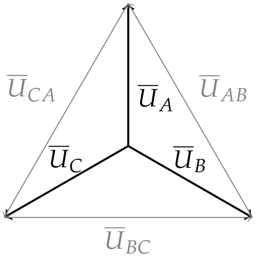
Sistemas Trifásicos

Teoría de Circuitos II

Oscar Perpiñán Lamigueiro

Tensiones de Fase y Línea



Tensiones de **Fase**: U_A, U_B, U_C
Tensiones de **Línea**: U_{AB}, U_{BC}, U_{CA}

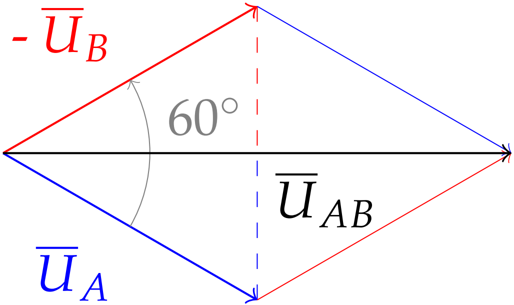
$$\overline{U}_{AB} = \overline{U}_A - \overline{U}_B$$
$$\overline{U}_{BC} = \overline{U}_B - \overline{U}_C$$
$$\overline{U}_{CA} = \overline{U}_C - \overline{U}_A$$

$$\overline{U}_{AB} + \overline{U}_{BC} + \overline{U}_{CA} = 0$$

Tensiones de Fase y Línea

$$\overline{U}_A = U_f \angle \theta_f$$
$$\overline{U}_B = U_f \angle \theta_f - 120^\circ$$

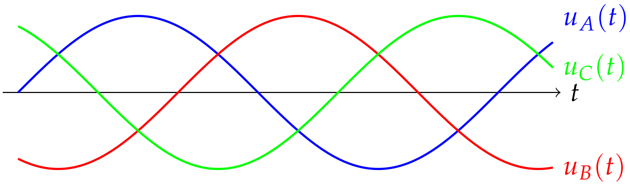
$$\begin{aligned}\overline{U}_{AB} = \overline{U}_A - \overline{U}_B &= \\ &= U_f \angle \theta_f - U_f \angle \theta_f - 120^\circ = \\ &= U_f \angle \theta_f + U_f \angle \theta_f + 60^\circ \\ &= 2 \cdot U_f \cdot \cos(30) \angle \theta_f + 30^\circ = \\ &= \sqrt{3} U_f \angle \theta_f + 30^\circ\end{aligned}$$



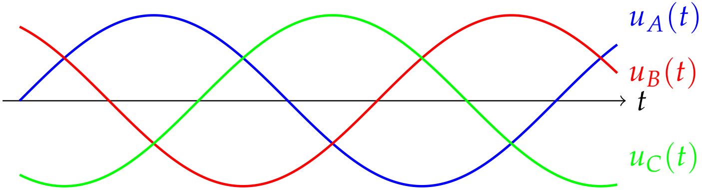
$$\boxed{\begin{aligned}U &= \sqrt{3} \cdot U_f \\ \theta_l &= \theta_f + 30^\circ\end{aligned}}$$

Secuencia de Fases

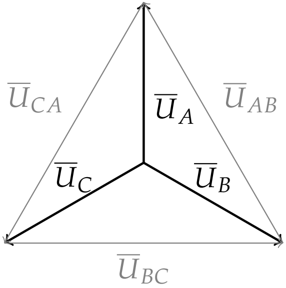
- Sentido en el que ocurren los máximos de cada fase.
- Secuencia de Fases Directa (**SFD**): ABC



- Secuencia de Fases Inversa (**SFI**): ACB



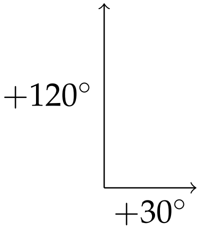
Secuencia de Fases Directa (SFD)



$$\overline{U}_A = \frac{U}{\sqrt{3}}\underline{/90^\circ}$$

$$\overline{U}_B = \frac{U}{\sqrt{3}}\underline{/-30^\circ}$$

$$\overline{U}_C = \frac{U}{\sqrt{3}}\underline{/-150^\circ}$$

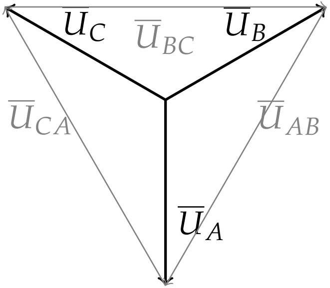


$$\overline{U}_{AB} = U\underline{/120^\circ}$$

$$\overline{U}_{BC} = U\underline{/0^\circ}$$

$$\overline{U}_{CA} = U\underline{/-120^\circ}$$

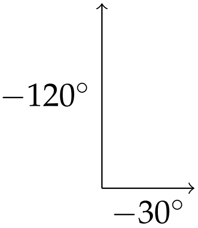
Secuencia de Fases Inversa (SFI)



