3.29

证明:

$\because OM \perp AB, ON \perp AC$

$\therefore AMON$ 四点共圆，圆心为 OA 中点

设圆心为 O_2

$\because Pow_O(A) = Pow_{O_2}(A) = 0$

$\Longleftarrow$ 只需证 AR 为圆 O 与圆 O_2 的根轴

$\Longleftarrow$ 只需证 $Pow_O(R) = Pow_{O_2}(R)$

$\Longleftarrow$ 只需证 $RM \cdot RN = RQ \cdot RP$

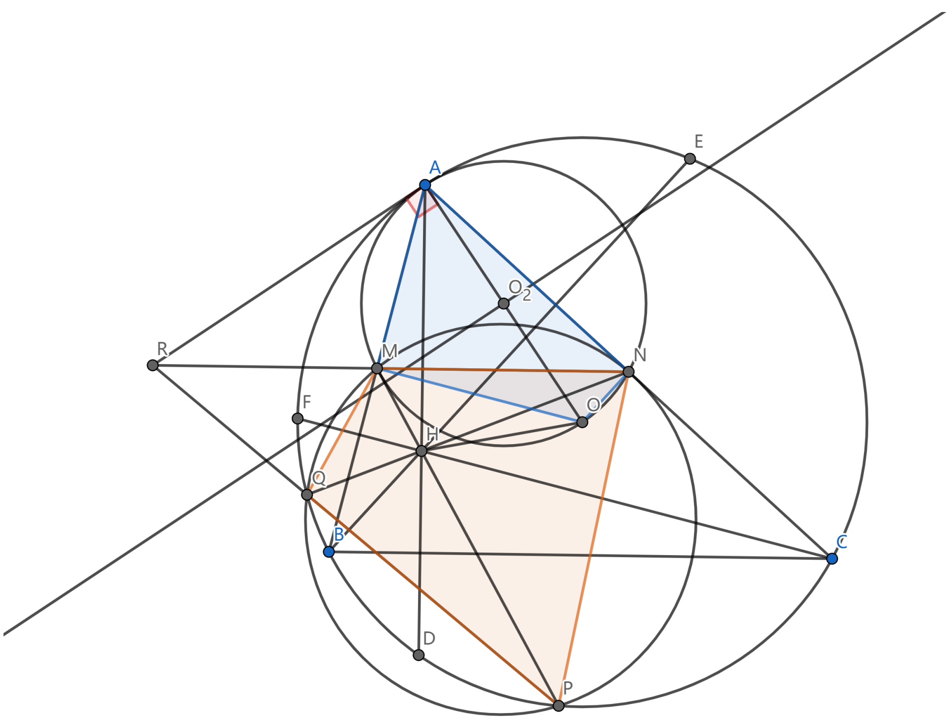
$\because MNPQ$ 四点共圆

$\therefore RM \cdot RN = RQ \cdot RP$

$\therefore AR$ 为圆 O 与圆 O_2 的根轴

$\therefore AR$ 为圆 O_2 切线，$\angle RAO = 90^\circ$

$Q.E.D$



4.13

证明:

连接 YX，则 $\angle YKD = 90^\circ$

$\Longrightarrow \triangle ADK \backsim \triangle DXY$，设 $\frac{AK}{XY} = \frac{p}{q}$

$\Longrightarrow I_A$ 为 XY 中点

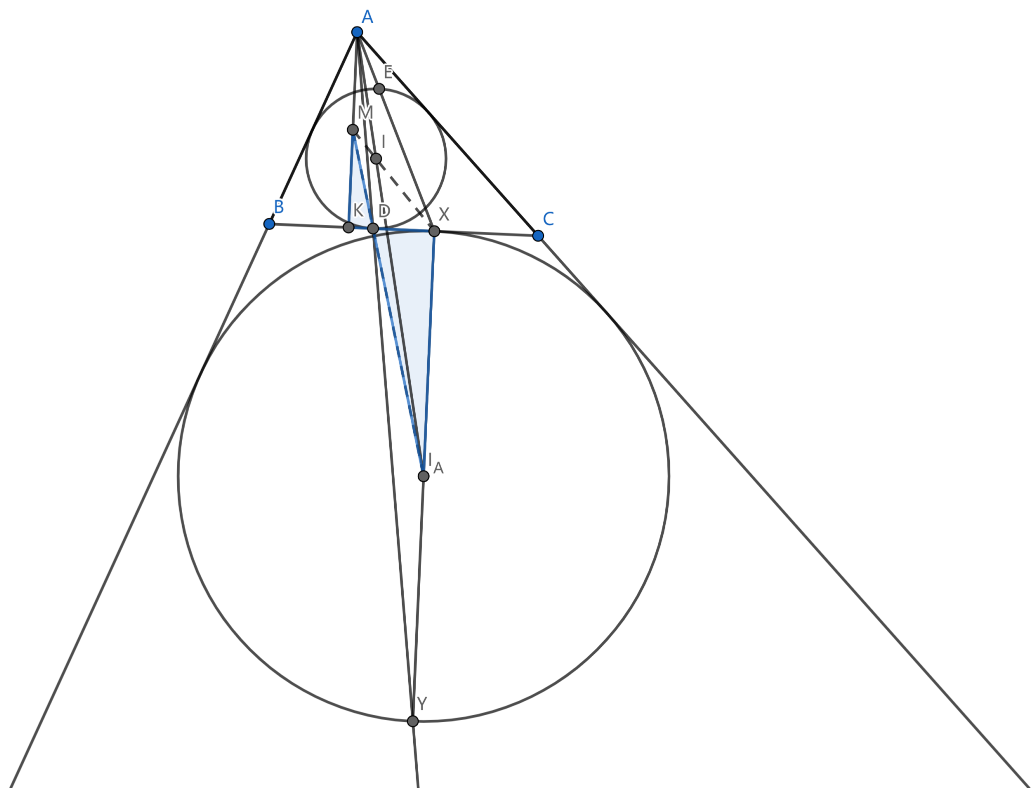
$\therefore \frac{MK}{XI_A} = \frac{\frac{p}{2}}{\frac{q}{2}} = \frac{p}{q} = \frac{KD}{DX}$

又 $\angle YXD = \angle MKD$

$\therefore \triangle MKD \backsim \triangle DKI_A \Longrightarrow \angle MDK = \angle XDO_A$

