$(0.3)\ (0.7) = 0.3087$

 # Pb A multiple choice test consists of 8 questions with 3 answers to each question (of which only one is correct). A student answers each question by rolling a balanced die and checking the first answer if he gets 1 (or) 2, the second answer if he gets 3 (or) 4 and the third answers if he gets 5 (or) 6. To get a distinction, the student must secure atleast 75% correct answers. If there is no negative marking what is the prob that the student secures a distinction.

Sol^n

$p = \frac{1}{3}$ so that $q = 1 - \frac{1}{3} = \frac{2}{3}$

Hence the required prob of securing a distinction (ie, getting correct answers to atleast 6 out of the 8 questions) is given by

$$P(6) + P(7) + P(8) = 8C_6 \left(\tfrac{1}{3}\right)^6 \left(\tfrac{2}{3}\right)^{8-6} + 8C_7 \left(\tfrac{1}{3}\right)^7 \left(\tfrac{2}{3}\right)^{8-7} + 8C_8 \left(\tfrac{1}{3}\right)^8 \left(\tfrac{2}{3}\right)^{8-8}$$

$$= \frac{1}{3^6}\left[28 \times \frac{4}{9} + 8 \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{9}\right] = \frac{129}{729 \times 9}$$

$$= 0.0197.$$

Poisson distribution: If n is large the evaluation of binomial prob can involve considerable computation. In such a case simple approximation to binomial property that is poisson distribution can be used.

Def: A random variable X is said to follow poisson distribution if it assumes only non-negative values and its prob Mass function is given by

$$P(X=x) = P(x,\lambda) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{\lfloor x}, & x=0,1,2,\dots \quad \lambda>0 \\ 0, & \text{elsewhere} \end{cases}$$

MGF

$$M_x(t) = \sum e^{tx} p(x)$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{\lfloor x} e^{tx}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{\lfloor x}$$

$$= e^{-\lambda}\left[1 + \frac{(\lambda e^t)^1}{1!} + \frac{(\lambda e^t)^2}{2!} + \frac{(\lambda e^t)^3}{3!} + \dots \infty\right]$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$$

$$M_x(t) = e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t-1)} \dots$$

Mean

$$M_x'(t) = e^{\lambda(e^t-1)} \cdot \lambda(e^t-0) = \underset{(u)}{e^{\lambda(e^t-1)}} \cdot \underset{(v)}{\lambda e^t} \quad \text{—— } (*)$$

$d(uv) = udv + vdu$

$$M_x''(t) = \lambda\left[e^{\lambda(e^t-1)} \cdot e^t + e^t e^{\lambda(e^t-1)} \lambda(e^t-0)\right] \quad \text{—— } (**)$$

Put $t=0$ in $(*)$

$$M_x'(0) = e^{\lambda(1-1)} \cdot \lambda e^0 = e^0 \cdot \lambda e^0 = \lambda$$

$$\text{Mean} = E(X) = \lambda$$

Put $t=0$ in (**)

$$M''_x(0) = \lambda\left[e^{\lambda(1-1)}(1) + e^0 e^{\lambda(1-1)}(1)\lambda\right]$$

$$E(x^2) = \lambda[1+\lambda] = \lambda+\lambda^2 \text{ (Second order) Moment)}$$

$$\text{variance} = E(x^2) - (Ex)^2$$
$$= \lambda+\lambda^2-(\lambda)^2$$
$$= \cancel{\lambda}+\lambda^2-\cancel{\lambda^2} = \lambda \quad \Rightarrow \text{variance} = \lambda$$

$$\boxed{\therefore \text{Mean} = \text{variance} = \lambda}$$

#Pb Find the prob that atmost five defection fuses will be found in a box of two hendred fuses, if experience shows that ~~2%~~ 2% of fuses are defective.

