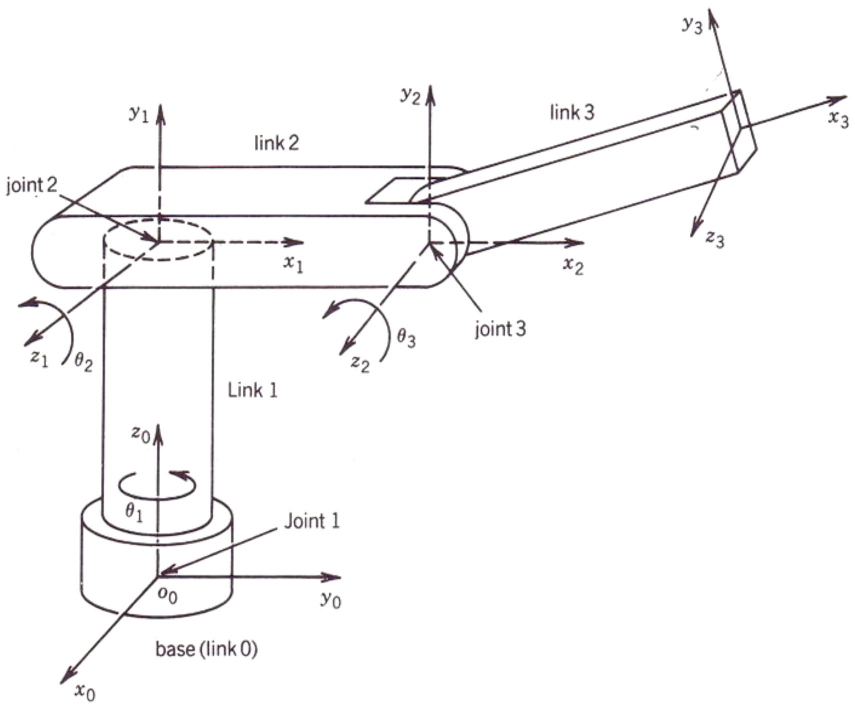
ETH zürich

IRIS Institute of Robotics and Intelligent Systems

Forward kinematics DH convention

Forward and Inverse Kinematics

- Kinematics: To describe the motion of the manipulator without consideration of the forces and torques causing the motion : A Geometric Description.



Forward Kinematics

To determine the position and orientation of the end effector with the given values for the joint variables.

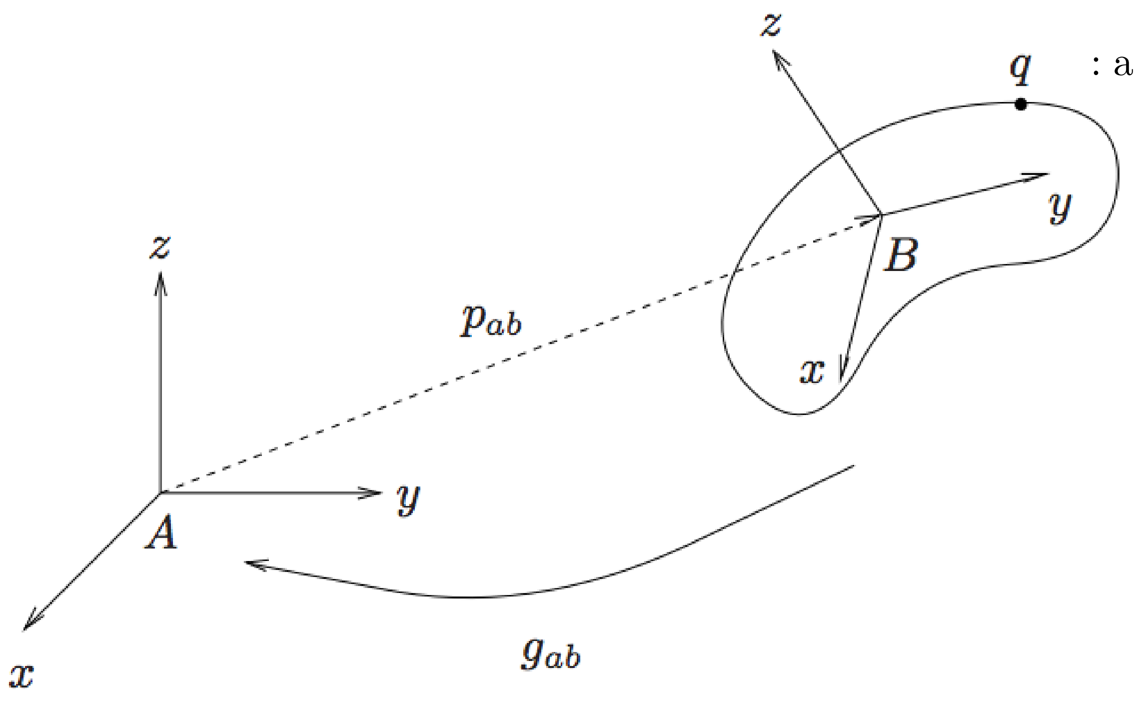
$$\theta_1, \theta_2, \theta_3 \rightarrow R_3^0$$