$\therefore D, I_A, M$ 共线

$Q.E.D$



4.16

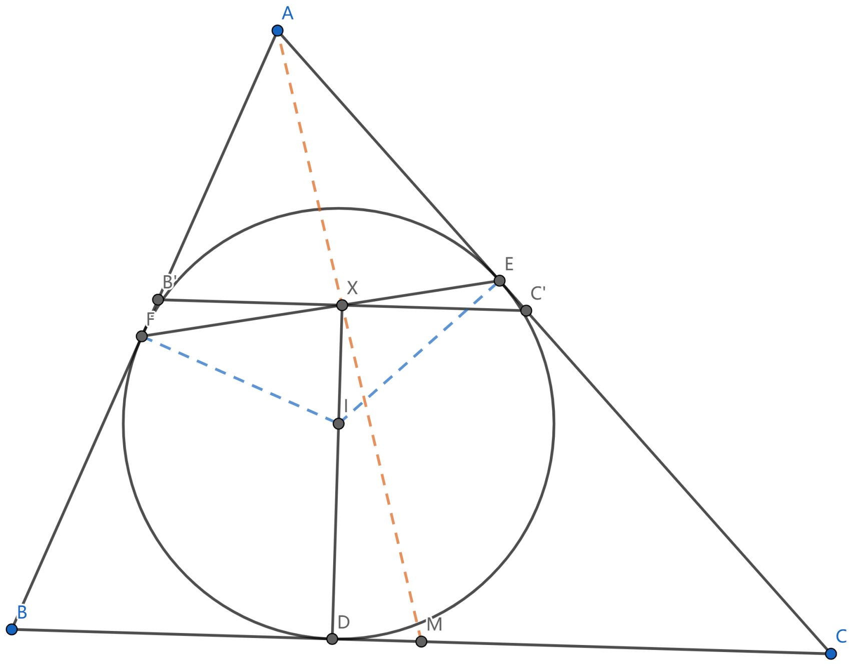
证明:

$\because AXM$ 共线，$\triangle ABC$ 位似于 $\triangle AB'C'$，位似中心为 A

$\therefore \frac{XB'}{XC'} = \frac{MB}{MC}$

$\therefore XB' = XC'$

$Q.E.D$

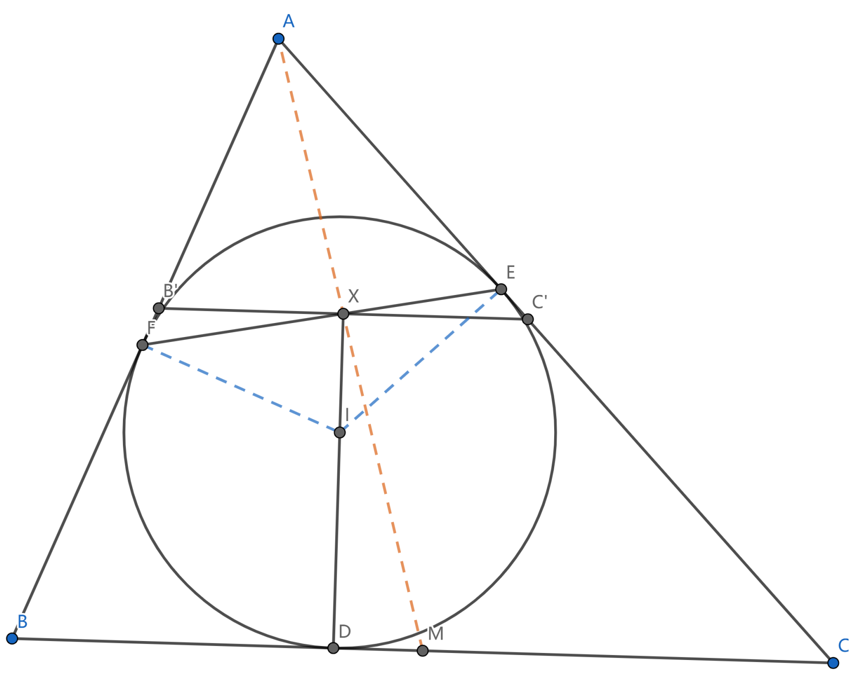


4.17

证明:

EF 过 X，AM 过 X，DI 过 X

$Q.E.D$



4.2

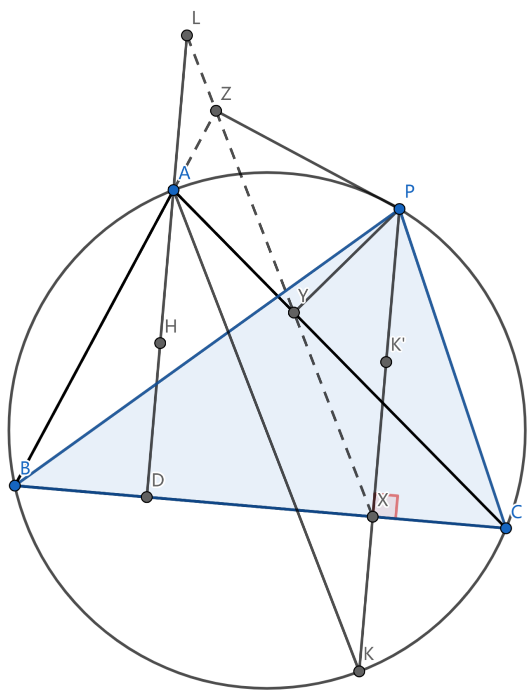
证明:

$\because P \in$ 圆(ABC)，AX 是 $\triangle ABC$ 的一条垂线

根据引理 1.17，可以得到 $\triangle APC$ 的垂心关于 BC 的对称点为 K

$\Longrightarrow K'$ 与垂心重合

$Q.E.D$



4.44

证明:

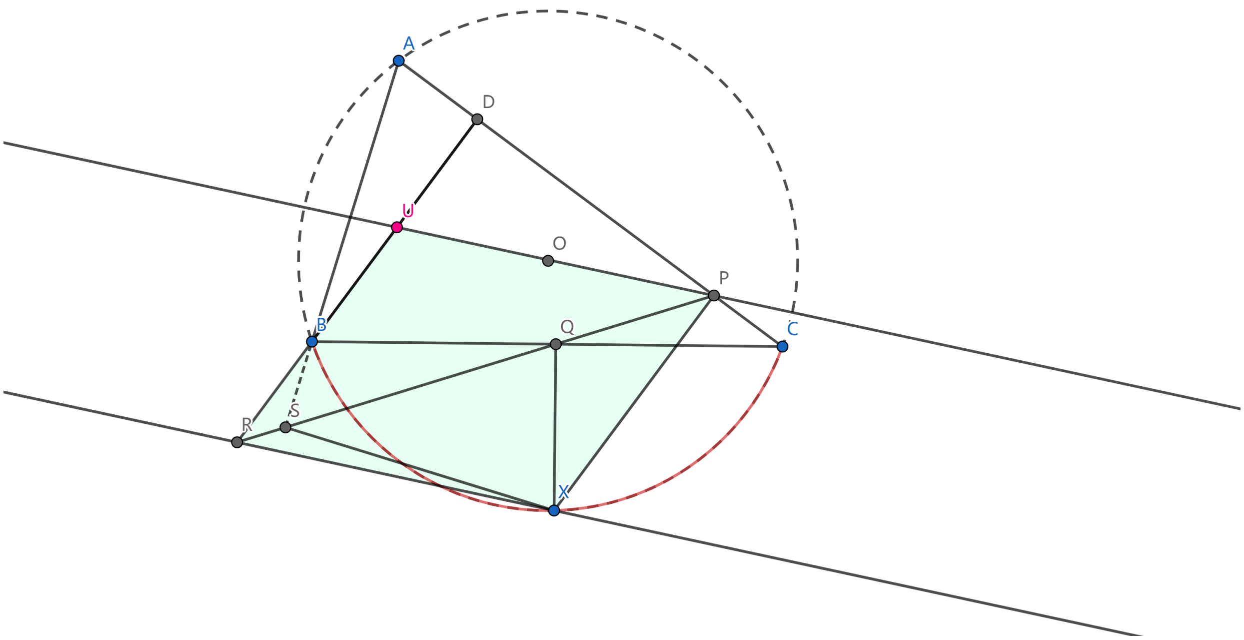
作 $XS \perp AB$ 于 S，设 $l \cap BD = U$

$\Longrightarrow PQS$ 是一条西姆松线

$\therefore UPXR$ 是平行四边形

$\Longrightarrow U$ 是一个定点

$Q.E.D$



4.51

证明:

$\because IP \perp MC$，$KQ \perp IC$

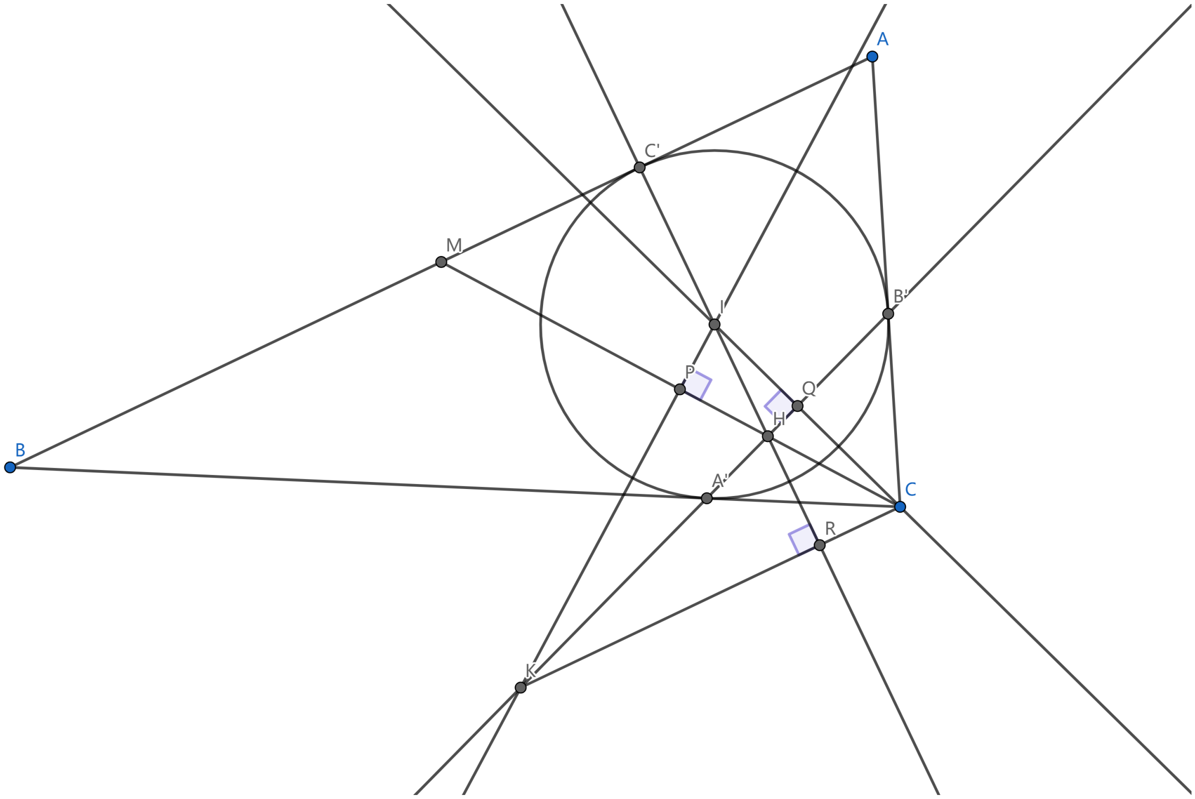
设 $IP \cap KQ = H$，则 H 为 $\triangle ICK$ 重心

作直线 $C'I \cap CK = R$，根据引理 4.17，$H \in C'R$

$\Longrightarrow C'R \perp KC$

又 $C'R \perp AB \Longrightarrow AB//KC$

$Q.E.D$



4.38

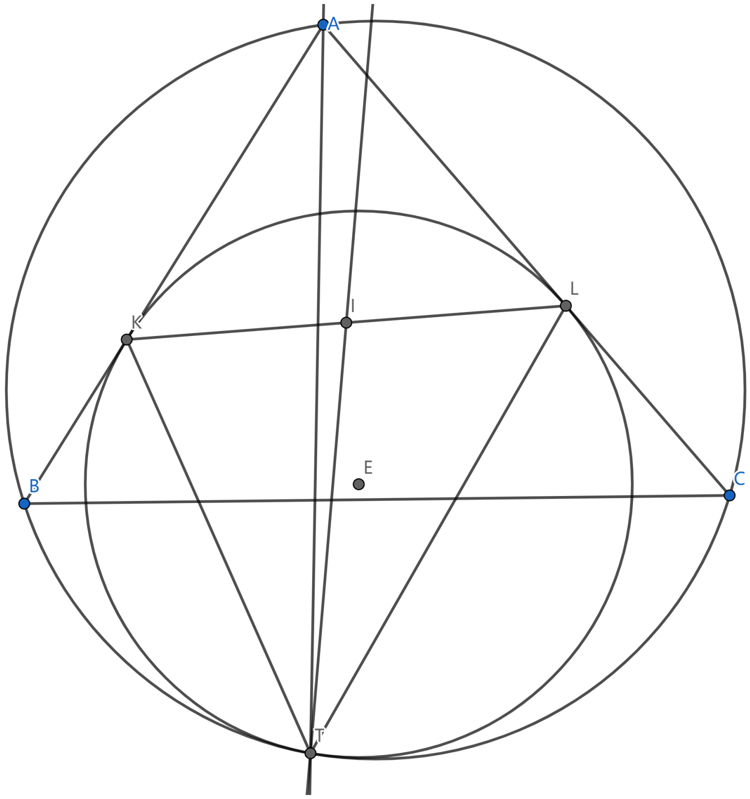
证明:

$\because AB, AC$ 分别切 ω 于 K, L 且 $AB \cap AC = A$

$\implies TA$ 是 $\triangle KLT$ 的类似中线，又 $KI = LI$

$\implies \angle ATK = \angle LTI$

$Q.E.D$



4.39

证明:

设 $TS \cap (KLT) = P$

作 d 切 (KLT) 于 P，作 BC 边的垂直平分线 l

设 $d \cap l = Q$，$TI \cap l = S'$，根据 4.32，$\triangle S'PQ \backsim \triangle STE$

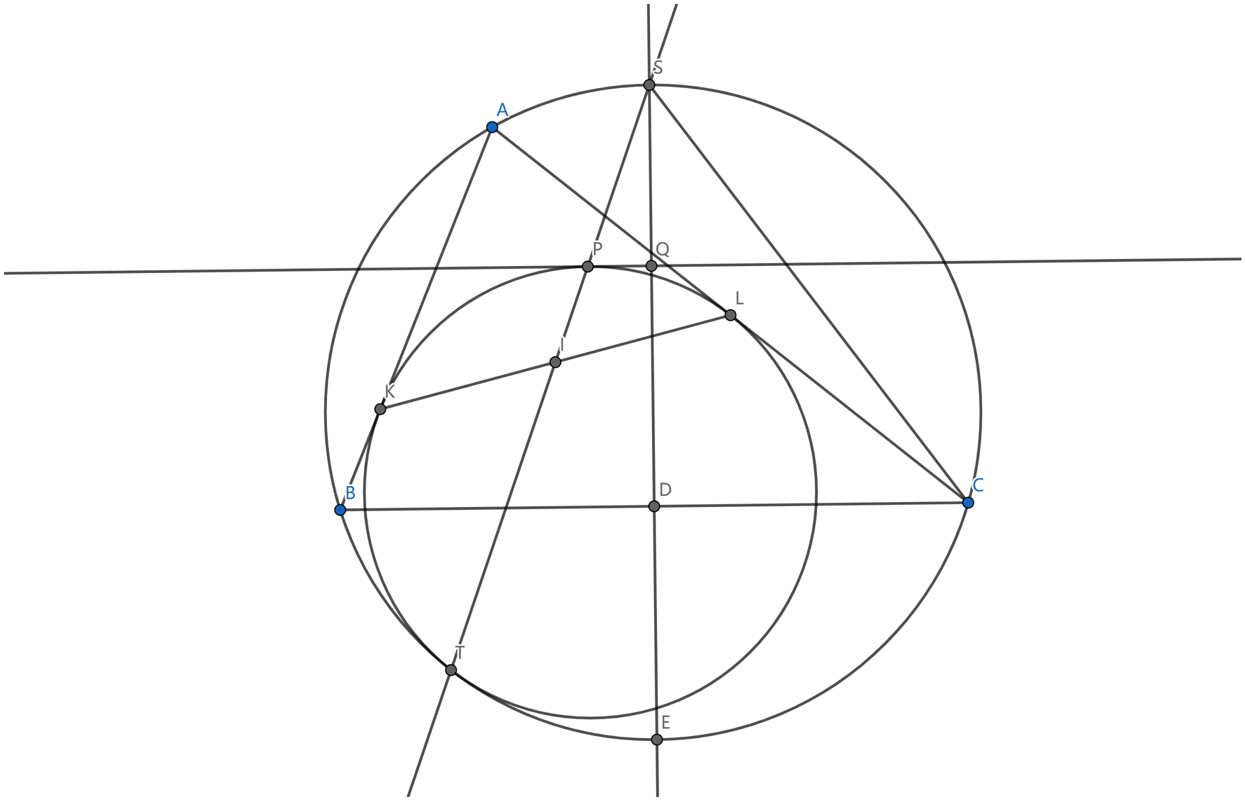
又有 $\triangle SPQ \backsim \triangle STE$，且 $PQ = PQ$

故 $\triangle SPQ \cong \triangle S'PQ$

$\Longrightarrow S = S'$，则 $S \in (ABC)$

故 S 为 弧 (ABC) 中点

$Q.E.D$



4.40(b)

证明:

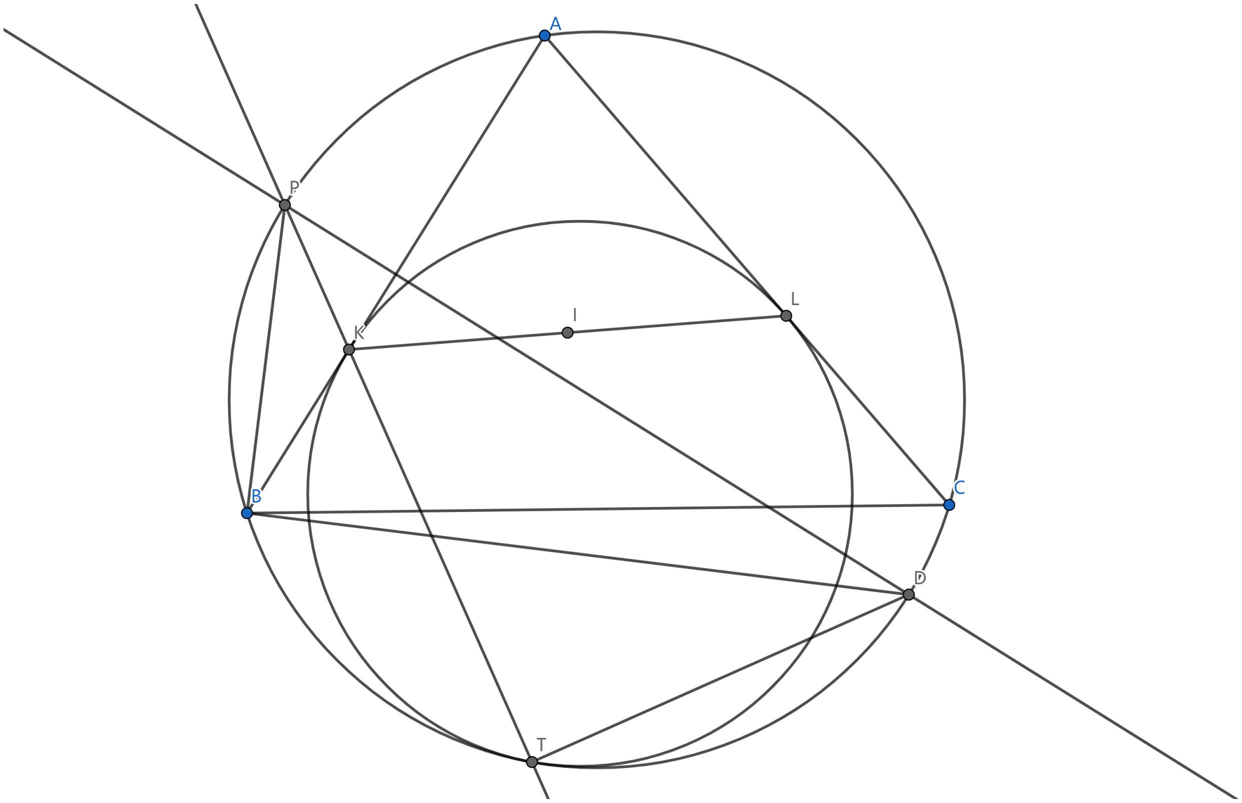
作 AB 的垂直平分线 $l \cap (ABC) = D$

则 PD 为直径

$\implies \angle PBD = \angle PDT$

故 P, A, B, C, D, T 共圆

$Q.E.D$



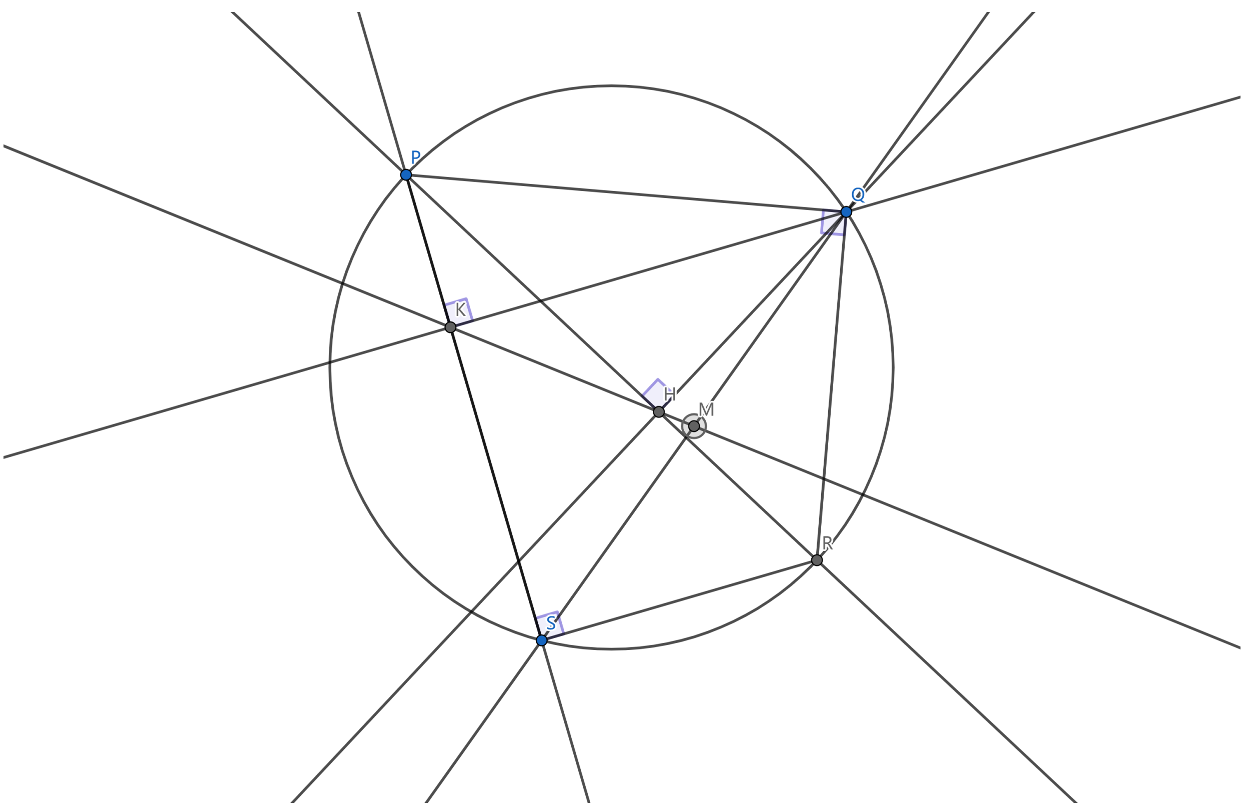
4.41

证明:

$\because Q \in (PRS)$，$QH \perp PR$，$QK \perp PS$

根据引理 4.4，M 为 QS 中点

$Q.E.D$



4.42

证明:

作 BC 的垂直平分线 $MF \cap (ABC) = F$

$\implies OMF$ 共线

设 $OF \cap (ABC)$ 于不同于 P 的点 Q

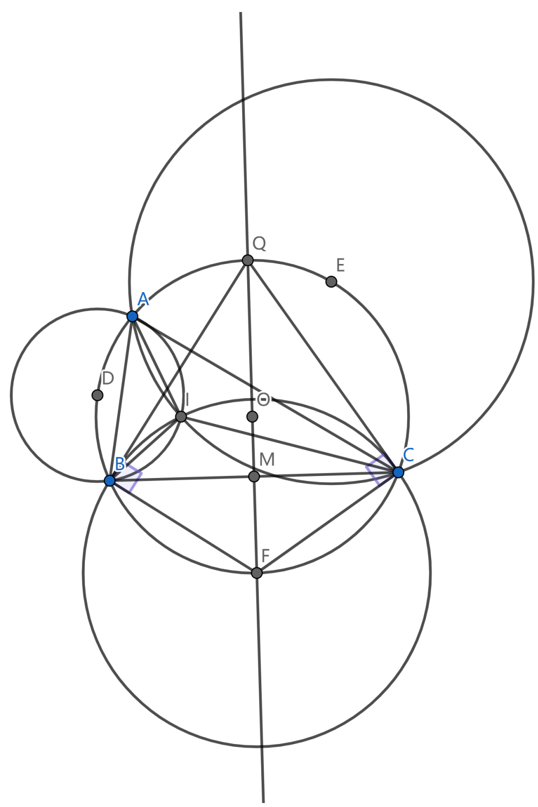
$\therefore \angle QBF = \angle QCF = 90^\circ$

$\therefore QBCF$ 四点共圆

同理 $D, E \subset (ABC)$

$(DEF) = (ABC)$

$Q.E.D$



4.40(e)

证明:

$\because \triangle ABT \backsim \triangle AEC$

$\therefore \angle ATB = \angle ACE$

延长 $TD \cap (ABC) = A_2$， A, A_2 关于 BC 的中垂线对称

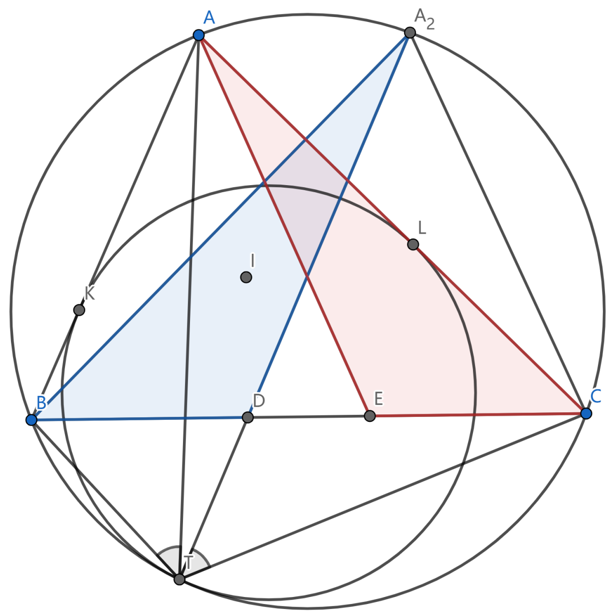
则 $\triangle AEC \cong \triangle A_2BD$

$\implies \angle ATB = \angle A_2BD$

又 $ABCT$ 四点共圆

$\therefore \angle ATB = \angle A_2TC$

$Q.E.D$



4.48

证明:

延长 $AO \cap (ABC) = D$

延长 $QP \cap (ABC) = E$，E 为 $\overset{\frown}{BC}$ 中点

$\triangle PXO \backsim \triangle EXD$，$EDAQ$ 四点共圆

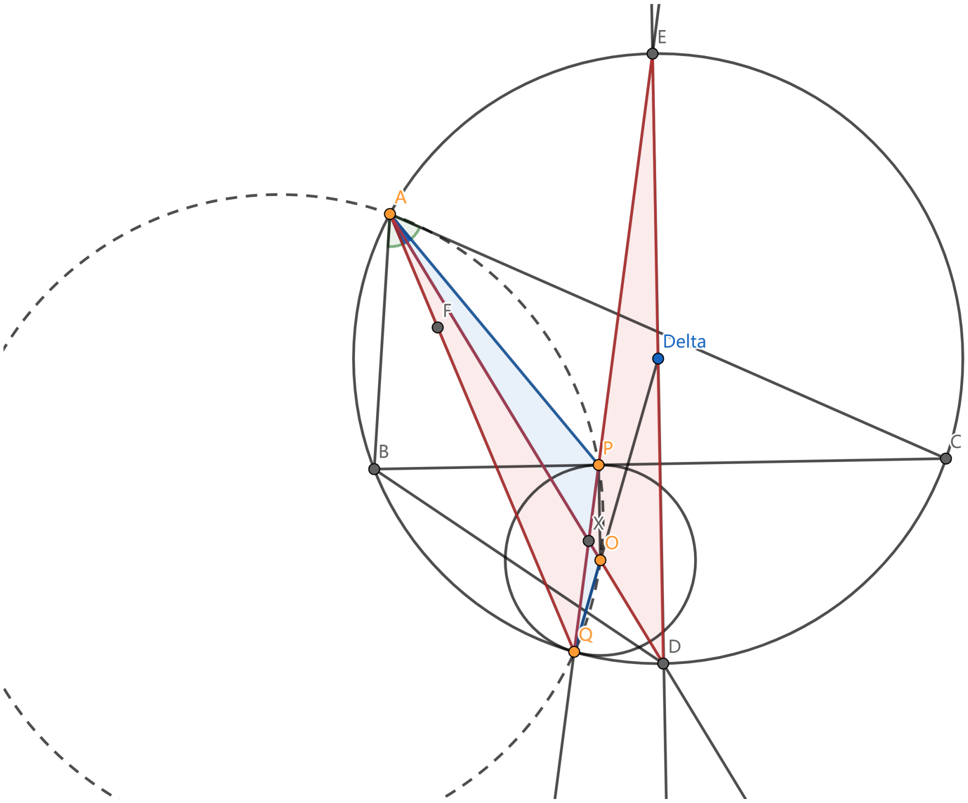
$\implies APOQ$ 四点共圆

$\angle PAO = \angle OQP = \angle QPO$

又 $OP = OQ$

$\therefore \angle PQO = \angle QPO = \angle XED = \angle QAD$

$Q.E.D$



5.5

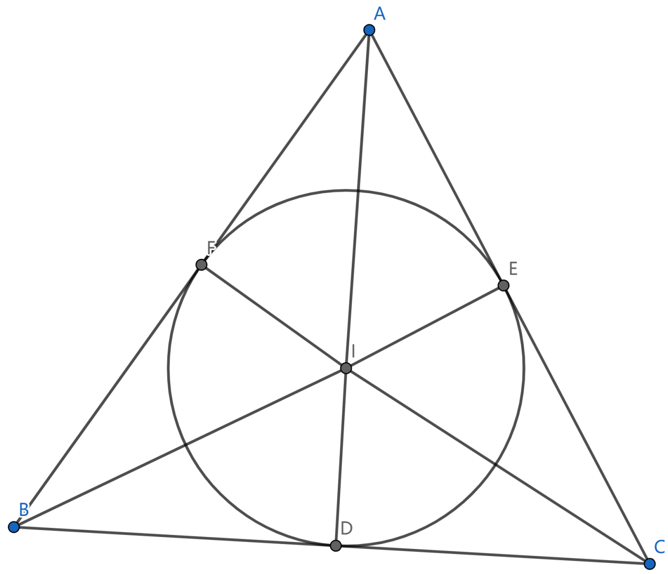
证明:

$[ABC] = \frac{1}{2}(AC \cdot r) + \frac{1}{2}(AB \cdot r) + \frac{1}{2}(BC \cdot r)$

$= \frac{1}{2}r(AB + BC + AC)$

$= sr$

$Q.E.D$



5.6

解:

$s_{\triangle ABC} = \frac{13+14+15}{2} = 21$

$S_{\triangle ABC} = \sqrt{21 \times (21-13) \times (21-14) \times (21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6}$

$= 84$

$h_a = \frac{84}{14} = 6$

$Q.E.D$

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5.9

证明:

设 $\angle DAC = \alpha, \angle BDC = \beta, \angle ACB = \theta, \angle ABD = \gamma$

令 $d_{(ABC)} = 1$

则

$$AB = \sin\theta$$
$$BC = \sin\beta$$
$$CD = \sin\alpha$$
$$AD = \sin\gamma$$
$$AC = \sin(\theta+\beta) = \sin(\alpha+\gamma)$$
$$BD = \sin(\theta+\gamma) = \sin(\alpha+\beta)$$

$\Longleftarrow \sin\theta\cdot\sin\beta+\sin\alpha\cdot\sin\gamma = \sin(\theta+\beta)\cdot\sin(\theta+\gamma)$

$\Longleftarrow \frac{\cos(\theta-\beta)-\cos(\theta+\beta)+\cos(\alpha-\gamma)-\cos(\alpha+\gamma)}{2} = \frac{\cos(\alpha-\gamma)-\cos(\alpha+\gamma+2\alpha)}{2}$

