GOVERNMENT OF TAMIL NADU

STANDARD EIGHT

TERM - I

VOLUME - 2

MATHEMATICS

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Department Of School Education

Untouchability is Inhuman and a Crime

Mathematics is a unique symbolic language in which the whole world works and acts accordingly. This text book is an attempt to make learning of Mathematics easy for the students community.

Mathematics is not about numbers, equations, computations or algorithms; it is about understanding

— William Paul Thurston

The main goal of Mathematics in School Education is to mathematise the child's thought process. It will be useful to know how to mathematise than to know a lot of Mathematics.

Contents

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1 RATIONAL NUMBERS

Learning Outcomes

- To understand the necessity for extending fractions to rational numbers.
- To represent rational numbers on the number line.
- To understand that between any two given rational numbers, there lies many rational numbers.
- To learn and perform the four basic operations on rational numbers.
- To solve the word problems on all the operations.
- To understand the properties, the additive identity and inverse and the multiplicative identity and inverse of rational numbers.
- To know how to simplify expressions with atmost three brackets.

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1.1 Introduction

Think about the situation

Observe the following conversation:

Pari : My dear friend Sethu, I have a doubt about fractions on the number line. Can you please clear that doubt?

Sethu : Tell me Pari, I will be happy to help you.

Pari : We know about fractions, right? Fractions like $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ are obviously clear, that $\frac{1}{4}$ represents 1 out of 4 parts, $\frac{1}{2}$ is 1 out of 2 parts and so on. But, where are they on the number line?

Sethu : Pari, it is easy to identify where they are on the number line. The fractions you have given here are proper fractions. Aren't they? As we know, proper fractons are greater than zero but definitely less than one.

Pari : Yes, I do agree to that, Sethu.

Sethu : Now, let me tell you where they lie on the number line. See the line that I have drawn for you.

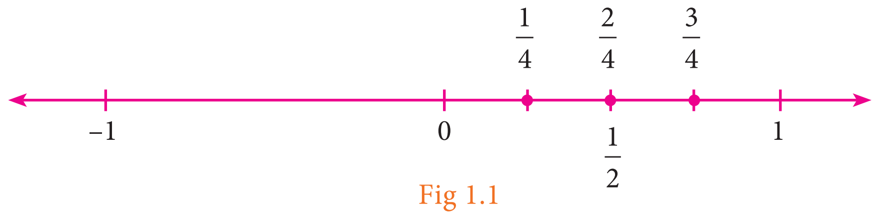


Fig 1.1

As you see here Pari, $\frac{1}{2}$ is exactly at the middle of 0 and 1 whereas $\frac{1}{4}$ is exactly at the middle of 0 and $\frac{1}{2}$. Also $\frac{3}{4}$ is exactly at the middle of $\frac{1}{2}$ and 1. Also, when we divide the distance between 0 and 1 roughly into 3 equal parts, the second part of it, represents $\frac{2}{3}$. Is it clear now?

Pari : Fine, it is very clear now. I think that these fractions $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and $\frac{2}{3}$ also correspond to the decimals 0.25, 0.50, 0.75 and 0.66. Am I correct, Sethu?

Sethu : Yes, you are correct.

Pari : By the way, I think that the improper fractions $\frac{13}{5}$, $\frac{10}{3}, \frac{31}{7}$ etc., should be converted into mixed fractions as $2\frac{3}{5}, 3\frac{1}{3}$, $4\frac{3}{7}$ respectively, so as to locate them easily on the number line as given below.

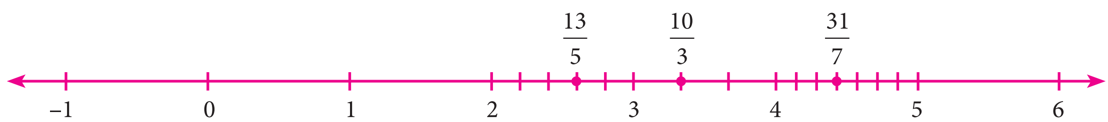


Fig 1.2

Sethu : Yes Pari, you are right. Here, it is clear that $\frac{13}{5}$ lies between 2 and 3, $\frac{10}{3}$ lies between 3 and 4 whereas $\frac{31}{7}$ lies between 4 and 5.

Pari : Sethu, let me ask you another question. If 50 students in a class contribute equally to a total of ₹ 35 for a cause, how much does each one contribute?

