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# microMathematics Plus

## version 2.20.0

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The microMathematics Plus is the mathematical calculator on Android oriented around a spreadsheet that allows live editing of mathematical identities combined with highly accurate computations.

It is based on a powerful touch-screen editor that allows users to create and manipulate naturally readable worksheets containing all basic mathematical notations.

The microMathematics Plus supports high school-level of mathematical calculations. This version has following mathematical limitations: it does not support special functions, vectors, matrices and many other things from high-level mathematics.

## 1 How To Use

This app is a powerful calculation software in a worksheet format. The worksheet can be freely edited, stored on SD card, opened from SD card and exported into an image or LaTeX format.

Worksheet is a mathematical document that contains text, formulas and plots. It supports live editing of typeset mathematical notations and its automatic computation.

Following objects can be inserted into worksheet: equations, result views, plots, text fragments and images. This document gives an overview on how to edit these objects.

### 1.1 Editing

Almost all available objects contain several editable fields. To edit the field use the symbols and functions on the tool bar.

All symbols can also be entered from the keyboard. To find out which keyboard symbol corresponds to the mathematical symbol you want to enter, read the hint by long click on the button of interest.

Using long click on a term you can select this term. The selected term can be deleted, copied to clipboard, pasted from clipboard or an other operator or function can be inserted after that term using buttons from the tool bar or keyboard.

The "Undo" command is available in the action bar. It erases the last change done to the document and reverts it to an older state:



### 1.2 Equation

An equation defines a numerical constant, an interval or a function. To create an equation, use the "New element" button in the action bar



or the "Add equation" button from the tool bar:



An equation with two empty fields appears. These fields shall be filled:

$$\square := \square$$

The equation name is given in the left field. The name shall contain letters or digits only and will be used in other objects in order to reference this equation.

From the action bar, you can open "Document settings" dialog window:



Depending on the parameter "Allow to re-define equations" in this dialog, there are two usage modes:

- if re-definition is not allowed, the equation name shall be unique within whole worksheet and the equation can be used both before and after its definition,
- if re-definition is allowed, you can define more than one equation with the same name. If such an equation is referenced, the last version defined before the caller equation will be used.

### 1.2.1 Constant

If the equation name does not contain any argument in brackets, it defines a constant or an interval:

$$N := 200 \quad Sq2 := \sqrt{100} \quad Pi2 := \frac{\pi}{2}$$

In the last example, a built-in constant pi was used. Currently, the following built-in constants are available:

$$\pi = 3.14159 \quad pi = 3.14159 \quad e = 2.71828$$

A previously defined constant can also be used:

$$NPi2 := N \cdot Pi2$$

You can also use the symbol "i" as imaginary unit in order to define a complex number:

$$z := 5 + 3i$$

### 1.2.2 Units

If you need a unit for the constant, you can put it from the keyboard into the same input field. The unit shall be separated from the number using space. You can use units both for real and complex constants:

$$r := 10m \quad a := 10m^2 \quad v := 10km/hr$$

$$\alpha := 45^\circ + 30' + 15'' \quad \varphi := 100 \cdot \frac{kg \cdot m}{s^2}$$

The document "units\_overview.mmt" contains the list of all supported units. This document is delivered with the app and stored in "Resources of microMathematics Plus".

### 1.2.3 Interval

An interval type equation defines a variable that is changed from a given minimum value up to a given maximum value with defined increment. This variable can be used as a function plot argument or as a parameter to build a function value table.

To define an interval, put a valid name on the left side of an empty equation. On the right side of this equation, put either a symbol ":", or click the button "Equidistant interval" from the tool bar:



Here, the first element is the interval start point, the next element is the second point, and the last element is the interval end point:

$$x := [0, 0.1 .. 10]$$

The interval elements shall be accessed by index:

$$x_0 = 0.0 \quad x_1 = 0.1 \quad x_{100} = 10.0$$

The increment is the difference between two neighbours values:

$$x_2 - x_1 = 0.1$$

For example, we can define an equidistant interval that contains N points distributed with increment "dy" where the interval start is zero as follows:

$$dy := 0.05 \quad y := [0, dy .. dy \cdot N]$$

### 1.2.4 Function

A function is a relation between one or more arguments and a set of permissible outputs with the property, that each argument value (real or complex) or arguments combination is related to exactly one output.

The function name and the function argument in brackets are given on the left side of an equation. It is not necessary to define the argument in the worksheet previously, you can define it as you want, but using letters or digits only:

$$f(t) := \sin(t) \cdot \cos(t) / 2$$

$$w(z) := e^{2i \cdot \pi \cdot z}$$

$$H(x, y) := \sqrt{x^2 + y^2}$$

$$g(x, y) := \frac{\sin(H(x, y))}{H(x, y/2) + 1}$$

The right side of the function contains a mathematical formula how to calculate the function. If this formula does not contain the declared function argument, such a function will be interpreted as a constant.

You can also use on the right side other built-in or previously defined functions. To insert a function enter its name, click the left bracket symbol "(" and then enter its argument. This argument can also be a formula, which contains any other operations and functions.

The document "functions\_overview.mmt", stored within the "Resources of microMathematic Plus", provides the list of all available functions.