$$\overline{U}_A = \frac{U}{\sqrt{3}}\angle{-90^\circ}$$

$$\overline{U}_B = \frac{U}{\sqrt{3}}\angle{30^\circ}$$

$$\overline{U}_C = \frac{U}{\sqrt{3}}\angle{150^\circ}$$



$$\overline{U}_{AB} = U\angle{-120^\circ}$$

$$\overline{U}_{BC} = U\angle{0^\circ}$$

$$\overline{U}_{CA} = U\angle{120^\circ}$$

1. Generadores
2. Receptores
3. Potencia en Sistemas Trifásicos
4. Compensación de Reactiva
5. Medida de Potencia en Sistemas Trifásicos
6. Equivalentes

Tipos de Receptores

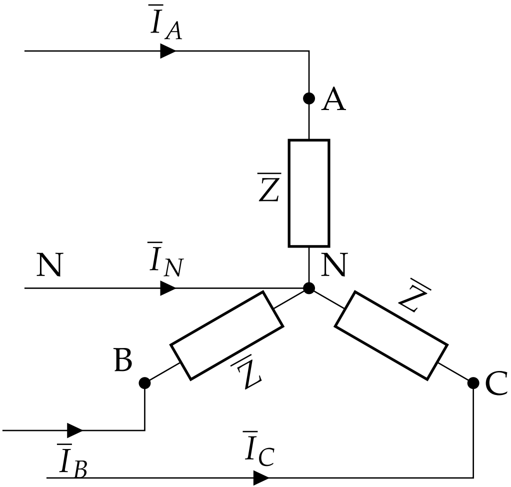
Conexión

- **Estrella** (punto común) Y
- **Triángulo** $\triangle$

Impedancias

- **Equilibrado** (las tres impedancias son idénticas en módulo **y** fase).
- **Desequilibrado**

Receptor en Estrella Equilibrado



$$\overline{I}_A = \frac{\overline{U}_A}{\overline{Z}} = \frac{U_f}{Z}\underline{/\pm 90^\circ - \theta}$$

$$\overline{I}_B = \frac{\overline{U}_B}{\overline{Z}} = \frac{U_f}{Z}\underline{/\mp 30^\circ - \theta}$$

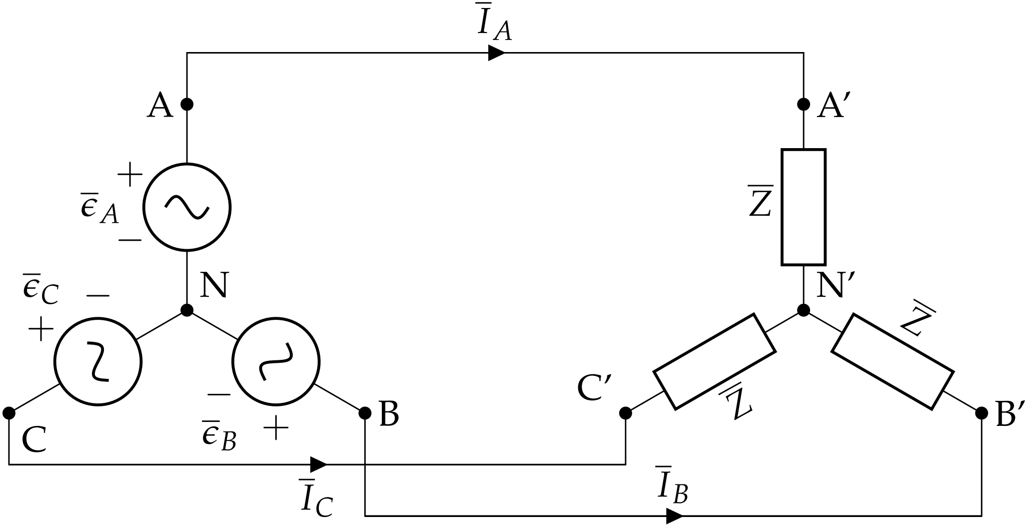
$$\overline{I}_C = \frac{\overline{U}_C}{\overline{Z}} = \frac{U_f}{Z}\underline{/\mp 150^\circ - \theta}$$

$$\boxed{|\overline{I}_A| = |\overline{I}_B| = |\overline{I}_C| = \frac{U_f}{Z}}$$

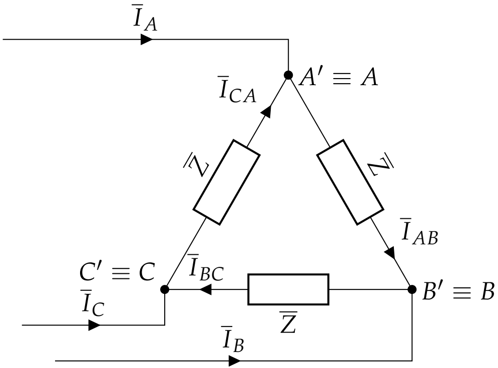
$$\overline{I}_A + \overline{I}_B + \overline{I}_C + \overline{I}_N = 0$$

$$\overline{I}_A + \overline{I}_B + \overline{I}_C = 0 \rightarrow \boxed{\overline{I}_N = 0}$$