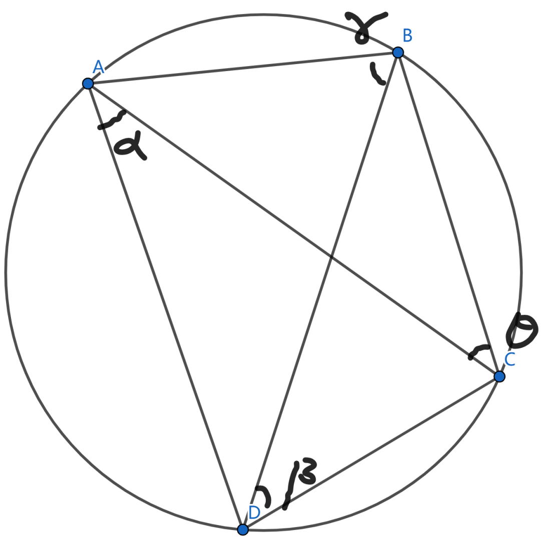
$\because \theta+\beta+\alpha+\gamma = 180^\circ$

$\therefore \cos(\theta+\beta)+\cos(\alpha+\gamma) = 0$

又 $\cos(\alpha+\gamma+2\alpha) = -\cos(\theta-\beta)$

则 $AB\cdot BC+CD\cdot AD = AC\cdot BD$

$Q.E.D$



5.16

证明:

如图，设 $\angle X_3A_1A_5 = \alpha_1$ ，$\angle X_4A_2A_1 = \alpha_2$，以此类推

设 $\triangle A_iA_{i+1}X_{i+3}$ 的外接圆半径为 R_i

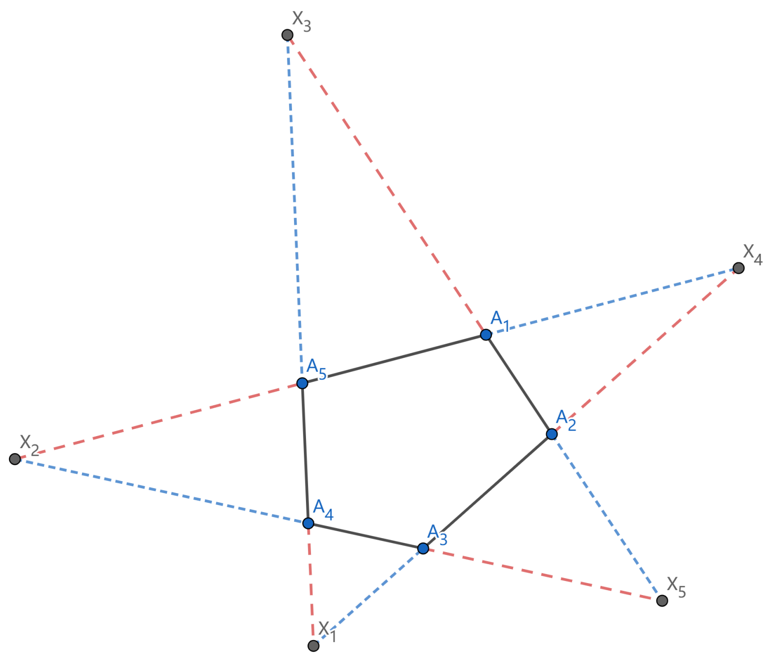
则

$$\prod_{i=1}^{5} X_iA_{i+2} = \prod_{i=1}^{5} \frac{R_{i+2}}{\sin \alpha_{i+3}}$$

$$\prod_{i=1}^{5} X_iA_{i+3} = \prod_{i=1}^{5} \frac{R_{i+3}}{\sin a_{i+4}}$$

则 $\prod_{i=1}^{5} X_iA_{i+2} = \prod_{i=1}^{5} X_iA_{i+3}$

$Q.E.D$



5.17

证明:

$$r \cdot r_a \cdot r_b \cdot r_c = \left(\prod_{x \in \{a,b,c\}} 2S_{\triangle ABC}/(a+b+c-2x) \right) \cdot 2S_{\triangle ABC}/(a+b+c)$$

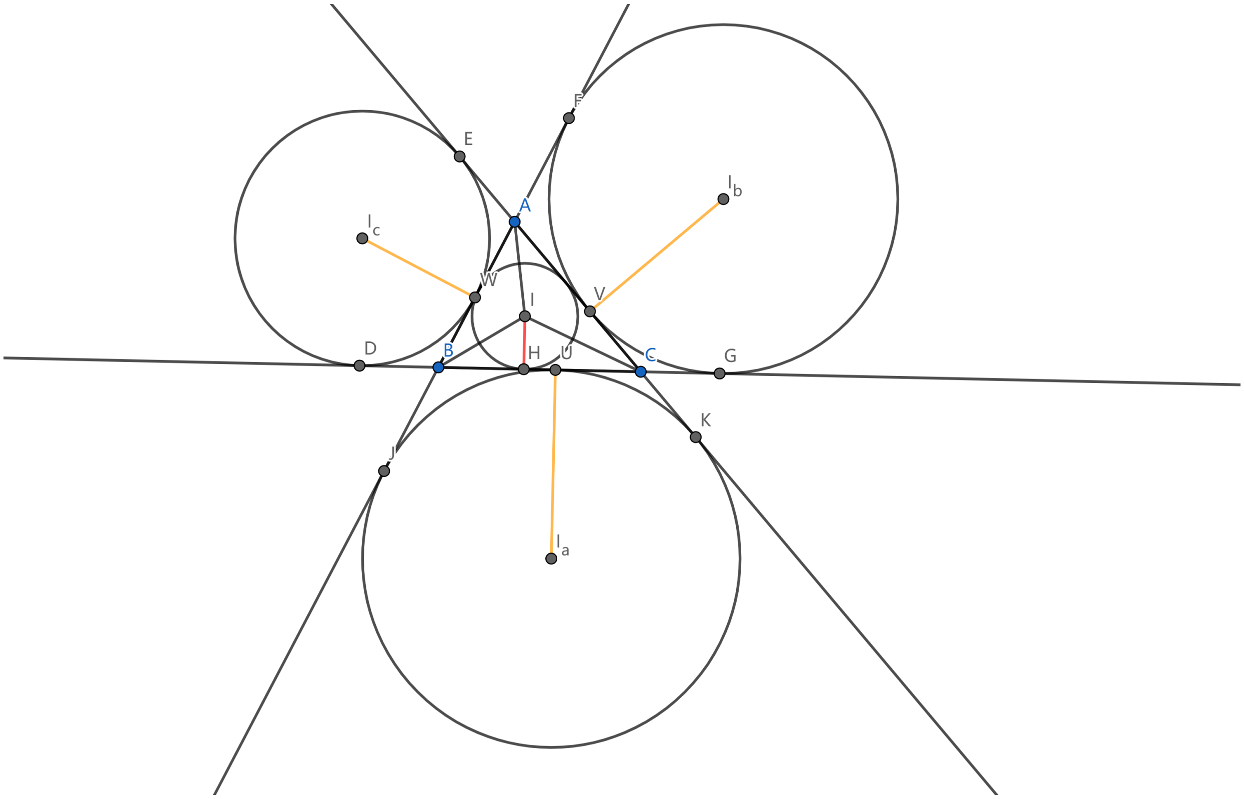
$$= 16 \left((S_{\triangle ABC})^4 \right) / \left(16 \cdot s \cdot (s-a) \cdot (s-b) \cdot (s-c)\right)$$

$$= 16 \left((S_{\triangle ABC})^4 \right) / 16 \left((S_{\triangle ABC})^2 \right)$$

$$= (S_{\triangle ABC})^2$$

则 $\sqrt{r \cdot r_a \cdot r_b \cdot r_c} = \sqrt{(S_{\triangle ABC})^2} = S_{\triangle ABC}$