Sethu : It is simple. Each one's contribution is $\frac{35}{50}$, simplified to $\frac{7}{10}$ of a rupee, which is 70 paise (or) ₹ 0.70. Why do you ask this question here, Pari?

Pari : Wait Sethu. Tell me, what if they (50 students) have a debt of ₹ 35? Shall I denote it by a negative sign as $\frac{-7}{10}$?

Sethu : I also think so! As we have seen the extension of whole numbers to integers, these negative fractions need to be accommodated somewhere on the number line.

Teacher : Pari and Sethu, I have been listening to your conversation for a while now. You have almost got everything correct! Now, we know that 0 acts as the mirror to the natural numbers (right of 0) to reflect negative integers (left of 0). By the same way, we can indicate the negative fractions on to the left of 0.

Sethu : Thank you, Teacher. We have now understood what you said and know how to mark negative fractions on the number line as under.

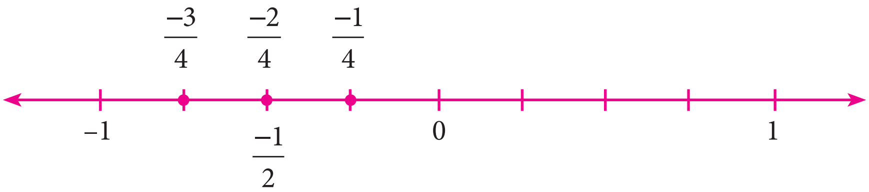


Fig 1.3

Observing the above conversation, one can see the need of negative fractions coming into the system of numbers that we have already know about.

Recap

Now let us recall about Fractions

1. Write the following fractions in the appropriate boxes.

$$\frac{-4}{5}, \frac{6}{7}, \frac{8}{3}, 4\frac{2}{3}, \frac{10}{7}, \frac{9}{12}, \frac{-12}{17}, 1\frac{4}{5}$$

Proper fraction	Improper fraction	Mixed fraction	Negative Fraction

2. Which of the following is not an equivalent fraction of $\frac{8}{12}$?

(A) $\frac{2}{3}$ (B) $\frac{16}{24}$ (C) $\frac{32}{60}$ (D) $\frac{24}{36}$

3. The simplest form of $\frac{125}{200}$ is __________.

4. Which is bigger $\frac{4}{5}$ or $\frac{8}{9}$?

5. Add the fractions : $\frac{3}{5}+\frac{5}{8}+\frac{7}{10}$

6. Simplify : $\frac{1}{8} - \left(\frac{1}{6}-\frac{1}{4}\right)$

7. Multiply $2\frac{3}{5}$ and $1\frac{4}{7}$.

8. Divide $\frac{7}{36}$ by $\frac{35}{81}$.

9. Fill in the boxes : $\dfrac{\square}{66} = \dfrac{70}{\square} = \dfrac{28}{44} = \dfrac{\square}{121} = \dfrac{7}{\square}$

10. In a city, $\dfrac{7}{20}$ of the population are women and $\dfrac{1}{4}$ are children. Find the fraction of the population of men.

1.1.1 Necessity for extending fractions to rational numbers

For the easy understanding and mathematical clarity, we shall introduce the rational numbers abstractly by focusing on two properties, namely every number has an opposite and every non-zero number has a reciprocal.

(i) Firstly, take the integers and form all possible 'fractions' where the numerators are integers and the denominators are non-zero integers. In this method, a rational number is defined as a 'ratio' of integers. The collection of rational numbers defined in this way will include the opposites of the fractions.

(ii) Secondly, we could take all the fractions together with their opposites. This would give us a new collection of numbers, called the fractions and numbers such as $\dfrac{-3}{4}, \dfrac{5}{-9}, \dfrac{-13}{2}$ etc.,

We know that, the fraction $\dfrac{4}{5}$ satisfies the equation $5x = 4$ since $5 \times \dfrac{4}{5} = 4$ and -2 satisfies the equation $x + 2 = 0$, since $-2 + 2 = 0$. However, there is neither a fraction nor an integer that satisfies the equation $5x + 2 = 0$.

We have studied about integers. We you add, subtract or multiply two or more integers, you will get only an integer. If we divide two integers, we will not always get an integer. For example, $\dfrac{-3}{5}$ and $\dfrac{-4}{-8}$ are not integers. These situations can be handled by extending the numbers to another collection of numbers called as rational numbers.

The following figure shows how rational numbers are an extension of the fractions and the integers.