### 1.2.5 Array

Arrays are special functions with following properties:

a) array elements can be assigned using an index given in [ ] instead of ( ). Any not assigned element is set by default to zero:

$$r[0] := 5 \quad r[3] := 6 \quad r[2] := -4$$

$$idx := [0, 1 \dots 3] \quad r_{idx} = \begin{bmatrix} 5.0 \\ 0.0 \\ -4.0 \\ 6.0 \end{bmatrix}$$

b) a previously defined interval can be used as array argument:

$$k := [0, 1 \dots 100] \quad m := [0, 1 \dots 200]$$

$$M[k, m] := \sin(k/10)^2 - 3 \cdot |\cos(m/10)|$$

c) array elements are calculated and stored in memory that reduces the access time to these values

d) array elements can be only accessed by using a lower index. To create lower index, put "]" after the array name:

$$M_{5,10} = -1.39106 \quad M_{10,5} = -1.92467$$

$$P[k, m] := \text{floor}(-10 \cdot M_{k, m})$$

e) if any array index is complex or negative or greater than the upper bound of the corresponding interval, the invalid number will be returned:

$$M_{10i, 100} = NaN \quad M_{90, 210} = NaN$$

### 1.3 Result View

This element is aimed to represent a calculation result as a number or a table. To add this element, use the "New element" button on the action bar or the "Add result view" button from the tool bar:



An equation with two fields appears, where the left field shall be filled:

$$\square = \square$$

The left term contains a formula to be calculated and the right term is the calculation result. The result will be shown when you press the floating button "Calculate".

Within the left term you can use any constants and functions defined previously as well as any built-in functions:

$$e^\pi \cdot f(NPi2) = 2.27286E - 14$$

If the left part does not contain any "interval-like" variables, the calculation result is just a real or complex number:

$$y_{N-1} - y_0 = 9.95$$

$$\Re(z) = 5.0 \quad \Im(z) = 3.0 \quad |z| = 5.83095$$

$$\sqrt{\sin\left(\frac{3}{2} \cdot \pi\right)} = 0.0 + 1.0i$$

If the left part contains any variable that has a dimensional unit, than the result can have dimension as well:

$$2 \cdot \alpha / 10s = 0.15884rad/s$$

If the left part contains an interval variable, the calculation result is a vector of values corresponding to this interval. Due to free space limit on the display, only the first six and the last elements of the vector will be displayed:

$$x = \begin{bmatrix} 0.0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ \dots \\ 10.0 \end{bmatrix} \quad y = \begin{bmatrix} 0.0 \\ 0.05 \\ 0.1 \\ 0.15 \\ 0.2 \\ 0.25 \\ \dots \\ 10.0 \end{bmatrix} \quad 2 \cdot y = \begin{bmatrix} 0.0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ \dots \\ 20.0 \end{bmatrix}$$

$$P_{k,m} = \begin{bmatrix} 30.0 & 29.0 & 29.0 & 28.0 & \dots & 12.0 \\ 29.0 & 29.0 & 29.0 & 28.0 & \dots & 12.0 \\ 29.0 & 29.0 & 29.0 & 28.0 & \dots & 11.0 \\ 29.0 & 28.0 & 28.0 & 27.0 & \dots & 11.0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 27.0 & 26.0 & 26.0 & 25.0 & \dots & 9.0 \end{bmatrix}$$

Number of displayed elements and the mode in which the result is displayed can be changed. Using the long click on the formula area and the context menu, select the whole formula. If the formula is selected, the floating button "Object properties" appears. If you click this button, the result properties dialog will be displayed:

$$x = [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, \dots, 10.0]$$

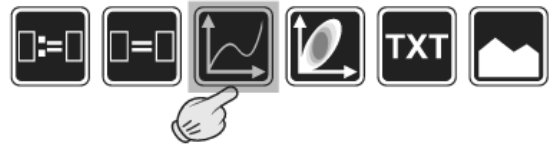


The second floating button, "Details", will also appear. If you click on this button, the "Details" dialog will be displayed, where you can observe all elements of the array.

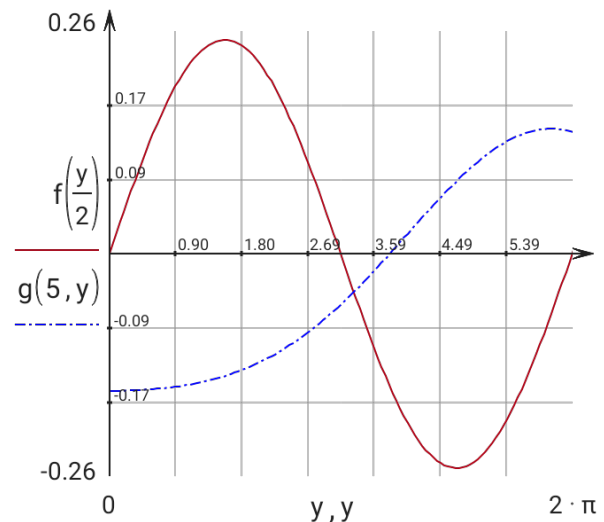
Note that the use of three or more "interval-like" variables on the left part of a result view is not allowed in this app version.

## 1.4 Function Plot

The function plot element displays a graph of a function, which depends on a single argument. To create a plot, use the "New element" button in the action bar or the "Add function plot" button from the tool bar:



Plot panel with six empty fields appears. The function to be plot shall be put in the middle-left field and the function argument in the middle-bottom field:



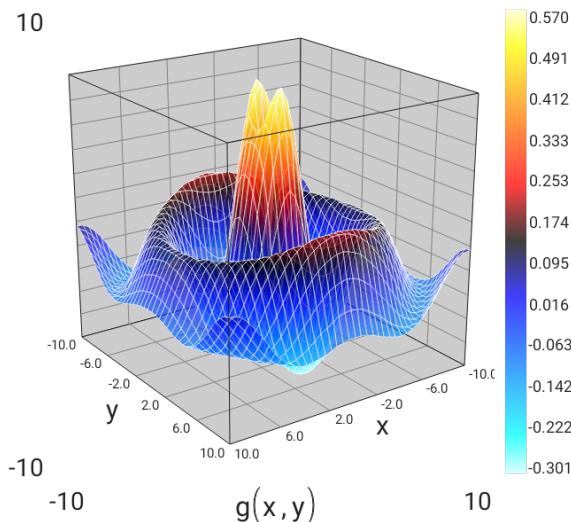
For more details see "Function Plot" and "Polar Function Plot" examples from the app navigation drawer.

