ETH zürich

IRIS
Institute of Robotics and Intelligent Systems

# Parallel & Redundant Robots
# Inverse Kinematics

# Redundant Robots

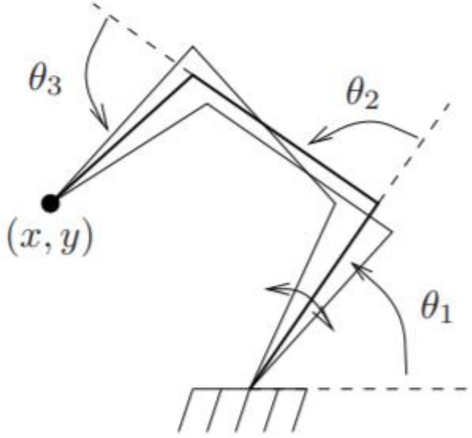
A kinematically redundant manipulator has more than the minimal number of DoFs required to complete a task

- can have multiple joint configurations which produce same end-effector configuration
    - extra DoFs to avoid obstacles and kinematics singularities
- Disadvantages: Loss of rigidity and increased complexity
    - Inverse Kinematics: there may exist infinitely many configurations

- Even if end-effector configuration is fixed, robot can still move

→ Self Motion Manifold

A motion along self-motion manifold is called internal motion




(a)

# Parallel Robots

A parallel manipulator is one in which two or more series chains connect the end-effector to the base of the robot

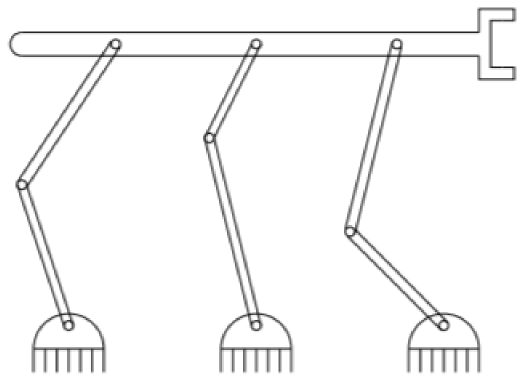
*Advantages*

- more rigidity of mechanism (better accuracy, speed, higher load carrying capabilities vs. open chain robots)
- Placement of actuators

*Disadvantages*

- Basic problem, two or more chains can fight against each other and apply forces which cause no net end-effector wrench. → complex Kinematics
- Generally smaller workspace

A set of joint torques which causes no net end-effector wrench is called an internal force.

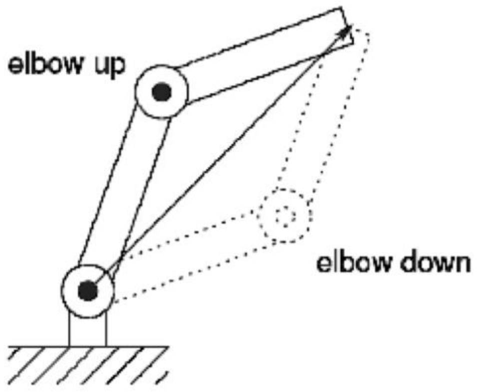


*Can be fully actuated by controlling Only the first link in each chain, No need to place motors at distal links*

# Inverse Kinematics

## Forward Kinematics

To determine the position and orientation of the end effector with the given values for the joint variables.

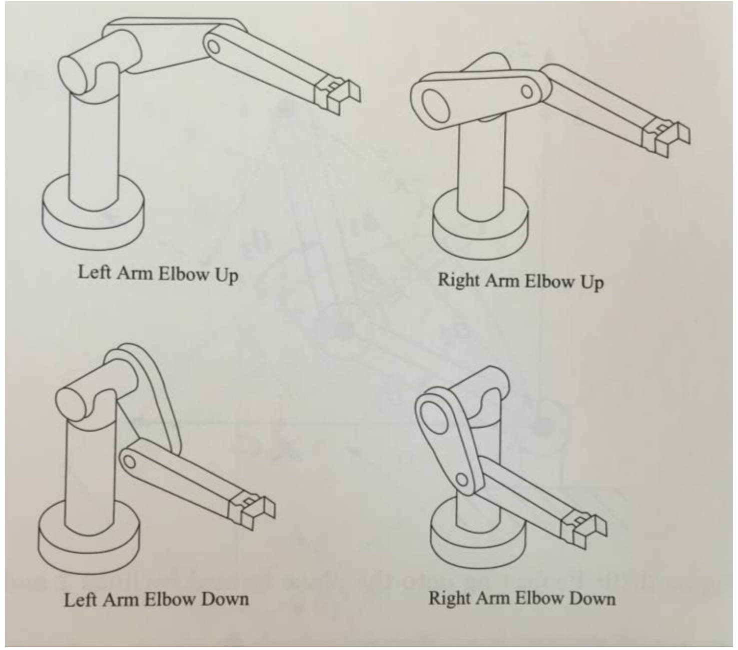



## Inverse Kinematics

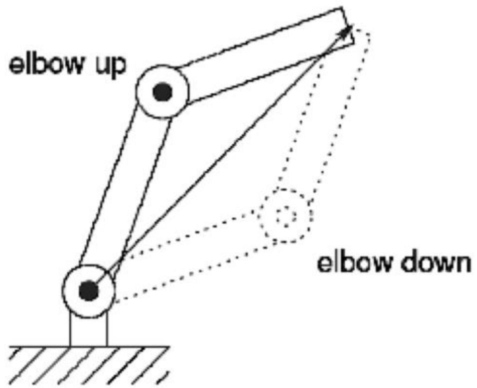
To determine the joint variables with the given end effector's position and orientation.

Solving the inverse kinematics problem, we are interested in finding a closed form (explicit solution) rather than a numerical one.

- There might be no analytic solution
- Multiple solutions or no solutions may exist



# Inverse Kinematics




2 classes of Inverse Kinematics Solutions:

- *Closed-form solutions (analytical solution exists)*
- *Numerical solutions (iterative approximation)*

*Closed form solutions*

- Fast and efficient: Real time calculations possible
- Might be too difficult to find – need to simplify description by decoupling sets of joints

$$\theta_k = f_k(h_{11}, \ldots, h_{34}), \qquad k = 1, \ldots, n$$

Numerical solutions

- High computational effort

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Instead of finding joint positions that achieve a desired end-effector configuration, we can try to find joint velocities that achieve desired end-effector twist → ***Inverse Velocity Kinematics***

$$\begin{bmatrix} v^0 \\ \omega^0 \end{bmatrix} = J_{6xN} \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_N \end{bmatrix} \qquad \dot{\boldsymbol{\theta}} = J^{-1} \begin{bmatrix} v^0 \\ \omega^0 \end{bmatrix}$$

# Assignment 6

- Define path (circle)

- Forward Kinematics: Look at the top view for $X, Y$ *(planar problem), state parametrized position P*

  $X$ = ...

  $Y$ = ...

- Analytical Jacobian $$J = \begin{bmatrix} \frac{\partial P_x}{\partial \theta_1} & \frac{\partial P_x}{\partial \theta_2} \\ \frac{\partial P_y}{\partial \theta_1} & \frac{\partial P_y}{\partial \theta_2} \end{bmatrix}$$

- Inverse Velocity Kinematics $$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J * \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

  Find V [m/s] = dP/(unit time step)

- Calculate new thetas: $$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J^{-1} * \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

  Use pseudoinverse of J (MATLAB: pinv(J), generalized inverse )