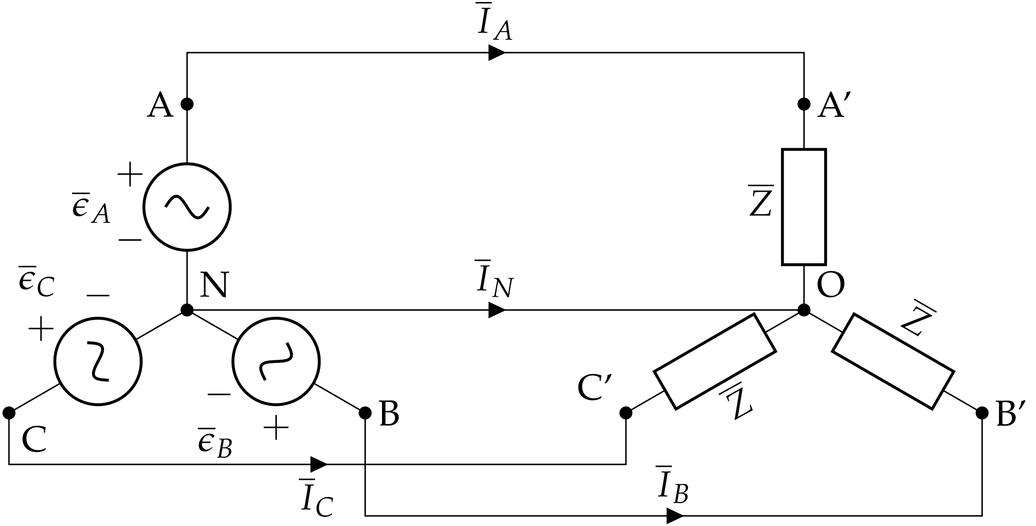
Sistemas Trifásicos

Oscar Perpiñán Lamigueiro

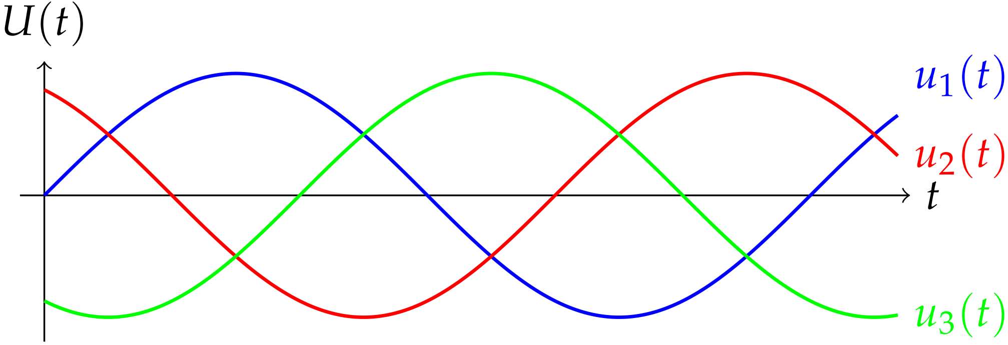


$\overline{I}_A$
A
A′
$\overline{\epsilon}_A$
+
−
$\overline{Z}$
$\overline{\epsilon}_C$
N
$\overline{I}_N$
O
−
+
$\overline{Z}$
C′
$\overline{Z}$
−
B
B′
C
$\overline{\epsilon}_B$
+
$\overline{I}_C$
$\overline{I}_B$

Motivación de los sistemas trifásicos

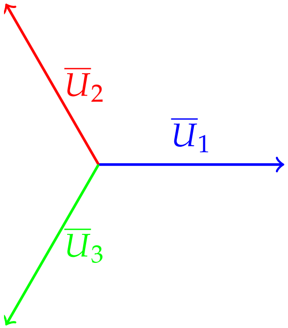
- En un sistema trifásico la potencia instantánea es constante, evitando vibraciones y esfuerzos en las máquinas. (*La potencia instantánea de un sistema monofásico es pulsante.*)
- La masa de conductor necesaria en un sistema trifásico es un 25 % inferior que en un monofásico para transportar la misma potencia.

Ondas Trifásicas



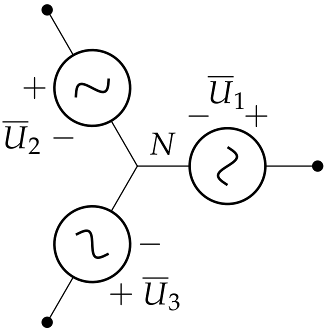
$$u_1(t) = U_0 \sin(\omega t)$$
$$u_2(t) = U_0 \sin(\omega t + 2\pi/3)$$
$$u_3(t) = U_0 \sin(\omega t - 2\pi/3)$$

Fasores de un sistema trifásico



$$\overline{U}_1 = U\underline{/0}$$
$$\overline{U}_2 = U\underline{/2\pi/3}$$
$$\overline{U}_3 = U\underline{/-2\pi/3}$$

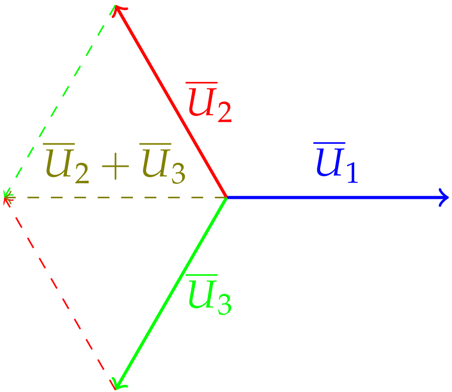
Conexión



$$u_1(t) = U_0 \sin(\omega t)$$
$$u_2(t) = U_0 \sin(\omega t + 2\pi/3)$$
$$u_3(t) = U_0 \sin(\omega t - 2\pi/3)$$

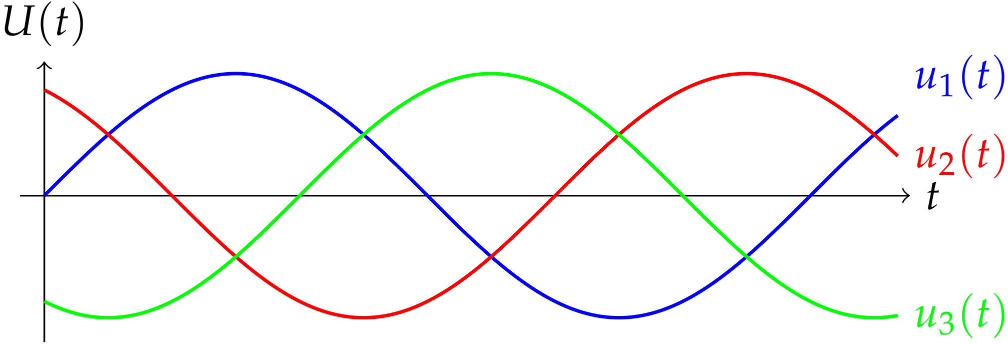
$$\overline{U}_1 = U\underline{/0}$$
$$\overline{U}_2 = U\underline{/2\pi/3}$$
$$\overline{U}_3 = U\underline{/-2\pi/3}$$

Las tensiones suman 0



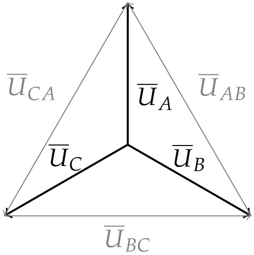
$$\boxed{\overline{U}_1 + \overline{U}_2 + \overline{U}_3 = 0}$$

Las tensiones suman 0



$$\boxed{u_1(t) + u_2(t) + u_3(t) = 0}$$

Tensiones de Fase y Línea



Tensiones de **Fase**: U_A, U_B, U_C
Tensiones de **Línea**: U_{AB}, U_{BC}, U_{CA}