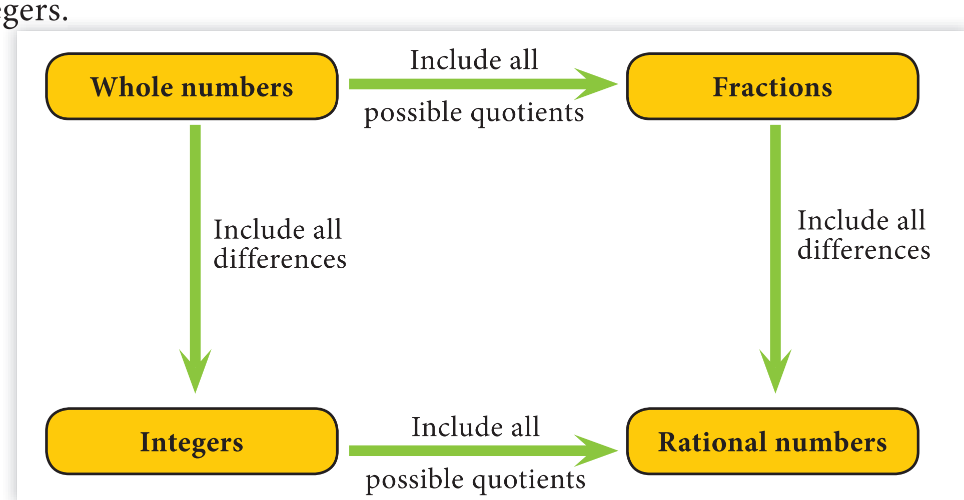
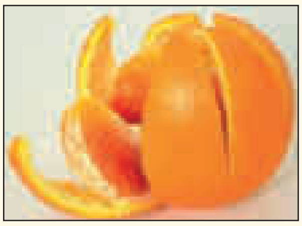


Fig 1.4

MATHEMATICS ALIVE **RATIONAL NUMBERS IN REAL LIFE**



If an orange is peeled off and 8 carpels are found, then one part of it is represents the rational number $\frac{1}{8}$.

An LPG domestic cylinder showing the weight of the tare and gas in decimal form.

1.2 Rational numbers - Definition

The collection of all numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ is called rational numbers which is denoted by the letter Q. Here, the top number a is called the numerator and the bottom number b is called the denominator.

Examples:

$\frac{1}{3}, \frac{6}{11}, \frac{-3}{5}$ and $\frac{-11}{-25}$ are some examples of rational numbers. Also, integers like 7, –4 and 0 are rational numbers as they can be written in the form $\frac{7}{1}, \frac{-4}{1}$ and $\frac{0}{1}$. Mixed numbers such as $-4\frac{2}{5} = \frac{-22}{5}$, $-5\frac{1}{3} = \frac{-16}{3}$, $3\frac{1}{2} = \frac{7}{2}$ etc., are also rational numbers. So, all integers as well as fractions are rational numbers. The decimal numbers too, like 0.75, 1.3, 0.888 etc., are also rational numbers since they can be written in fractions form as :

$$0.75 = \frac{75}{100} = \frac{3}{4}$$

$$1.3 = \frac{13}{10} \quad \text{and}$$

$$0.888 = \frac{888}{1000} = \frac{222}{250} = \frac{111}{125}$$

In banks, home loans are given for a pre-determined interest rate as given above in decimal percentages which can be converted into rational numbers.

Note

The word 'ratio' in math refers to comparison of the sizes of two different quantities of any kind. For example, if there is one teacher for every 20 students in a class, then the ratio of teachers to students is 1:20. Ratios are often written as fractions and so $1:20 = \frac{1}{20}$. For this reason, numbers in the fractions form are called rational numbers.

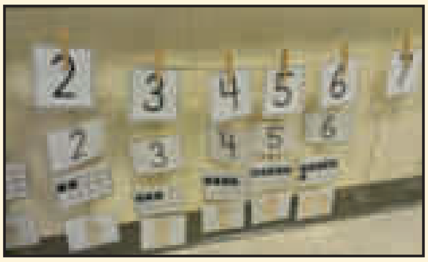
Try these

(i) Is the number $\frac{7}{-5}$ a fraction or a rational number ? Why?

(ii) Write any 6 rational numbers of your choice.

