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TERM - I

VOLUME 2

MATHEMATICS

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Department Of School Education

SYMBOLS

$=$	equal to
$\neq$	not equal to
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\approx$	equivalent to
$\cup$	union
$\cap$	intersection
$\mathbb{U}$	universal Set
$\in$	belongs to
$\notin$	does not belong to
$\subset$	proper subset of
$\subseteq$	subset of or is contained in
$\not\subset$	not a proper subset of
$\not\subseteq$	not a subset of or is not contained in
A' (or) A^c	complement of A
$\varnothing$ (or) $\{\ \}$	empty set or null set or void set
$n(A)$	number of elements in the set A
$P(A)$	power set of A
$\|\|\|^{ly}$	similarly

Δ	symmetric difference
$\mathbb{N}$	natural numbers
$\mathbb{W}$	whole numbers
$\mathbb{Z}$	integers
$\mathbb{R}$	real numbers
$\triangle$	triangle
$\angle$	angle
$\perp$	perpendicular to
$\parallel$	parallel to
$\Longrightarrow$	implies
$\therefore$	therefore
$\because$	since (or) because
$\lvert\ \rvert$	absolute value
$\simeq$	approximately equal to
$\mid$ (or) $:$	such that
$\equiv$ (or) $\cong$	congruent
$\equiv$	identically equal to
π	pi
$\pm$	plus or minus

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CONTENTS

Captions used in this Textbook

எண்ணென்ப ஏனை எழுத்தென்ப இவ்விரண்டும்
கண்ணென்ப வாழும் உயிர்க்கு – குறள் 392

Numbers and letters, they are known as
eyes to humans, they are. Kural 392

Learning Outcomes

To transform the classroom processes into learning centric with a set of bench marks

Note

To provide additional inputs for students in the content

Activity

To encourage students to involve in activities to learn mathematics

ICT Corner

To encourage learner's understanding of content through application of technology

Thinking Corner

To kindle the inquisitiveness of students in learning mathematics. To make the students to have a diverse thinking

Points to Remember

To recall the points learnt in the topic

Multiple Choice Questions

To provide additional assessment items on the content

Progress Check

Self evaluation of the learner's progress

Exercise

To evaluate the learners' in understanding the content

Let's use the QR code in the text books! How?

- Download the QR code scanner from the Google PlayStore/ Apple App Store into your smartphone
- Open the QR code scanner application
- Once the scanner button in the application is clicked, camera opens and then bring it closer to the QR code in the text book.
- Once the camera detects the QR code, a url appears in the screen.Click the url and go to the content page.

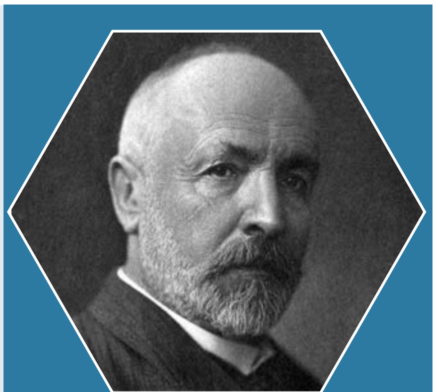
1 SET LANGUAGE

A set is a many that allows itself to thought of as a one

-Georg Cantor

E3RQRH

The theory of sets was developed by German mathematician Georg Cantor. Today it is used in almost every branch of Mathematics. In Mathematics, sets are convenient because all mathematical structures can be regarded as sets.



Georg Cantor
(1845 - 1918)

Learning Outcomes

- To describe a set.
- To represent sets in descriptive form, set builder form and roster form.
- To identify different types of sets.
- To understand and perform set operations.
- To use Venn diagrams to represent sets and set operations.
- To solve life oriented simple word problems.

1.1 Introduction

In our daily life, we often deal with collection of objects like books, stamps, coins, etc. Set language is a mathematical way of representing a collection of objects.