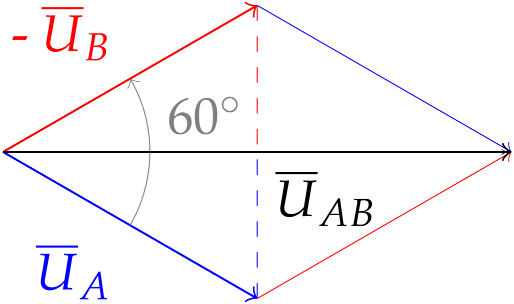
$$\overline{U}_{AB} = \overline{U}_A - \overline{U}_B$$
$$\overline{U}_{BC} = \overline{U}_B - \overline{U}_C$$
$$\overline{U}_{CA} = \overline{U}_C - \overline{U}_A$$

$$\overline{U}_{AB} + \overline{U}_{BC} + \overline{U}_{CA} = 0$$

Tensiones de Fase y Línea

$$\overline{U}_A = U_f \angle \theta_f$$
$$\overline{U}_B = U_f \angle \theta_f - 120^\circ$$

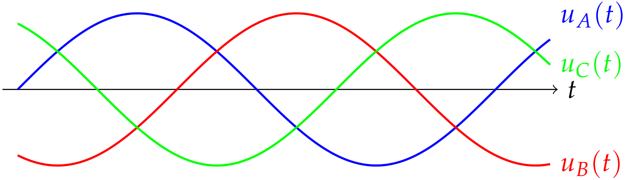
$$\begin{aligned}
\overline{U}_{AB} = \overline{U}_A - \overline{U}_B &= \\
&= U_f \angle \theta_f - U_f \angle \theta_f - 120^\circ = \\
&= U_f \angle \theta_f + U_f \angle \theta_f + 60^\circ \\
&= 2 \cdot U_f \cdot \cos(30) \angle \theta_f + 30^\circ = \\
&= \sqrt{3} U_f \angle \theta_f + 30^\circ
\end{aligned}$$



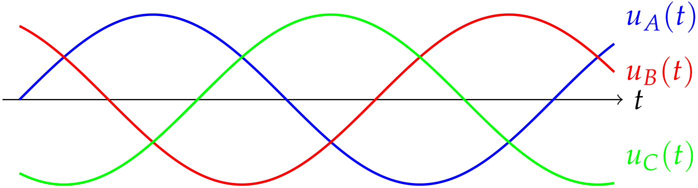
$$\boxed{\begin{aligned} U &= \sqrt{3} \cdot U_f \\ \theta_l &= \theta_f + 30^\circ \end{aligned}}$$

Secuencia de Fases

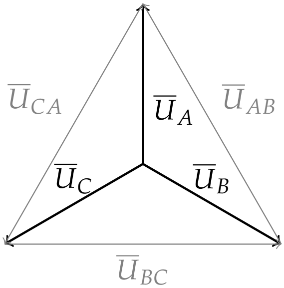
- Sentido en el que ocurren los máximos de cada fase.
- Secuencia de Fases Directa (**SFD**): ABC



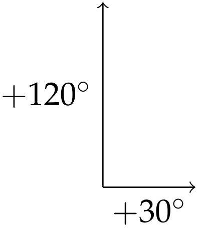
- Secuencia de Fases Inversa (**SFI**): ACB



Secuencia de Fases Directa (SFD)

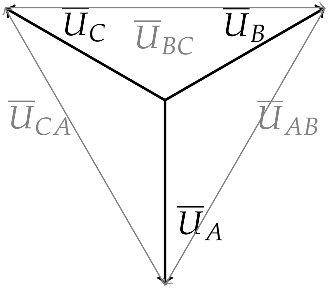


$$\overline{U}_A = \frac{U}{\sqrt{3}}\underline{/90^\circ}$$
$$\overline{U}_B = \frac{U}{\sqrt{3}}\underline{/-30^\circ}$$
$$\overline{U}_C = \frac{U}{\sqrt{3}}\underline{/-150^\circ}$$

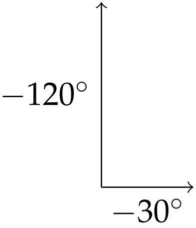


$$\overline{U}_{AB} = U\underline{/120^\circ}$$
$$\overline{U}_{BC} = U\underline{/0^\circ}$$
$$\overline{U}_{CA} = U\underline{/-120^\circ}$$

Secuencia de Fases Inversa (SFI)



$$\overline{U}_A = \frac{U}{\sqrt{3}}\underline{/-90^\circ}$$
$$\overline{U}_B = \frac{U}{\sqrt{3}}\underline{/30^\circ}$$
$$\overline{U}_C = \frac{U}{\sqrt{3}}\underline{/150^\circ}$$



$$\overline{U}_{AB} = U\underline{/-120^\circ}$$
$$\overline{U}_{BC} = U\underline{/0^\circ}$$
$$\overline{U}_{CA} = U\underline{/120^\circ}$$

Tipos de Receptores

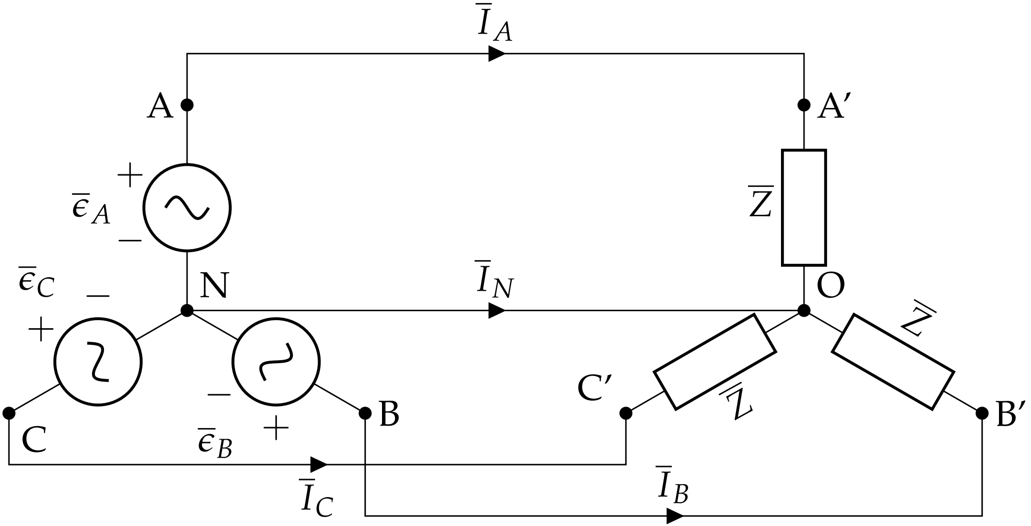
Conexión

- **Estrella** (punto común) Y
- **Triángulo** $\triangle$

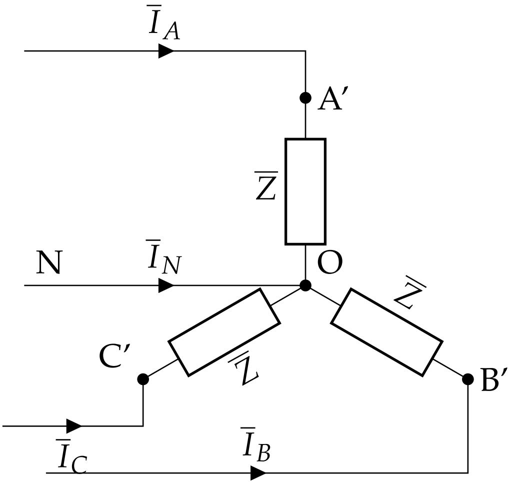
Impedancias

- **Equilibrado** (las tres impedancias son idénticas en módulo **y** fase).
- **Desequilibrado**

Receptor en Estrella Equilibrado



Receptor en Estrella Equilibrado



$$\bar{I}_A = \frac{\overline{U}_A}{\overline{Z}} = \frac{U_f}{Z}\underline{/\pm 90^\circ - \varphi}$$

$$\bar{I}_B = \frac{\overline{U}_B}{\overline{Z}} = \frac{U_f}{Z}\underline{/\mp 30^\circ - \varphi}$$