$Q.E.D$



5.19

证明:

$BE^2 = 4b^2 + c^2 - 4bc\cos\angle BAE$

$AD^2 = a^2 + b^2 - ab\cos\angle ACD$

又 $a^2 = b^2 + c^2 + bc\cos\angle BAE$

$c^2 = a^2 + b^2 + ab\cos\angle ACD$

则 $BE^2 = -4a^2 + 8b^2 + 5c^2$

$AD^2 = -c^2 + 2a^2 + 2b^2$

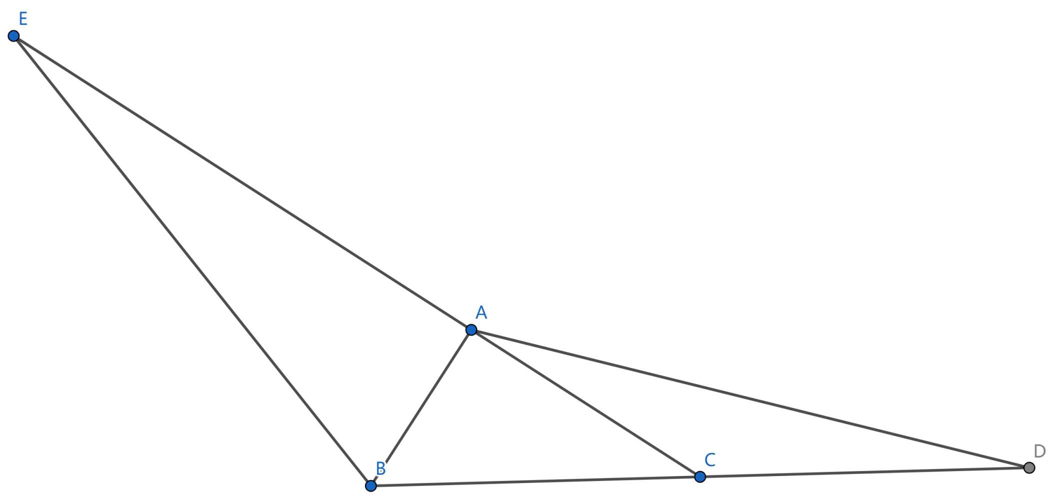
$\because BE = AD$

$-4a^2 + 8b^2 + 5c^2 = -c^2 + 2a^2 + 2b^2$

则 $a^2 = b^2 + c^2$

$\therefore \triangle ABC$ 是 $Rt\triangle$

$Q.E.D$



5.20

证明:

根据引理 1.18 ，有 $DI = DB = DC$

则只需证 $CD = AI$ 即可

设 $AB = c$ ，$AC = b$ ，$BD = w$

则 $CD = DI = BD = w$

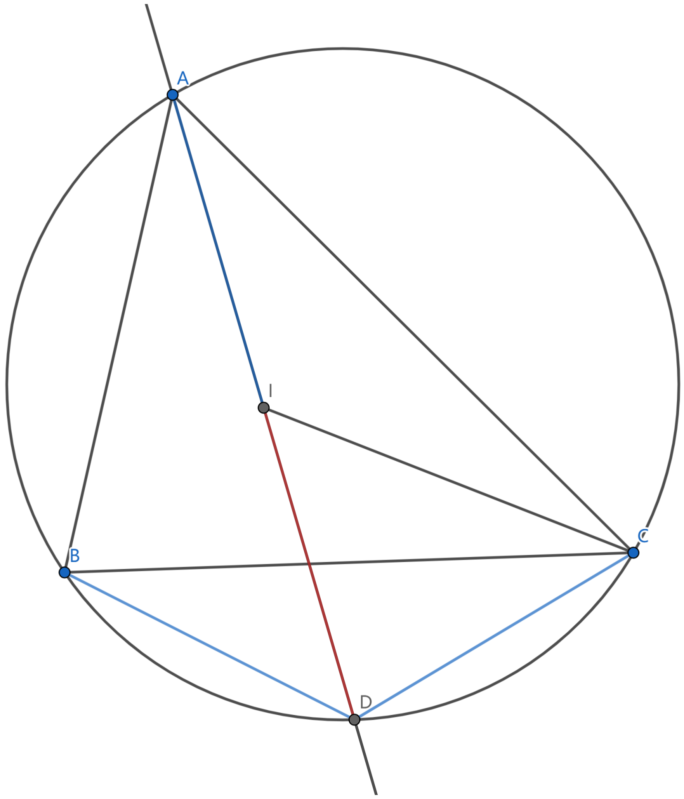
根据定理 5.9 ，有 $bw + cw = \frac{(b+c)}{2} \cdot AD$

$\therefore AD = 2w$

又 $DI = w \implies AI = w$

$\therefore CD = AI$

$Q.E.D$



5.18

证明:

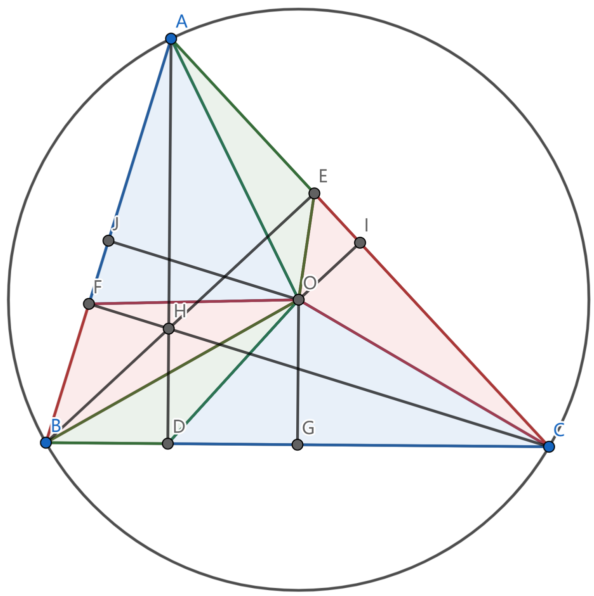
只需证 $S_{\triangle BQD}=S_{\triangle AOE}$

又 $AE\cos\angle OEA=DO\cos\angle BOD$

则 $2S_{\triangle BQD}=R\cdot AE\cos\angle OEA$, $2S_{\triangle AOE}=R\cdot AE\cos\angle BOD$

则 $S_{\triangle BQD}=S_{\triangle AOE}$

$Q.E.D$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