Study the problem: 16 students play only Cricket, 18 students play only Volley ball and 3 students play both Cricket and Volley ball, while 2 students play neither Cricket nor Volley ball. Totally 39 students are there in a class.

We can describe this pictorially as follows:

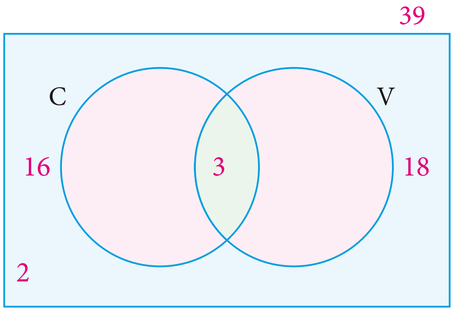


Fig. 1.1

What do the circles in the picture represent? They are **collections** of students who play games. We do not need to draw 39 students, or even 39 symbols, or use different colours to distinguish those who play Cricket from those who play Volleyball etc. Simply calling the collections C and V is enough; we can talk of those in both C and V, in neither and in one but not the other. This is **the language of sets**. A great deal of mathematics is written in this language and hence we are going to study it.

But **why** is this language so important, why should mathematicians want to use this language? One reason is that everyday language is imprecise and can cause confusion. For example, if I write 1, 2, 3,… what do the three dots at the end mean? You say,"Of course, they mean the list of natural numbers". What if I write 1, 2, 4,…? What comes next ? It could be 7, and then 11, and so on. (Can you see why ?) Or it could be continued as 8, 16, and so on. If we explicitly say, "The collection of numbers that are powers of 2", then we know that the latter is meant. So, in general, when we are talking of collections of numbers, we may refer to some collection in some short form, but writing out the collection may be difficult. It is here that the language of sets is of help. We can speak of the powers of 2 as a **set** of numbers.

Note, this is a list that goes on forever, so it is an **infinite** set. By now, we have come across several infinite sets: the set of natural numbers, the set of integers, the set of rational numbers, and many more. We also know the set of prime numbers, again an infinite set. But we know many **finite** sets too. The number of points of intersection of three lines on the plane is an example.

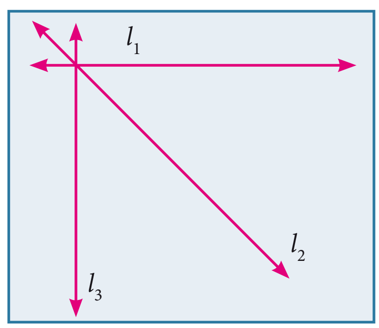


Fig. 1.2

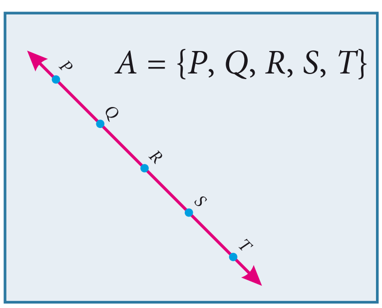


Fig. 1.3

We talk of a point being on a line and we know that a line contains inifinite number of points. For example, five points P,Q,R,S and T which lie on a line can be denoted by a set $A = \{P, Q, R, S, T\}$.

We can already see that many of these sets are important in whatever algebra and geometry we have learnt, and we expect that there will be more important sets coming along as we learn mathematics. That is why we are going to learn the language of sets. For now, we will work with small finite sets and learn its language.

Let us look at the following pictures. What do they represent?

Here, Fig.1.4 represents a collection of fruits and Fig. 1.5 represents a collection of house- hold items.

We observe in the above cases, our attention turns from one individual object to a collection of objects based on their characteristics. Any such collection is called a set.

Fig. 1.4

Fig. 1.5

1.2 Set

A set is a well-defined collection of objects.

Here "well-defined collection of objects" means that given a specific object it must be possible for us to decide whether the object is an element of the given collection or not.

The objects of a set are called its members or elements.

For example,

1. The collection of all books in a District Central Library.
2. The collection of all colours in a rainbow.
3. The collection of prime numbers.

