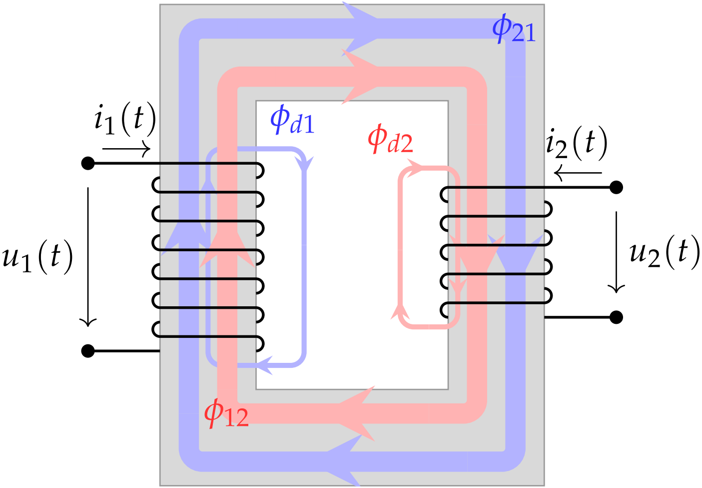
Transformadores

Teoría de Circuitos II

Oscar Perpiñán Lamigueiro

Recordatorio



$$L_1 = N_1 \frac{\phi_{11}}{i_1}$$

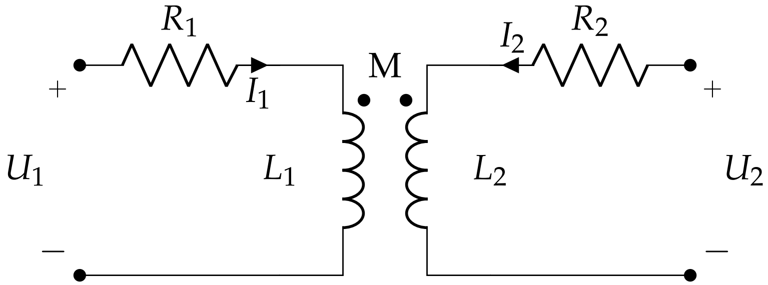
$$L_2 = N_2 \frac{\phi_{22}}{i_2}$$

$$M = N_1 \frac{\phi_{12}}{i_2}$$

$$= N_2 \frac{\phi_{21}}{i_1}$$

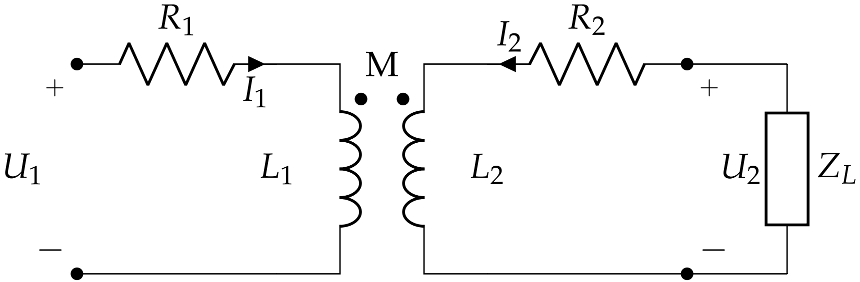
$$M = k\sqrt{L_1 \cdot L_2}$$

Ecuaciones del Transformador Real



$$\overline{U}_1 = (R_1 + j\omega L_1) \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega M \cdot \bar{I}_1 + (R_2 + j\omega L_2) \cdot \bar{I}_2$$

Ejemplo: impedancia de entrada desde primario



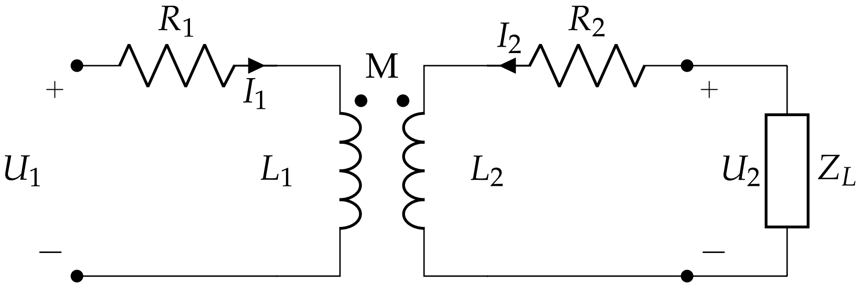
Ecuaciones del transformador

$$\overline{U}_1 = (R_1 + j\omega L_1) \cdot \overline{I}_1 + j\omega M \cdot \overline{I}_2$$
$$\overline{U}_2 = j\omega M \cdot \overline{I}_1 + (R_2 + j\omega L_2) \cdot \overline{I}_2$$

Ecuación de la carga

$$\overline{U}_2 = -\overline{I}_2 \cdot \overline{Z}_L$$

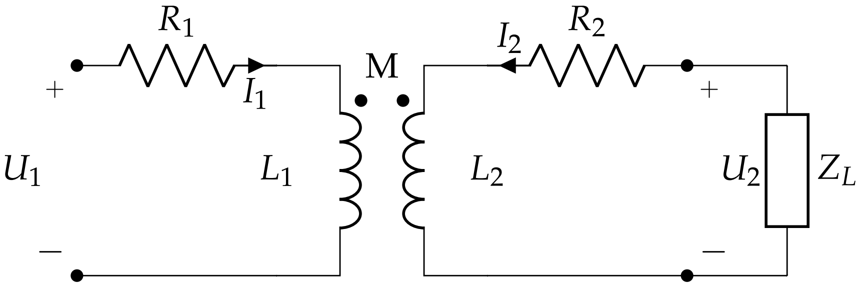
Ejemplo: impedancia de entrada desde primario



Combinando la ecuación del secundario con la ecuación de la carga:

$$\bar{I}_2 = -\frac{j\omega M}{(R_2 + j\omega L_2) + \overline{Z}_L} \cdot \bar{I}_1$$

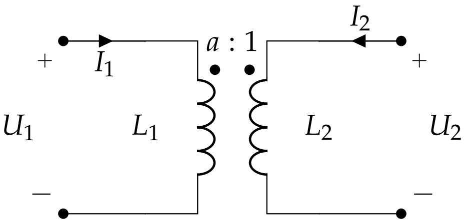
Ejemplo: impedancia de entrada desde primario



Combinando con la ecuación del primario:

$$\overline{Z}_{in} = \frac{\overline{U}_1}{\overline{I}_1} = (R_1 + j\omega L_1) + \frac{\omega^2 M^2}{(R_2 + j\omega L_2) + \overline{Z}_L} = \overline{Z}_1 + \frac{\omega^2 M^2}{\overline{Z}_2 + \overline{Z}_L}$$

Definición



Las pérdidas resistivas son despreciables.

$$R_1 = R_2 = 0$$

El acoplamiento es perfecto.

$$k = 1 \rightarrow \begin{cases} \phi_{12} & = \phi_{22} \\ \phi_{21} & = \phi_{11} \end{cases}$$