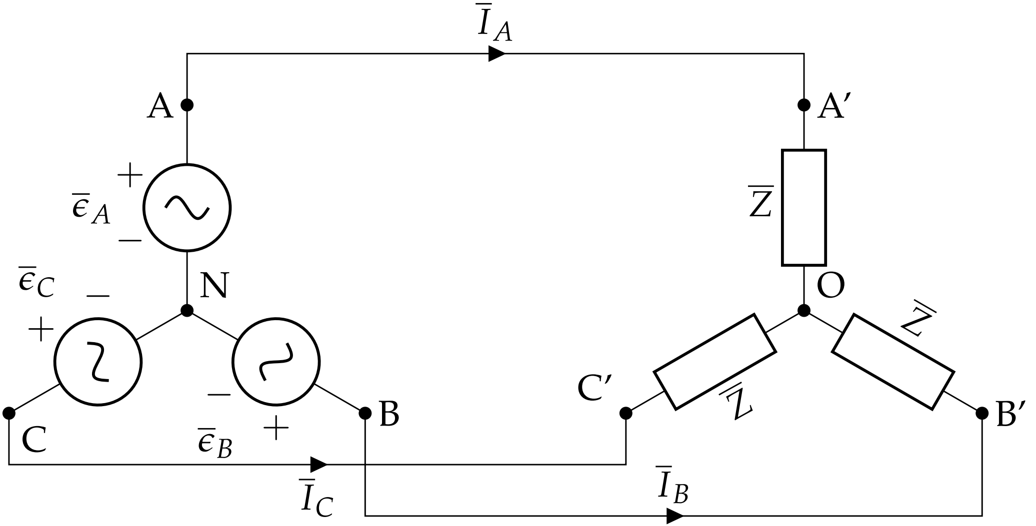
$$\bar{I}_C = \frac{\overline{U}_C}{\overline{Z}} = \frac{U_f}{Z}\underline{/\mp 150^\circ - \varphi}$$

$$\boxed{|\bar{I}_A| = |\bar{I}_B| = |\bar{I}_C| = \frac{U_f}{Z}}$$

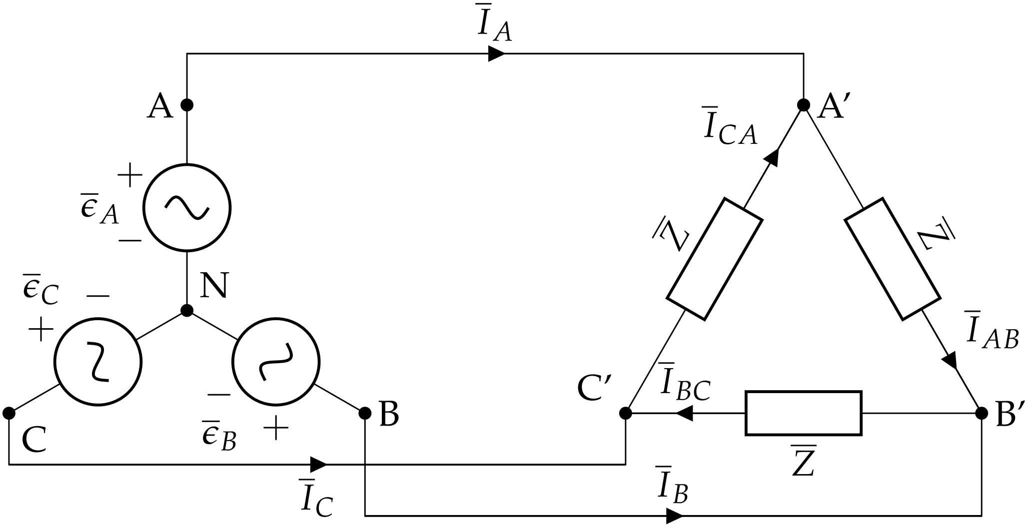
$$\bar{I}_A + \bar{I}_B + \bar{I}_C + \bar{I}_N = 0$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0 \rightarrow \boxed{\bar{I}_N = 0}$$

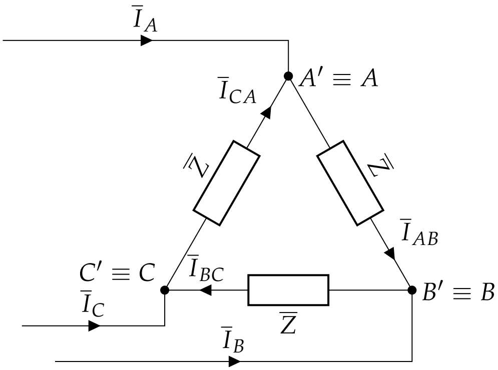
Receptor en Estrella Equilibrado



Receptor en Triángulo Equilibrado



Receptor en Triángulo Equilibrado



$$\bar{I}_{AB} = \frac{\overline{U}_{AB}}{\overline{Z}} = \frac{U}{Z}\underline{/\pm 120^\circ - \varphi}$$

$$\bar{I}_{BC} = \frac{\overline{U}_{BC}}{\overline{Z}} = \frac{U}{Z}\underline{/0 - \varphi}$$

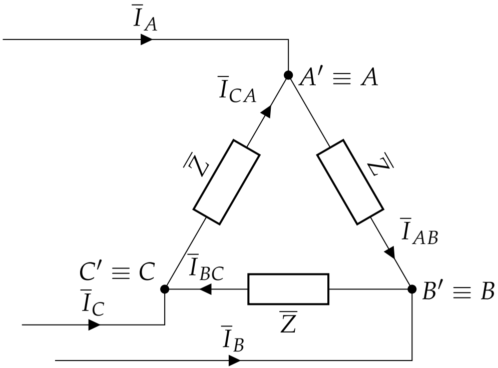
$$\bar{I}_{CA} = \frac{\overline{U}_{CA}}{\overline{Z}} = \frac{U}{Z}\underline{/\mp 120^\circ - \varphi}$$

$$\bar{I}_{AB} + \bar{I}_{BC} + \bar{I}_{CA} = 0$$

Corriente de Fase:

$$\boxed{I_f = |\bar{I}_{AB}| = |\bar{I}_{BC}| = |\bar{I}_{CA}| = \frac{U}{Z}}$$

Receptor en Triángulo Equilibrado



$$\overline{I}_A = \overline{I}_{AB} - \overline{I}_{CA} = \sqrt{3} \cdot \frac{U}{Z} \underline{/ \pm 90^\circ - \varphi}$$

$$\overline{I}_B = \overline{I}_{BC} - \overline{I}_{AB} = \sqrt{3} \cdot \frac{U}{Z} \underline{/ \mp 30^\circ - \varphi}$$

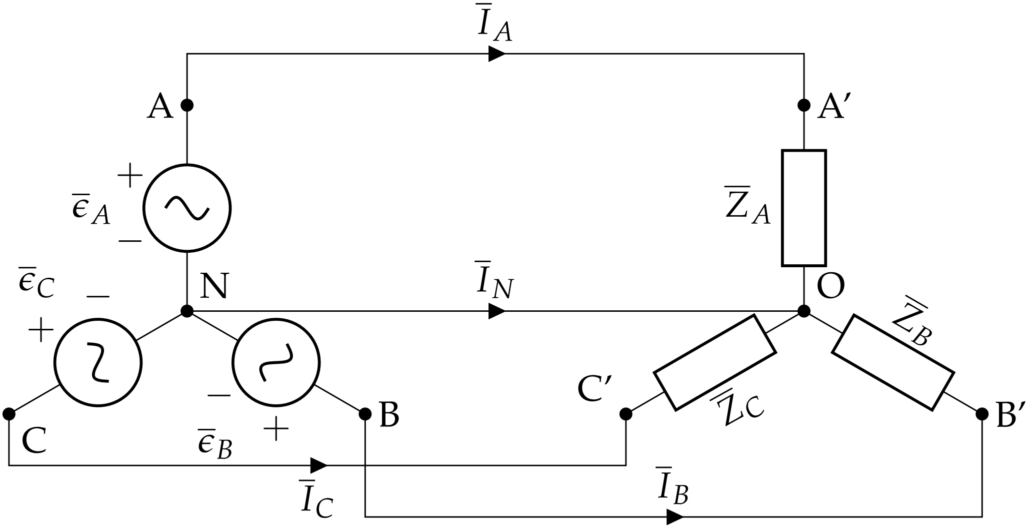
$$\overline{I}_C = \overline{I}_{CA} - \overline{I}_{BC} = \sqrt{3} \cdot \frac{U}{Z} \underline{/ \mp 150^\circ - \varphi}$$