## 1.5 Three-dimensional Plot

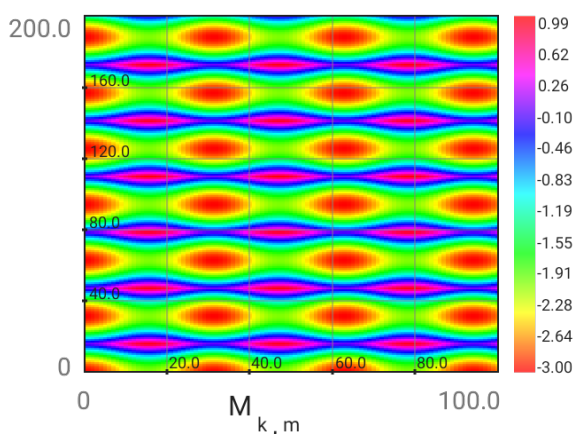
The 3D plot element displays a graph of a single function that depends on two arguments. To create such a plot, use the "New element" button in the action bar or the "Add 3D plot" button from the tool bar:



$$x := [-10, -9.5 .. 10] \quad y := [-10, -9.5 .. 10]$$



In the center-bottom field, put the function name or an equation that contains exactly two previously defined intervals. The use of an array is also possible:



For more details see "3D Plot" example from the app navigation drawer.

## 1.6 Text Fragment

The text fragment element displays simple text like this one. To add a text fragment, use the "New element" button in the action bar or "Add text fragment" button from the tool bar:



If the whole text within a fragment is selected using the context menu "Select all", a floating button "Object properties" appears in the bottom-right of the screen.

If you click on this button, the "Text properties" dialog will be displayed, where you can select the

text style and activate the numbering. For example, the titles in this document have the style "Subsection" with activated numbering.

## 1.7 Image

You can also insert an image from the image file. To do it, use the "New element" button from the action bar or the "Add image from file" button from the tool bar:



The "Image settings" dialog will appear. There you can select a file with the image to be inserted and set the necessary image size.

The following image formats are supported: png, bmp, gif, jpeg, svg.

If you activate the "Embedded image" flag in the "Image settings" dialog, then the image will be embedded directly in your document. Embedded image results in stand-alone, but larger document.

If the "Embedded image" flag is not set, the image file will be just referenced rather than embedded, i.e. your document references the image file outside the document. In case you move your document please do not forget to move the image file as well.

You can change the properties of an already existing image. Long click on the image area until the floating button "Object properties" appears. If you press this button, a dialog with image properties will be displayed.

## 2 Example: Function Plot

This example demonstrates how to prepare and adjust a graphical representation of a function. For example, we want to plot three different functions:

$$f(x) := 25 + 10 \cdot \sin(\sqrt{|x|})$$

$$g(x) := \frac{2}{e^{|x|/15}} \cdot f(x \cdot 50)$$

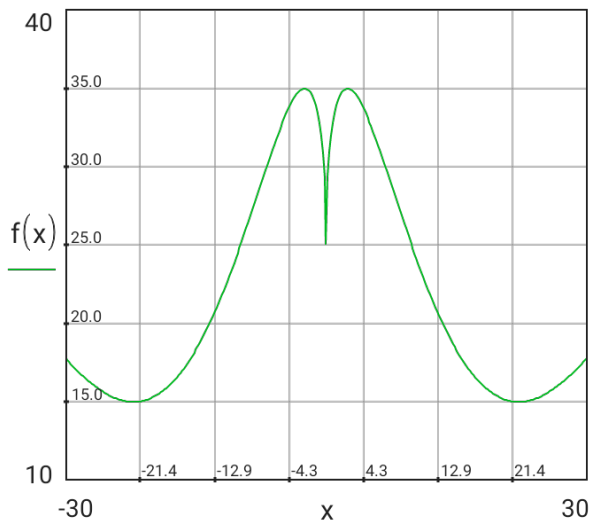
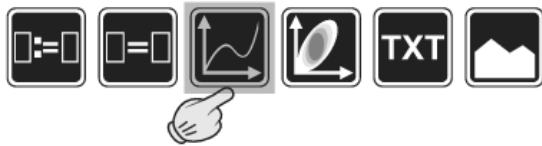
$$h(x) := \min(f(x), g(x))$$

The function argument that represents the x-values will be taken for N points within the interval  $[x_1, x_2]$ :

$$N := 300 \quad x_1 := -30 \quad x_2 := 30$$

$$x := [x1, x1 + (x2 - x1) / N .. x2]$$

After the functions and their arguments are defined, you can add the plot box using the "New element" button in the action bar or "Add function plot" button from the tool bar:

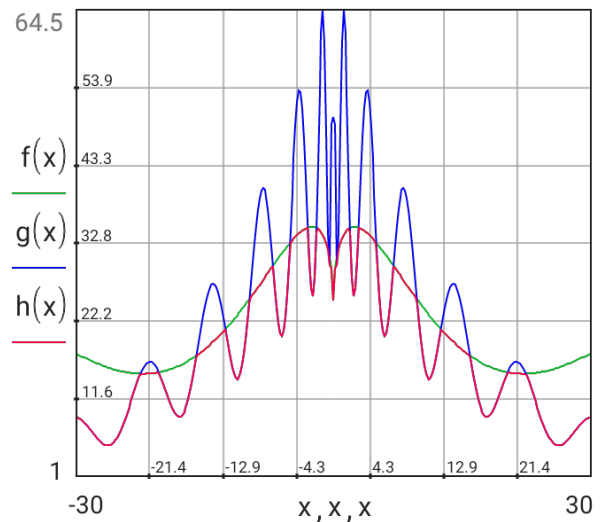


The function to be plotted will be put in the middle-left field. It can also be a built-in or previously declared function as well as a mathematical expression that contains any other operators and functions.

