Repaso de trigonometría y números complejos

1. Trigonometría

Ecuación fundamental

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

Cuadratura

$$\sin(\theta + \pi/2) = \cos(\theta) \qquad \sin(\theta - \pi/2) = -\cos(\theta)$$
$$\cos(\theta + \pi/2) = -\sin(\theta) \qquad \cos(\theta - \pi/2) = \sin(\theta)$$

Suma y resta de ángulos

$$\begin{aligned}
\cos(\alpha - \beta) &= \cos(\alpha)\cdot\cos(\beta) + \sin(\alpha)\cdot\sin(\beta)\\
\cos(\alpha + \beta) &= \cos(\alpha)\cdot\cos(\beta) - \sin(\alpha)\cdot\sin(\beta)\\
\sin(\alpha - \beta) &= \sin(\alpha)\cdot\cos(\beta) - \cos(\alpha)\cdot\sin(\beta)\\
\sin(\alpha + \beta) &= \sin(\alpha)\cdot\cos(\beta) + \cos(\alpha)\cdot\sin(\beta)
\end{aligned}$$

Productos y ángulo doble

$$\begin{aligned}
\cos(\alpha)\cdot\cos(\beta) &= \frac{1}{2}\cdot[\cos(\alpha+\beta) + \cos(\alpha-\beta)]\\
\cos(2\alpha) &= 2\cdot\cos^2(\alpha) - 1\\
\cos(2\alpha) &= 1 - 2\cdot\sin^2(\alpha)\\
\sin(2\alpha) &= 2\cdot\sin(\alpha)\cdot\cos(\alpha)
\end{aligned}$$

Derivadas e integrales

$$\begin{aligned}
\frac{\mathrm{d}\sin(\omega t+\theta)}{\mathrm{d}t} &= \omega\cdot\cos(\omega t+\theta)\\
\frac{\mathrm{d}\cos(\omega t+\theta)}{\mathrm{d}t} &= -\omega\cdot\sin(\omega t+\theta)\\
\int\sin(\omega t+\theta)\,\mathrm{d}t &= -\frac{1}{\omega}\cdot\cos(\omega t+\theta) + k\\
\int\cos(\omega t+\theta)\,\mathrm{d}t &= \frac{1}{\omega}\cdot\sin(\omega t+\theta) + k
\end{aligned}$$

Aprovechando las relaciones de cuadratura, podemos comprobar que **las derivadas adelantan $\pi/2$**:

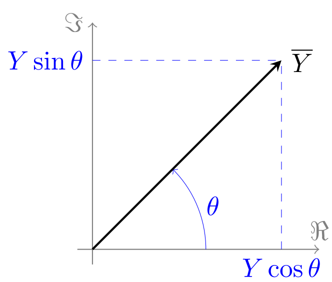
$$\begin{aligned}
\frac{\mathrm{d}\sin(\omega t+\theta)}{\mathrm{d}t} &= \omega\cdot\sin(\omega t+\theta+\pi/2)\\
\frac{\mathrm{d}\cos(\omega t+\theta)}{\mathrm{d}t} &= \omega\cdot\cos(\omega t+\theta+\pi/2)
\end{aligned}$$

Y **las integrales retrasan $\pi/2$**:

$$\begin{aligned}
\int\sin(\omega t+\theta)\,\mathrm{d}t &= \frac{1}{\omega}\cdot\sin(\omega t+\theta-\pi/2) + k\\
\int\cos(\omega t+\theta)\,\mathrm{d}t &= \frac{1}{\omega}\cdot\cos(\omega t+\theta-\pi/2) + k
\end{aligned}$$

2. Números complejos

Definición



Forma polar:

$$\begin{aligned}
\overline{Y} &= Y\cdot e^{\mathrm{j}\theta}\\
&= Y\underline{/\theta}
\end{aligned}$$

Forma binómica:

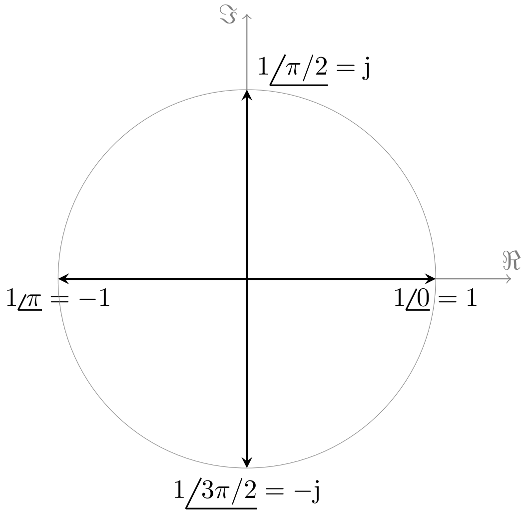
$$\begin{aligned}
\overline{Y} &= Y\cdot[\cos(\theta) + \mathrm{j}\cdot\sin(\theta)]\\
&= a_Y + \mathrm{j}\,b_Y
\end{aligned}$$

$$\begin{aligned}
|\overline{Y}| &= \sqrt{a_Y^2 + b_Y^2}\\
&= \sqrt{Y^2\cos^2(\theta) + Y^2\sin^2(\theta)}\\
&= Y
\end{aligned}$$

Fórmula de Euler

$$e^{\mathrm{j}\,\theta} = \cos(\theta) + \mathrm{j} \cdot \sin(\theta)$$

Complejo unitario



$$\begin{aligned} 1\angle 0 &= e^{\mathrm{j}\,0} = 1 \\ 1\angle \pi/2 &= e^{\mathrm{j}\,\pi/2} = \mathrm{j} \\ 1\angle \pi &= e^{\mathrm{j}\,\pi} = -1 \\ 1\angle 3\pi/2 &= e^{\mathrm{j}\,3\pi/2} = -\,\mathrm{j} \end{aligned}$$

$$\begin{aligned} \mathrm{j}^2 &= e^{\mathrm{j}\,\pi/2} \cdot e^{\mathrm{j}\,\pi/2} \\ &= e^{\mathrm{j}\,\pi} \\ &= -1 \end{aligned}$$

$$\begin{aligned} 1/\,\mathrm{j} &= 1/e^{\mathrm{j}\,\pi/2} \\ &= e^{-\,\mathrm{j}\,\pi/2} \\ &= e^{\mathrm{j}\,3\pi/2} \\ &= -\,\mathrm{j} \end{aligned}$$

Operaciones

$$\begin{aligned} \overline{Y} + \overline{Z} &= \Big[Y\cos(\theta_Y) + Z\cos(\theta_Z)\Big] + \mathrm{j} \cdot \Big[Y\sin(\theta_Y) + Z\sin(\theta_Z)\Big] \\ \overline{Y} - \overline{Z} &= \Big[Y\cos(\theta_Y) - Z\cos(\theta_Z)\Big] + \mathrm{j} \cdot \Big[Y\sin(\theta_Y) - Z\sin(\theta_Z)\Big] \end{aligned}$$

$$\begin{aligned} \overline{Y} \cdot \overline{Z} &= (Y \cdot Z) \cdot e^{\theta_Y + \theta_Z} \\ &= (Y \cdot Z)\angle \theta_Y + \theta_Z \end{aligned}$$

$$\overline{Y}^2 = Y^2 \angle 2\theta_Y$$

$$\begin{aligned} \frac{\overline{Y}}{\overline{Z}} &= \frac{Y}{Z} \cdot e^{\theta_Y - \theta_Z} \\ &= \frac{Y}{Z} \angle \theta_Y - \theta_Z \end{aligned}$$

Conjugado

$$\begin{aligned} \overline{Y}^* &= Y \cdot e^{-\,\mathrm{j}\,\theta} \\ &= Y\angle -\theta \\ &= Y \cdot [\cos(\theta) - \mathrm{j} \cdot \sin(\theta)] \\ &= a_Y - \mathrm{j}\, b_Y \end{aligned}$$

$$\overline{Y} \cdot \overline{Y}^* = |Y|^2$$