Relación de Transformación

Retomamos las ecuaciones de $M_{12} = M_{21} = M$:

$$N_1 \frac{\phi_{12}}{i_2} = N_2 \frac{\phi_{21}}{i_1}$$

Con la condición $k = 1$ escribimos:

$$N_1 \frac{\phi_{22}}{i_2} = N_2 \frac{\phi_{11}}{i_1}$$

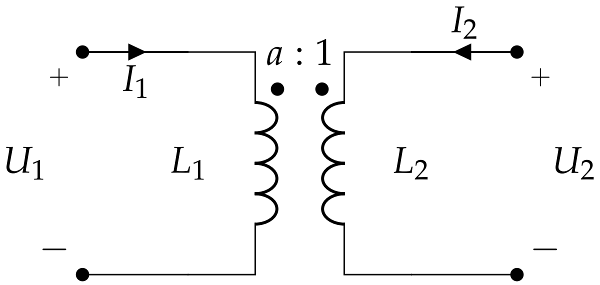
Y con las definiciones de L_1 y L_2:

$$N_1 \frac{L_2}{N_2} = N_2 \frac{L_1}{N_1}$$

Obtenemos la relación de transformación:

$$\boxed{\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 = a^2}$$

Ecuaciones del Transformador Perfecto



$$\overline{U}_1 = j\omega L_1 \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega M \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2$$

Relación de Tensiones

Dividiendo las ecuaciones:

$$\frac{\overline{U}_1}{\overline{U}_2} = \frac{j\omega L_1 \cdot \bar{I}_1 + j\omega M \cdot \bar{I}_2}{j\omega M \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2}$$

Empleando la relación de transformación:

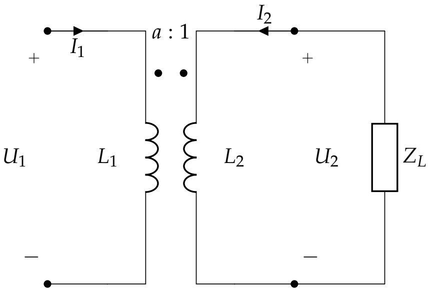
$$\frac{L_1}{L_2} = a^2 \rightarrow \begin{cases} L_1 & = a^2 \cdot L_2 \\ M & = a \cdot L_2 \end{cases}$$

Obtenemos:

$$\frac{\overline{U}_1}{\overline{U}_2} = \frac{a^2 L_2 \cdot \bar{I}_1 + a L_2 \cdot \bar{I}_2}{a L_2 \cdot \bar{I}_1 + L_2 \cdot \bar{I}_2}$$

$$\boxed{\frac{\overline{U}_1}{\overline{U}_2} = a = \frac{N_1}{N_2}}$$

Ejemplo: Impedancia de Entrada



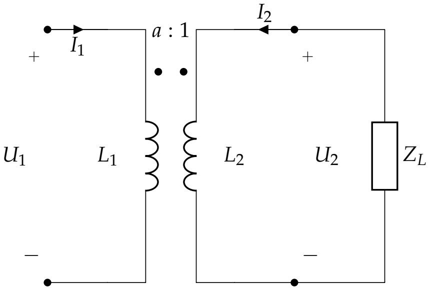
Ecuaciones del transformador:

$$\overline{U}_1 = j\omega L_1 \cdot \overline{I}_1 + j\omega M \cdot \overline{I}_2$$
$$\overline{U}_2 = j\omega M \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_2$$

Ecuación de la impedancia:

$$\overline{U}_2 = -\overline{Z}_L \cdot \overline{I}_2$$

Ejemplo: Impedancia de Entrada



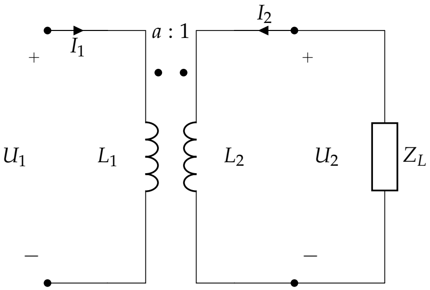
Despejamos I_2:

$$\overline{I}_2 = -\frac{j\omega M}{j\omega L_2 + \overline{Z}_L} \cdot \overline{I}_1$$

Y sustituimos:

$$\overline{Z}_{in} = \frac{\overline{U}_1}{\overline{I}_1} = j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + \overline{Z}_L}$$

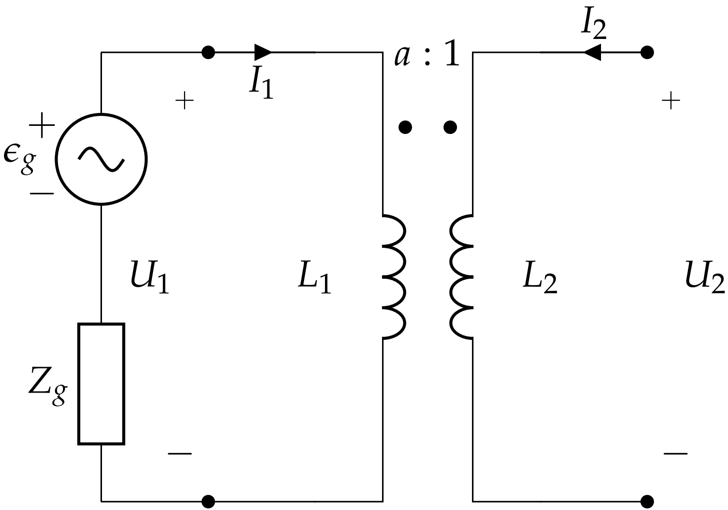
Ejemplo: Impedancia de Entrada



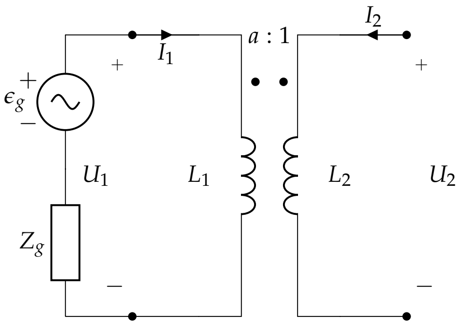
Teniendo en cuenta la relación entre L_1, L_2 y M:

$$\overline{Z}_{in} = j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + \overline{Z}_L} = \frac{j\omega L_1 \overline{Z}_L}{j\omega L_2 + \overline{Z}_L} \rightarrow \boxed{\overline{Z}_{in} = a^2 \cdot \frac{j\omega L_2 \cdot \overline{Z}_L}{j\omega L_2 + \overline{Z}_L} = \frac{j\omega L_1 \cdot a^2 \cdot \overline{Z}_L}{j\omega L_1 + a^2 \cdot \overline{Z}_L}}$$

Ejemplo: Equivalente de Thévenin desde secundario



Tensión de Thévenin



Ecuación del generador:

$$\overline{U}_1 = \overline{\epsilon}_g - \bar{I}_1 \cdot \overline{Z}_g$$

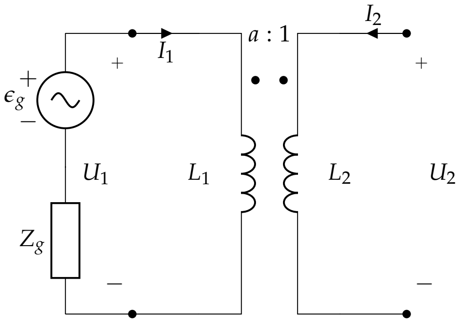
Ecuaciones del transformador ($\bar{I}_2 = 0$)

$$\overline{U}_1 = j\omega L_1 \cdot \bar{I}_1$$
$$\overline{U}_2 = j\omega M \cdot \bar{I}_1$$