We see that in the adjacent box, statements (1), (2), and (4) are well defined and therefore they are sets. Whereas (3) and (5) are not well defined because the words good and beautiful are difficult to agree on. I might consider a student to be good and you may not. I might consider Malligai is beautiful but you may not. So we will consider only those collections to be sets where there is no such ambiguity.

Which of the following are sets ?

1. Collection of Natural numbers.
2. Collection of English alphabets.
3. Collection of good students in a class.
4. Collection of States in our country.
5. Collection of beautiful flowers in a garden.

Therefore (3) and (5) are not sets.

Note

(i) Elements of a set are listed only once.

(ii) The order of listing the elements of the set does not change the set.

Both these conditions are natural. The collection 1,2,3,4,5,6,7,8, ... as well as the collection 1, 3, 2, 4, 5, 7, 6, 8, ... are the same though listed in different order. Since it is necessary to know whether an object is an element in the set or not, we do not want to list that element many times.

Activity-1

Discuss and give as many examples of collections from your daily life situations, which are sets and which are not sets.

Notation

A set is usually denoted by capital letters of the English Alphabets A, B, P, Q, X, Y, etc.

The elements of a set are denoted by small letters of the English alphabets a, b, p, q, x, y, etc.

The elements of a set is written within curly brackets "{ }"

If x is an element of a set A or x belongs to A, we write $x \in A$.

If x is not an element of a set A or x does not belongs to A, we write $x \notin A$.

For example,

Consider the set $A = \{2,3,5,7\}$ then

2 is an element of A; we write $2 \in A$

5 is an element of A; we write $5 \in A$

6 is not an element of A; we write $6 \notin A$

Example 1.1

Consider the set A = {Ashwin, Muralivijay, Vijay Shankar, Badrinath }.

Fill in the blanks with the appropriate symbol $\in$ or $\notin$.

(i) Muralivijay ____ A. (ii) Ashwin ______ A. (iii) Badrinath ______A.

(iv) Ganguly _____ A. (v) Tendulkar _____ A

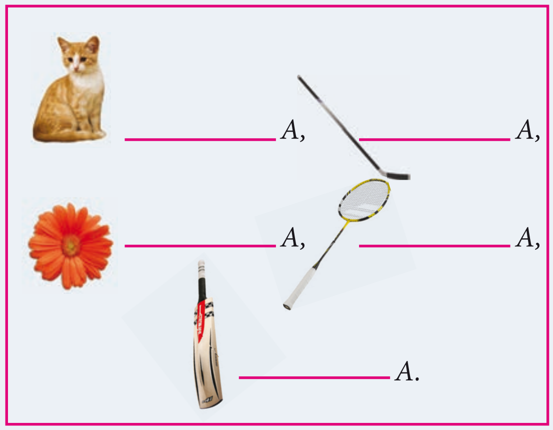
Solution

(i) Muralivijay $\in A$. (ii) Ashwin $\in A$ (iii) Badrinath $\in A$

(iv) Ganguly $\notin A$. (v) Tendulkar $\notin A$.

Activity-2

Insert the appropriate symbol $\in$ (belongs to) or $\notin$ (does not belong to) in the blanks.



1.3 Representation of a Set

The collection of odd numbers can be described in many ways:

(1) "The set of odd numbers" is a fine description, we understand it well.

(2) It can be written as {1, 3, 5, …} and you know what I mean.

(3) Also, it can be said as the collection of all numbers x where x is an odd number.

All of them are equivalent and useful. For instance,the two descriptions "The collection of all solutions to the equation $x-5 = 3$" and {8} refer to the same set.

A set can be represented in any one of the following three ways or forms:

(i) Descriptive Form.

(ii) Set-Builder Form or Rule Form.

(iii) Roster Form or Tabular Form.

1.3.1 Descriptive Form

In descriptive form, a set is described in words.

For example,

(i) The set of all vowels in English alphabets.

(ii) The set of whole numbers.