Inverse Kinematics

To determine the joint variables with the given the end effector's position and orientation.

$$R_3^0 \rightarrow \theta_1, \theta_2, \theta_3$$

Rigid Body Motion



: a point attached to the rigid body

$$g_{ab}(t) = \begin{bmatrix} R_{ab}(t) & p_{ab}(t) \\ 0 & 1 \end{bmatrix}$$

the rigid body motion of the frame B attached to the body, relative to a fixed or inertial frame A

Forwards Kinematics with Screw Theory: POE

- Forward Kinematics defines a transformation between the joint space and the task space

$$g_{st}(\theta) = g_{sl_1}(\theta_1) g_{l_1 l_2}(\theta_2) \cdots g_{l_{n-1} l_n}(\theta_n) g_{l_n t}.$$

- Joint Space:
  - Defined by the independent angles θ
  - Configuration of robot joints
- Task Space:
  - Defined by position and orientation of end-effector
  - Cartesian space

Forwards Kinematics with Screw Theory: POE

- General forward kinematics map

$$g_{st}(\theta) = g_{sl_1}(\theta_1) g_{l_1 l_2}(\theta_2) \cdots g_{l_{n-1} l_n}(\theta_n) g_{l_n t}.$$

- Written using the product of exponentials formula:

$$g_{st}(\theta_1, \theta_2, \ldots, \theta_n) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} \ldots e^{\hat{\xi}_n \theta_n} g_{st}(0)$$

$$g_{st}(\theta) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} \ldots e^{\hat{\xi}_n \theta_n} g_{st}(0)$$

- Product of exponentials uses only two frames!
  - Base frame S and tool frame T

Forwards Kinematics with Screw Theory: Example

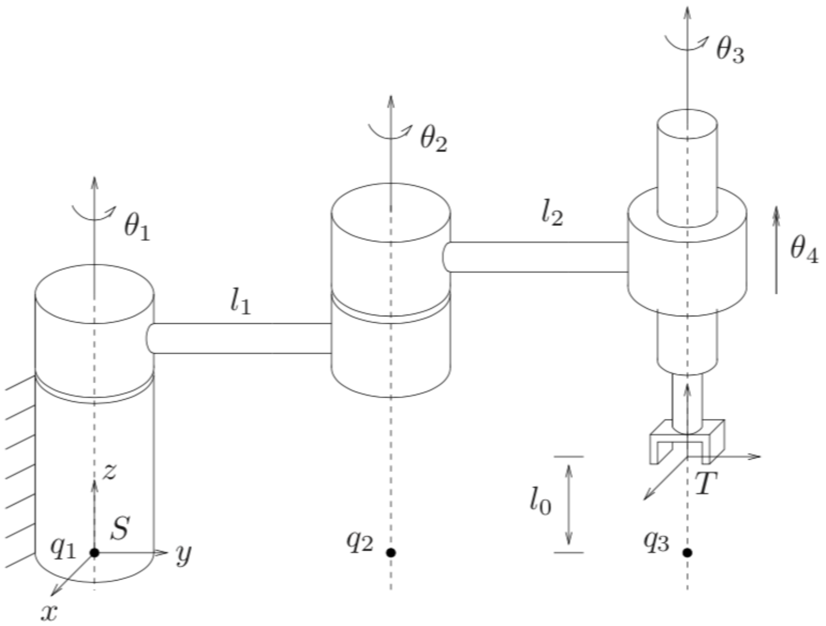
- Start from the general formula:

$$g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} \dots e^{\hat{\xi}_n\theta_n} g_{st}(0)$$

- Find $g_{st}(0)$ and twist coordinates and calculate exponentials:

$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \xi_2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \xi_3 = \begin{bmatrix} l_1+l_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad q_2 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix} \qquad q_3 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix} \qquad \xi_4 = \begin{bmatrix} v_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



- Compute forward kinematics :

$$g_{st}(\theta) = e^{\widehat{\xi}_1\theta_1} e^{\widehat{\xi}_2\theta_2} e^{\widehat{\xi}_3\theta_3} e^{\widehat{\xi}_4\theta_4} g_{st}(0) = \begin{bmatrix} R(\theta) & p(\theta) \\ 0 & 1 \end{bmatrix}$$

Example taken from: A Mathematical Introduction to Robotic Manipulation, page 88f

The Denavit-Hartenberg Convention

- In general, we would need 6 independent parameters to define the transformation between two "neighboring" coordinate frames
- The D-H convention reduces the problem to 4 parameters by a clever choice of the origin and orientation for the coordinate frames
  - Cancellations occur!

- **Each homogeneous transformation A_i can be represented as a product of four basic transformations in a fixed sequence**

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

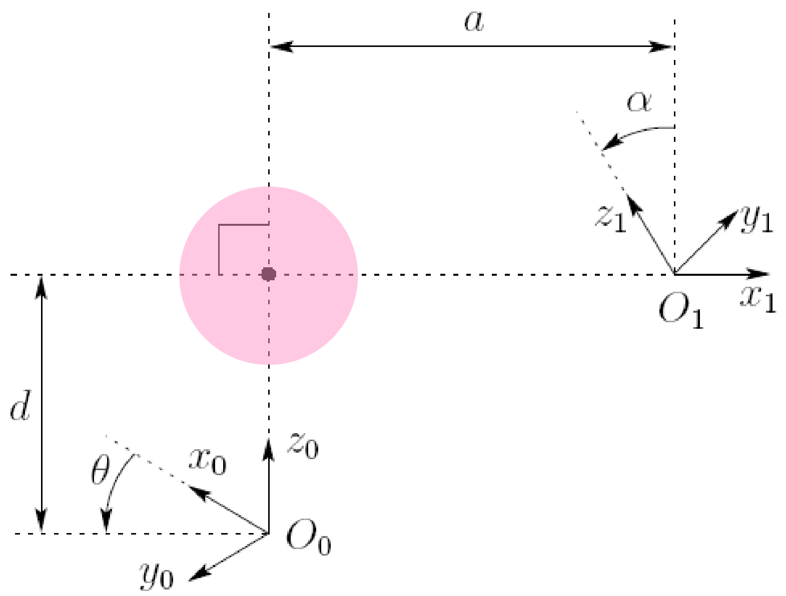
The Denavit-Hartenberg Convention

- Assume three features!

  DH1: The axis X_i is perpendicular to Z_{i-1}

  DH2: The axis X_i intersects the axis Z_{i-1}

  DH3: Z axes are along rotational axes
  or translational direction of the joints



θ_i : joint angle
- angle from x_{i-1} to x_i measured in a plane normal to z_{i-1}

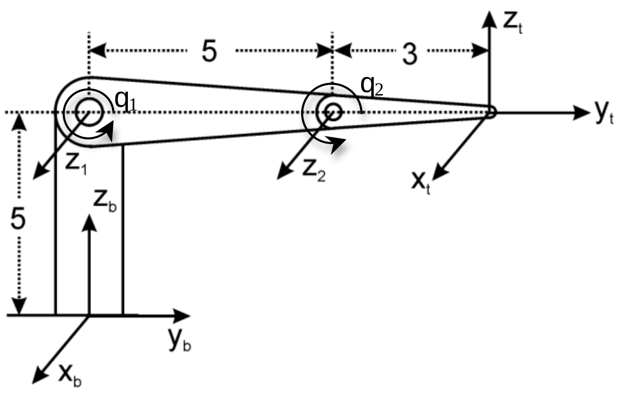
d_i : link offset
- distance from o_{i-1} to intersection of x_i and z_{i-1} measured along z_{i-1}