Despejamos I_1:

$$\bar{I}_1 = \frac{\overline{\epsilon_g}}{\overline{Z}_g + j\omega L_1}$$

Tensión de Thévenin



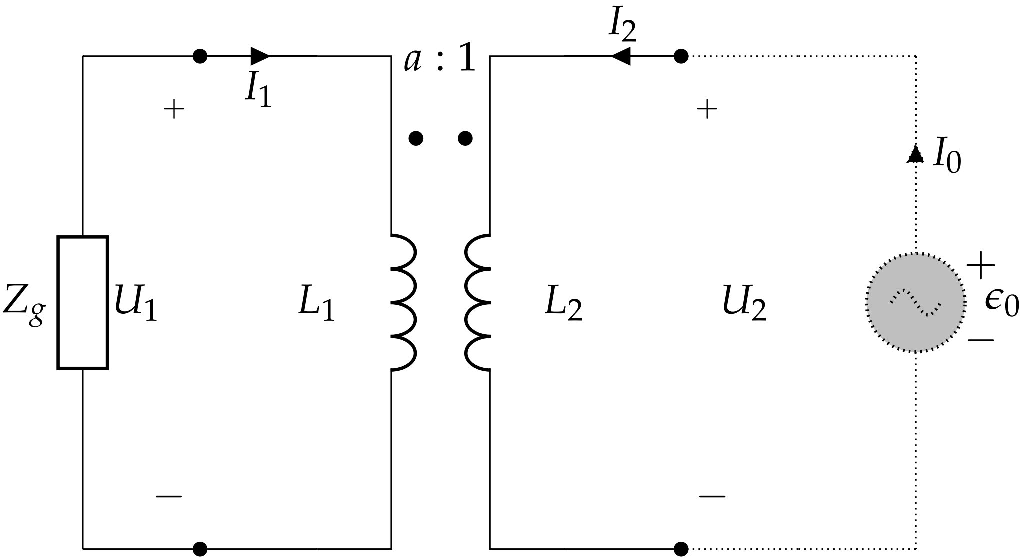
Tensión en abierto:

$$\overline{\epsilon}_{th} = \overline{U}_2 = \frac{j\omega M}{j\omega L_1 + \overline{Z}_g} \cdot \overline{\epsilon}_g$$

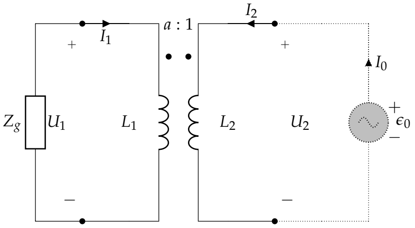
Teniendo en cuenta que $M = L_1/a$:

$$\overline{\epsilon}_{th} = \frac{1}{a} \cdot \left(\frac{j\omega L_1}{j\omega L_1 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$

Impedancia de Thévenin



Impedancia de Thévenin



Ecuaciones del transformador:

$$\overline{U}_1 = j\omega L_1 \cdot \overline{I}_1 + j\omega M \cdot \overline{I}_0$$

$$\overline{\epsilon}_0 = j\omega M \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_0$$

Ecuación de la impedancia:

$$\overline{U}_1 = -\overline{Z}_g \cdot \overline{I}_1$$

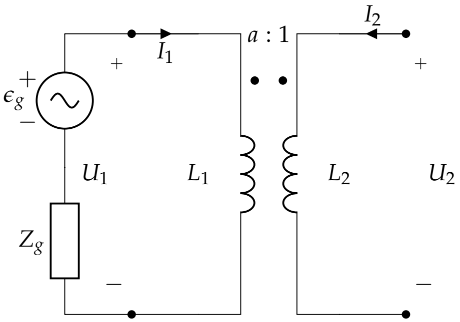
Impedancia de Thévenin:

$$\overline{Z}_{th} = \frac{\overline{\epsilon}_0}{\overline{I}_0} = j\omega L_2 + \frac{\omega^2 M^2}{j\omega L_1 + \overline{Z}_g}$$

Con $L_2 = L_1/a^2$ y $M = L_1/a$:

$$\overline{Z}_{th} = \frac{1}{a^2} \cdot \frac{j\omega L_1 \cdot \overline{Z}_g}{j\omega L_1 + \overline{Z}_g}$$

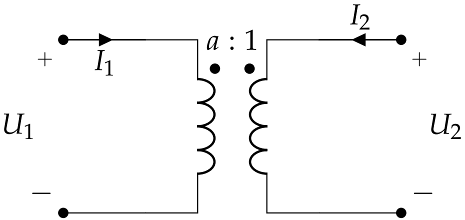
Resumen: Equivalente de Thévenin



$$\overline{Z}_{th} = \frac{1}{a^2} \cdot \frac{j\omega L_1 \cdot \overline{Z}_g}{j\omega L_1 + \overline{Z}_g}$$

$$\overline{\epsilon}_{th} = \frac{1}{a} \cdot \left(\frac{j\omega L_1}{j\omega L_1 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$

Definición



Las pérdidas resistivas son despreciables.

$$R_1 = R_2 = 0$$

El acoplamiento es perfecto.

$$k = 1$$

Las bobinas tienen un número muy elevado de espiras.

$$N_1 \to \infty$$

$$N_2 \to \infty$$

El flujo en cada bobina es nulo

Para que las tensiones inducidas sean finitas…

$$\overline{U}_1 = N_1\overline{\phi}_1$$
$$\overline{U}_2 = N_2\overline{\phi}_2$$

…los flujos (fasoriales) que los atraviesan deben ser nulos.

$$\overline{\phi}_1 \to 0$$
$$\overline{\phi}_2 \to 0$$

Siendo:

$$\overline{\phi}_1 = \overline{\phi}_{11} + \overline{\phi}_{12}$$
$$\overline{\phi}_2 = \overline{\phi}_{22} + \overline{\phi}_{21}$$

El flujo mutuo es nulo

Teniendo en cuenta que el acoplamiento es perfecto, $k = 1$:

$$\left.\begin{array}{ll} \phi_{12} & = \phi_{22} \\ \phi_{21} & = \phi_{11} \end{array}\right\} \rightarrow \left\{\begin{array}{ll} 0 & = \overline{\phi}_{21} + \overline{\phi}_{12} \\ 0 & = \overline{\phi}_{12} + \overline{\phi}_{21} \end{array}\right.$$

O también:

$$\boxed{\overline{\phi}_{11} + \overline{\phi}_{22} = 0}$$

Relación de Transformación

Hemos obtenido:

$$\overline{\phi}_{11} + \overline{\phi}_{22} = 0$$

Con las definiciones de L_1, L_2:

$$L_1 = N_1 \frac{\phi_{11}}{I_1}; \quad L_2 = N_2 \frac{\phi_{22}}{I_2}$$

Podemos escribir:

$$\frac{L_1 \bar{I}_1}{N_1} + \frac{L_2 \bar{I}_2}{N_2} = 0$$

Y con la relación entre ambas obtenemos

$$L_1 = L_2 \cdot \left(\frac{N_1}{N_2}\right)^2 \rightarrow \frac{N_1 L_2 \bar{I}_1}{N_2^2} + \frac{L_2 \bar{I}_2}{N_2} = 0 \rightarrow \boxed{\frac{\bar{I}_1}{\bar{I}_2} = \mp \frac{1}{a} = \mp \frac{N_2}{N_1}}$$