1.3.2 Set Builder Form or Rule Form

In set builder form, all the elements are described by a rule.

For example,

(i) $A = \{x : x$ is a vowel in English alphabets$\}$

(ii) $B = \{x | x$ is a whole number$\}$

Note

The symbol ':' or '|' stands for "such that".

1.3.3 Roster Form or Tabular Form

A set can be described by listing all the elements of the set.

For example,

(i) $A = \{a, e, i, o, u\}$

(ii) $B = \{0,1,2,3,\ldots\}$

Note

Three dots (…) in the example (ii) is called ellipsis. It indicates that the pattern of the listed elements continues in the same manner.

Can this form of representation be possible always?

Activity-3

Write the following sets in respective forms.

S.No.	Descriptive Form	Set Builder Form	Roster Form
1	The set of all natural numbers less than 10		
2		$\{x : x$ is a multiple of 3, $x \in \mathbb{N}\}$	
3			$\{2,4,6,8,10\}$
4	The set of all days in a week.		
5			$\{\ldots-3,-2,-1,0,1,2,3\ldots\}$

Example 1.2

Write the set of letters of the following words in Roster form

(i) ASSESSMENT (ii) PRINCIPAL

Solution

(i) ASSESSMENT

$A = \{A, S, E, M, N, T\}$

(ii) PRINCIPAL

$B = \{P, R, I, N, C, A, L\}$

Exercise 1.1

1. Which of the following are sets?

   (i) The Collection of prime numbers upto 100.

   (ii) The Collection of rich people in India.

   (iii) The Collection of all rivers in India.

   (iv) The Collection of good Hockey players.

2. List the set of letters of the following words in Roster form.

   (i) INDIA (ii) PARALLELOGRAM

   (iii) MISSISSIPPI (iv) CZECHOSLOVAKIA

3. Consider the following sets $A = \{0, 3, 5, 8\}$, $B = \{2, 4, 6, 10\}$ and $C = \{12, 14, 18, 20\}$.

   (a) State whether True or False:

   (i) $18 \in C$ (ii) $6 \notin A$ (iii) $14 \notin C$ (iv) $10 \in B$

   (v) $5 \in B$ (vi) $0 \in B$

   (b) Fill in the blanks:

   (i) $3 \in$ _____ (ii) $14 \in$ ______ (iii) 18 _____ B (iv) 4 ______ B

4. Represent the following sets in Roster form.

   (i) A = The set of all even natural numbers less than 20.

   (ii) $B = \{y : y = \frac{1}{2n}, n \in \mathbb{N}, n \le 5\}$

   (iii) $C = \{x : x$ is perfect cube, $27 < x < 216\}$

   (iv) $D = \{x : x \in \mathbb{Z}, -5 < x \le 2\}$

5. Represent the following sets in set builder form.

   (i) B = The set of all Cricket players in India who scored double centuries in One Day Internationals.

   (ii) $C = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}$

   (iii) D = The set of all tamil months in a year.

   (iv) E = The set of odd Whole numbers less than 9.

6. Represent the following sets in descriptive form.

   (i) P ={ January, June, July}

   (ii) $Q = \{7,11,13,17,19,23,29\}$

   (iii) $R = \{x : x \in \mathbb{N}, x < 5\}$

   (iv) $S = \{x : x$ is a consonant in English alphabets$\}$

1.4 Types of Sets

There is a very special set of great interest: the empty collection ! Why should one care about the empty collection? Consider the set of solutions to the equation $x^2+1 = 0$. It has no elements at all in the set of Real Numbers. Also consider all rectangles with one angle greater than 90 degrees. There is no such rectangle and hence this describes an empty set.

So, the empty set is important, interesting and deserves a special symbol too.

1.4.1 Empty Set or Null Set

A set consisting of no element is called the *empty set* or *null set* or *void set*.

It is denoted by $\varnothing$ or { }.

Thinking Corner

Are the sets {0} and {$\varnothing$} empty sets?