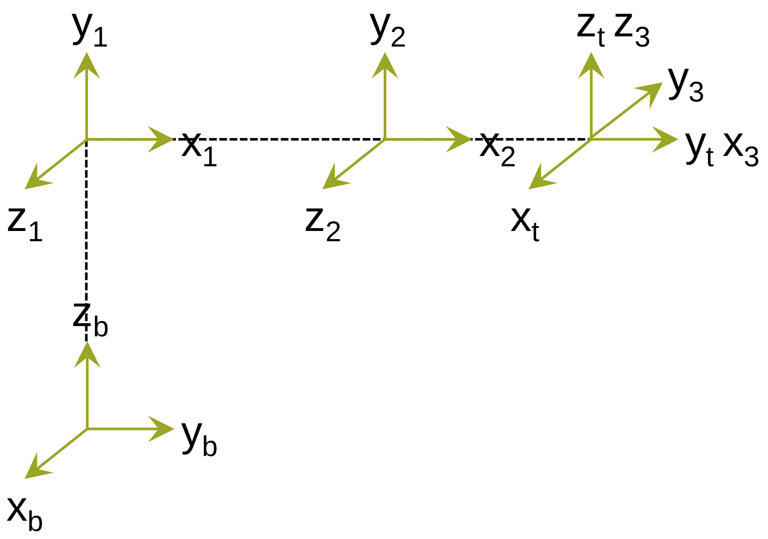
a_i : link length
- distance between z_{i-1} and z_i measured along x_i

α_i : link twist
- angle between z_{i-1} and z_i measured in a plane normal to x_i

The Denavit-Hartenberg Convention: Example

- Forward Kinematics with Denavit-Hartenberg convention





DH1: The axis X_i is perpendicular to Z_{i-1}

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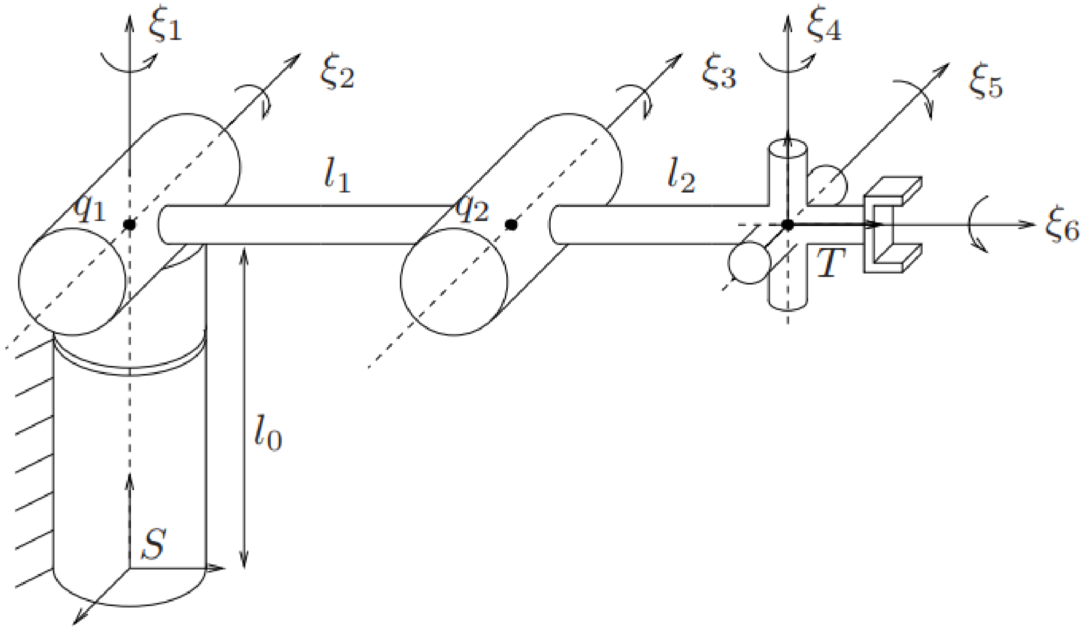
DH3: Z axes are along rotational axes or translational direction of the joints

Link	a	α	d	θ
B to 1	0	90	5	90
1 to 2	5	0	0	θ_1
2 to 3	3	-90	0	θ_2
3 to t	0	0	0	-90

- θ_i : joint angle — angle from x_{i-1} to x_i measured in a plane normal to z_{i-1}
- d_i : link offset — distance from o_{i-1} to intersection of x_i and z_{i-1} measured along z_{i-1}
- a_i : link length — distance between z_{i-1} and z_i measured along x_i
- α_i : link twist — angle between z_{i-1} and z_i measured in a plane normal to x_i

DH Convention: Example (How to place coordinate frames)