Receptor en Estrella Equilibrado



Receptor en Triángulo Equilibrado



$$\bar{I}_{AB} = \frac{\bar{U}_{AB}}{\bar{Z}} = \frac{U}{Z}\underline{/\pm 120^\circ - \theta}$$

$$\bar{I}_{BC} = \frac{\bar{U}_{BC}}{\bar{Z}} = \frac{U}{Z}\underline{/0 - \theta}$$

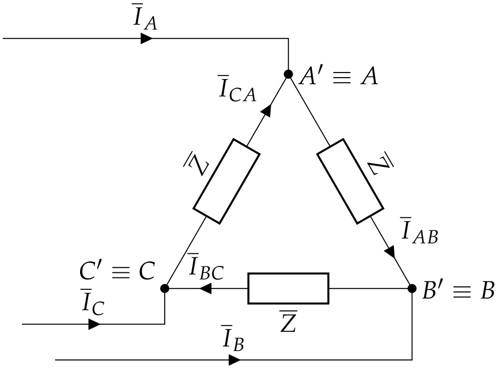
$$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}} = \frac{U}{Z}\underline{/\mp 120^\circ - \theta}$$

$$\bar{I}_{AB} + \bar{I}_{BC} + \bar{I}_{CA} = 0$$

Corriente de Fase:

$$\boxed{I_f = |\bar{I}_{AB}| = |\bar{I}_{BC}| = |\bar{I}_{CA}| = \frac{U}{Z}}$$

Receptor en Triángulo Equilibrado



$$\overline{I}_A = \overline{I}_{AB} - \overline{I}_{CA} = \sqrt{3} \cdot \frac{U}{Z} \underline{/\pm 90^\circ - \theta}$$

$$\overline{I}_B = \overline{I}_{BC} - \overline{I}_{AB} = \sqrt{3} \cdot \frac{U}{Z} \underline{/\mp 30^\circ - \theta}$$

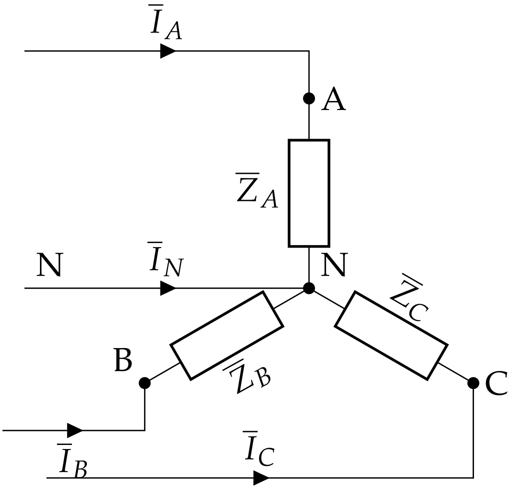
$$\overline{I}_C = \overline{I}_{CA} - \overline{I}_{BC} = \sqrt{3} \cdot \frac{U}{Z} \underline{/\mp 150^\circ - \theta}$$

Corriente de Línea:

$$\boxed{I = |\overline{I}_A| = |\overline{I}_B| = |\overline{I}_C| = \sqrt{3} \cdot \frac{U}{Z}}$$

$$\boxed{I = \sqrt{3} \cdot I_f}$$

Receptor en Estrella Desequilibrado con Neutro



$$\bar{I}_A = \frac{\bar{U}_A}{\bar{\bar{Z}}_A}$$

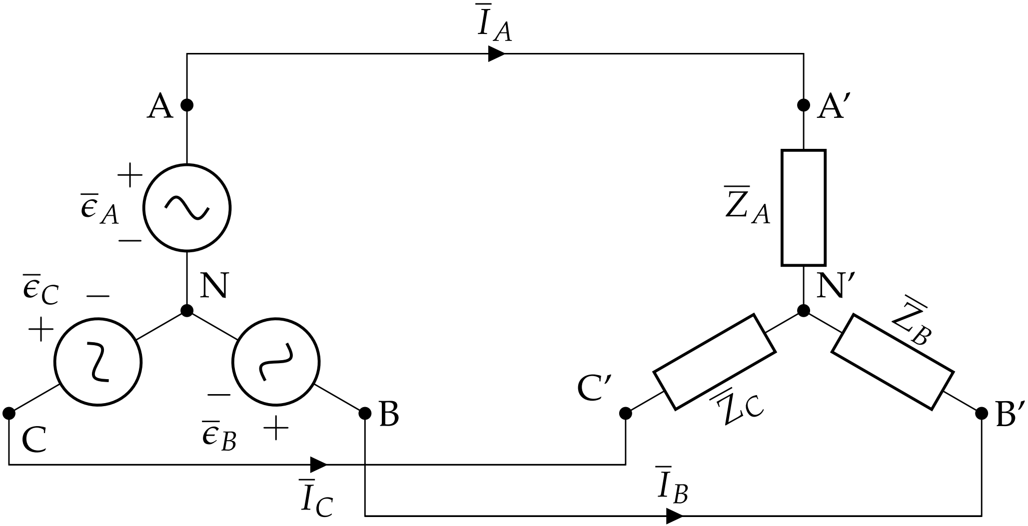
$$\bar{I}_B = \frac{\bar{U}_B}{\bar{\bar{Z}}_B}$$