Un transformador ideal no consume potencia

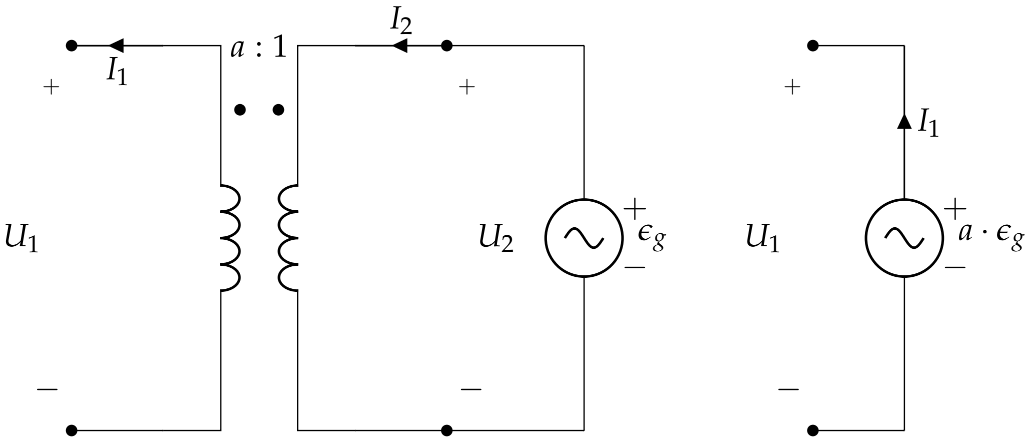
$$\overline{S}_1 = \overline{U}_1 \cdot \overline{I}_1^*$$

$$\overline{S}_2 = \overline{U}_2 \cdot \overline{I}_2^*$$

$$\overline{U}_2 \cdot \overline{I}_2^* = \frac{1}{a} \cdot \overline{U}_1 \cdot a \cdot \overline{I}_1^* = \overline{S}_1$$

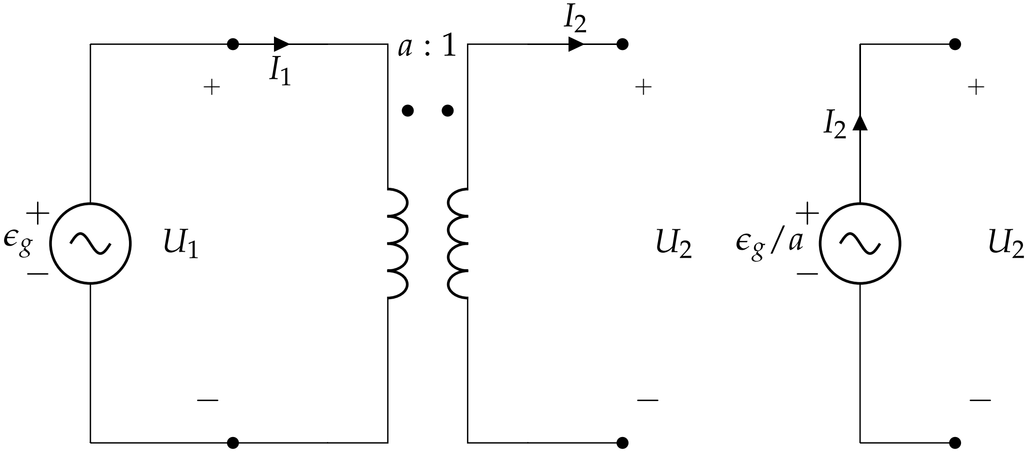
$$\boxed{\overline{S}_1 = \overline{S}_2} \quad \boxed{P_1 = P_2} \quad \boxed{Q_1 = Q_2}$$

Fuente de Tensión de Secundario a Primario



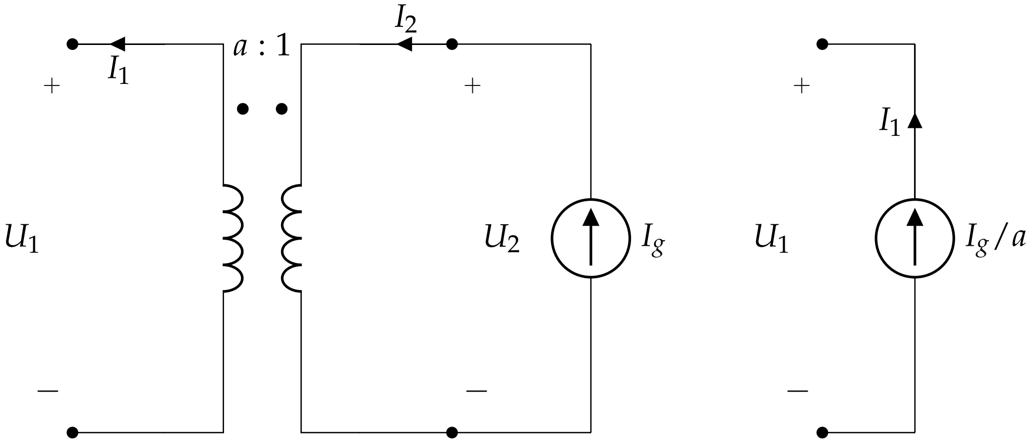
$$\overline{U}_1 = a \cdot \overline{U}_2 \rightarrow \boxed{\overline{\epsilon}_{g1} = a \cdot \overline{\epsilon}_g}$$

Fuente de Tensión de Primario a Secundario



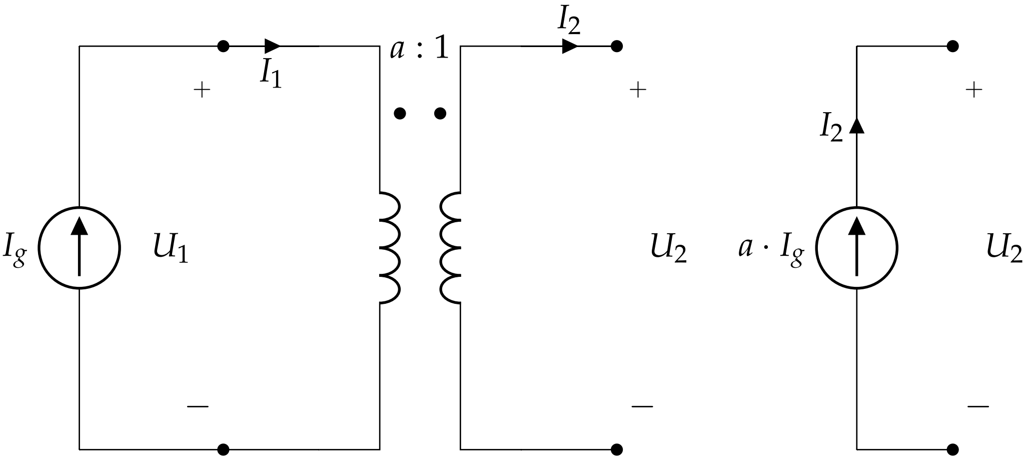
$$\overline{U}_2 = \overline{U}_1/a \rightarrow \boxed{\overline{\epsilon}_{g2} = \overline{\epsilon}_g/a}$$

Fuente de Corriente de Secundario a Primario



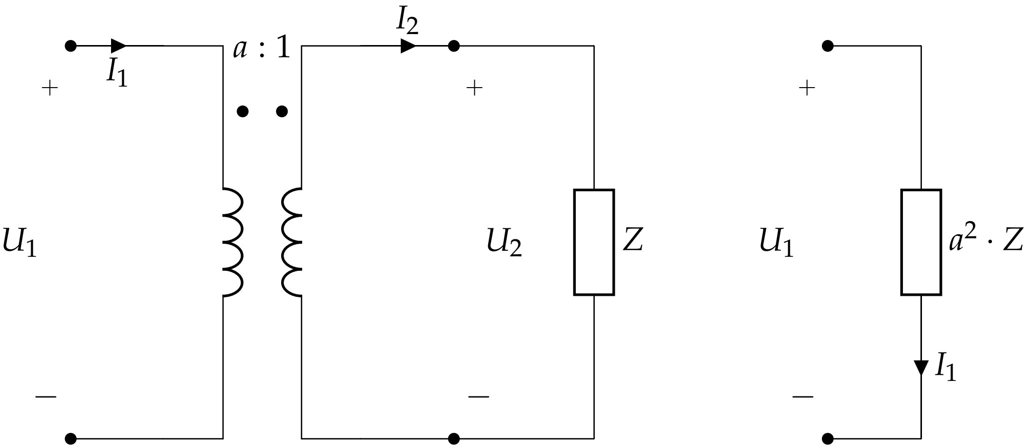
$$\bar{I}_1 = \bar{I}_2/a \rightarrow \boxed{\bar{I}_{g1} = \bar{I}_g/a}$$

