ETH zürich

System Modeling – HS 2020

Exercise Set 2 Discussion

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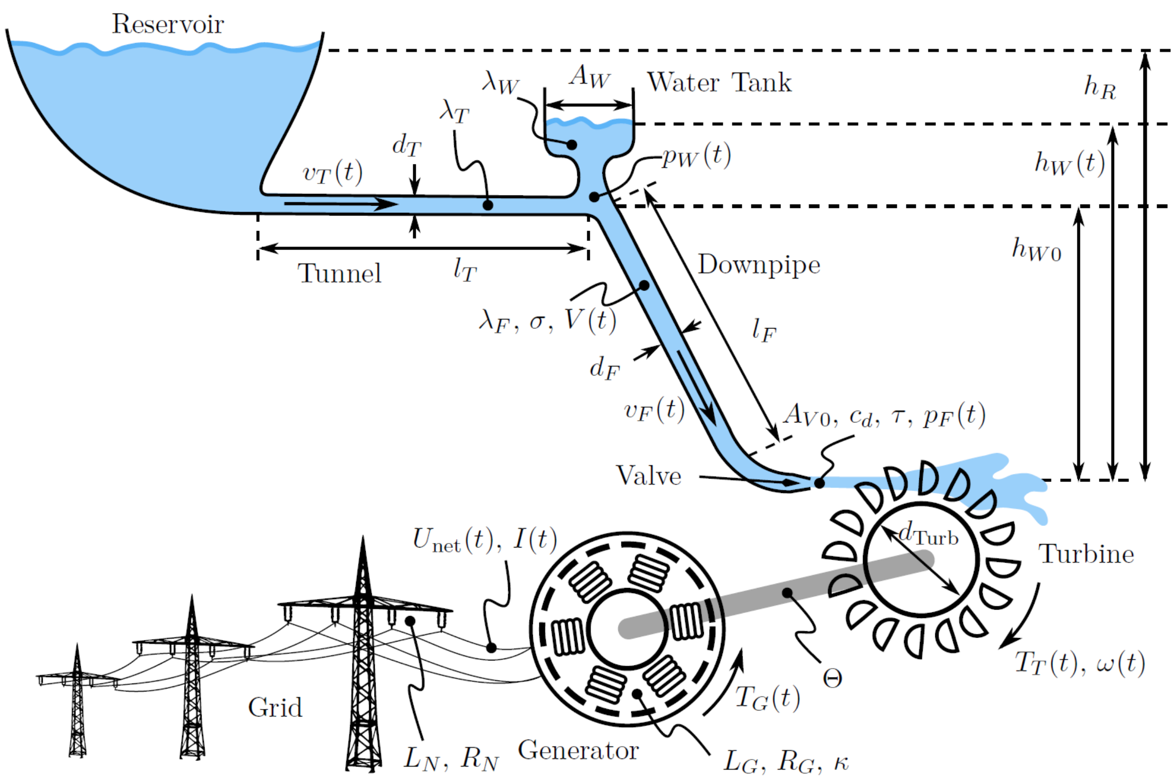
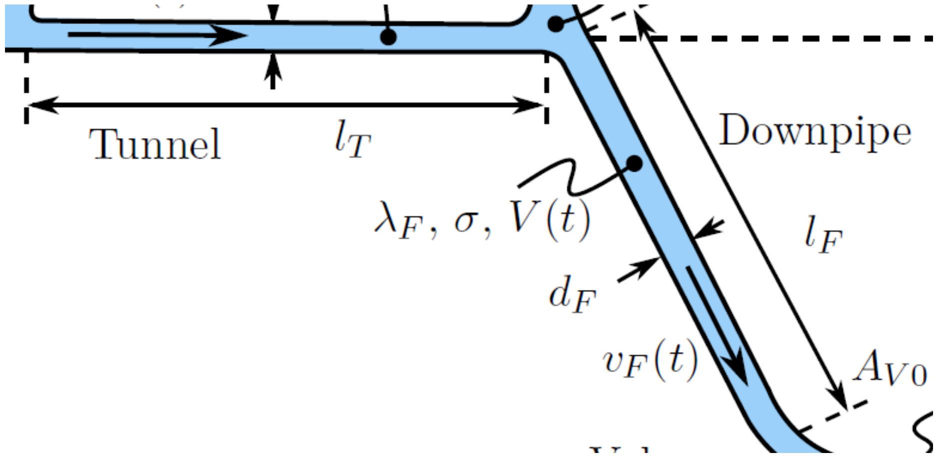


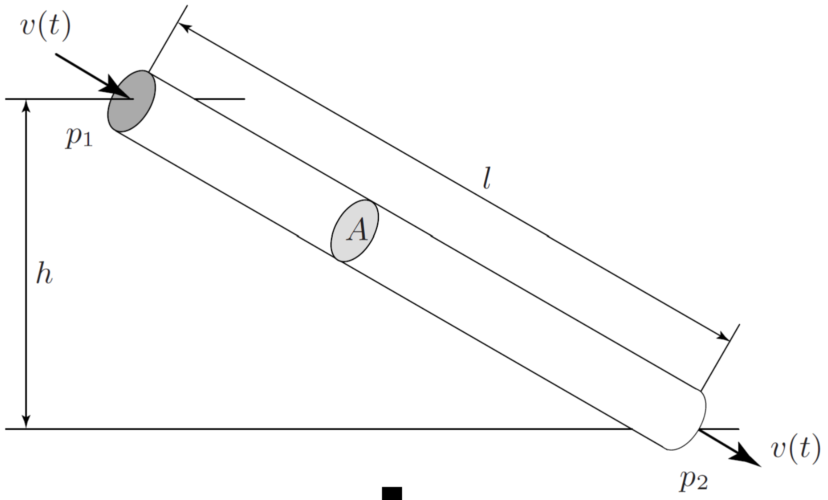
Figure 1: Schematic description of the hydroelectric powerplant