Activity-1

Use a string as a number line and fix it on the wall, for the length of the class room. Just mark the integers spaciously and ask the students to pick the rational number cards from a box and fix it at the right place on the number line string. This can be played between teams and the team which fixes more number of cards correctly (by marking) will be the winner.



1.2.1 Rational numbers on a number line

We know how the integers are represented on a number line. The same way, rational numbers can also be represented on a number line.

Now, let us represent the number $\frac{-3}{4}$ on the number line. Being negative, $\frac{-3}{4}$ would be marked to the left of 0 and it is between 0 and –1.

We know that in integers, 1 and –1 are equidistant from 0 and so are the pairs 2 and –2, 3 and -3 from 0. This remains the same for rational numbers too. Now, as we mark $\frac{3}{4}$ to the right of zero, at 3 parts out of 4 between 0 and 1, the same way, we mark $\frac{-3}{4}$ to the left of zero, at 3 parts out of 4 between 0 and –1 as shown below.

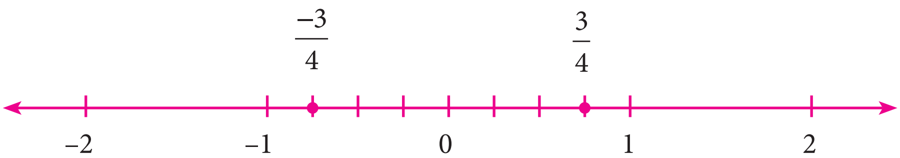


Fig. 1.5

Similarly, it is easy to say that $\frac{-5}{2}$ lies between –2 and -3 as $\frac{-5}{2} = -2\frac{1}{2}$.

Remember that all proper rational numbers lie between 0 and 1 (or) 0 and –1 just like the fractions.

Now, where do these rational numbers $\frac{18}{5}$ and $-\frac{32}{7}$ lie on a number line?

Here, $\frac{18}{5} = 3\frac{3}{5}$ and $-\frac{32}{7} = -4\frac{4}{7}$

Now, $\frac{18}{5}$ lies between 3 and 4 on the number line. The unit part between 3 and 4 is divided into 5 equal parts and the third part is marked as $\frac{3}{5}$. Thus, the arrow mark indicates $3\frac{3}{5}=\frac{18}{5}$. Also, it is clear that the rational number $-\frac{32}{7}$ which is $-4\frac{4}{7}$ lies between -4 and -5 on the number line. Here, the unit part between -4 and -5 is divided to 7 equal parts and fourth part is marked as $\frac{4}{7}$. Thus the arrow mark indicates $-4\frac{4}{7}=-\frac{32}{7}$. These rational numbers are shown on the number line as shown below.

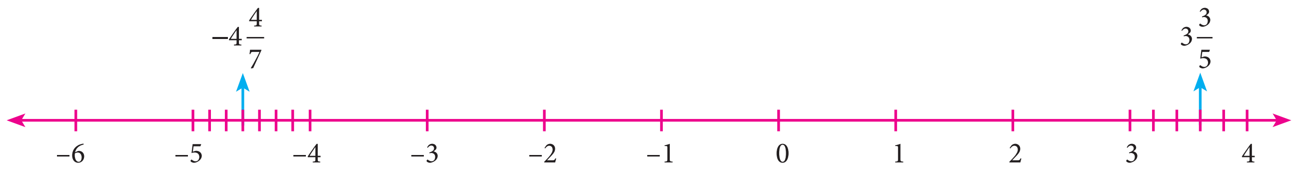


Fig. 1.6

1.2.2 Writing a decimal as a rational number

Every terminating or repeating decimal can be written in the rational form as shown in the following example.

Example 1.1

Write the following decimal numbers as rationals.

(i) 3.0 (ii) 0.25 (iii) 0.666… (iv) −5.8 (v) 1.15

Solution:

(i) $3.0=\frac{30}{10}=\frac{3}{1}$ (ii) $0.25=\frac{25}{100}=\frac{1}{4}$

(iii) $0.666...=\frac{2}{3}$ (check… you will know how in IX std)

(iv) $-5.8=\frac{-58}{10}=\frac{-29}{5}=-5\frac{4}{5}$

(v) $1.15=\frac{115}{100}=\frac{23}{20}=1\frac{3}{20}$

Note

There are decimal numbers which are non-terminating and non-repeating, such as $\pi=3.1415926535...$, 3.01002000400005...etc., They are not rational numbers because they cannot be written in $\frac{a}{b}$ form.

Try this

Explain why the following statements are true?