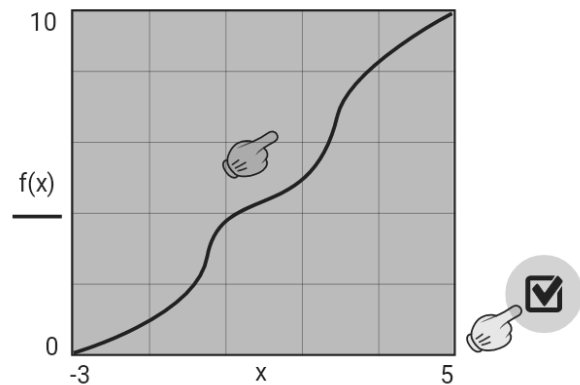
The function input, which represents the x-values will be put in the middle-bottom field. It can be a variable of interval type or a mathematical expression that contains an interval variable.

All other four fields describe the plot boundaries. If these elements remain empty, the program will calculate corresponding values automatically. However, you can edit these fields at any time and put there the values you want.

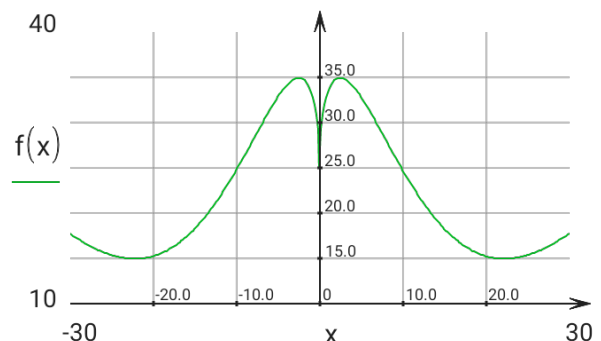
You can plot several functions on the same plot view. To add an other function, select the function (by long click in the middle-left field) after which an other function shall be added and press "Add new argument" button from the tool bar:



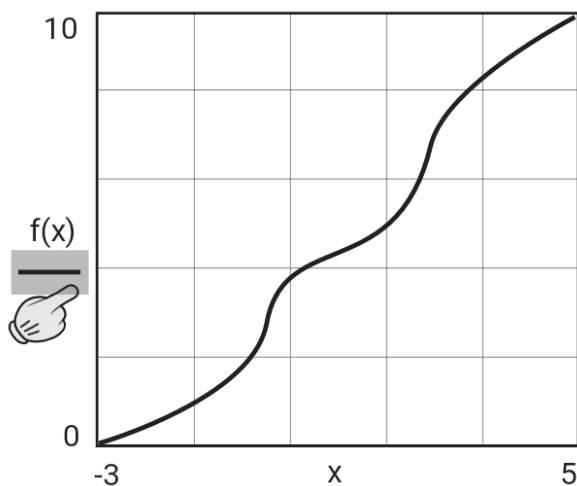
By long click on the middle of plot area, the context menu and the floating button "Object properties" will appear.



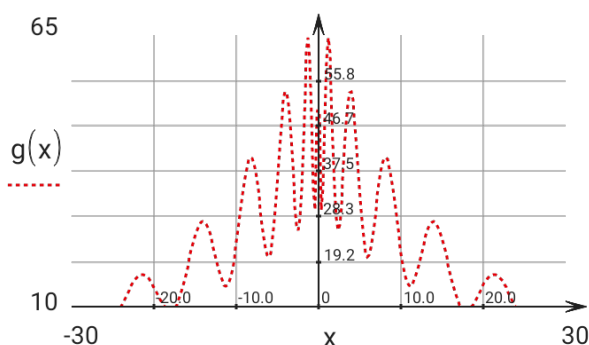
If you press this floating button, the "Plot Settings" dialog will be displayed. Here, you can change size and style of the plot area. For example, the crossed graph looks like this:



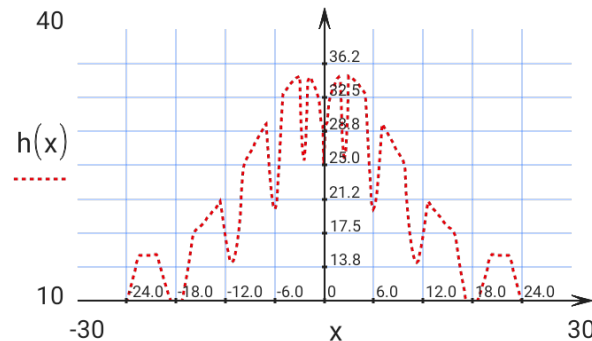
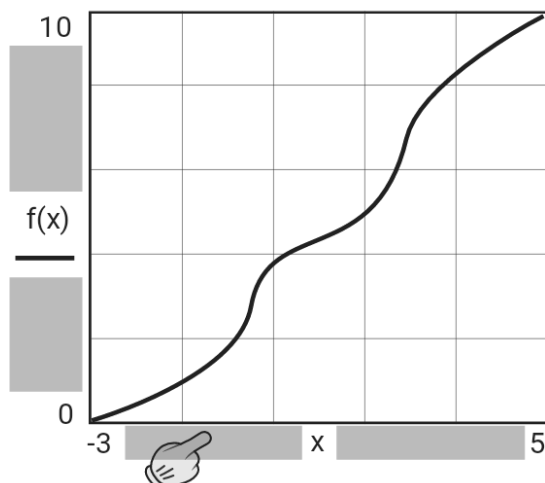
You can also change the plot line color, width and style in the "Line Settings" dialog. It appears by long click on the line marker below the function name on the left of plot area:



For example, we can use dotted lines:



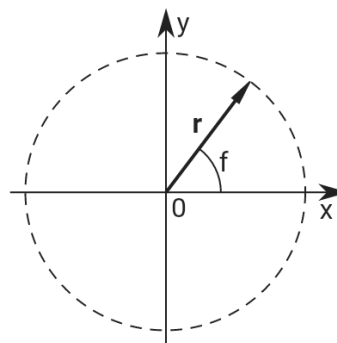
The number of axis labels and grid line color can be changed in the "Grid Settings" dialog. It appears by long click on the free space between the x minimum value (-30) and the argument (x) symbol or between the x symbol and the x maximum value (30) below the plot area:



To hide grid completely just set the number of grid lines to zero for both vertical and horizontal axes.

### 3 Example: Polar Function Plot

Now we plot several functions given in the polar coordinate system. Each point in this system is determined by a distance  $r$  from the origin and the angle  $f$  from the x-axis.



The angle  $f$  is our independent variable that is changed as follows:

$$f := [0.01, 0.05 \dots 300]$$

The distance  $r(f)$  is our dependent variable. Having a pair of  $f$  and  $r$ , we can transform it to the Cartesian coordinates  $x$  and  $y$  using sine and cosine functions:

$$x(r) := r \cdot \cos(f) \quad y(r) := r \cdot \sin(f)$$

#### 3.1 A snail

We will define our polar function in three steps. The first expression defines a "wheel":

$$A := 1.1 \quad B := 1.271 \quad q := 2$$

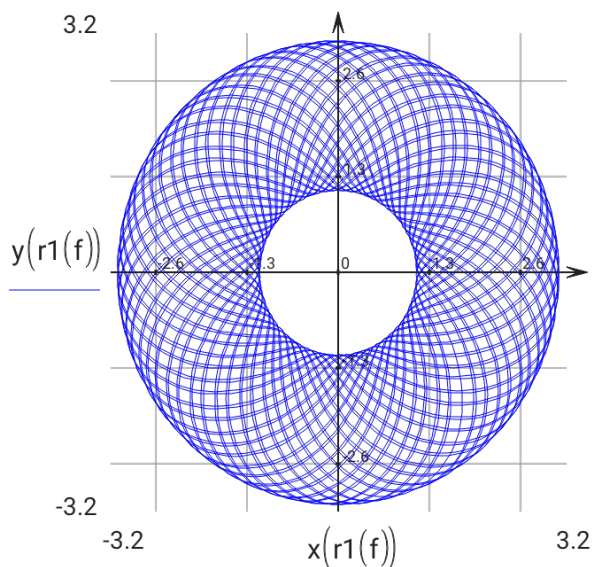
$$r1(f) := A + 2 \cdot \sin(B \cdot f)^q$$

To plot this function, we add the plot box using the "New element" button in the action bar or "Add function plot" button from the tool bar:



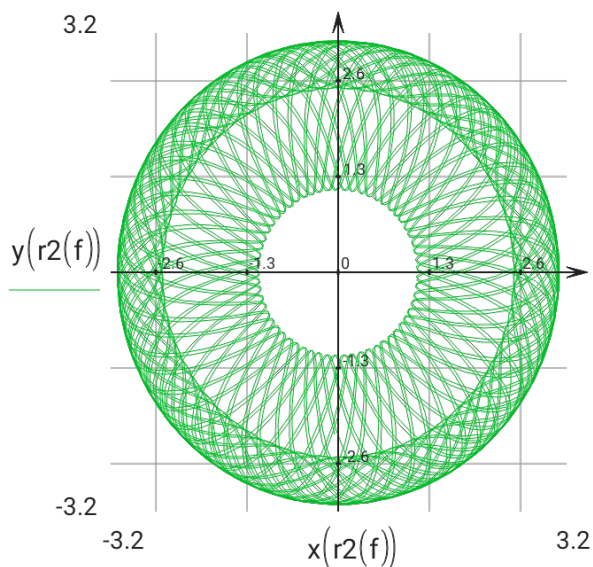


Instead of  $f$  and  $r$ , we use here previously defined rules for  $x$  and  $y$  transformation, where  $r1(f)$  is used as a symbolic argument for these rules:



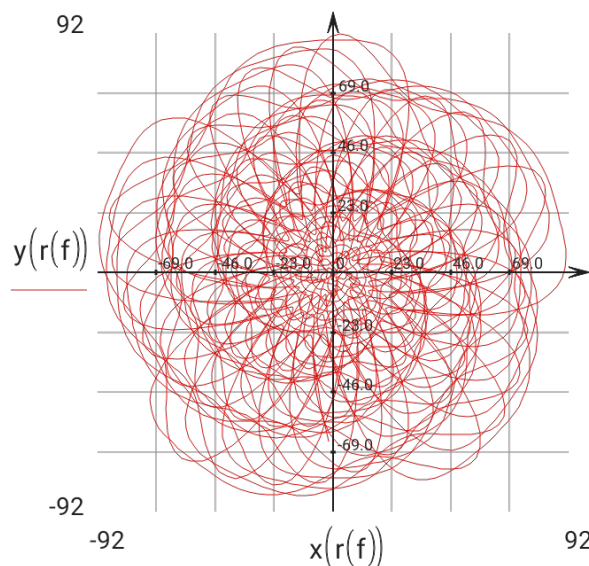
Next, we can modify this wheel as follows:

$$r2(f) := A + 2 \cdot \sin(B \cdot f + 1 \cdot r1(f))^q$$



Finally, we scale the last function  $r2(f)$  using a float to integer conversion that looks like a step function. As a result, we obtain a nice snail:

$$r(f) := r2(f) \cdot \text{floor}(f) / 10$$



### 3.2 Japanese Maple

Japanese Maple is well known for its attractive leaf shapes and colors. Such a leaf can be described mathematically and plotted as a curve in the polar coordinate system:

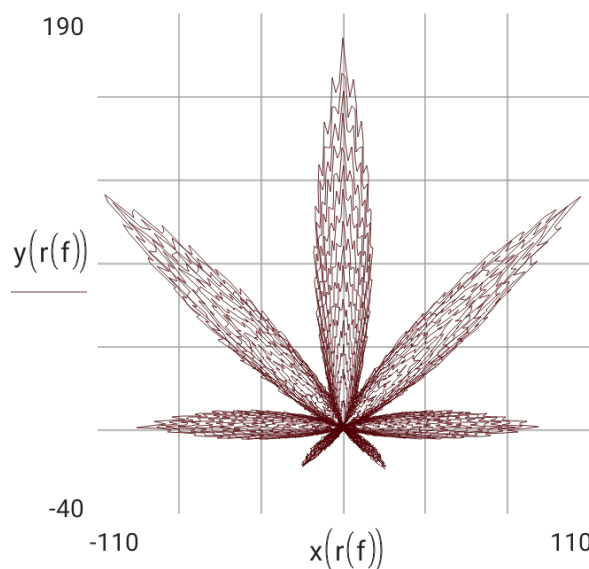
$$f := [0.01, 0.02 \dots 100]$$

$$x(r) := r \cdot \cos(f) \quad y(r) := r \cdot \sin(f)$$

$$s1(f) := (1 + \sin(f)) \cdot (1 - 0.9 \cdot |\sin(4 \cdot f)|)$$

$$s2(f) := 0.9 + 0.05 \cdot \cos(200 \cdot f)$$

$$r(f) := \text{floor}(f) \cdot s1(f) \cdot s2(f) + \text{random}(2) - 1$$



[http://en.wikipedia.org/wiki/Acer\\_palmatum](http://en.wikipedia.org/wiki/Acer_palmatum)



## 4 Example: 3D Plot

This example demonstrates 3D plots for three different functions of two variables.

First, we define intervals for both x and y arguments. The interval for the x-axis depends on the number of points along the x-axis and the minimum and maximum values, x1 and x2:

$$N := 300 \quad x1 := -2 \quad x2 := 2$$

$$x := [x1, x1 + |x2 - x1| / N .. x2]$$

The interval for the y-axis is defined analogously:

$$M := 300 \quad y1 := -3 \quad y2 := 3$$

$$y := [y1, y1 + |y2 - y1| / M .. y2]$$

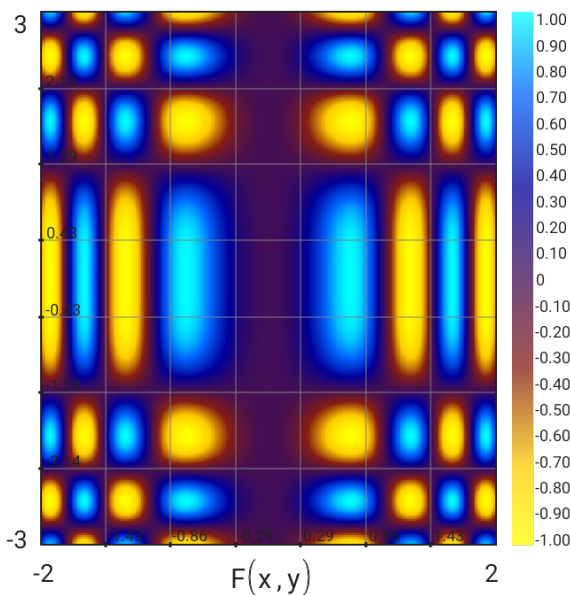
For example, let us plot a trigonometric function that is a product of sine and cosine:

$$F(x, y) := \sin(3 \cdot x^2) \cdot \cos(y^2)$$

To create a 3D plot view, click on the "New element" button from the action bar or "Add 3D plot" button from the tool bar:



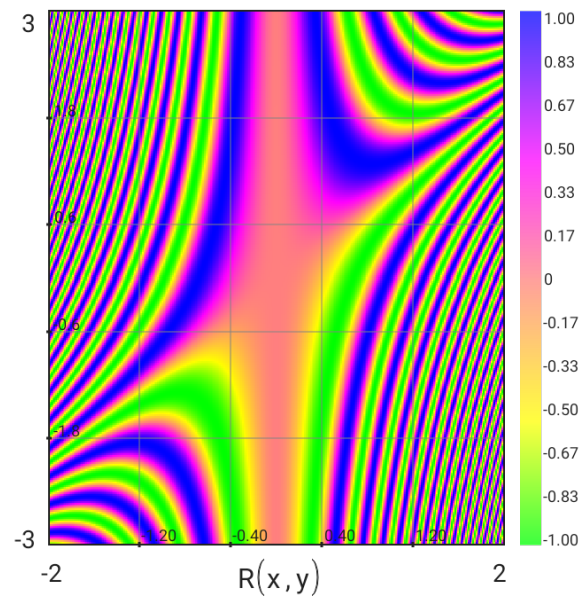
Put the function name  $F(x,y)$  into the center-bottom field:



The plot boundaries, plot size and style, labels and grid can be adjusted by analogy with the function plot using the plot settings dialog (see "Function Plot" example from the app navigation drawer for more details). To open this dialog, long click on the plot area until the floating button "Object properties" will appear, and then click this button.