Fuente de Corriente de Primario a Secundario



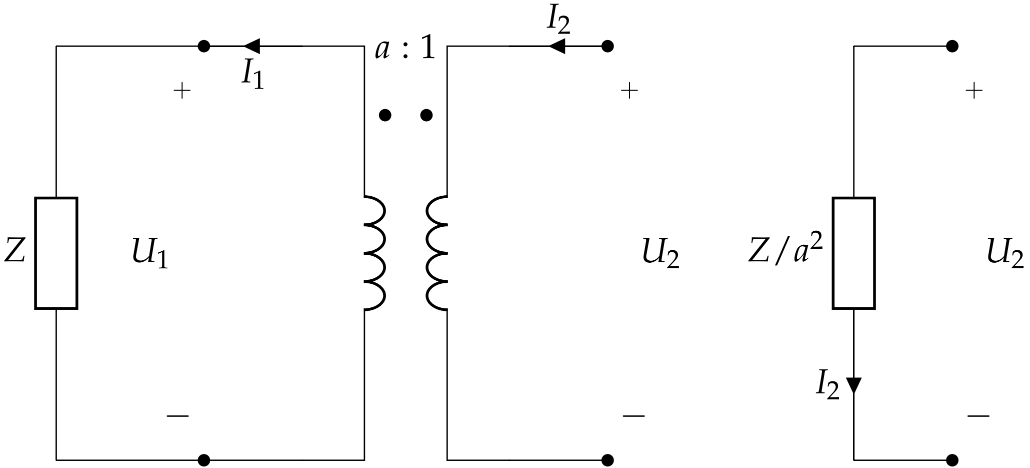
$$\bar{I}_2 = a \cdot \bar{I}_1 \rightarrow \boxed{\bar{I}_{g2} = a \cdot \bar{I}_g}$$

Impedancia de Secundario a Primario



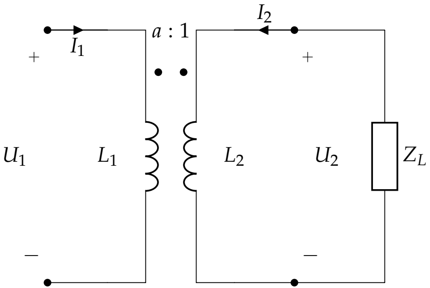
$$\left.\begin{aligned} \overline{U}_1 &= a \cdot \overline{U}_2 \\ \overline{I}_1 &= \overline{I}_2 / a \end{aligned}\right\} \rightarrow \boxed{\overline{Z}_1 = \frac{\overline{U}_1}{\overline{I}_1} = a^2 \cdot \overline{Z}}$$

Impedancia de Primario a Secundario



$$\left.\begin{aligned} \overline{U}_2 &= \overline{U}_1/a \\ \overline{I}_2 &= a \cdot \overline{I}_1 \end{aligned}\right\} \rightarrow \boxed{\overline{Z}_2 = \frac{\overline{U}_2}{\overline{I}_2} = \overline{Z}/a^2}$$

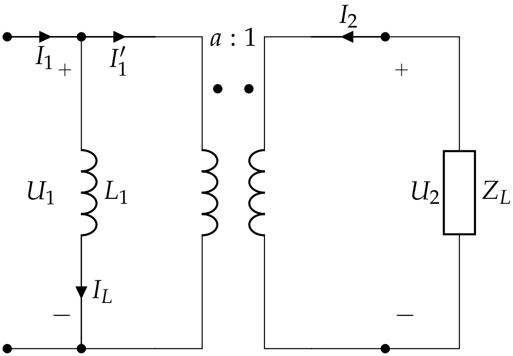
Recordatorio: impedancia de entrada de un T. Perfecto

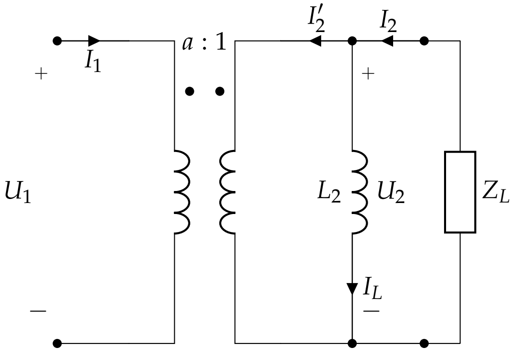


$$\boxed{\overline{Z}_{in} = \frac{j\omega L_1 \cdot (a^2 \overline{Z}_L)}{j\omega L_1 + (a^2 \cdot \overline{Z}_L)}}$$

$$\boxed{\overline{Z}_{in} = a^2 \cdot \frac{j\omega L_2 \cdot \overline{Z}_L}{j\omega L_2 + \overline{Z}_L}}$$

Circuito equivalente con transformador ideal

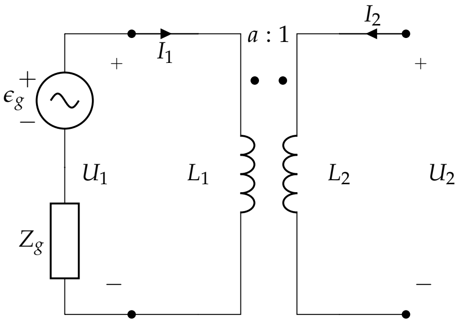




$$\boxed{\overline{Z}_{in} = \frac{j\omega L_1 \cdot (a^2 \overline{Z}_L)}{j\omega L_1 + (a^2 \cdot \overline{Z}_L)}}$$

$$\boxed{\overline{Z}_{in} = a^2 \cdot \frac{j\omega L_2 \cdot \overline{Z}_L}{j\omega L_2 + \overline{Z}_L}}$$

Recordatorio: Equivalente de Thévenin



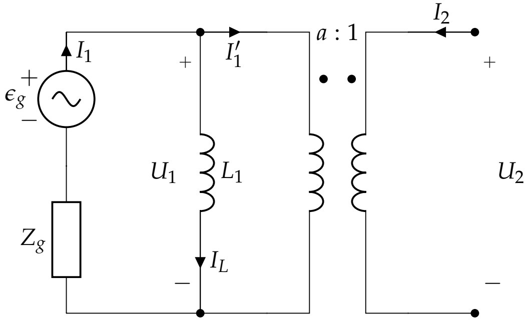
$$\overline{Z}_{th} = \frac{1}{a^2} \cdot \frac{j\omega L_1 \cdot \overline{Z}_g}{j\omega L_1 + \overline{Z}_g}$$

$$\overline{\epsilon}_{th} = \frac{1}{a} \cdot \left(\frac{j\omega L_1}{j\omega L_1 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$

Equivalente en primario con transformador ideal

$$\overline{Z}_{th} = \frac{1}{a^2} \cdot \frac{j\omega L_1 \cdot \overline{Z}_g}{j\omega L_1 + \overline{Z}_g}$$