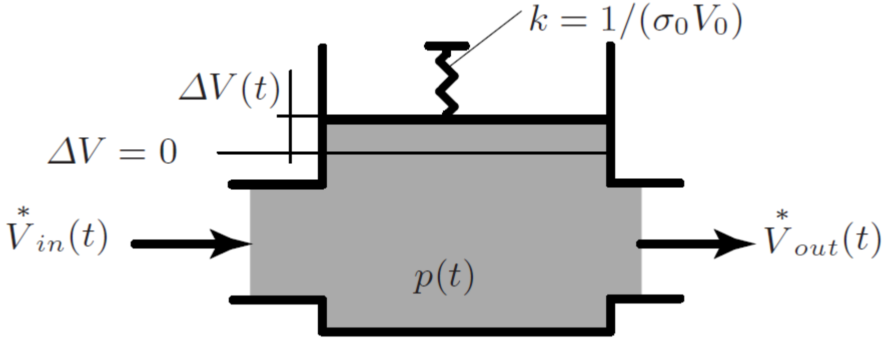
Exercise 2 – Hydraulic Systems

Duct, Compressibility, Reservoir

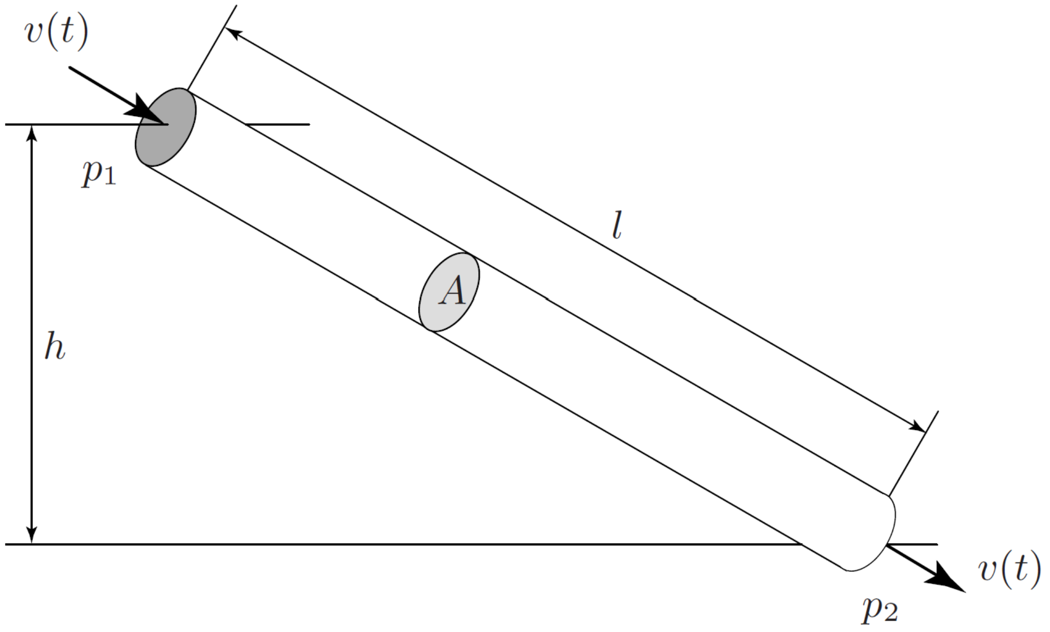
Pipe Modeling – Lumped Parameter







Duct



$$m \cdot \frac{d}{dt} v(t) = \sum Forces$$

$$F_{pressure}(t) = A \cdot p(t)$$

$$F_{gravity} = A \cdot \rho g h$$

$$F_{fric}(t) = A\rho l \cdot \lambda\big(v(t)\big) \cdot \frac{1}{2d} v(t)^2 \cdot sign(v)$$

$$m = A\rho l$$

$$\Rightarrow \quad \boxed{m \cdot \frac{d}{dt} v(t) = A \cdot \big(p_1(t) - p_2(t)\big) + \mathrm{A}\rho \cdot gh - F_{fric}(t)}$$

Duct – Useful Extras

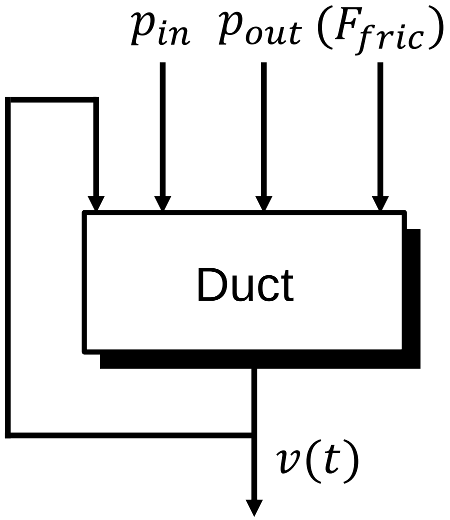
$$\boxed{m \cdot \frac{d}{dt} v(t) = A \cdot \big(p_1(t) - p_2(t)\big) + A\rho \cdot gh - F_{fric}(t)}$$

$\div\, m = \div\, A\rho l$

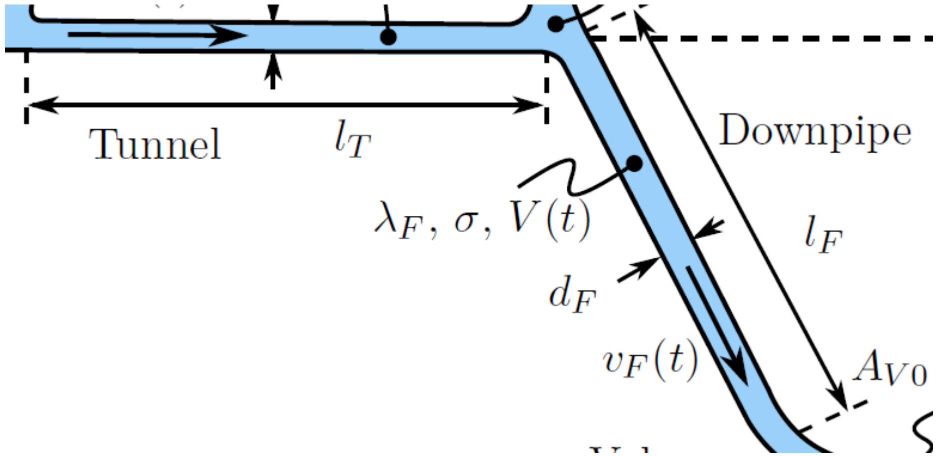
$$\boxed{\frac{d}{dt} v(t) = \frac{\big(p_1(t) - p_2(t)\big)}{\rho l} + \frac{gh}{l} - \frac{F_{fric}(t)}{m}}$$

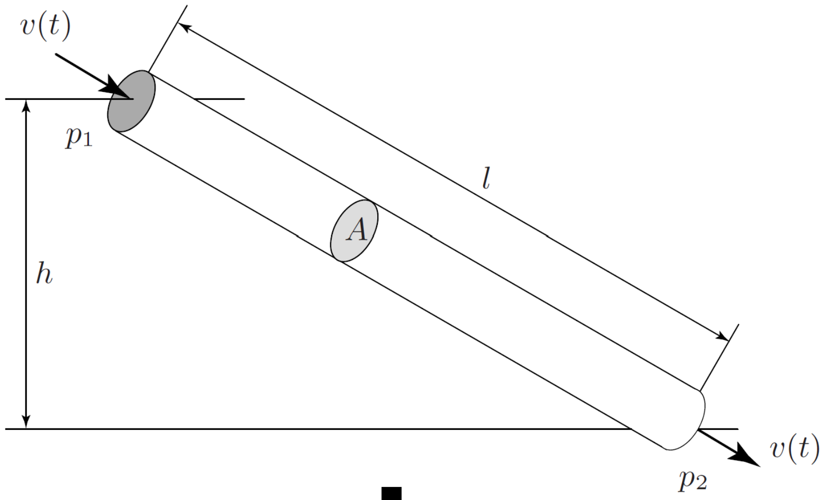
with $\frac{F_{fric}(t)}{m} = \lambda\big(v(t)\big) \cdot \frac{1}{2} v(t)^2$