Additionally, you can change the number of labels along z-axis and choose the color palette in the "Color Map Settings" dialog. This dialog appears by long click on the z-axis bar on the right of main graph area.

$$R(x, y) := \sin(5 \cdot x^2 \cdot (y - x))$$



A function of two arguments can be also plotted as a surface in 3D space. This mode can be activated in the "Plot Settings" dialog that appears if you click floating button "Object properties" after long clicking on the plot area. Let us plot the following function, using arrays in order to improve calculation time:

$$N := 100 \quad n := [0, 1 .. N] \quad x1 := -15 \quad x2 := 15$$

$$M := 100 \quad m := [0, 1 .. M] \quad y1 := -15 \quad y2 := 15$$

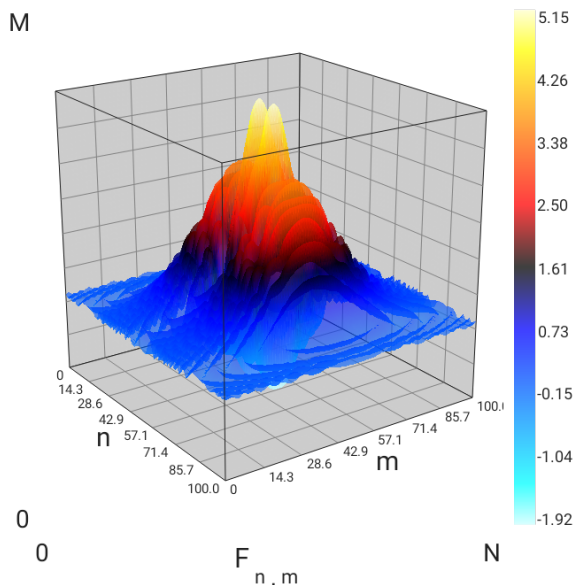
$$x[n] := (x1 + (x2 - x1) \cdot n / N)^2$$

$$y[m] := (y1 + (y2 - y1) \cdot m / M)^2$$

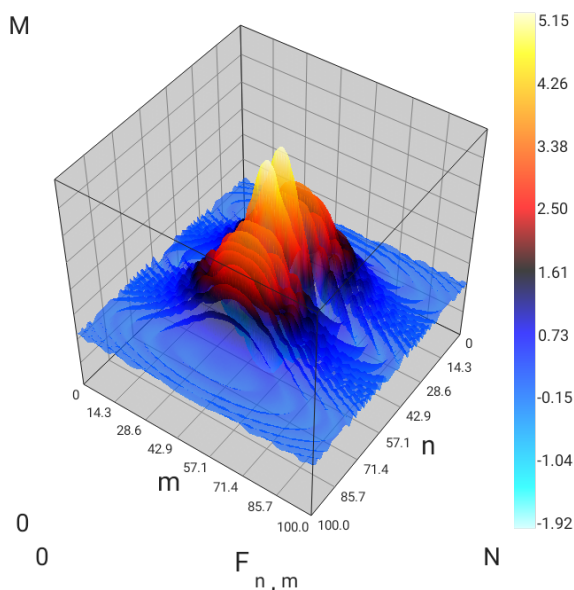
$$r[n, m] := 0.04 \cdot x_n + 0.02 \cdot y_m$$

$$t[n, m] := (x_n + 0.05 \cdot y_m) \cdot \exp(1 - r_{n,m})$$

$$F[n, m] := \frac{\sin(x_n + 0.1 \cdot y_m)}{0.15 + r_{n,m}} + \frac{t_{n,m}}{10}$$



For the surface plot, there are additional settings presented in the "Plot Settings" dialog. You can choose whether the mesh lines shall be shown, select the opacity for mesh color, define the rotation and elevation angles of the plot box. For example, the previous surface plotted with other rotation and elevation angles looks like this:



## 5 Example: Series and Integrals

This example demonstrates how to calculate series and integrals.

### 5.1 Taylor series

In mathematics, Taylor series is a representation of a function as an infinite sum of terms that are calcu-

lated from the values of the function's derivatives at a single point.