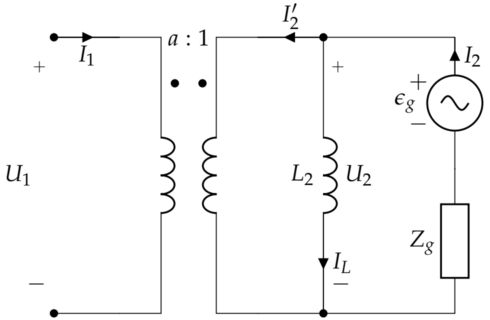
$$\overline{\epsilon}_{th} = \frac{1}{a} \cdot \left(\frac{j\omega L_1}{j\omega L_1 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$



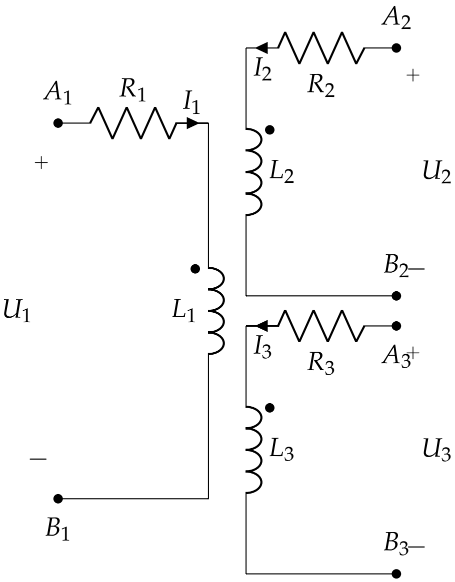
Equivalente en secundario con transformador ideal

$$\overline{Z}_{th} = a^2 \cdot \frac{j\omega L_2 \cdot \overline{Z}_g}{j\omega L_2 + \overline{Z}_g}$$

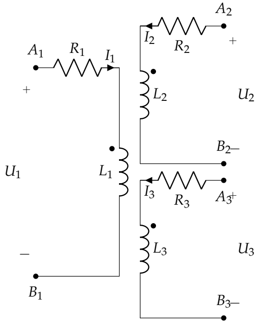
$$\overline{\epsilon}_{th} = a \cdot \left(\frac{j\omega L_2}{j\omega L_2 + \overline{Z}_g} \right) \cdot \overline{\epsilon}_g$$



Transformador Real de Varios Devanados

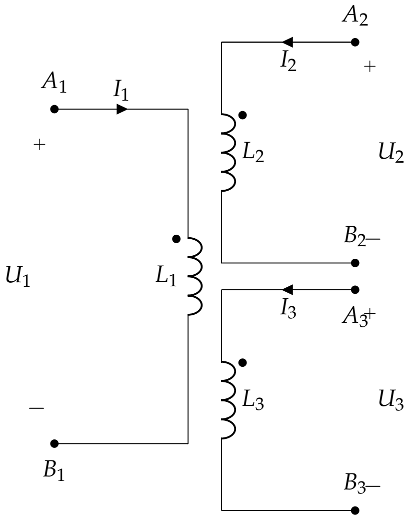


Ecuaciones del Transformador Real



$$\overline{U}_1 = (R_1 + j\omega L_1)\cdot \bar{I}_1 + j\omega M_{12}\cdot \bar{I}_2 + j\omega M_{13}\cdot \bar{I}_3$$
$$\overline{U}_2 = j\omega M_{12}\cdot \bar{I}_1 + (R_2 + j\omega L_2)\cdot \bar{I}_2 + j\omega M_{23}\cdot \bar{I}_3$$
$$\overline{U}_3 = j\omega M_{13}\cdot \bar{I}_1 + j\omega M_{12}\cdot \bar{I}_2 + (R_3 + j\omega L_3)\cdot \bar{I}_3$$

Transformador Perfecto



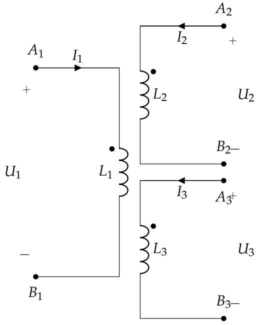
Las pérdidas resistivas son despreciables.

$$R_1 = R_2 = R_3 = 0$$

El acoplamiento es perfecto.

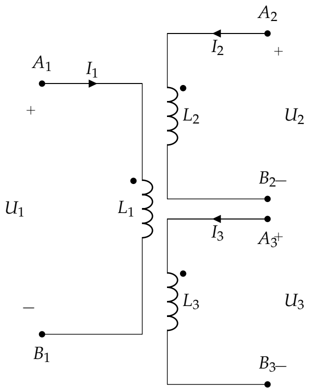
$$k_{12} = k_{13} = k_{23} = 1$$

Ecuaciones del Transformador Perfecto



$$\overline{U}_1 = j\omega L_1 \cdot \overline{I}_1 + j\omega M_{12} \cdot \overline{I}_2 + j\omega M_{13} \cdot \overline{I}_3$$
$$\overline{U}_2 = j\omega M_{12} \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_2 + j\omega M_{23} \cdot \overline{I}_3$$
$$\overline{U}_3 = j\omega M_{13} \cdot \overline{I}_1 + j\omega M_{12} \cdot \overline{I}_2 + j\omega L_3 \cdot \overline{I}_3$$

Relaciones de Transformación



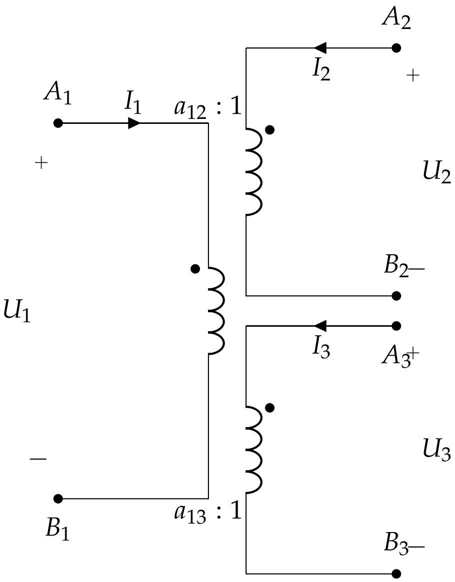
$$\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 = a_{12}^2$$

$$\frac{L_1}{L_3} = \left(\frac{N_1}{N_3}\right)^2 = a_{13}^2$$

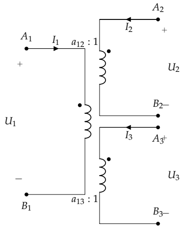
$$\frac{\overline{U}_1}{\overline{U}_2} = a_{12}$$

$$\frac{\overline{U}_1}{\overline{U}_3} = a_{13}$$

Transformador Ideal



Relación de Transformación del Transformador Ideal



Debido a las condiciones de idealidad:

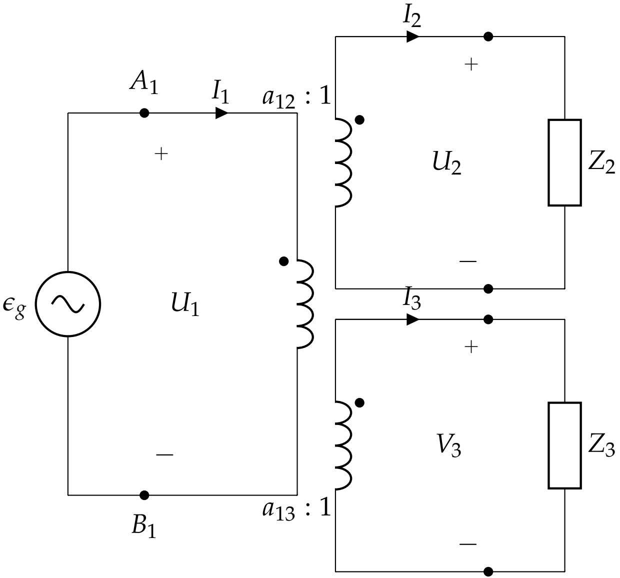
$$\overline{\phi}_{11} \pm \overline{\phi}_{22} \pm \overline{\phi}_{33} = 0$$

$$N_1\bar{I}_1 \pm N_2\bar{I}_2 \pm N_3\bar{I}_3 = 0$$

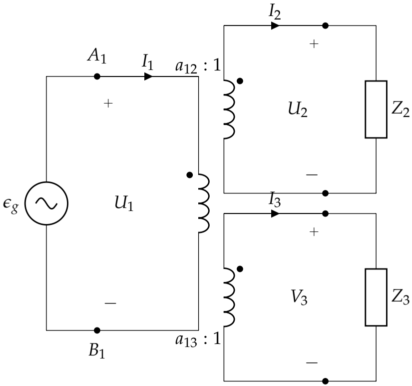
En términos de corriente:

$$\boxed{\bar{I}_1 = \mp 1/a_{12} \cdot \bar{I}_2 \mp 1/a_{13} \cdot \bar{I}_3}$$