- $\dot{m} = v \cdot \rho \cdot A$, for a tube: $\dot{m} = v \cdot \rho \cdot \pi \frac{D^2}{4}$
- $dm = \rho \cdot A \cdot dl$
- $\sin(\alpha) = \frac{dh}{dl}$ shape factor: $\frac{l}{d}$



Pipe Modeling





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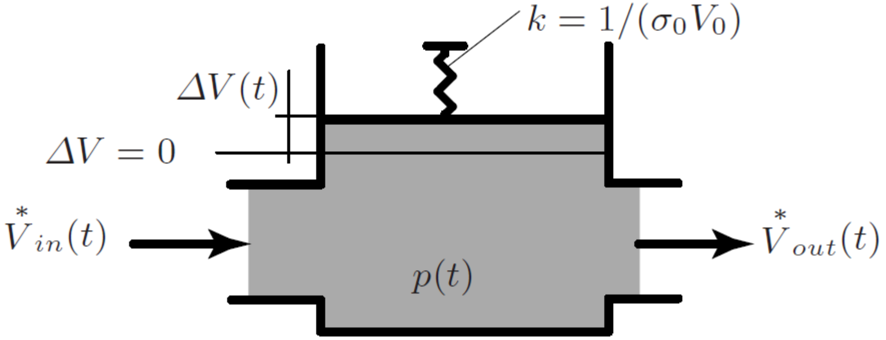
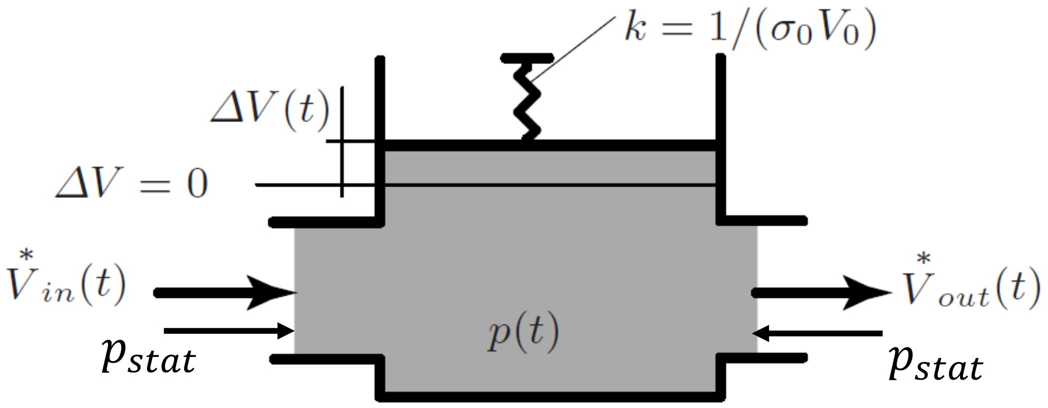


Compressibility



Model elasticity of the pipe → Compressible element at the end

$$\frac{d}{dt}V(t) = \dot V_{in}(t) - \dot V_{out}(t) = A_{in} \cdot v_{in}(t) - A_{out} \cdot v_{out}(t)$$

$$\mathrm{p}(t) = k\Delta V(t) + p_{stat} = \frac{1}{\sigma_0}\frac{\Delta V(t)}{V_0} + p_{stat}$$

- σ_0: compressibility $[\frac{1}{Pa}]$
- $k_0 = \frac{1}{\sigma_0}$: elasticity constant $[Pa]$
- σ_0 must be determined experimentally
- $V_o = l \cdot A$: Volume of the duct

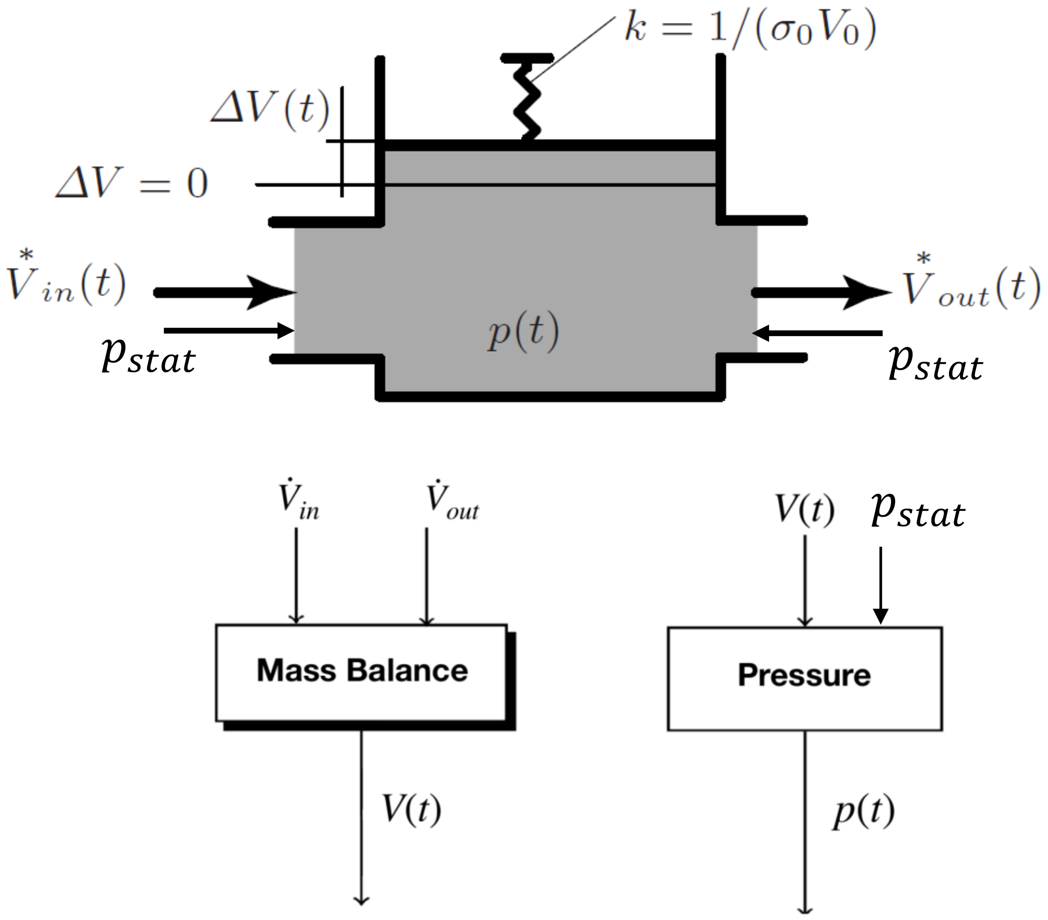
With:

$\Delta V = V(t) - V_0$

$p_{stat} = p_0$ If open-end duct

$p_{stat} = \rho g h_{duct}$ If closed-end duct (has a valve)

