghETH. Your focus is to serve fresh pasta with different sauces everyday. In order not to run into delays during your activity, you aim to have hot water right after parking and therefore you decide to warm up the water while driving. To find the optimal way to that goal, you start by formulating a model of the system.

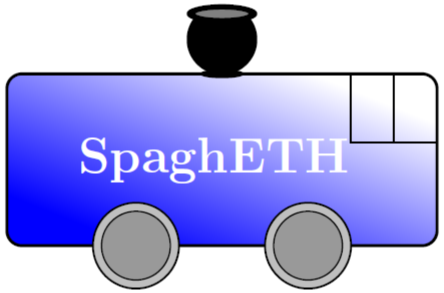

Figure 3: Sketch of the system.

The truck is modeled as a point mass of mass m. The propulsive power acting on the truck is given by

$$P_\mathrm{p}(\theta) = P_\mathrm{max} \cdot (1 - \exp(-c_1 \theta)) ,$$

where $\theta(t)$ is the (normalized) angular position of the pedal and c_1 and P_max are known constants. The truck is also subject to the aerodynamic drag force

$$F_\mathrm{drag} = \frac{1}{2} c_2 v^2,$$

where c_2 is a known constant and v is the speed of the truck. Clearly, your truck is equipped with a pot of known area A and volume V. Your cooker allows you to set the heat flow given to the water. The heat transfer coefficient α between the water surface