Impedancia de Entrada



Impedancia de Entrada



Ecuaciones del transformador:

$$\overline{U}_1 = \overline{U}_2 \cdot a_{12}$$
$$\overline{U}_1 = \overline{U}_3 \cdot a_{13}$$
$$\overline{I}_1 = 1/a_{12} \cdot \overline{I}_2 + 1/a_{13} \cdot \overline{I}_3$$

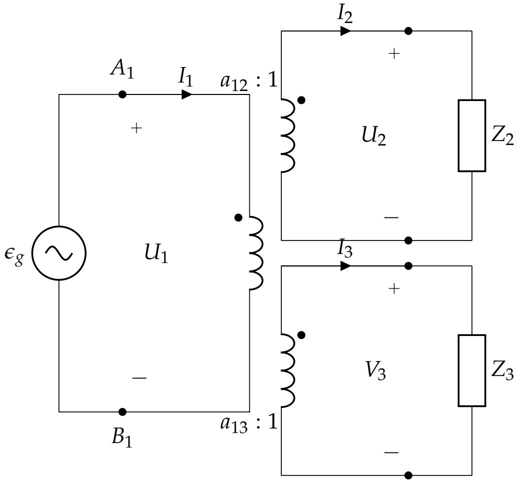
Ecuaciones Terminales

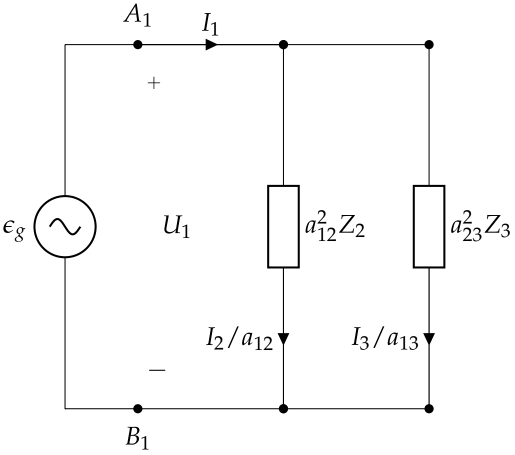
$$\overline{U}_2 = \overline{Z}_2 \cdot \overline{I}_2$$
$$\overline{U}_3 = \overline{Z}_3 \cdot \overline{I}_3$$

Resultado:

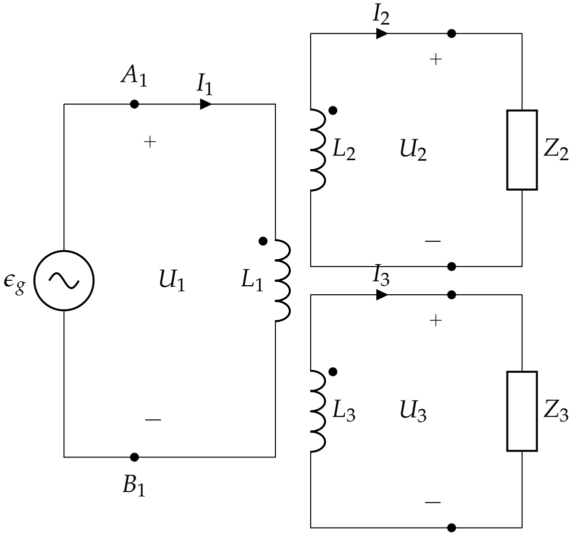
$$\frac{\overline{I}_1}{\overline{U}_1} = \boxed{\overline{Y}_{in} = \frac{1}{a_{12}^2 \overline{Z}_2} + \frac{1}{a_{13}^2 \overline{Z}_3}}$$