Compressibility



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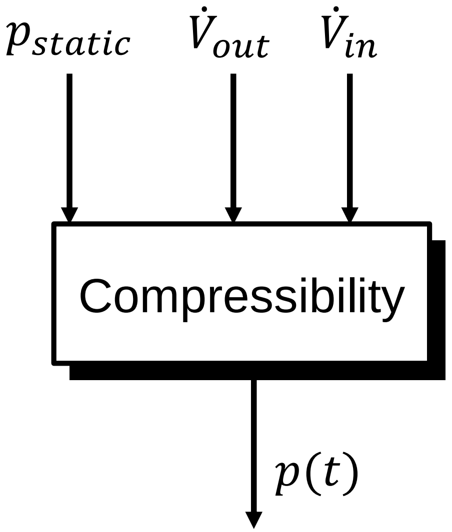
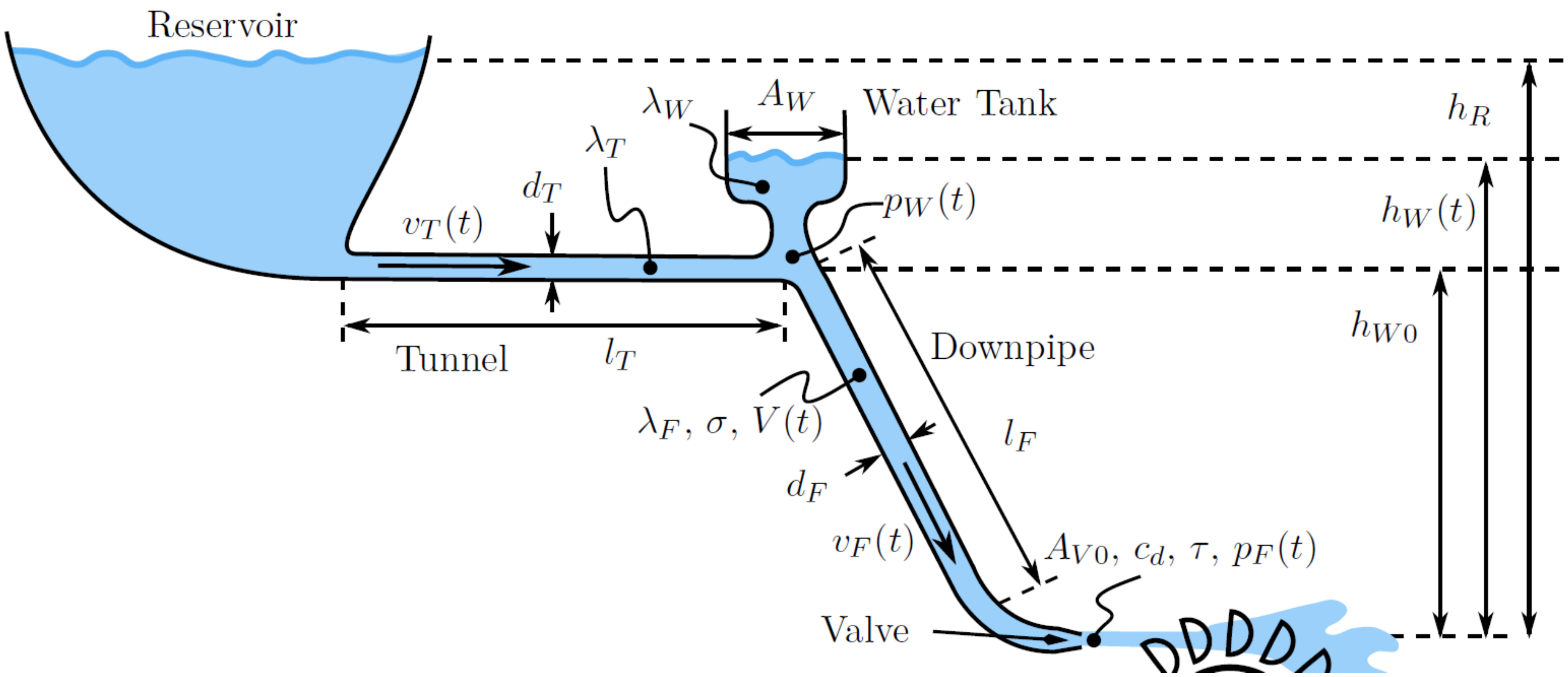
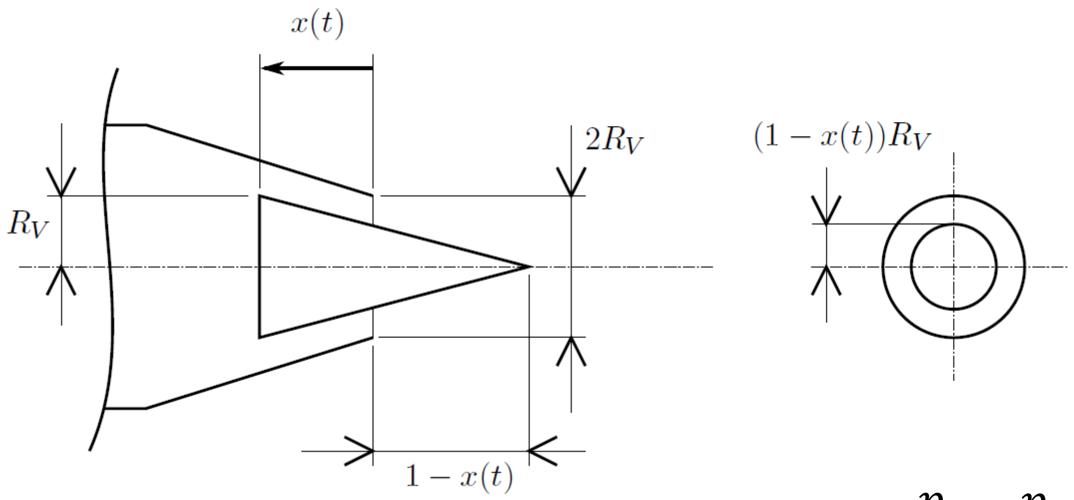


Overview



Reservoir
Water Tank
λ_W
A_W
h_R
λ_T
d_T
$v_T(t)$
$p_W(t)$
$h_W(t)$
h_{W0}
Tunnel
l_T
Downpipe
λ_F, σ, $V(t)$
l_F
d_F
$v_F(t)$
A_{V0}, c_d, τ, $p_F(t)$
Valve