Corriente de Línea:

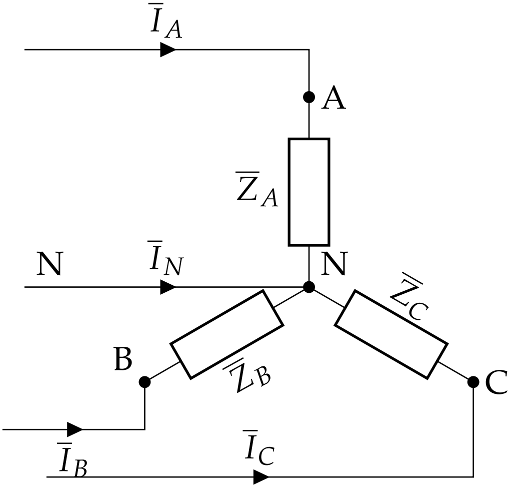
$$\boxed{I = |\overline{I}_A| = |\overline{I}_B| = |\overline{I}_C| = \sqrt{3} \cdot \frac{U}{Z}}$$

$$\boxed{I = \sqrt{3} \cdot I_f}$$

Receptor en Estrella Desequilibrado con Neutro



Receptor en Estrella Desequilibrado con Neutro



$$\overline{I}_A = \frac{\overline{U}_A}{\overline{\overline{Z}}_A}$$

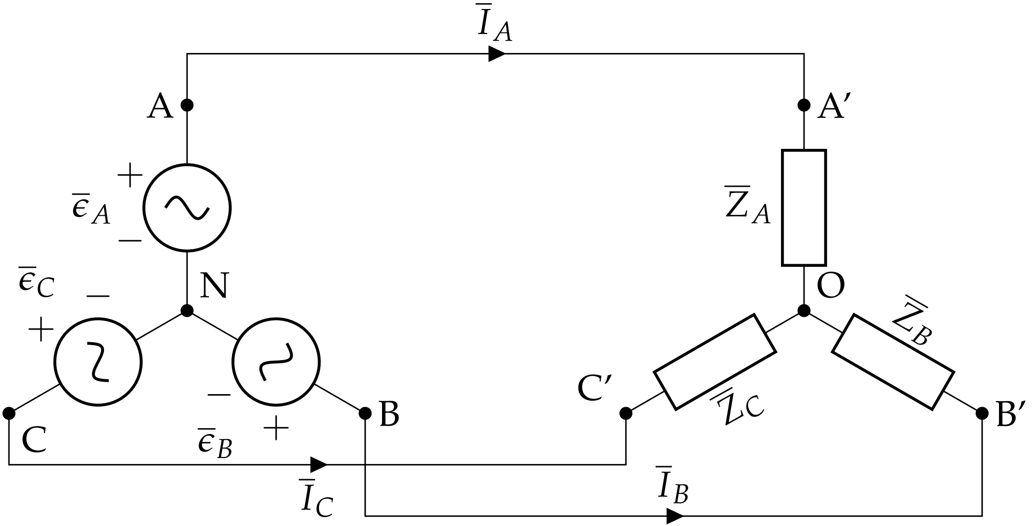
$$\overline{I}_B = \frac{\overline{U}_B}{\overline{\overline{Z}}_B}$$

$$\overline{I}_C = \frac{\overline{U}_C}{\overline{\overline{Z}}_C}$$

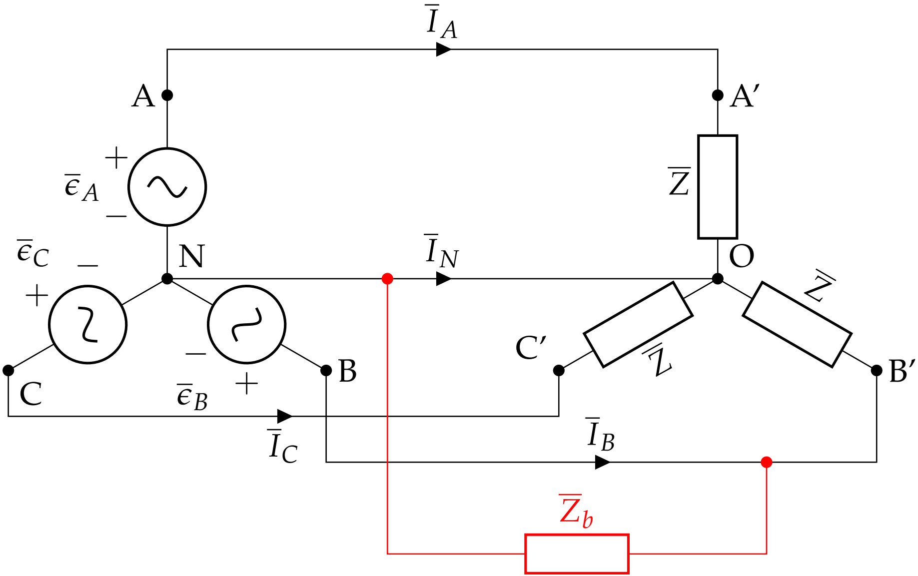
$$\overline{I}_A + \overline{I}_B + \overline{I}_C + \overline{I}_N = 0$$

$$\overline{I}_A + \overline{I}_B + \overline{I}_C \neq 0 \rightarrow \boxed{\overline{I}_N \neq 0}$$

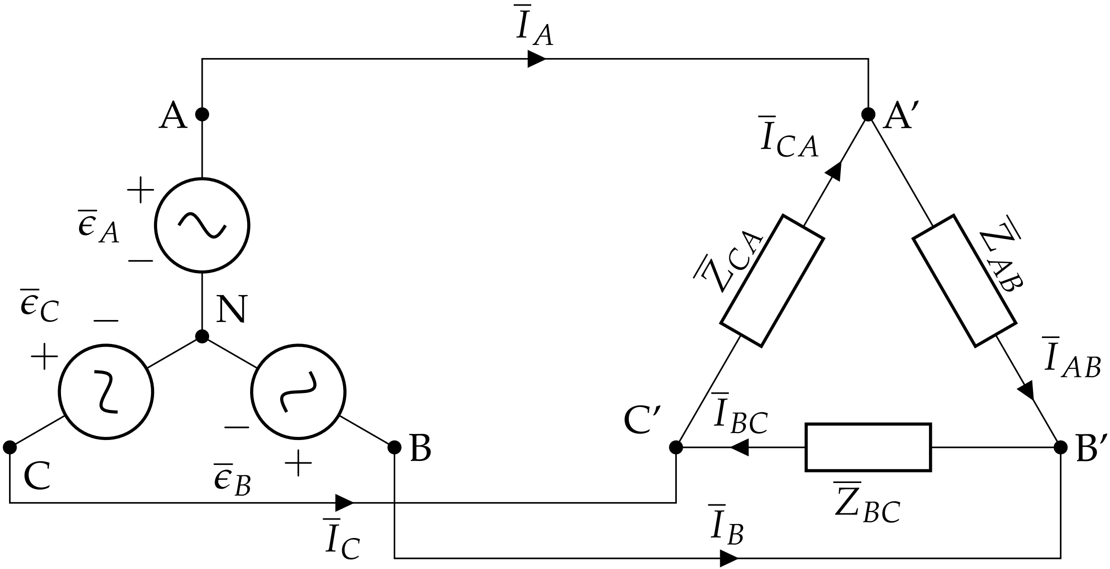
Receptor en Estrella Desequilibrado sin Neutro



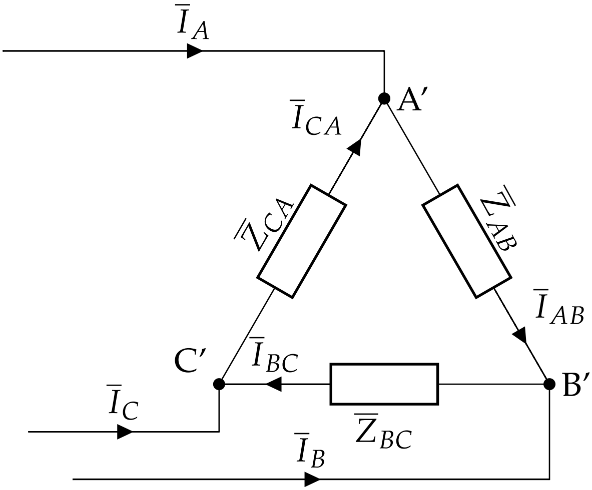
Receptor en Estrella con Carga Monofásica