Circuito Equivalente

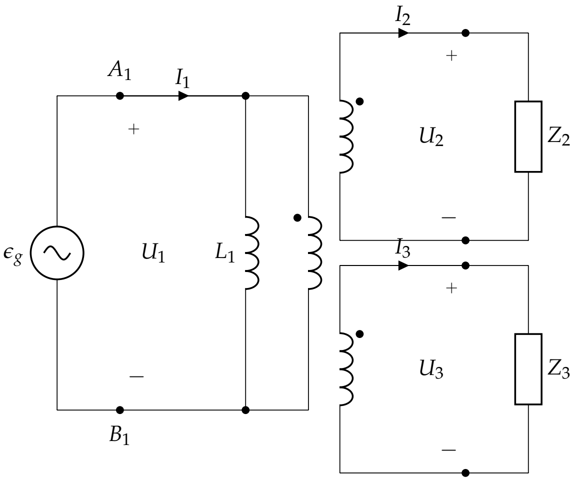




Circuito Equivalente de un Transformador Perfecto

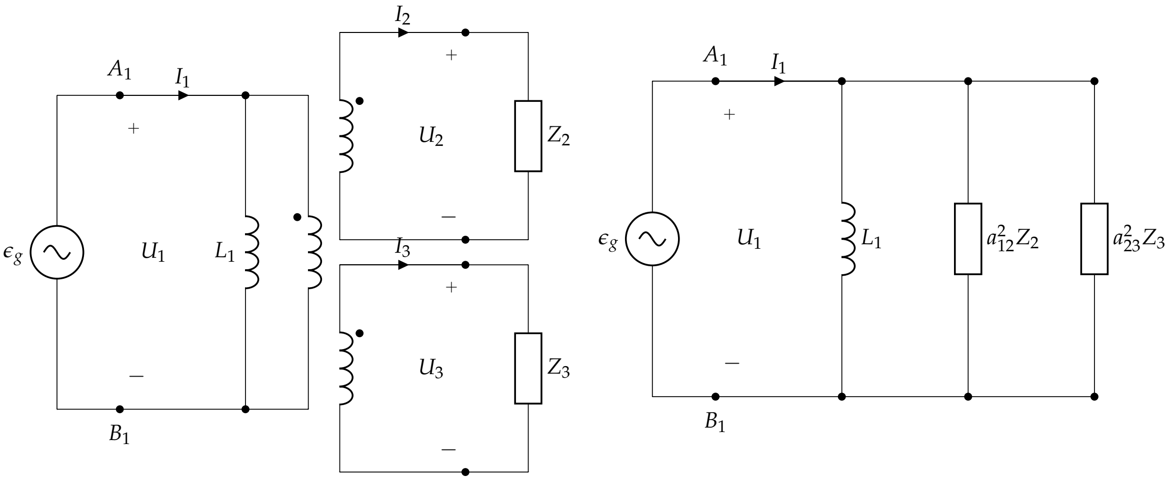


Transformador Perfecto

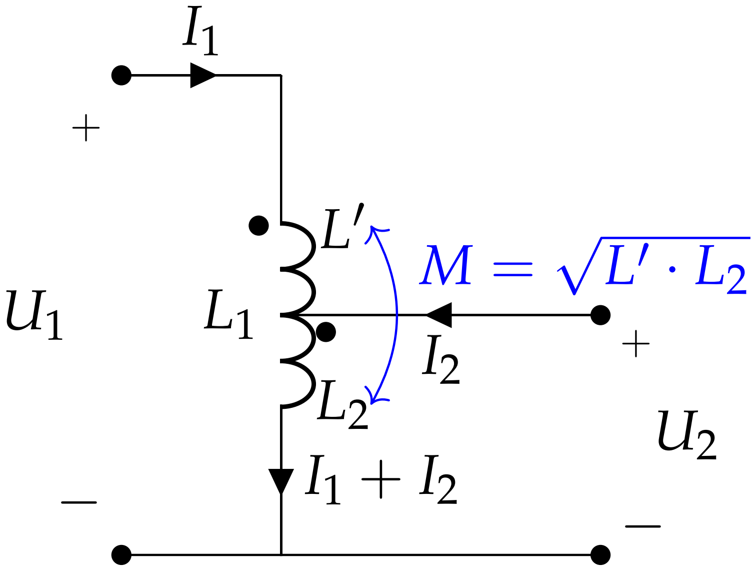


Equivalente Ideal

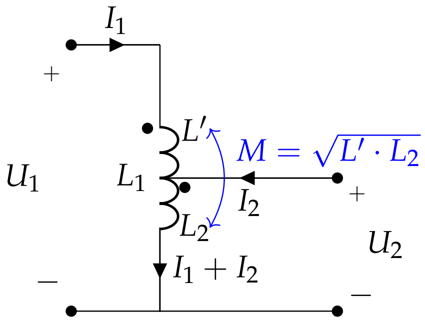
Circuito Equivalente de un Transformador Perfecto



Autotransformador Perfecto

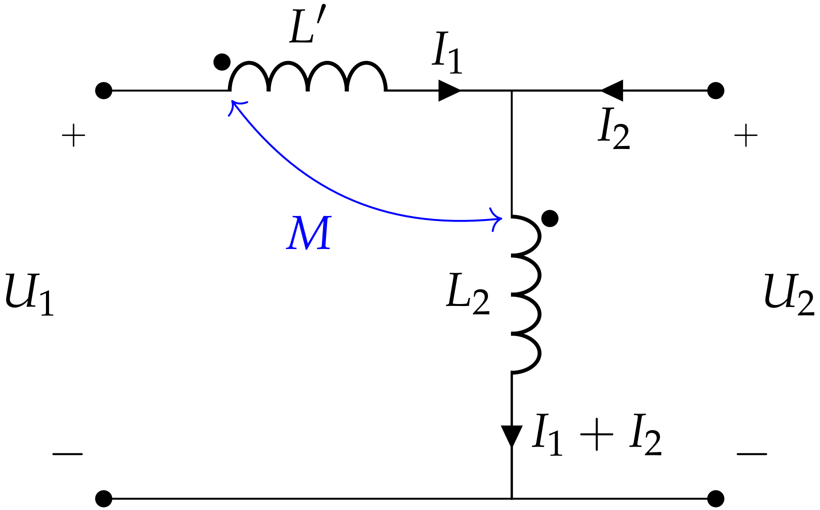


Ecuaciones del Autotransformador Perfecto

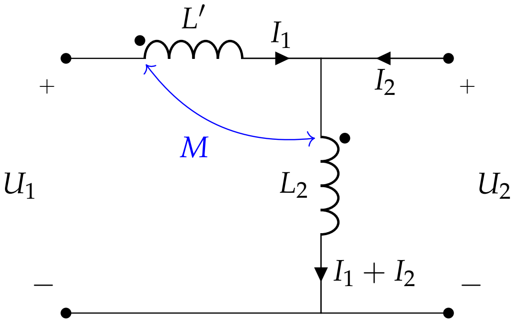


$$\overline{U}_1 = j\omega L_1 \cdot \bar{I}_1 + j\omega(M + L_2) \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega(M + L_2) \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2$$

Circuito Alternativo del Autotransformador Perfecto

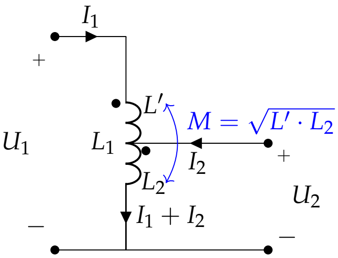


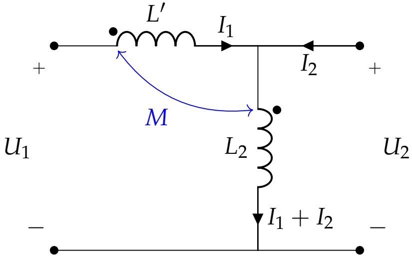
Ecuaciones del Circuito Alternativo



$$\overline{U}_1 = j\omega(L' + L_2 + 2M) \cdot \bar{I}_1 + j\omega(L_2 + M) \cdot \bar{I}_2$$
$$\overline{U}_2 = j\omega(L_2 + M) \cdot \bar{I}_1 + j\omega L_2 \cdot \bar{I}_2$$

Ecuaciones Comparadas



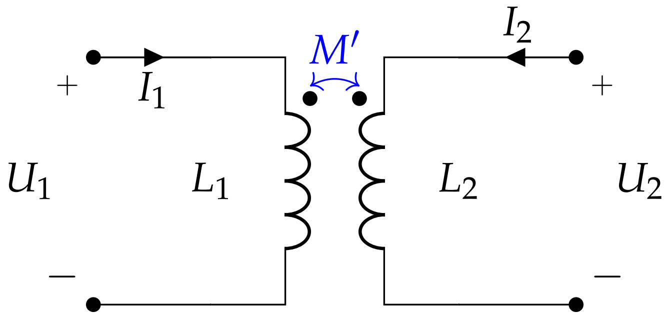


$$\overline{U}_1 = j\omega L_1 \cdot \overline{I}_1 + j\omega(M + L_2) \cdot \overline{I}_2 = j\omega(L' + L_2 + 2M) \cdot \overline{I}_1 + j\omega(M + L_2) \cdot \overline{I}_2$$

$$\overline{U}_2 = j\omega(L_2 + M) \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_2 = j\omega(L_2 + M) \cdot \overline{I}_1 + j\omega L_2 \cdot \overline{I}_2$$

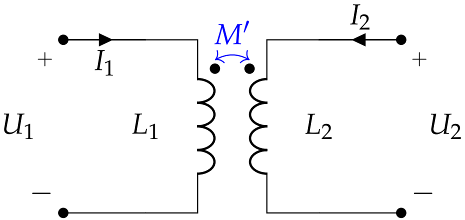
$$L_1 = L' + L_2 + 2M$$
$$M' = M + L_2$$

Transformador Perfecto Equivalente



$$L_1 = L' + L_2 + 2M$$
$$M = \sqrt{L' \cdot L_2}$$
$$M' = M + L_2$$

Transformador Perfecto Equivalente



Comprobamos:

$$
\begin{aligned}
M' &= \sqrt{L_1 \cdot L_2} = \\
&= \sqrt{(L' + L_2 + 2M)L_2} = \\
&= \sqrt{L'L_2 + L_2^2 + 2ML_2} = \\
&= \sqrt{M^2 + L_2^2 + 2ML_2} = \\
&= M + L_2
\end{aligned}
$$

Autotransformador Ideal