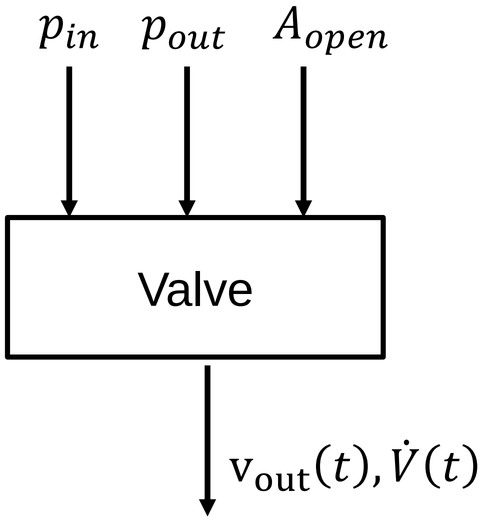
Valve – Algebraic Block



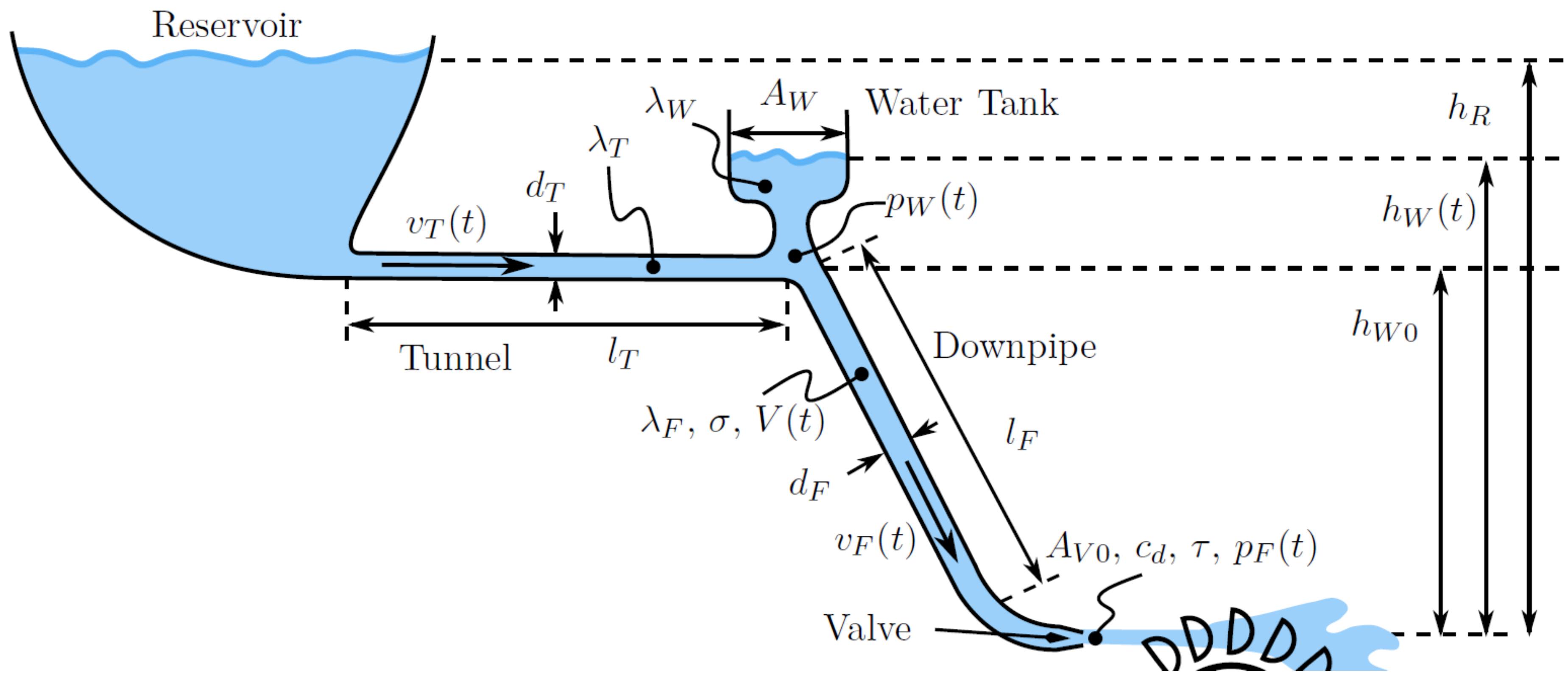
Often interested in velocity and mass/volume-flow:

- Find function for the open area of the valve
- Bernouilli for the velocity:

$$v_{out}(t) = c_d\sqrt{\frac{2}{\rho}\cdot\left(p_{in}(t) - p_{out}\right)}\ , \qquad v_{out} \gg v_{in}$$



Overview



Reservoir
λ_W
A_W
Water Tank
h_R
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$v_T(t)$
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λ_F, σ, $V(t)$
l_F
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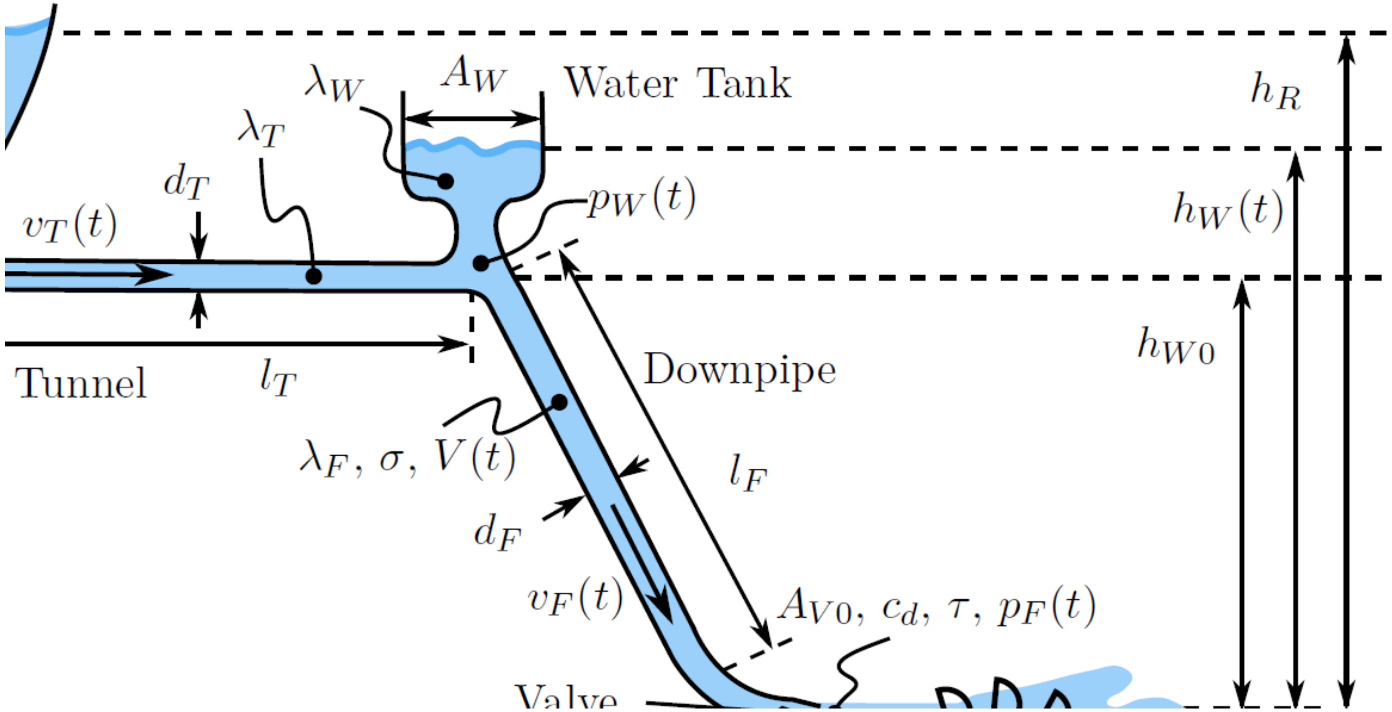
Valve