Solⁿ Given $n=200$ $p=0.02$ $\lambda = np = 200\times 0.02 = 4$

$$\Rightarrow P(x\le 5) = \frac{e^{-4}4^0}{\underline{|0}} + \frac{e^{-4}4^1}{\underline{|1}} + \frac{e^{-4}4^2}{\underline{|2}} + \frac{e^{-4}4^3}{\underline{|3}} + \frac{e^{-4}4^4}{\underline{|4}} + \frac{e^{-4}4^5}{\underline{|5}}$$

$$= e^{-4}\left[\frac{4^0}{\underline{|0}} + \frac{4^1}{\underline{|1}} + \frac{4^2}{\underline{|2}} + \frac{4^3}{\underline{|3}} + \frac{4^4}{\underline{|4}} + \frac{4^5}{\underline{|5}}\right]$$

$$= e^{-4}\left[\frac{1}{1} + \frac{4}{1} + \frac{16}{2} + \frac{64}{6} + \frac{256}{24} + \frac{1024}{120}\right]$$

$$= e^{-4}\left[1+4+8+\frac{32}{3}+\frac{32}{3}+8.53\right]$$

$$= e^{-4}\left[1+4+8+10.66+10.66+8.53\right]$$

$$= e^{-4}[42.863] = 0.018315\times 42.863 = 0.785$$

If 3% of electric bulbs manufactured by a company are defective. Find the prob that in a sample of 100 bulbs exactly 5 are defective.

Sol^n $n = 100 \quad p = 0.03 \quad \lambda = np = 100 \times 0.03 = 3$

$$P(x=5) = \frac{e^{-3} 3^5}{\lfloor 5} = \frac{e^{-3}(243)}{120} = \frac{0.049 \times 243}{120} = \frac{12.09}{120}$$

$$= 0.1008$$

An insurance company insures 4000 people against loss of both eyes in a car accident based on previous data, the rates were computed on the assumption that on the average 10 persons in 1,00,000 will have car accident each year that result in this type of injury. What is the probability that more than 3 of the insured will collect on their policy in a given year?

Sol^n Gn $n = 4000$ and $p =$ prob of loss of both eyes in a car accident $= \frac{10}{100,000} = 0.0001$

Since p is very small and n is large we can use poisson distribution

$$\lambda = np = 4000 \times 0.0001 = 0.4$$

$$P(x>3) = 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$$

$$= 1 - \left[\frac{e^{-0.4}(0.4)^0}{\lfloor 0} + \frac{e^{-0.4}(0.4)^1}{\lfloor 1} + \frac{e^{-0.4}(0.4)^2}{\lfloor 2} + \frac{e^{-0.4}(0.4)^3}{\lfloor 3}\right]$$

$$= 1 - e^{-0.4}\left((0.4)^0 + (0.4)^1 + \frac{(0.4)^2}{2} + \frac{(0.4)^3}{6}\right]$$

$$= 1 - 0.6703(1 + 0.4 + 0.08 + 0.0107] = 1 - 0.6703 \times 1.4907$$

$$= 0.0008$$

Pb A Manufacturer, who produces medicine bottles find that 0.1% of the bottles are defective. The bottles are packed in boxes; each Containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottle. using poisson distribution, find how many boxes will contain.

(i) no defective

(ii) at most 3 defective

(iii) at least two defective

$$\boxed{e^{-0.5} = 0.6065}$$

Sol^n

$N=100, \quad n=500 \quad p = 0.001$ and $\lambda = np = 500 \times 0.001 = 0.5$

$$\Rightarrow P(X=x) = \frac{e^{-0.5}(0.5)^x}{\lfloor x}, \quad x = 0, 1, 2, \ldots$$

Hence in Consignment of 100 boxes, the frequency number of boxes containing x defective bottle is

$$f(x) = N \times P(X=x) = \frac{100 \times e^{-0.5}(0.5)^x}{\lfloor x} \quad x = 0, 1, 2 \ldots 500$$

(i) No defective $\Rightarrow 100 \times P(X=0) = 100 \times 0.6065 = 60.65 \simeq 61$ bottles

(ii) atmost 3 defective $= 100 \times P[x \leq 3] = 100 \times [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 100 \times 0.6065 [1 + 0.5 + 0.125 + 0.0208]$

$= 100 \times 0.6065 \times 1.6458 = 100 \times 0.9981 \approx 100$ bottles

(iii) atleast 2 defective $= 100 \times [P(x \geq 2)] = 100 (1 - [P(x=0) + P(x=1)])$

$= 100 [1 - 0.6065 - 0.6065 \times 0.5]$

$= 100 \times 0.09025 \approx 9$ bottles