(i) $0.8=\frac{4}{5}$ (ii) $1.4>\frac{1}{4}$ (iii) $0.74<\frac{3}{4}$

(iv) $0.4>0.386$ (v) $0.096<0.24$ (vi) $0.128=0.1280$

1.2.3 Equivalent rational numbers

If the numerator and denominator of a rational number (say $\frac{a}{b}$) is multiplied by a non-zero integer (say c), we obtain another rational number which is equivalent to the given rational number. This is exactly the same way of getting equivalent fractions.

For example, take $\frac{a}{b} = \frac{-4}{7}$ and $c = 5$

Now, $\frac{a}{b} \times \frac{c}{c} = \frac{a \times c}{b \times c} = \frac{-4 \times 5}{7 \times 5} = \frac{-20}{35}$ is an equivalent rational number to $\frac{-4}{7}$ and if c is taken as 2,3,-4 etc., the corresponding rational numbers are $\frac{-8}{14}, \frac{-12}{21}, \frac{6}{-28}$ respectively.

1.2.4 Rational numbers in standard form

If in a rational number $\frac{a}{b}$, the only common factor of a and b is 1 and b is positive, then the rational number is said to be in standard form.

The rational numbers $\frac{4}{5}, \frac{-3}{7}, \frac{1}{6}, \frac{-4}{13}, \frac{-50}{51}$ etc., are all said to be in standard form.

If a rational number is not in the standard form, then it can be simplified to get the standard form.

Note

The quotient of two numbers with different signs is a negative number.

That is, $\frac{-4}{5} = -\frac{4}{5}$ and $\frac{4}{-5} = -\frac{4}{5}$ and so $\frac{-4}{5} = \frac{4}{-5} = -\frac{4}{5}$

Also, if the two numbers are of the same sign, then the quotient is a positive number.

That is, $\frac{+3}{+4} = \frac{3}{4}$ and $\frac{-3}{-4} = \frac{3}{4}$.

0 is neither a positive nor a negative rational number.

DO YOU KNOW?

The collection of rational numbers is denoted by the letter Q because it is formed by considering all quotients, except those involving division by 0. Decimal numbers can be put in quotient form and hence they are also rational numbers.

Example 1.2

Reduce to the standard form (i) $\frac{48}{-84}$ (ii) $\frac{-18}{-42}$

Solution:

(i) **Method 1:**

$$\frac{48}{-84}=\frac{48\div(-2)}{-84\div(-2)}=\frac{-24\div 2}{42\div 2}=\frac{-12\div 3}{21\div 3}=\frac{-4}{7}$$ (dividing by –2,2 and 3 successively)

Method 2:

The HCF of 48 and 84 is 12 (Find it). Thus, we can get its standard form by dividing it by –12.

$$=\frac{48\div(-12)}{-84\div(-12)}=\frac{-4}{7}$$

(ii) **Method 1:**

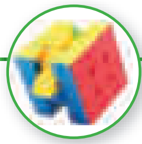
$$\frac{-18}{-42}=\frac{-18\div(-2)}{-42\div(-2)}=\frac{9\div 3}{21\div 3}=\frac{3}{7}$$ (dividing by –2 and 3 successively)

Method 2:

The HCF of 18 and 42 is 6 (Find it). Thus, we can get its standard form by dividing it by 6.

$$\frac{-18}{-42}=\frac{-18\times(-1)}{-42\times(-1)}=\frac{18}{42}=\frac{18\div 6}{42\div 6}=\frac{3}{7}$$

Try these



1. Which of the following pairs represents equivalent rational numbers?

   (i) $\frac{-6}{4}, \frac{18}{-12}$ (ii) $\frac{-4}{-20}, \frac{1}{-5}$ (iii) $\frac{-12}{-17}, \frac{60}{85}$,

2. Find the standard form of

   (i) $\frac{36}{-96}$ (ii) $\frac{-56}{-72}$ (iii) $\frac{27}{18}$

3. Mark the following rational numbers on a number line.

   (i) $\frac{-2}{3}$ (ii) $\frac{-8}{-5}$ (iii) $\frac{5}{-4}$

1.2.5 Comparison of rational numbers

You know how to compare integers and fractions taking two at a time and say which is smaller or greater. Now you will learn how to compare a pair of rational numbers.

- Two positive rational numbers, say $\frac{3}{5}$ and $\frac{5}{6}$ can be compared as studied earlier in comparison of two fractions.