Water Tank



λ_W
A_W
Water Tank
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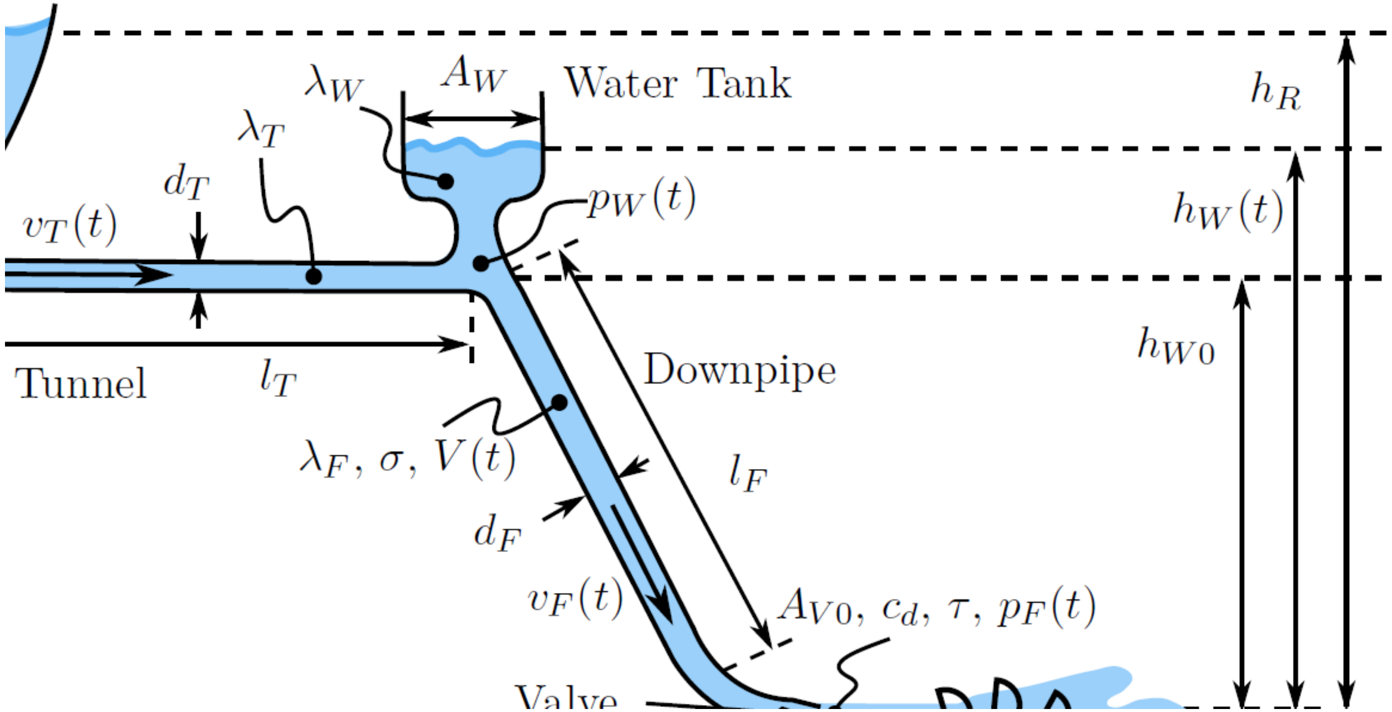
Water Tank



λ_W
A_W
Water Tank
h_R
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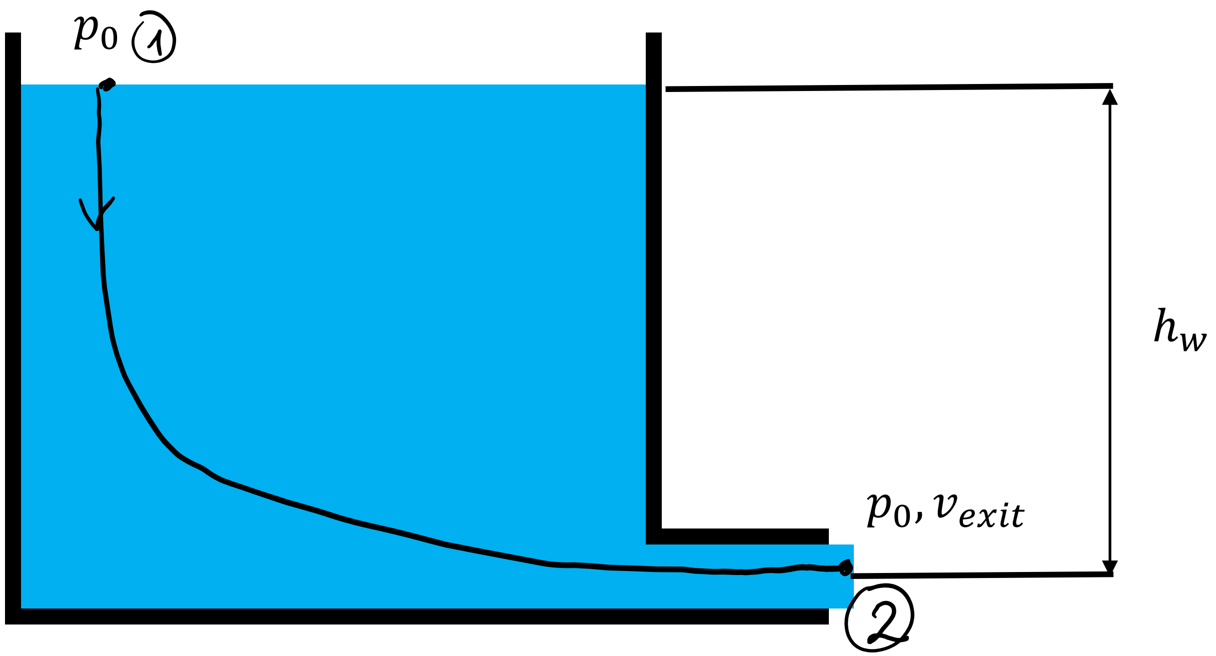
Valve

Bernoulli

① → ②



Incompressible Fluid, no friction, along a streamline:

$$\int_1^2 \frac{\partial v}{\partial t} dS + \left[\frac{p}{\rho} + \frac{1}{2} v^2 + g \cdot h(s)\right]_1^2 = 0$$

Simplified for stationary flow:

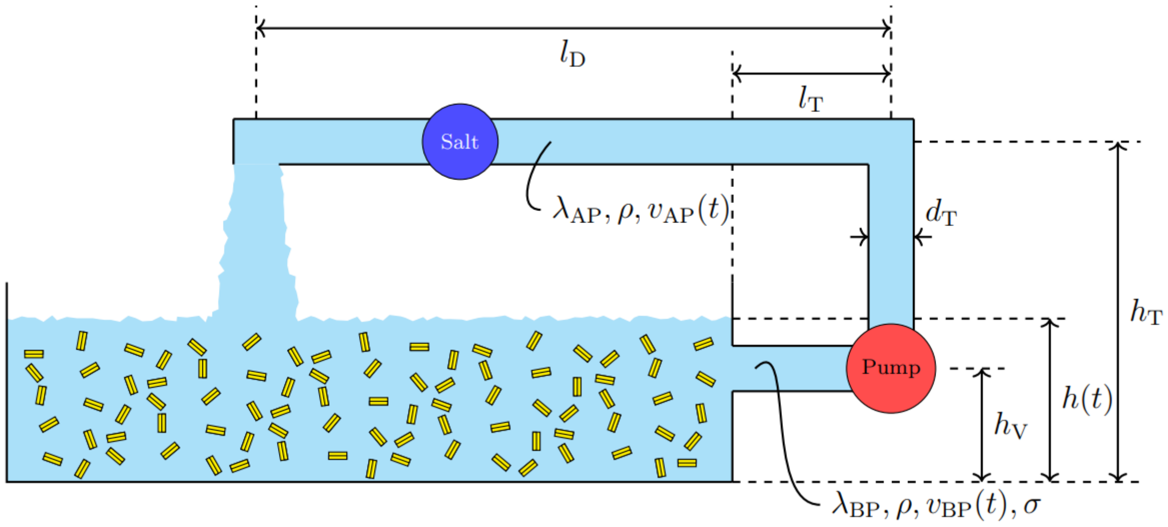
$$v_1^2 + \frac{p_1}{\rho} + g h_1 = v_2^2 + \frac{p_2}{\rho} + g \cdot h_2$$

Here:

$$0^2 + \frac{p_0}{\rho} + g h_w = v_{exit}^2 + \frac{p_0}{\rho} + g \cdot 0$$

$$v_{exit} = \sqrt{g h_w}$$

Example

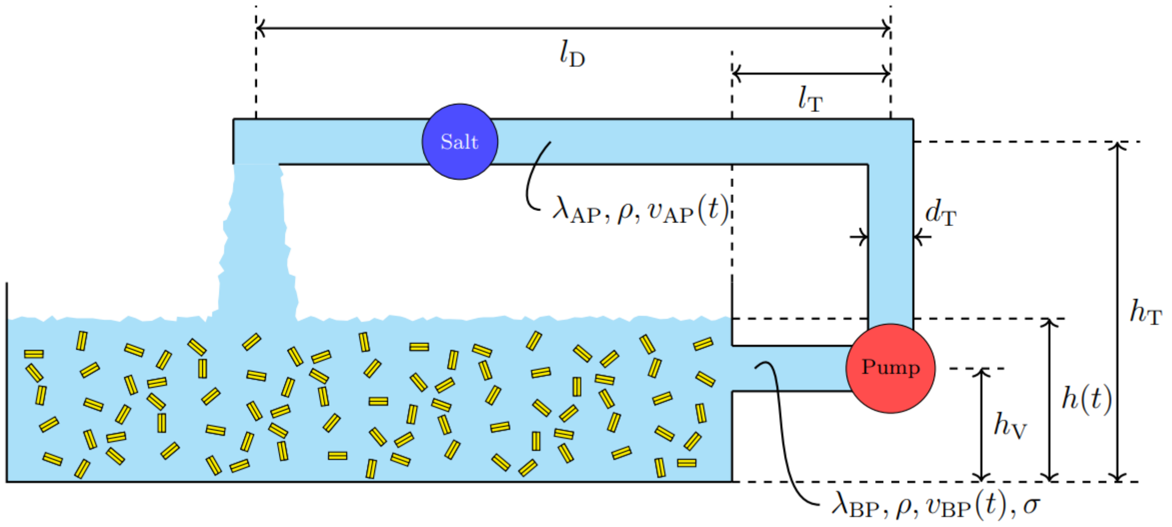


Your SpaghETH is growing every week more and although no particular production issues occur you are concerned about ecology. Since each tank of pasta you cook needs water and a correct salt seasoning for it to taste that delicious, you need a lot of salt and water, which are often wasted. For this reason, you open a research branch in your startup which decides to design a duct-hydraulic system to counteract the waste of water and salt. The tank where the pasta cooks is connected to a duct (diameter d_T), the *Tunnel*. Before the water enters a second duct (diameter d_T), the *Seasoner*, a pump increases its pressure by Δp. Note that only compressibility effects of the *Tunnel* should be taken into account. The pressure at the water's surface p_∞ is assumed to be known. Assume a circular tunnel, whose area reads $A_T = \pi d_T^2/4$. The area of the water tank is A_W (with $A_W \gg A_T$). A sketch of the system with the relative parameters is shown in Figure 4.

1. List all the reservoirs and the relative state variables.
2. Find the pressure $p_1(t)$ at the beginning of the *Tunnel* as a function of the velocity in the *Tunnel* $v_{BP}(t)$.
3. Formulate the differential equation for $v_{BP}(t)$ as function of the pressure right before the pump $p_2(t)$.
4. Exploiting the compressibility of the *Tunnel*, find the pressure $p_2(t)$ explicitly.
5. Formulate the differential equation for $v_{AP}(t)$.
6. Formulate the differential equation for $h(t)$.