Receptor en Triángulo Desequilibrado



Receptor en Triángulo Desequilibrado



$$\overline{I}_{AB} = \frac{\overline{U}_{AB}}{\overline{\overline{Z}}_{AB}}$$

$$\overline{I}_{BC} = \frac{\overline{U}_{BC}}{\overline{\overline{Z}}_{BC}}$$

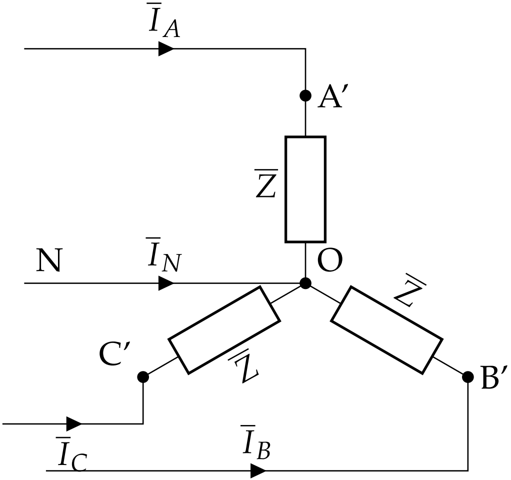
$$\overline{I}_{CA} = \frac{\overline{U}_{CA}}{\overline{\overline{Z}}_{CA}}$$

$$\overline{I}_A = \overline{I}_{AB} - \overline{I}_{CA}$$

$$\overline{I}_B = \overline{I}_{BC} - \overline{I}_{AB}$$

$$\overline{I}_C = \overline{I}_{CA} - \overline{I}_{BC}$$

Receptor en Estrella Equilibrado

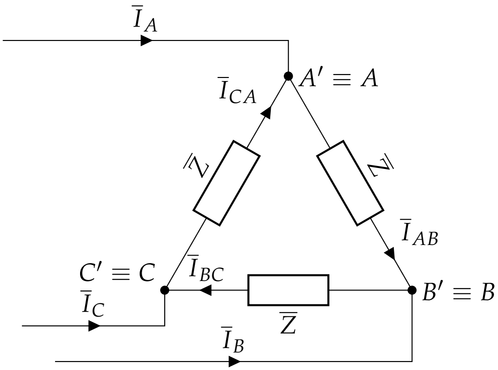


$$P = 3 \cdot P_Z = 3 \cdot U_Z I_Z \cos(\varphi)$$
$$Q = 3 \cdot Q_Z = 3 \cdot U_Z I_Z \sin(\varphi)$$

$$I_Z = I$$
$$U_Z = U_F$$

$$P = 3U_F I \cos(\varphi) = \sqrt{3}UI\cos(\varphi)$$
$$Q = 3U_F I \sin(\varphi) = \sqrt{3}UI\sin(\varphi)$$
$$S = \sqrt{P^2 + Q^2} = \sqrt{3}UI$$

Receptor en Triángulo Equilibrado

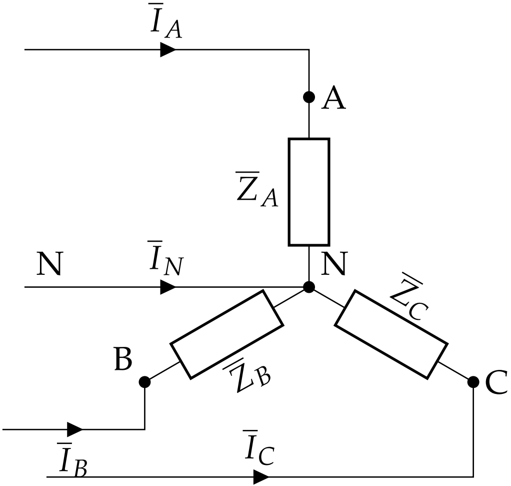


$$P = 3 \cdot P_Z = 3 \cdot U_Z I_Z \cos(\varphi)$$
$$Q = 3 \cdot Q_Z = 3 \cdot U_Z I_Z \sin(\varphi)$$

$$I_Z = I_F$$
$$U_Z = U$$

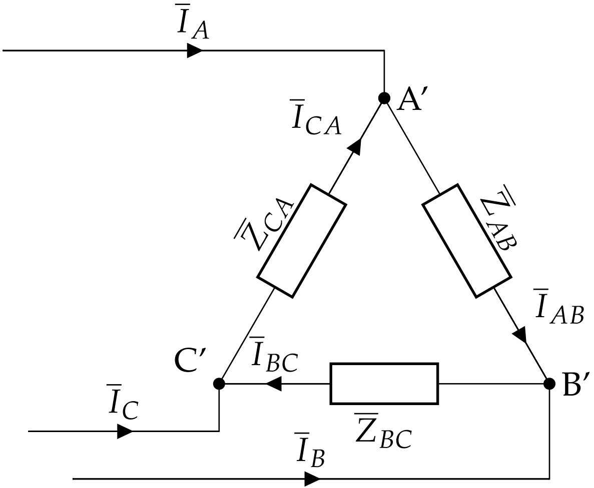
$$P = 3UI_F \cos(\varphi) = \sqrt{3}UI \cos(\varphi)$$
$$Q = 3UI_F \sin(\varphi) = \sqrt{3}UI \sin(\varphi)$$
$$S = \sqrt{P^2 + Q^2} = \sqrt{3}UI$$

Receptor en Estrella Desequilibrado



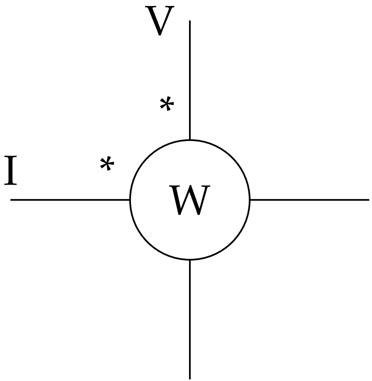
$$P = P_A + P_B + P_C$$
$$Q = Q_A + Q_B + Q_C$$
$$\overline{S} = P + jQ$$

Receptor en Triángulo Desequilibrado



$$P = P_{AB} + P_{BC} + P_{CA}$$
$$Q = Q_{AB} + Q_{BC} + Q_{CA}$$
$$\overline{S} = P + jQ$$

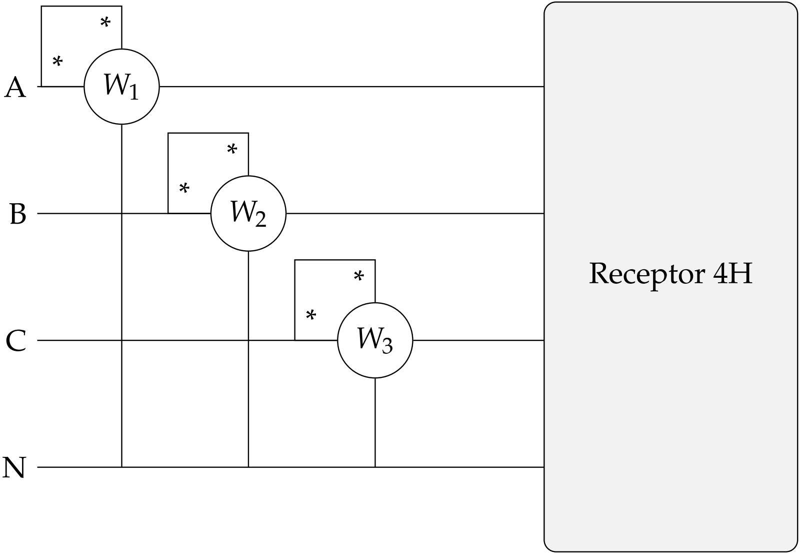
Recordatorio: vatímetro



Vatímetro: equipo de medida de 4 terminales (1 par para tensión, 1 par para corriente)

