Repaso de trigonometría y números complejos

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1. Trigonometría

Ecuación Fundamental

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

Cuadratura

$$\begin{aligned}
\sin(\theta + \pi/2) &= \cos(\theta) \\
\cos(\theta + \pi/2) &= -\sin(\theta) \\
\sin(\theta - \pi/2) &= -\cos(\theta) \\
\cos(\theta - \pi/2) &= \sin(\theta)
\end{aligned}$$

Suma y resta de ángulos

$$\begin{aligned}
\cos(\alpha - \beta) &= \cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta) \\
\cos(\alpha + \beta) &= \cos(\alpha) \cdot \cos(\beta) - \sin(\alpha) \cdot \sin(\beta) \\
\sin(\alpha - \beta) &= \sin(\alpha) \cdot \cos(\beta) - \cos(\alpha) \cdot \sin(\beta) \\
\sin(\alpha + \beta) &= \sin(\alpha) \cdot \cos(\beta) + \cos(\alpha) \cdot \sin(\beta)
\end{aligned}$$

Ángulo doble

$$\begin{aligned}
\cos(2\alpha) &= 2 \cdot \cos^2(\alpha) - 1 \\
\cos(2\alpha) &= 1 - 2 \cdot \sin^2(\alpha)
\end{aligned}$$

$$\sin(2\alpha) = 2 \cdot \sin(\alpha) \cdot \cos(\alpha)$$

Derivadas e Integrales

$$\begin{aligned}
\frac{\mathrm{d}\sin(\omega t + \theta)}{\mathrm{d}t} &= \omega \cdot \cos(\omega t + \theta) \\
\frac{\mathrm{d}\cos(\omega t + \theta)}{\mathrm{d}t} &= -\omega \cdot \sin(\omega t + \theta)
\end{aligned}$$

$$\begin{aligned}
\int \sin(\omega t + \theta)\,\mathrm{d}t &= -\frac{1}{\omega} \cdot \cos(\omega t + \theta) + k \\
\int \cos(\omega t + \theta)\,\mathrm{d}t &= \frac{1}{\omega} \cdot \sin(\omega t + \theta) + k
\end{aligned}$$

Aprovechando las relaciones de cuadratura podemos comprobar que las derivadas adelantan $\pi/2$:

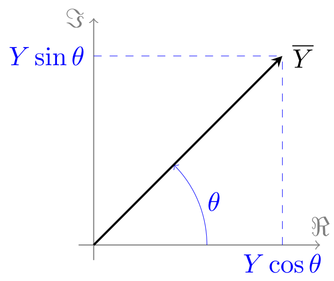
$$\frac{\mathrm{d}\sin(\omega t+\theta)}{\mathrm{d}t} = \omega\cdot\sin(\omega t+\theta+\pi/2)$$
$$\frac{\mathrm{d}\cos(\omega t+\theta)}{\mathrm{d}t} = \omega\cdot\cos(\omega t+\theta+\pi/2)$$

Y las integrales retrasan $\pi/2$:

$$\int \sin(\omega t+\theta)\,\mathrm{d}t = \frac{1}{\omega}\cdot\sin(\omega t+\theta-\pi/2)+k$$
$$\int \cos(\omega t+\theta)\,\mathrm{d}t = \frac{1}{\omega}\cdot\cos(\omega t+\theta-\pi/2)+k$$

2. Números complejos

Definición



Euler/Polar:

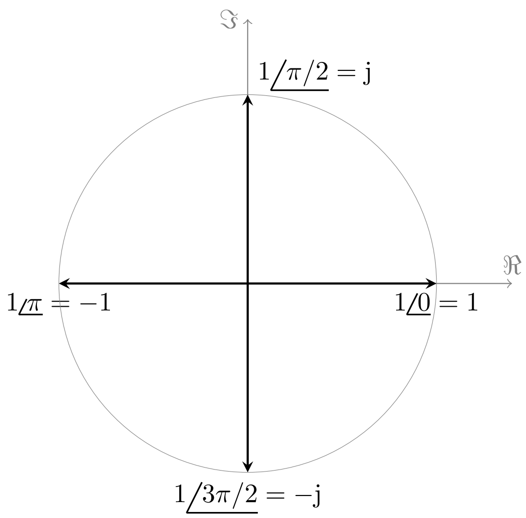
$$\begin{aligned}\overline{Y} &= Y\cdot e^{\mathrm{j}\,\theta}\\ &= Y\angle\underline{\theta}\end{aligned}$$

Binómica:

$$\begin{aligned}\overline{Y} &= Y\cdot\Big(\cos(\theta)+\mathrm{j}\cdot\sin(\theta)\Big)\\ &= a_Y+\mathrm{j}\,b_Y\end{aligned}$$

$$\begin{aligned}|Y| &= \sqrt{a_Y^2+b_Y^2}\\ &= \sqrt{Y^2\cos^2(\theta)+Y^2\sin^2(\theta)}\\ &= Y\end{aligned}$$

Complejo unitario



$$\begin{aligned}1\angle\underline{0} &= e^{\mathrm{j}\,0} = 1\\ 1\angle\underline{\pi/2} &= e^{\mathrm{j}\,\pi/2} = \mathrm{j}\\ 1\angle\underline{\pi} &= e^{\mathrm{j}\,\pi} = -1\\ 1\angle\underline{3\pi/2} &= e^{\mathrm{j}\,3\pi/2} = -\mathrm{j}\end{aligned}$$

$$\begin{aligned}\mathrm{j}^2 &= e^{\mathrm{j}\,\pi/2}\cdot e^{\mathrm{j}\,\pi/2}\\ &= e^{\mathrm{j}\,\pi}\\ &= -1\end{aligned}$$

$$\begin{aligned}1/\mathrm{j} &= 1/e^{\mathrm{j}\,\pi/2}\\ &= e^{-\mathrm{j}\,\pi/2}\\ &= e^{\mathrm{j}\,3\pi/2}\\ &= -\mathrm{j}\end{aligned}$$

Operaciones

$$
\begin{aligned}
\overline{Y} + \overline{Z} &= \Big(Y\cos(\theta_Y) + Z\cos(\theta_Z)\Big) + \mathrm{j}\cdot\Big(Y\sin(\theta_Y) + Z\sin(\theta_Z)\Big)\\
\overline{Y} - \overline{Z} &= \Big(Y\cos(\theta_Y) - Z\cos(\theta_Z)\Big) + \mathrm{j}\cdot\Big(Y\sin(\theta_Y) - Z\sin(\theta_Z)\Big)
\end{aligned}
$$

$$
\begin{aligned}
\overline{Y}\cdot\overline{Z} &= (Y\cdot Z)\cdot e^{\theta_Y+\theta_Z}\\
&= (Y\cdot Z)\underline{/\theta_Y+\theta_Z}
\end{aligned}
$$

$$
\overline{Y}^2 = Y^2\underline{/2\theta_Y}
$$

$$
\begin{aligned}
\frac{\overline{Y}}{\overline{Z}} &= \frac{Y}{Z}\cdot e^{\theta_Y-\theta_Z}\\
&= \frac{Y}{Z}\underline{/\theta_Y-\theta_Z}
\end{aligned}
$$

Conjugado

$$
\begin{aligned}
\overline{Y}^* &= Y\cdot e^{-\mathrm{j}\theta}\\
&= Y\underline{/-\theta}\\
&= Y\cdot\Big(\cos(\theta) - \mathrm{j}\cdot\sin(\theta)\Big)\\
&= a_Y - \mathrm{j}\,b_Y
\end{aligned}
$$

$$
\overline{Y}\cdot\overline{Y}^* = |Y|^2
$$