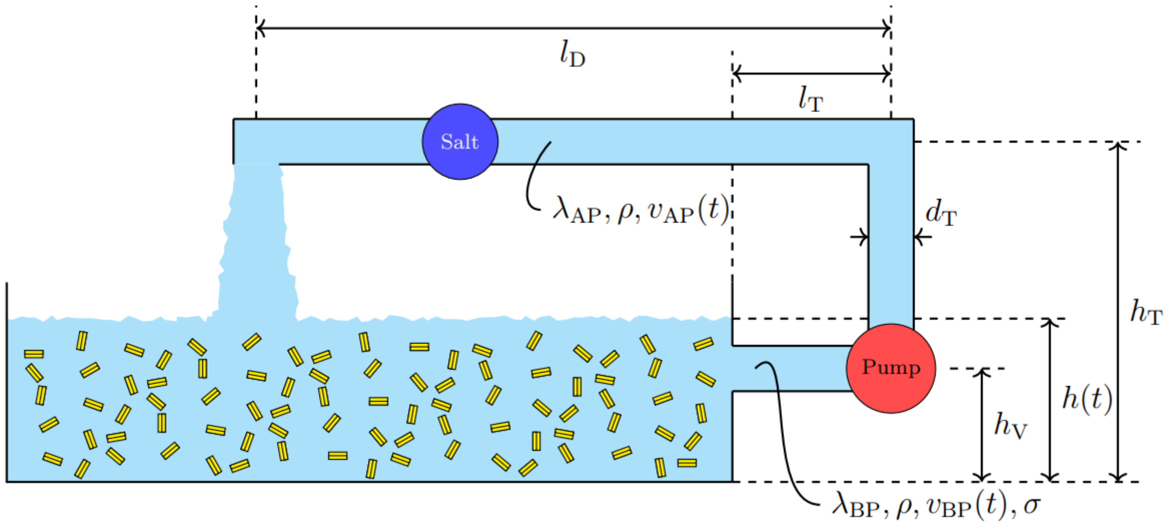
Example

1. List all the reservoirs and the relative state variables.



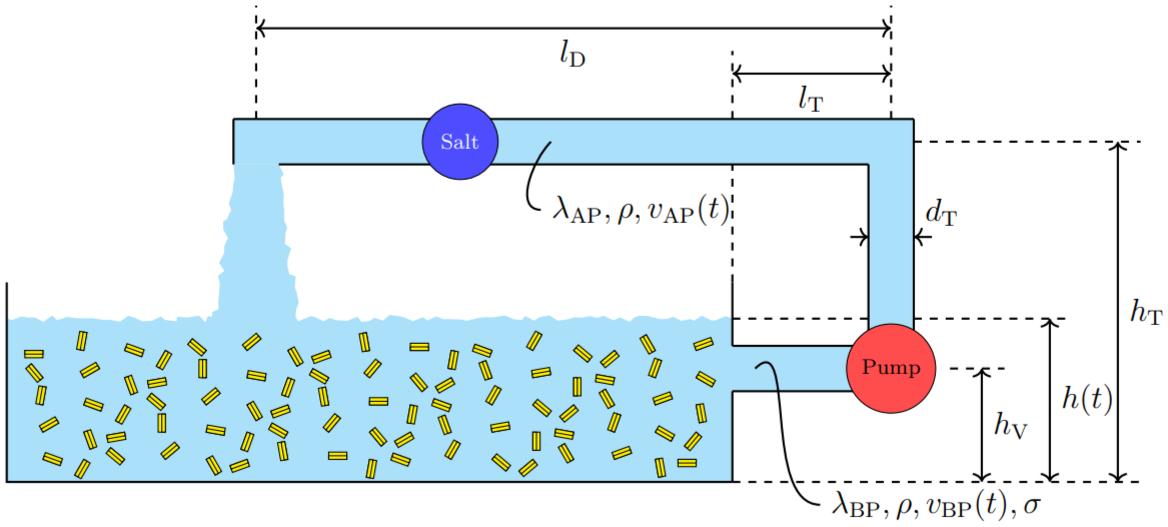
Example

2. Find the pressure $p_1(t)$ at the beginning of the *Tunnel* as a function of the velocity in the *Tunnel* $v_{\mathrm{BP}}(t)$.



Example

3. Formulate the differential equation for $v_{\mathrm{BP}}(t)$ as function of the pressure right before the pump $p_2(t)$.



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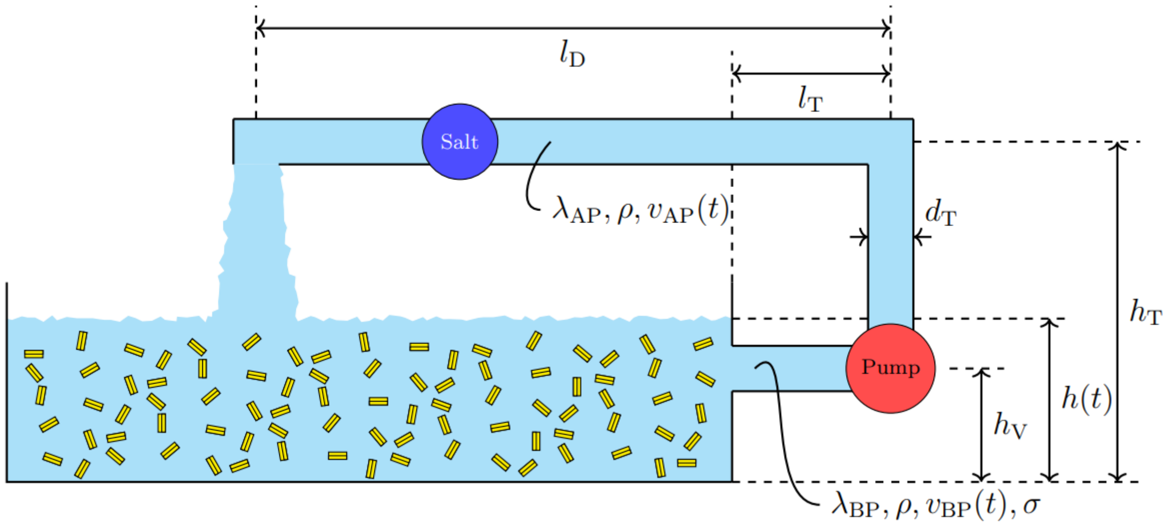
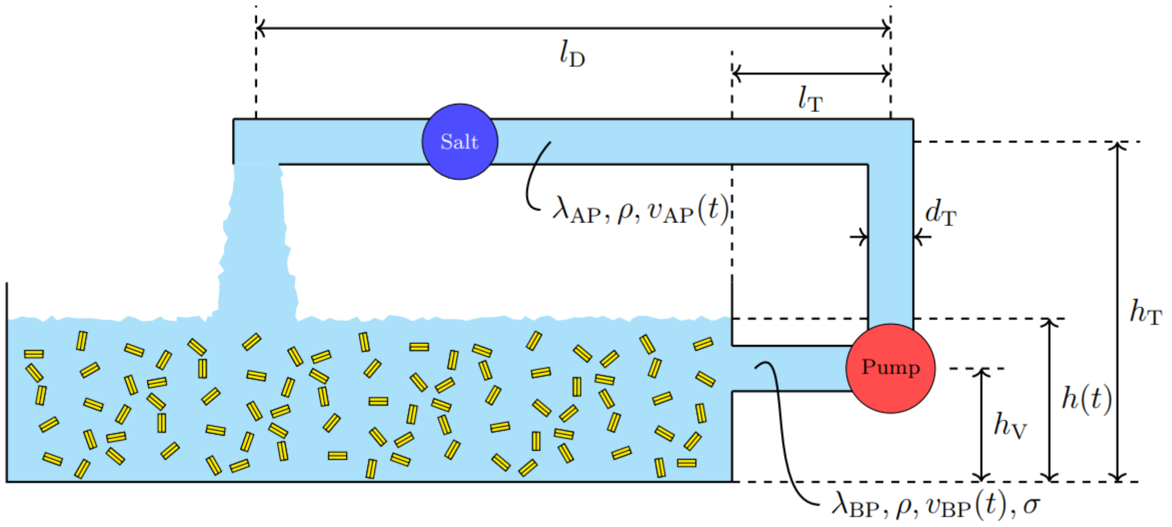
Example

4. Exploiting the compressibility of the *Tunnel*, find the pressure $p_2(t)$ explicitly.



Example

5. Formulate the differential equation for $v_{\mathrm{AP}}(t)$.



Example

6. Formulate the differential equation for $h(t)$.