For example,  $Ts(x, N)$  is the Taylor expansion as a function of argument  $x$  and the number of terms  $N$ :

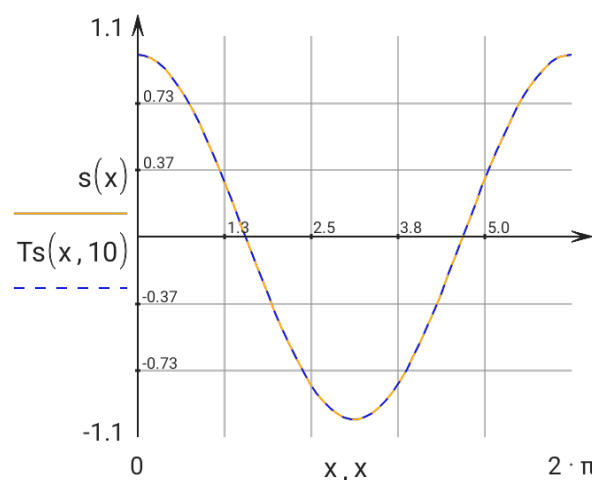
$$Ts(x, N) := \sum_{n=0}^N \frac{(-1)^n}{(2 \cdot n)!} \cdot x^{2 \cdot n}$$

This expansion approximates the cosine function:

$$s(x) := \cos(x)$$

If we plot both functions together for the same interval, they look both equal:

$$x := [0, 0.1 \dots 2 \cdot \pi]$$

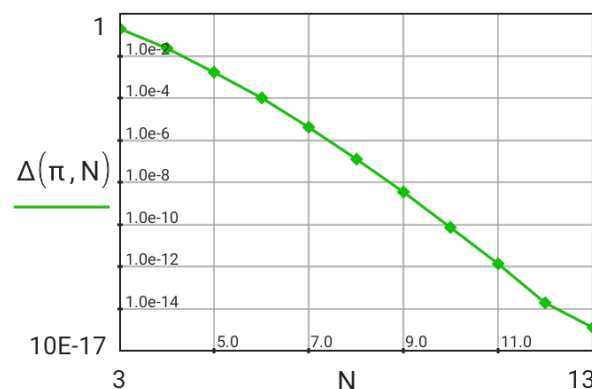


However, there is a numerical error due to limited number of approximation terms  $N$ . The following function  $\Delta(x, N)$  describes this error:

$$\Delta(x, N) := |s(x) - Ts(x, N)|$$

We can plot this function in logarithmic coordinates and see that the numerical error will be decreased if we get more terms into the Taylor summation:

$$N := [3, 4 \dots 13]$$



## 5.2 Binomial series

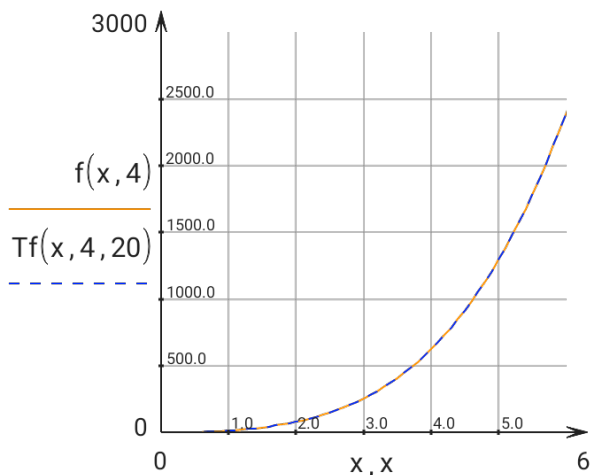
Let us consider this power function:

$$f(x, \alpha) := (1 + x)^\alpha$$

This function can be approximated using Binomial series:

$$Tf(x, \alpha, N) := \sum_{n=0}^N \left( \prod_{k=1}^n \frac{\alpha - k + 1}{k} \right) \cdot x^n$$

We can also plot both functions (the given power function and its approximation) together on the same plot:



## 5.3 Integrals

It is also possible to calculate a definite integral numerically using Simpson method. For example, we can calculate the integral using "Result View" element:

$$\int_0^{3\pi/2} \cos\left(\frac{2 \cdot x}{9}\right)^{-2} dx = 7.79423$$

The analytical solution is

$$I := \frac{9\sqrt{3}}{2}, \quad I = 7.79423$$

Numerical error can be calculated as:

$$\int_0^{3\pi/2} \cos\left(\frac{2 \cdot x}{9}\right)^{-2} dx - I = 4.26681E - 9$$

This error depends on the value "Significant digits in result" that can be changed in the "Document Settings" dialog available from the action bar:



If this value increased, the threshold that controls the Simpson method precision will also be increased.

## 6 About

### 6.1 Authors

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### 6.2 The app icon

The app icon is generated from the following function defined in the polar coordinate system:

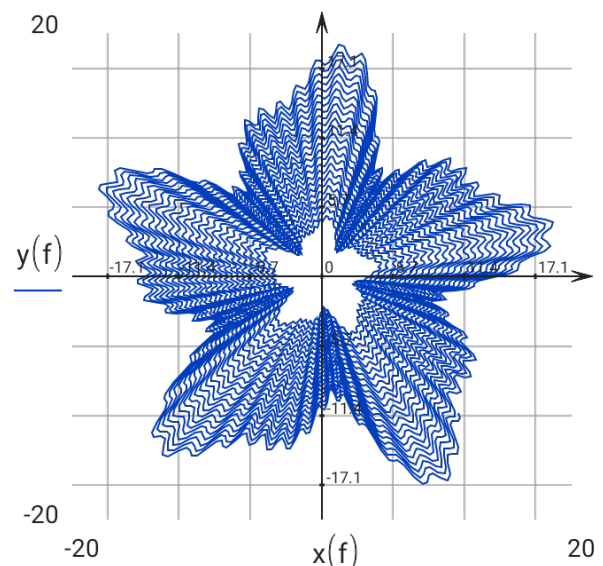
$$f := [0.01, 0.03 \dots 150]$$

$$s(f) := 4 + \sin(5 \cdot f) + \frac{\sin(10 \cdot f)}{2} + \frac{\sin(60 \cdot f)}{6}$$

$$r(f) := 0.9 \cdot (1 + f/50) \cdot s(f)$$

$$x(f) := r(f) \cdot \cos(f)$$

$$y(f) := r(f) \cdot \sin(f)$$



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