$$\boxed{W = \Re(\overline{U} \cdot \overline{I}^{*})}$$

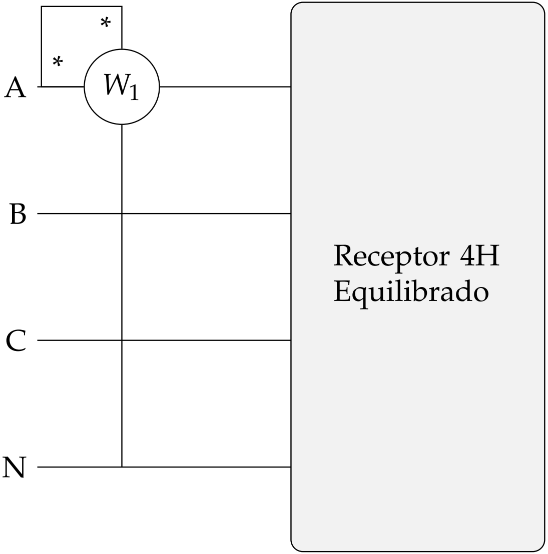
Sistema de 4 Hilos



$$W_1 = \Re(\overline{U}_A \cdot \overline{I}_A^*) = P_A$$
$$W_2 = \Re(\overline{U}_B \cdot \overline{I}_B^*) = P_B$$
$$W_3 = \Re(\overline{U}_C \cdot \overline{I}_C^*) = P_C$$

$$\boxed{P = W_1 + W_2 + W_3}$$

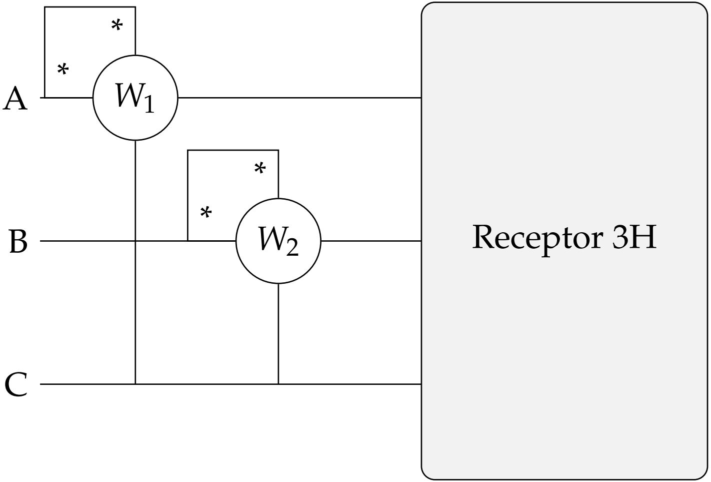
Sistema de 4 Hilos Equilibrado



$$P_A = P_B = P_C$$

$$\boxed{P = 3 \cdot W_1}$$

Sistema de 3 Hilos



Montaje de Aron

$$W_1 = \Re(\overline{U}_{AC} \cdot \overline{I}_A^*)$$

$$W_2 = \Re(\overline{U}_{BC} \cdot \overline{I}_B^*)$$

$$W_1 + W_2 = ?$$

Sistema de 3 Hilos

Desarrollamos las dos expresiones usando corrientes de fase y obviando el operador $\Re$:

$$\overline{U}_{AC} \cdot \overline{I}_A^* = \overline{U}_{AC} \cdot (\overline{I}_{AB}^* - \overline{I}_{CA}^*)$$
$$\overline{U}_{BC} \cdot \overline{I}_B^* = \overline{U}_{BC} \cdot (\overline{I}_{BC}^* - \overline{I}_{AB}^*)$$

Sumamos las dos expresiones:

$$\overline{U}_{AC} \cdot \overline{I}_A^* + \overline{U}_{BC} \cdot \overline{I}_B^* = \overline{U}_{AC} \cdot \overline{I}_{AB}^* - \overline{U}_{AC} \cdot \overline{I}_{CA}^* + \overline{U}_{BC} \cdot \overline{I}_{BC}^* - \overline{U}_{BC} \cdot \overline{I}_{AB}^*$$

Y agrupamos, teniendo en cuenta que $\overline{U}_{AB} + \overline{U}_{BC} + \overline{U}_{CA} = 0$:

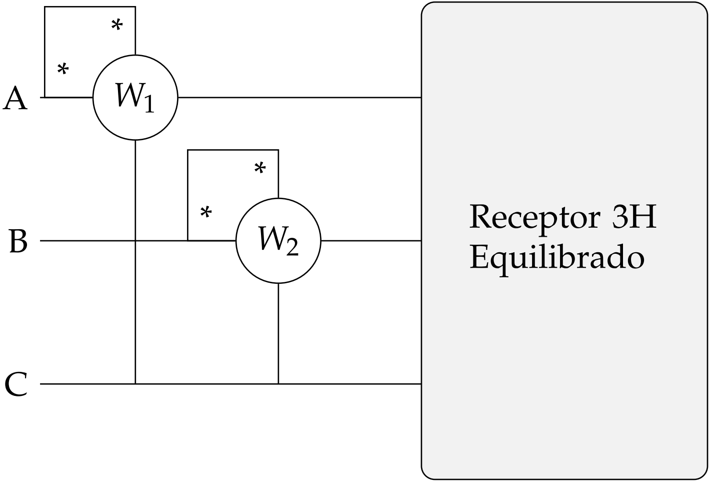
$$\overline{U}_{CA} \cdot \overline{I}_{CA}^* + \overline{U}_{BC} \cdot \overline{I}_{BC}^* + (\overline{U}_{AC} \cdot \overline{I}_{AB}^* - \overline{U}_{BC} \cdot \overline{I}_{AB}^*) = \overline{S}_{CA} + \overline{S}_{BC} + \overline{S}_{AB}$$

Extrayendo la parte real de este resultado obtenemos:

$$\Re(\overline{S}_{AB} + \overline{S}_{BC} + \overline{S}_{CA}) = P \rightarrow \boxed{W_1 + W_2 = P}$$

Sistema de 3 Hilos Equilibrado

Cuando el sistema es equilibrado, las lecturas individuales de los vatímetros tienen significado.



$$W_1 = \Re(\overline{U}_{AC} \cdot \overline{I}_A^*) = ?$$
$$W_2 = \Re(\overline{U}_{BC} \cdot \overline{I}_B^*) = ?$$

Sistema de 3 Hilos Equilibrado

Supongamos SFD:

$$\begin{aligned}
\overline{U}_{AC} &= -\overline{U}_{CA} = U\underline{/-120^\circ + 180^\circ} = U\underline{/60^\circ} \\
\overline{I}_A &= I\underline{/90^\circ - \varphi} \\
\overline{U}_{AC} \cdot \overline{I}_A^* &= UI\underline{/\varphi - 30^\circ} \rightarrow \boxed{W_1 = UI\cos(\varphi - 30^\circ)}
\end{aligned}$$

$$\begin{aligned}
\overline{U}_{BC} &= U\underline{/0} \\
\overline{I}_B &= I\underline{/-30^\circ - \varphi} \\
\overline{U}_{BC} \cdot \overline{I}_B^* &= UI\underline{/\varphi + 30^\circ} \rightarrow \boxed{W_2 = UI\cos(\varphi + 30^\circ)}
\end{aligned}$$

Sistema de 3 Hilos Equilibrado

Desarrollamos los dos cosenos:

$$\cos(30^\circ - \varphi) = \cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi$$
$$\cos(30^\circ + \varphi) = \cos 30^\circ \cos\varphi - \sin 30^\circ \sin\varphi$$