$$\bar{I}_C = \frac{\bar{U}_C}{\bar{\bar{Z}}_C}$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C + \bar{I}_N = 0$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C \neq 0 \rightarrow \boxed{\bar{I}_N \neq 0}$$

Receptor en Estrella Desequilibrado sin Neutro



Método del desplazamiento del neutro

Ecuaciones del receptor:

$$
\begin{aligned}
\overline{U}_{A'N'} &= \overline{I}_A \cdot \overline{Z}_A \\
\overline{U}_{B'N'} &= \overline{I}_B \cdot \overline{Z}_B \\
\overline{U}_{C'N'} &= \overline{I}_C \cdot \overline{Z}_C
\end{aligned}
$$

Ecuación del nudo N':

$$\overline{I}_A + \overline{I}_B + \overline{I}_C = 0$$

Método del desplazamiento del neutro

Relacionamos las tensiones en el receptor con las tensiones del generador:

$$\begin{aligned}
\overline{U}_{A'N'} &= \overline{U}_{AN} - \overline{U}_{NN'} \\
\overline{U}_{B'N'} &= \overline{U}_{BN} - \overline{U}_{NN'} \\
\overline{U}_{C'N'} &= \overline{U}_{CN} - \overline{U}_{NN'}
\end{aligned}$$

Despejamos las corrientes teniendo en cuenta estas relaciones:

$$\begin{aligned}
\bar{I}_A &= \frac{\overline{U}_{AN} - \overline{U}_{NN'}}{\overline{Z}_A} \\
\bar{I}_B &= \frac{\overline{U}_{AN} - \overline{U}_{NN'}}{\overline{Z}_B} \\
\bar{I}_C &= \frac{\overline{U}_{AN} - \overline{U}_{NN'}}{\overline{Z}_C}
\end{aligned}$$

Método del desplazamiento del neutro

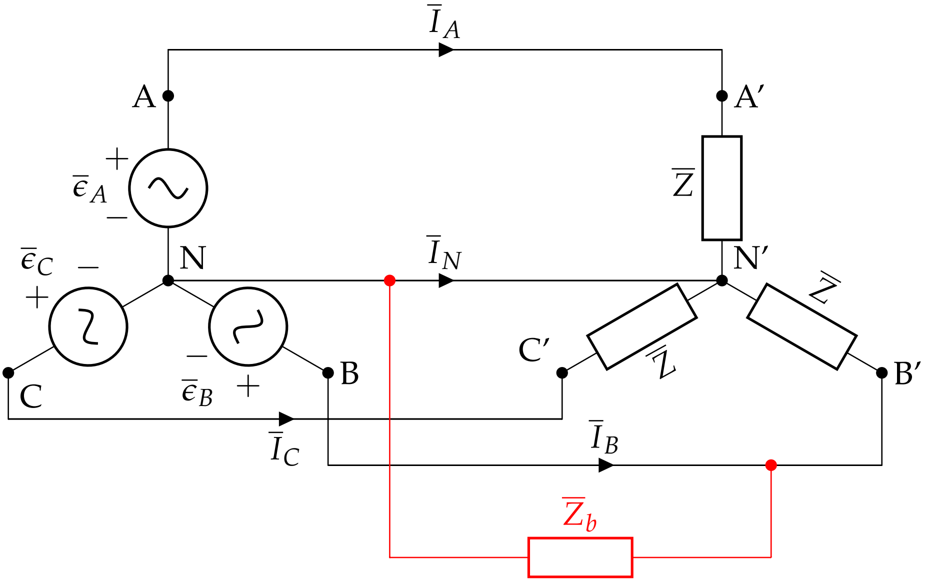
Finalmente, usando la ecuación del nudo N' despejamos la tensión $U_{N'N}$ (tensión de desplazamiento del neutro)*:

$$\boxed{\overline{U}_{N'N} = \frac{\overline{U}_{AN} \cdot \overline{Y}_A + \overline{U}_{BN} \cdot \overline{Y}_B + \overline{U}_{CN} \cdot \overline{Y}_C}{\overline{Y}_A + \overline{Y}_B + \overline{Y}_C}}$$