DH1: The axis X_i is perpendicular to Z_{i-1}

DH2: The axis X_i intersects the axis Z_{i-1}

DH3: Z axes are along rotational axes or translational direction of the joints

Step 1: Place Z axes

(along rotational or translational axes)

Step 2: Place X axes

(following DH convention)

Step 3: Complete CSs with Y axes

(according to right hand rule)

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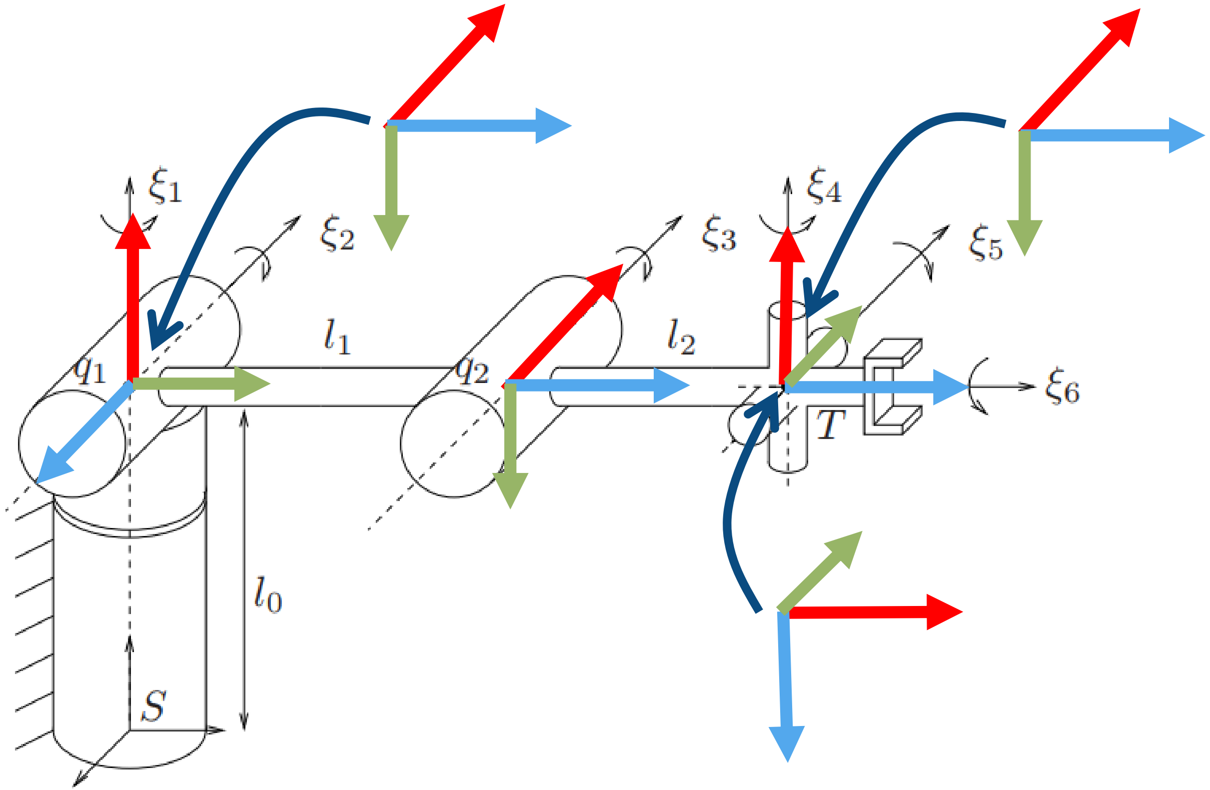

DH1: The axis X_i is perpendicular to Z_{i-1}

DH2: The axis X_i intersects the axis Z_{i-1}

DH3: Z axes are along rotational axes or translational direction of the joints

Step 1: Place Z axes

(along rotational or translational axes)

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(following DH convention)

Step 3: Complete CSs with Y axes

(according to right hand rule)

Assignment 3

a) $g_{0t}(0)$ (by inspection)

b) Screw parameters
h, l, M

c) $\hat{\xi}_i \quad \xi_i$

d) $g_{b0}(0)$

e) $g_{bt}(\theta) = g_{b0} g_{0t}(\theta)$

$$= g_{b0} e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} g_{0t}(0)$$

g) D-H convention, try to find smallest set of CS
Hint: CS do not have to be placed directly on joints