Si sumamos obtenemos la potencia activa:

$$\boxed{W_1 + W_2 = \sqrt{3}UI\cos\varphi = P}$$

Si restamos obtenemos la potencia reactiva (salvo un factor):

$$\boxed{W_1 - W_2 = UI\sin\varphi = \frac{Q}{\sqrt{3}}}$$

Por tanto, también podemos calcular el ángulo del receptor:

$$\boxed{\tan\varphi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}}$$

Sistema de 3 Hilos Equilibrado

Repetimos el desarrollo con SFI:

$$\overline{U}_{AC} = -\overline{U}_{CA} = U\underline{/120^\circ + 180^\circ} = U\underline{/-60^\circ}$$

$$\bar{I}_A = I\underline{/-90^\circ - \varphi}$$

$$\overline{U}_{AC} \cdot \bar{I}_A^* = UI\underline{/\varphi + 30^\circ} \rightarrow \boxed{W_1 = UI\cos(\varphi + 30^\circ)}$$

$$\overline{U}_{BC} = U\underline{/0}$$

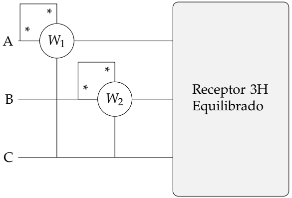
$$\bar{I}_B = I\underline{/30^\circ - \varphi}$$

$$\overline{U}_{BC} \cdot \bar{I}_B^* = UI\underline{/\varphi - 30^\circ} \rightarrow \boxed{W_2 = UI\cos(\varphi - 30^\circ)}$$

$$\boxed{W_1 + W_2 = \sqrt{3}UI\cos\varphi = P}$$

$$\boxed{W_1 - W_2 = -UI\sin\varphi = -\frac{Q}{\sqrt{3}}}$$

Sistema de 3 Hilos Equilibrado



SFD

$$W_1 = UI\cos(\varphi - 30^\circ)$$
$$W_2 = UI\cos(\varphi + 30^\circ)$$

$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_1 - W_2)$$
$$\tan\varphi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}$$

SFI

$$W_1 = UI\cos(\varphi + 30^\circ)$$
$$W_2 = UI\cos(\varphi - 30^\circ)$$

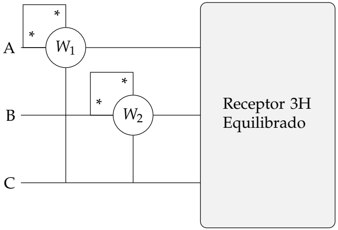
$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_2 - W_1)$$
$$\tan\varphi = \sqrt{3}\frac{W_2 - W_1}{W_1 + W_2}$$

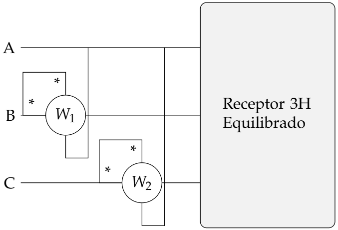
Otras conexiones: 3H SFD

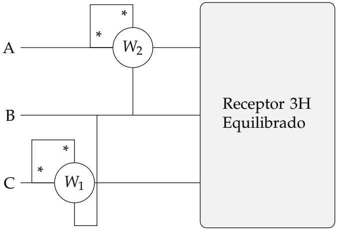
$$\boxed{(ABC) :: A \triangleright B \triangleright C \Longrightarrow \{AB, BC, CA\}}$$

$$W_1 = UI\cos(\varphi - 30^\circ) \qquad P = W_1 + W_2$$

$$W_2 = UI\cos(\varphi + 30^\circ) \qquad Q = \sqrt{3}(W_1 - W_2)$$







$$W_1 : AC \notin SFD$$
$$W_2 : BC \in SFD$$

$$W_1 : BA \notin SFD$$
$$W_2 : CA \in SFD$$

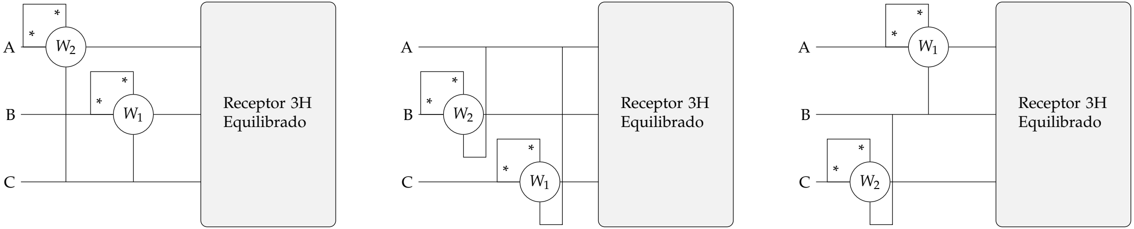
$$W_1 : CB \notin SFD$$
$$W_2 : AB \in SFD$$

Otras conexiones: 3H SFI

$$\boxed{(ACB) :: A \triangleright C \triangleright B \Longrightarrow \{AC, CB, BA\}}$$

$$W_1 = UI\cos(\varphi - 30^\circ) \qquad P = W_1 + W_2$$

$$W_2 = UI\cos(\varphi + 30^\circ) \qquad Q = \sqrt{3}(W_1 - W_2)$$



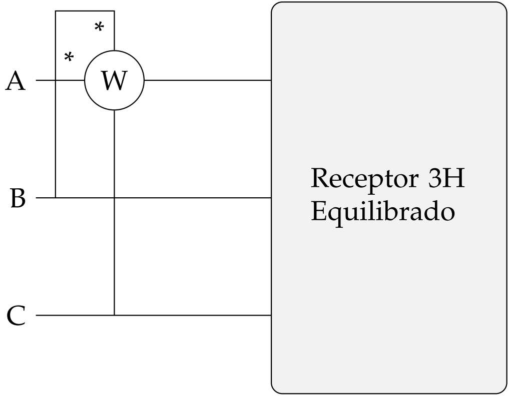
$W_1 : BC \notin SFI$ $\quad$ $W_1 : CA \notin SFI$ $\quad$ $W_1 : AB \notin SFI$

$W_2 : AC \in SFI$ $\quad$ $W_2 : BA \in SFI$ $\quad$ $W_2 : CB \in SFI$

Medida de Reactiva con un Vatímetro

Cuando el sistema está equilibrado, es posible medir la potencia reactiva con un único vatímetro.

Supongamos **SFD**:



$$W = \Re(\overline{U}_{BC} \cdot \overline{I}_A^*)$$

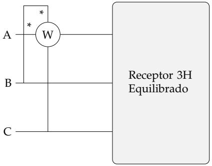
$$\overline{U}_{BC} = U\underline{/0}$$

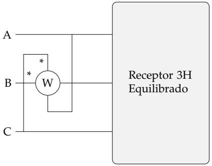
$$\overline{I}_A = I\underline{/90^\circ - \varphi}$$

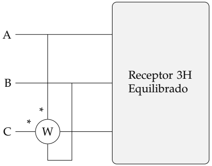
$$W = \Re(UI\underline{/\varphi - 90^\circ}) = \\ = UI\sin(\varphi)$$

$$\boxed{W = \frac{Q}{\sqrt{3}}}$$

Conexiones para medida de reactiva







$$W = \Re(\overline{U}_{BC} \cdot \overline{I}_A^*)$$

$$W = \Re(\overline{U}_{CA} \cdot \overline{I}_B^*)$$

$$W = \Re(\overline{U}_{AB} \cdot \overline{I}_C^*)$$

$$BC \in SFD$$
$$BC \notin SFI$$

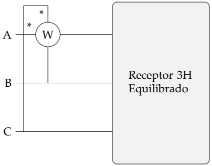
$$CA \in SFD$$
$$CA \notin SFI$$

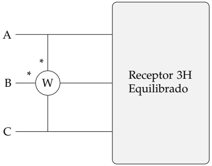
$$AB \in SFD$$
$$AB \notin SFI$$

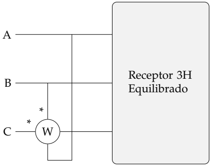
$$SFD \rightarrow \boxed{W = \frac{Q}{\sqrt{3}}}$$