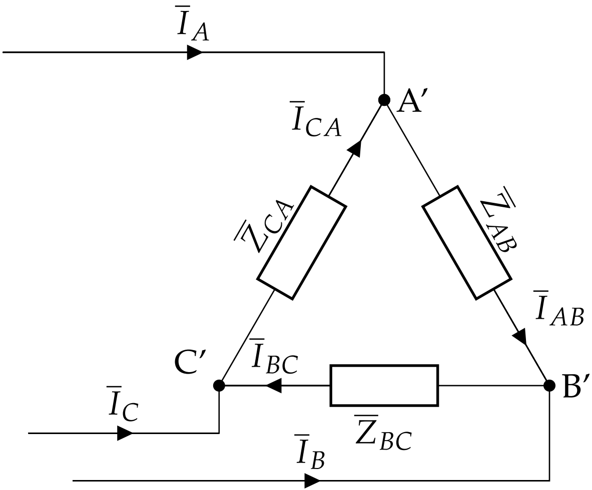
Una vez calculada esta tensión $\overline{U}_{N'N}$ se pueden calcular las corrientes de línea.

*Se puede llegar a este mismo resultado aplicando el teorema de Millman.

Receptor en Estrella con Carga Monofásica



Receptor en Triángulo Desequilibrado



$$\overline{I}_{AB} = \frac{\overline{U}_{AB}}{\overline{Z}_{AB}}$$

$$\overline{I}_{BC} = \frac{\overline{U}_{BC}}{\overline{Z}_{BC}}$$

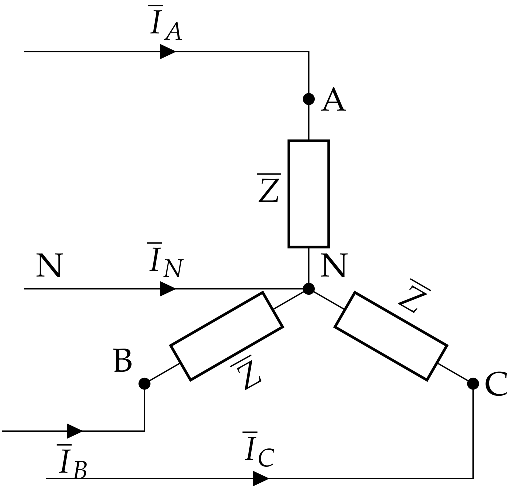
$$\overline{I}_{CA} = \frac{\overline{U}_{CA}}{\overline{Z}_{CA}}$$

$$\overline{I}_A = \overline{I}_{AB} - \overline{I}_{CA}$$

$$\overline{I}_B = \overline{I}_{BC} - \overline{I}_{AB}$$

$$\overline{I}_C = \overline{I}_{CA} - \overline{I}_{BC}$$

Receptor en Estrella Equilibrado

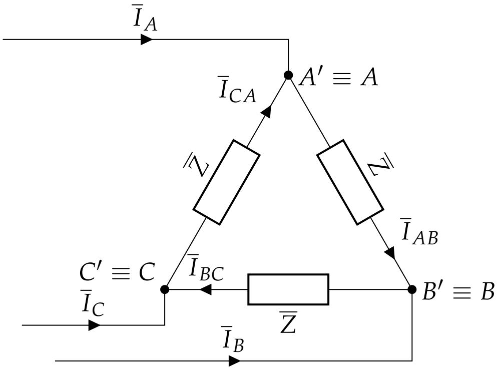


$$P = 3 \cdot P_Z = 3 \cdot U_Z I_Z \cos(\theta)$$
$$Q = 3 \cdot Q_Z = 3 \cdot U_Z I_Z \sin(\theta)$$

$$I_Z = I$$
$$U_Z = U_F$$

$$P = 3U_F I \cos(\theta) = \sqrt{3} UI \cos(\theta)$$
$$Q = 3U_F I \sin(\theta) = \sqrt{3} UI \sin(\theta)$$
$$S = \sqrt{P^2 + Q^2} = \sqrt{3} UI$$

Receptor en Triángulo Equilibrado

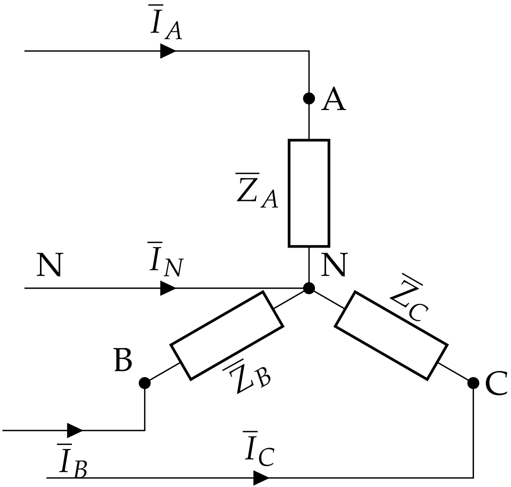


$$P = 3 \cdot P_Z = 3 \cdot U_Z I_Z \cos(\theta)$$
$$Q = 3 \cdot Q_Z = 3 \cdot U_Z I_Z \sin(\theta)$$

