Test answer

法 1

证明

设 A 关于 MN 的对称点为 A''

设 $A'B, A'C$ 的中垂线交 BC 于 P, Q

设 MN 交 $A'A''$ 于 R

$\because \angle MPA' = \angle MRA' = 90^\circ, \angle NQA' = \angle NRA' = 90^\circ$

$PMRA', NQRA'$ 分别共圆

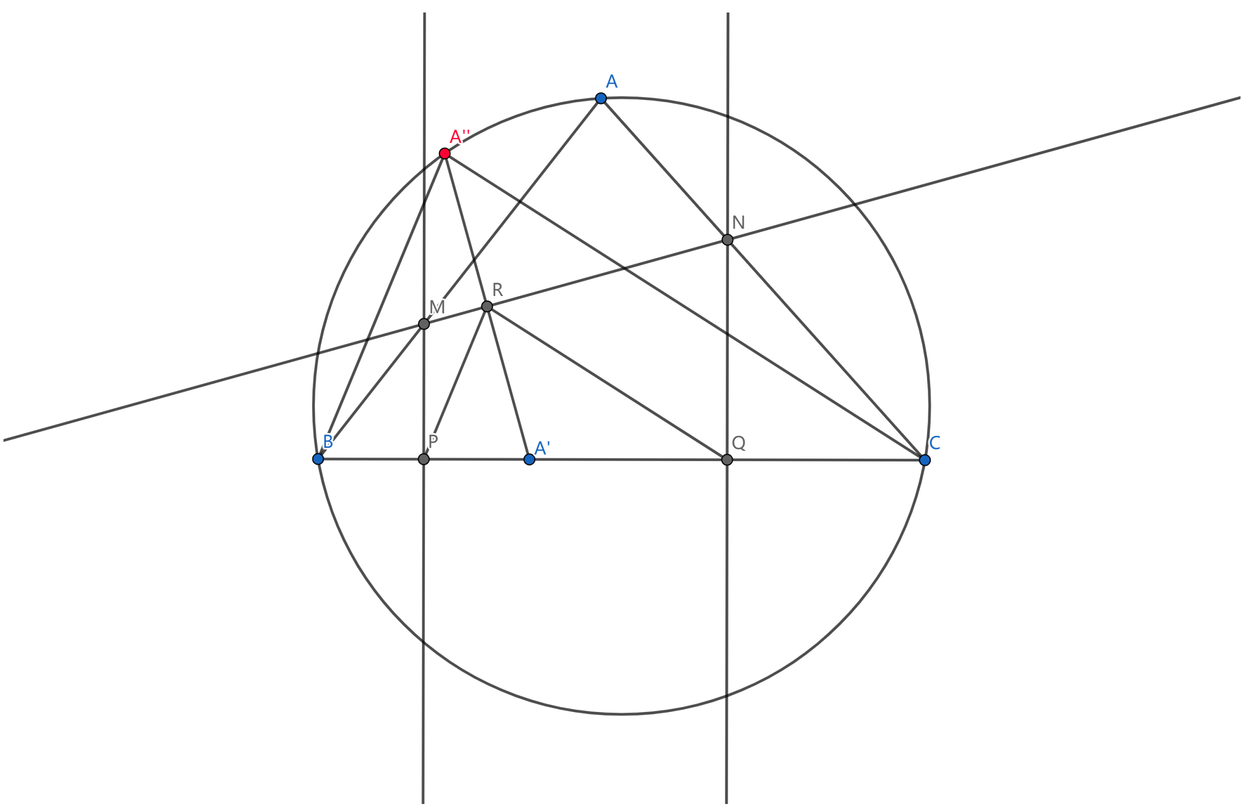
则有 $\angle PRQ = \angle BAC$

又 $PR // BA'', QR // CA''$

则 $\angle PRQ = \angle BA''C$

则 $ABCA''$ 四点共圆

$Q.E.D$



法 2

证明:

$\angle MXN = \angle MAN = \angle A$

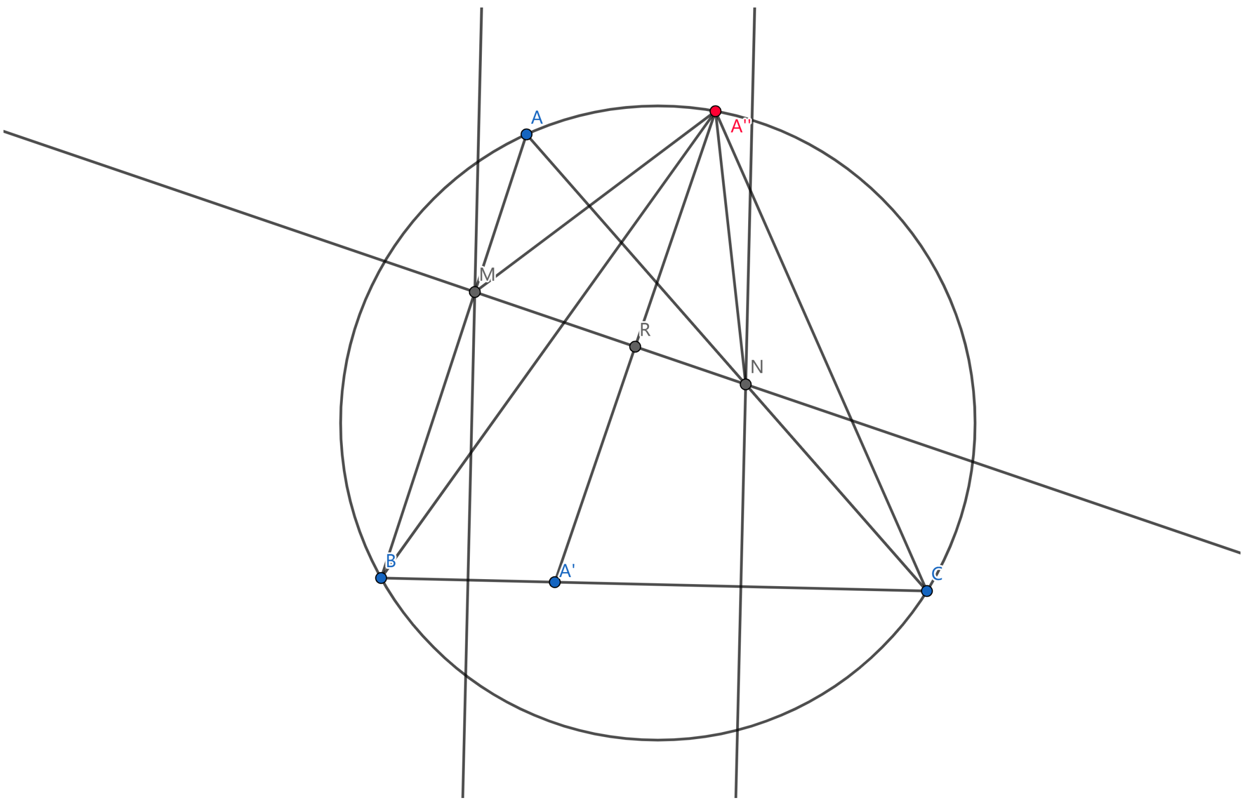
$\therefore$ 只需证 $\angle MA''B = \angle CA''N$

$\therefore$ 只需证 $\triangle A''MB \backsim \triangle A''CN$

$\because \angle BMA'' = 180^\circ - \angle AMA'' = 180^\circ - \angle ANA'' = \angle A''NC$

$\triangle A''MB \backsim \triangle A''CN$

$Q.E.D$



法 3

M 为 $\triangle A'A''B$ 外心，N 为 $\triangle A'A''C$ 外心

设 $\angle BA''A' = \alpha, \angle CA'A'' = \beta$

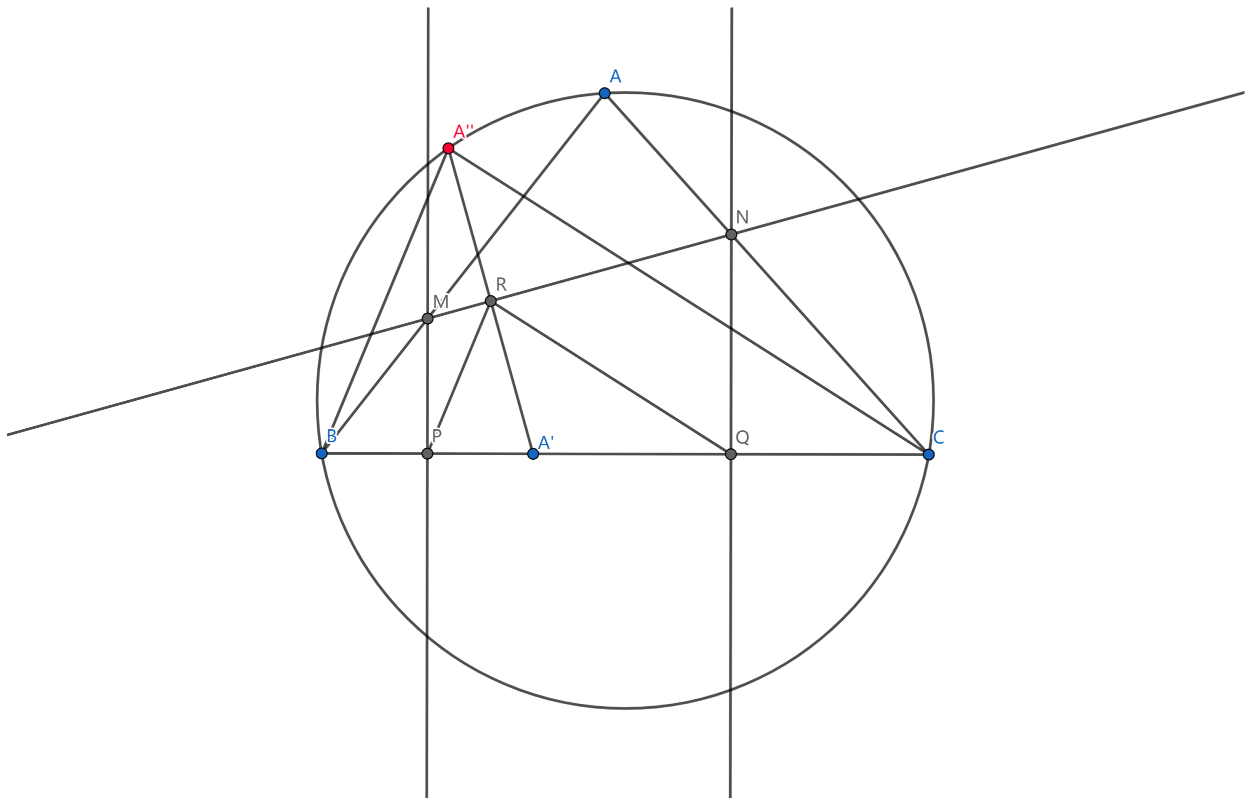
则 $\angle PNA' = \alpha, \angle QNA' = \beta, \angle BMA' = 2\alpha, \angle CNA' = 2\beta$