$$SFI \rightarrow \boxed{W = -\frac{Q}{\sqrt{3}}}$$

Conexiones para medida de reactiva







$$W = \Re(\overline{U}_{CB} \cdot \overline{I}_A^*)$$

$$CB \notin SFD$$

$$CB \in SFI$$

$$W = \Re(\overline{U}_{AC} \cdot \overline{I}_B^*)$$

$$AC \notin SFD$$

$$AC \in SFI$$

$$W = \Re(\overline{U}_{BA} \cdot \overline{I}_C^*)$$

$$BA \notin SFD$$

$$BA \in SFI$$

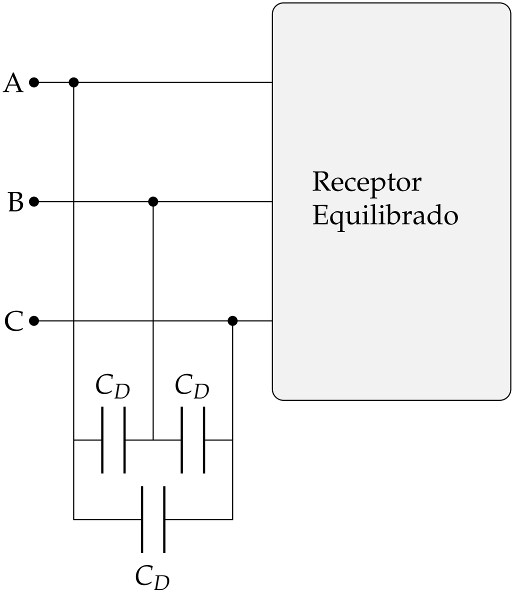
$$SFD \rightarrow \boxed{W = -\frac{Q}{\sqrt{3}}}$$

$$SFI \rightarrow \boxed{W = \frac{Q}{\sqrt{3}}}$$

Objetivo

- Sea un receptor **equilibrado inductivo** del que conocemos P, Q y, por tanto, su factor de potencia $\cos\varphi$.
- Para reducir la potencia reactiva del sistema debemos instalar un **banco de condensadores** que suministrarán una potencia reactiva Q_c.
- Como **resultado**, la potencia reactiva y el factor de potencia del sistema serán $Q' = Q - Q_c$ y $\cos\varphi' > \cos\varphi$.
- En trifásica existen dos posibilidades:
  - Conexión en triángulo: $C_\triangle$
  - Conexión en estrella: C_Y.

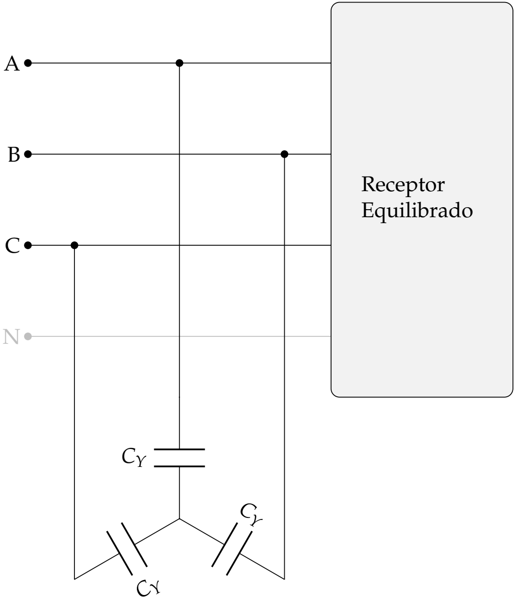
Conexión en Triángulo



$$
\begin{aligned}
Q &= P \tan \varphi \\
Q' &= P \tan \varphi' = \\
&= Q - Q_c \\
Q_c &= 3 \cdot \omega C_\triangle \cdot U^2
\end{aligned}
$$

$$
\boxed{C_\triangle = \frac{P(\tan \varphi - \tan \varphi')}{3\omega U^2}}
$$

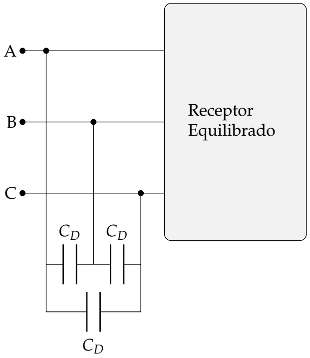
Conexión en Estrella



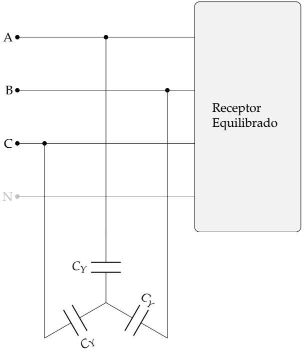
$$Q = P\tan\varphi$$
$$Q' = P\tan\varphi' =$$
$$= Q - Q_c$$
$$Q_c = 3\cdot\omega C_Y\cdot U_f^2$$

$$\boxed{C_Y = \frac{P(\tan\varphi - \tan\varphi')}{\omega U^2}}$$

Comparación Estrella-Triángulo



$$\boxed{C_\triangle = \frac{P(\tan\varphi - \tan\varphi')}{3\omega U^2}}$$



$$\boxed{C_Y = \frac{P(\tan\varphi - \tan\varphi')}{\omega U^2}}$$

Dado que $C_Y = 3 \cdot C_\triangle$ la **configuración recomendada** es **triángulo**.

Potencia Instantánea en Sistemas Equilibrados

Supongamos un receptor equilibrado en estrella con SFD:

$$u_A(t) = \sqrt{2}U_f \cos(\omega t + 90^\circ)$$
$$u_B(t) = \sqrt{2}U_f \cos(\omega t - 30^\circ)$$
$$u_C(t) = \sqrt{2}U_f \cos(\omega t - 150^\circ)$$

$$i_A(t) = \sqrt{2}I \cos(\omega t + 90^\circ - \varphi)$$
$$i_B(t) = \sqrt{2}I \cos(\omega t - 30^\circ - \varphi)$$
$$i_C(t) = \sqrt{2}I \cos(\omega t - 150^\circ - \varphi)$$

$$p_A(t) = u_A(t) \cdot i_A(t)$$
$$p_B(t) = u_B(t) \cdot i_B(t)$$
$$p_C(t) = u_C(t) \cdot i_C(t)$$

$$p(t) = p_A(t) + p_B(t) + p_C(t)$$

Potencia Instantánea en Sistemas Equilibrados

$$
\begin{aligned}
p(t) = {} & \sqrt{2}U_f \cos(\omega t + 90^\circ) \cdot \sqrt{2}I \cos(\omega t + 90^\circ - \varphi) + \\
& + \sqrt{2}U_f \cos(\omega t - 30^\circ) \cdot \sqrt{2}I \cos(\omega t - 30^\circ - \varphi) + \\
& + \sqrt{2}U_f \cos(\omega t - 150^\circ) \cdot \sqrt{2}I \cos(\omega t - 150^\circ - \varphi)
\end{aligned}
$$

$$
\cos(\alpha) \cdot \cos(\beta) = \frac{1}{2} \cdot (\cos(\alpha + \beta) + \cos(\alpha - \beta))
$$

$$
\begin{aligned}
p(t) = {} & U_f I[\cos(2\omega t + 180^\circ - \varphi) + \cos(\varphi)] + \\
& + U_f I[\cos(2\omega t - 60^\circ - \varphi) + \cos(\varphi)] + \\
& + U_f I[\cos(2\omega t - 300^\circ - \varphi) + \cos(\varphi)]
\end{aligned}
$$

$$
\boxed{p(t) = 3 \cdot U_f \cdot I \cdot \cos(\varphi) = \sqrt{3} \cdot U \cdot I \cdot \cos(\varphi)}
$$

Masa de conductor

Comparemos un sistema monofásico y un sistema trifásico (3H) que transmiten la **misma potencia activa** y funcionan a la **misma tensión entre líneas**.

$$UI_1 \cos\varphi = P_1 = P_3 = \sqrt{3}UI_3 \cos\varphi \rightarrow \boxed{I_1 = \sqrt{3}I_3}$$

Las **pérdidas en la línea** deben ser **iguales** para salvar la **misma distancia**:

$$2R_1 I_1^2 = P_{1l} = P_{3l} = 3R_3 I_3^2$$

Sustituyendo la relación de corrientes y teniendo en cuenta la relación entre resistencia y sección:

$$2 \cdot R_1 \cdot 3I_3^2 = 3 \cdot R_3 I_3^2 \rightarrow R_1 = \frac{1}{2}R_3 \rightarrow \boxed{S_1 = 2 \cdot S_3}$$

Finalmente, la relación entre masas de conductor es:

$$\frac{m_3}{m_1} = \frac{3 \cdot S_3}{2 \cdot S_1} = \boxed{\frac{3}{4}}$$