$$I_Z = I_F$$
$$U_Z = U$$

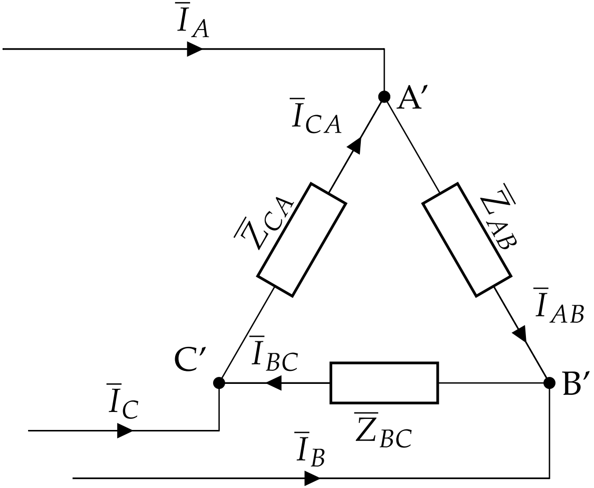
$$P = 3UI_F \cos(\theta) = \sqrt{3}UI \cos(\theta)$$
$$Q = 3UI_F \sin(\theta) = \sqrt{3}UI \sin(\theta)$$
$$S = \sqrt{P^2 + Q^2} = \sqrt{3}UI$$

Receptor en Estrella Desequilibrado



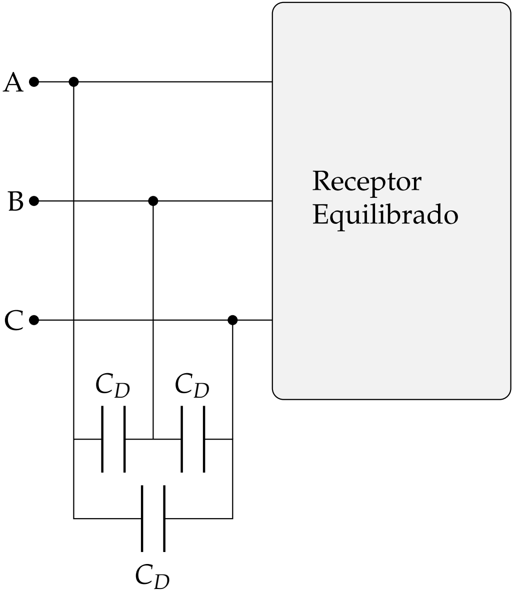
$$P = P_A + P_B + P_C$$
$$Q = Q_A + Q_B + Q_C$$
$$\overline{S} = P + jQ$$

Receptor en Triángulo Desequilibrado



$$P = P_{AB} + P_{BC} + P_{CA}$$
$$Q = Q_{AB} + Q_{BC} + Q_{CA}$$
$$\overline{S} = P + jQ$$

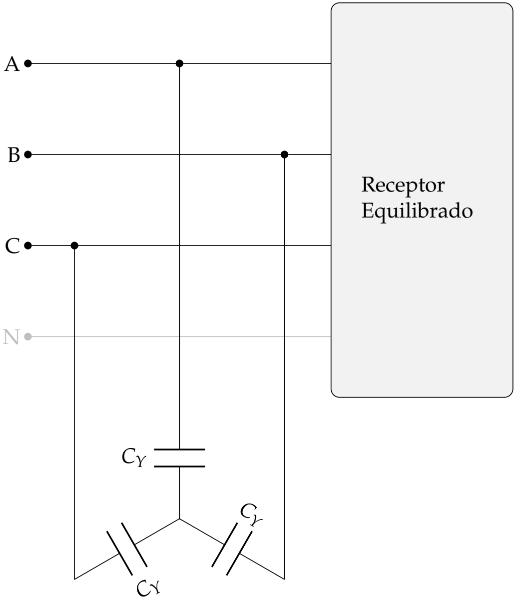
Conexión en Triángulo



$$\begin{aligned} Q &= P\tan\theta \\ Q' &= P\tan\theta' = \\ &= Q - Q_c \\ Q_c &= 3\cdot\omega C_\triangle \cdot U^2 \end{aligned}$$

$$\boxed{C_\triangle = \frac{P(\tan\theta - \tan\theta')}{3\omega U^2}}$$

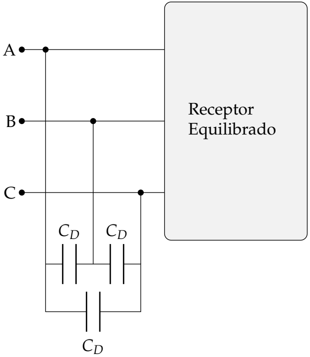
Conexión en Estrella



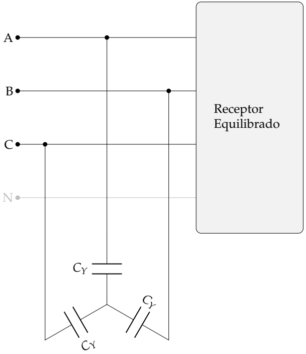
$$
\begin{aligned}
Q &= P\tan\theta \\
Q' &= P\tan\theta' = \\
&= Q - Q_c \\
Q_c &= 3\cdot\omega C_Y \cdot U_f^2
\end{aligned}
$$