又 $MP // NQ$

$\therefore \angle MA'N = \alpha + \beta$

$\angle BAC = \alpha + \beta = \angle MA'N$

$Q.E.D$



法4

证明:

过 A' 作 $A'R \perp MN$ 交 (ABC) 于 A''

即证 MN 垂直平分 AA''

即证 $A'R = A''R$

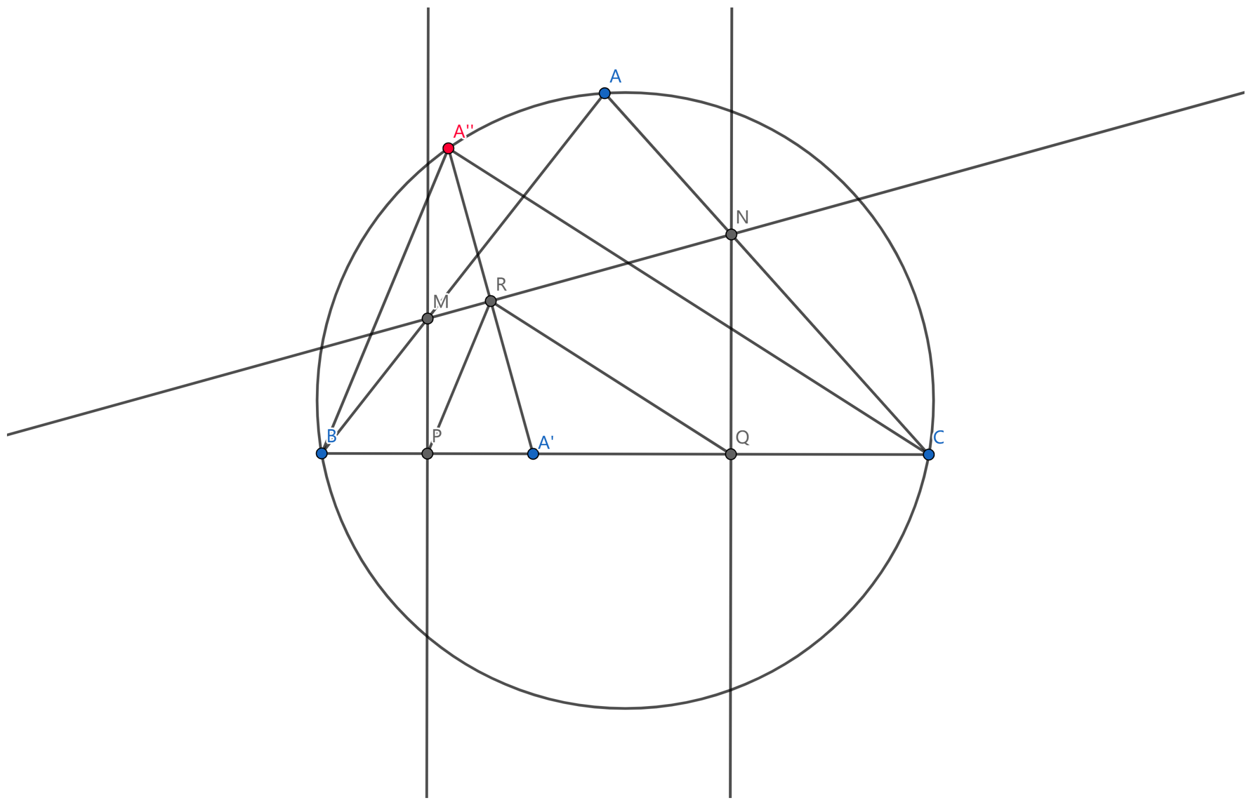
$\angle NCQ = \angle NFQ, \angle MBP = \angle MFP$

$\angle EPQ = \angle BAC = \angle BA''C$

$\therefore \triangle DEF$ 与 $\triangle CA''B$ 位似，$k = \frac{1}{2}$

则 $A'R = A''R$

$Q.E.D$



A
A''
N
R
M
B
P
A'
Q
C