$$
\boxed{C_Y = \frac{P(\tan\theta - \tan\theta')}{\omega U^2}}
$$

Comparación Estrella-Triángulo



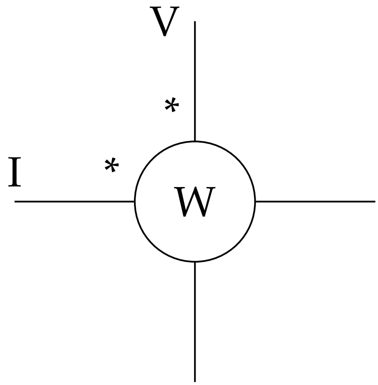
$$\boxed{C_{\triangle} = \frac{P(\tan\theta - \tan\theta')}{3\omega U^2}}$$



$$\boxed{C_{Y} = \frac{P(\tan\theta - \tan\theta')}{\omega U^2}}$$

Dado que $C_Y = 3 \cdot C_{\triangle}$ la **configuración recomendada** es **triángulo**.

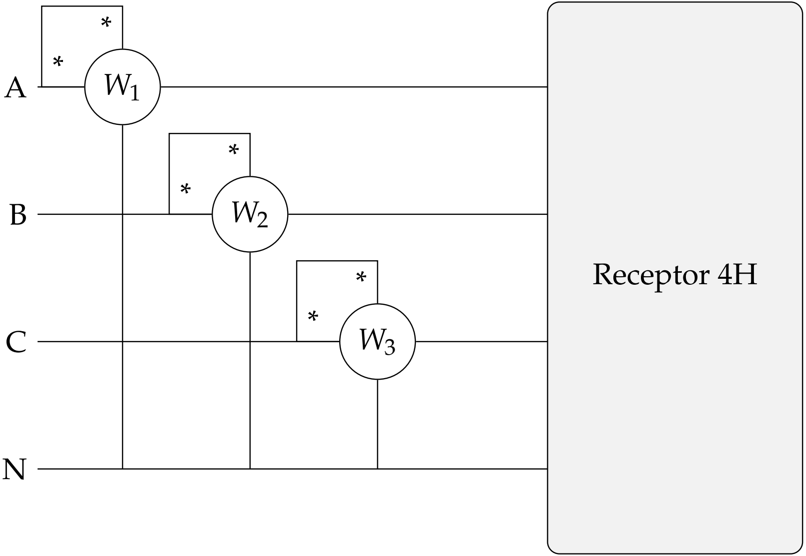
Recordatorio: vatímetro



Vatímetro: equipo de medida de 4 terminales (1 par para tensión, 1 par para corriente)

$$\boxed{W = \Re(\overline{U} \cdot \overline{I}^*)}$$

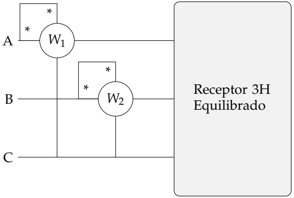
Sistema de 4 Hilos



$$W_1 = \Re(\overline{U}_A \cdot \overline{I}_A^*) = P_A$$
$$W_2 = \Re(\overline{U}_B \cdot \overline{I}_B^*) = P_B$$
$$W_3 = \Re(\overline{U}_C \cdot \overline{I}_C^*) = P_C$$

$$\boxed{P = W_1 + W_2 + W_3}$$

Sistema de 3 Hilos Equilibrado



SFD

$$W_1 = UI\cos(\theta - 30^\circ)$$
$$W_2 = UI\cos(\theta + 30^\circ)$$

$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_1 - W_2)$$
$$\tan\theta = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}$$

SFI

$$W_1 = UI\cos(\theta + 30^\circ)$$
$$W_2 = UI\cos(\theta - 30^\circ)$$

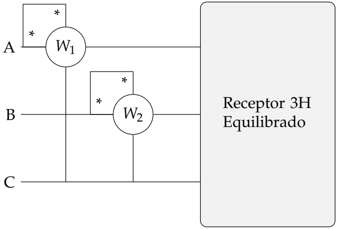
$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_2 - W_1)$$
$$\tan\theta = \sqrt{3}\frac{W_2 - W_1}{W_1 + W_2}$$

Otras conexiones: 3H SFD

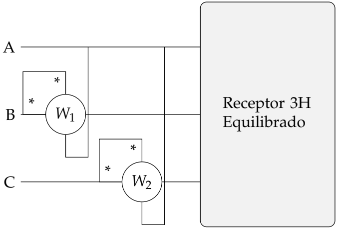
$$\boxed{(ABC) :: A \triangleright B \triangleright C \Longrightarrow \{AB, BC, CA\}}$$

$$W_1 = UI\cos(\theta - 30^\circ) \qquad P = W_1 + W_2$$

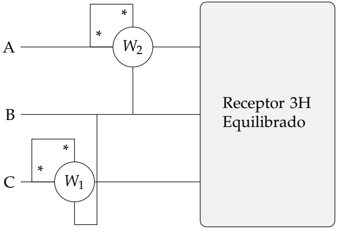
$$W_2 = UI\cos(\theta + 30^\circ) \qquad Q = \sqrt{3}(W_1 - W_2)$$



$$W_1 : AC \notin SFD$$
$$W_2 : BC \in SFD$$



$$W_1 : BA \notin SFD$$
$$W_2 : CA \in SFD$$



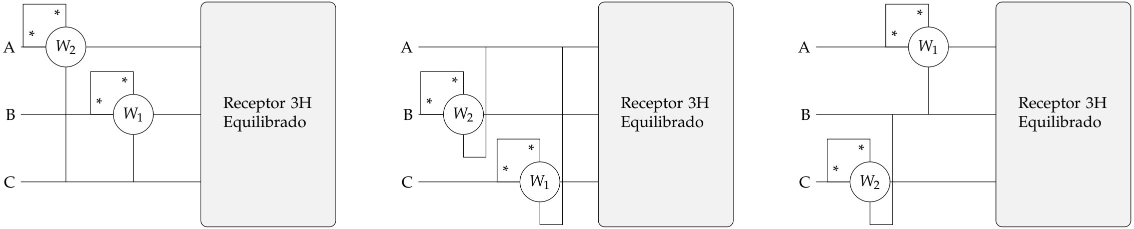
$$W_1 : CB \notin SFD$$
$$W_2 : AB \in SFD$$

Otras conexiones: 3H SFI

$$\boxed{(ACB) :: A \triangleright C \triangleright B \Longrightarrow \{AC, CB, BA\}}$$

$$W_1 = UI\cos(\theta - 30^\circ) \qquad P = W_1 + W_2$$

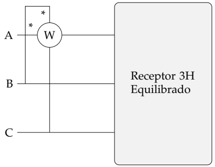
$$W_2 = UI\cos(\theta + 30^\circ) \qquad Q = \sqrt{3}(W_1 - W_2)$$

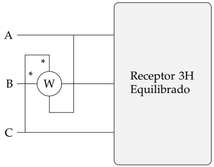


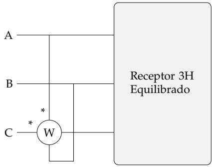
$$W_1 : BC \notin SFI \qquad W_1 : CA \notin SFI \qquad W_1 : AB \notin SFI$$

$$W_2 : AC \in SFI \qquad W_2 : BA \in SFI \qquad W_2 : CB \in SFI$$

Conexiones para medida de reactiva







$$W = \Re(\overline{U}_{BC} \cdot \overline{I}_A^*)$$

$$BC \in SFD$$

$$BC \notin SFI$$

$$W = \Re(\overline{U}_{CA} \cdot \overline{I}_B^*)$$

$$CA \in SFD$$

$$CA \notin SFI$$

$$W = \Re(\overline{U}_{AB} \cdot \overline{I}_C^*)$$

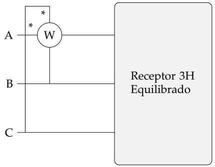
$$AB \in SFD$$

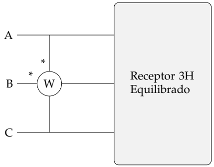
$$AB \notin SFI$$

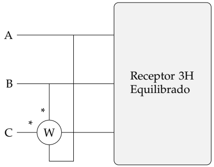
$$SFD \rightarrow \boxed{W = \frac{Q}{\sqrt{3}}}$$

$$SFI \rightarrow \boxed{W = -\frac{Q}{\sqrt{3}}}$$

Conexiones para medida de reactiva







$$W = \Re(\overline{U}_{CB} \cdot \overline{I}_A^*)$$

$$W = \Re(\overline{U}_{AC} \cdot \overline{I}_B^*)$$

$$W = \Re(\overline{U}_{BA} \cdot \overline{I}_C^*)$$

$$CB \notin SFD \qquad AC \notin SFD \qquad BA \notin SFD$$

$$CB \in SFI \qquad AC \in SFI \qquad BA \in SFI$$

$$SFD \rightarrow \boxed{W = -\frac{Q}{\sqrt{3}}}$$

$$SFI \rightarrow \boxed{W = \frac{Q}{\sqrt{3}}}$$

Medida de la reactiva con receptor desequilibrado

Equivalente de un motor trifásico

Equivalencia de fuentes

Estrella -> Triángulo

Equivalencia de fuentes

Triángulo -> Estrella

Estudio generalizado de los sistemas trifásicos