

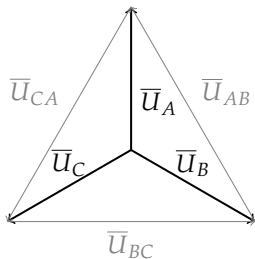
Sistemas Trifásicos

Teoría de Circuitos II

Oscar Perpiñán Lamigueiro

- ① Generadores
- ② Receptores
- ③ Potencia en Sistemas Trifásicos
- ④ Compensación de Reactiva
- ⑤ Medida de Potencia en Sistemas Trifásicos
- ⑥ Conversión de Fuentes Reales
- ⑦ Estudio generalizado de los sistemas trifásicos

Tensiones de Fase y Línea



Tensiones de **Fase**: U_A, U_B, U_C

Tensiones de **Línea**: U_{AB}, U_{BC}, U_{CA}

$$\bar{U}_{AB} = \bar{U}_A - \bar{U}_B$$

$$\bar{U}_{BC} = \bar{U}_B - \bar{U}_C$$

$$\bar{U}_{CA} = \bar{U}_C - \bar{U}_A$$

$$\bar{U}_{AB} + \bar{U}_{BC} + \bar{U}_{CA} = 0$$

Tensiones de Fase y Línea

$$\bar{U}_A = U_f \angle \theta_f$$

$$\bar{U}_B = U_f \angle \theta_f - 120^\circ$$

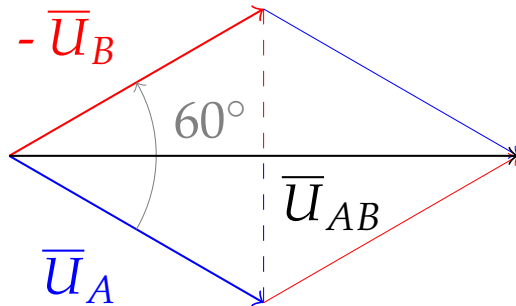
$$\bar{U}_{AB} = \bar{U}_A - \bar{U}_B =$$

$$= U_f \angle \theta_f - U_f \angle \theta_f - 120^\circ =$$

$$= U_f \angle \theta_f + U_f \angle \theta_f + 60^\circ$$

$$= 2 \cdot U_f \cdot \cos(30^\circ) \angle \theta_f + 30^\circ =$$

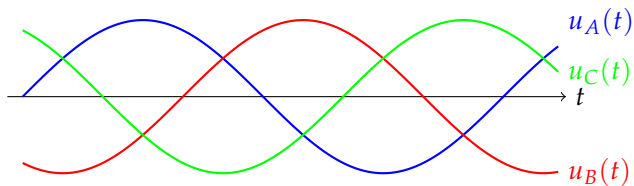
$$= \sqrt{3} U_f \angle \theta_f + 30^\circ$$



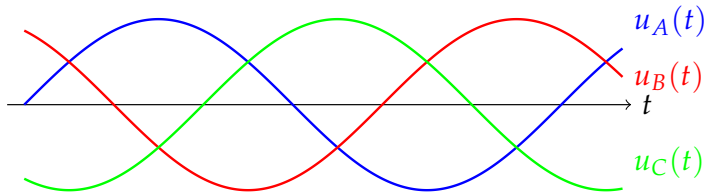
$$\begin{aligned} U &= \sqrt{3} \cdot U_f \\ \theta_l &= \theta_f + 30^\circ \end{aligned}$$

Secuencia de Fases

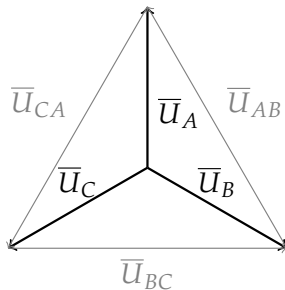
- Sentido en el que ocurren los máximos de cada fase.
- Secuencia de Fases Directa (**SFD**): ABC



- Secuencia de Fases Inversa (**SFI**): ACB



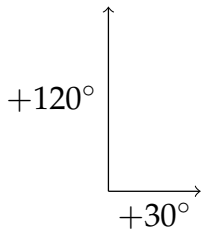
Secuencia de Fases Directa (SFD)



$$\bar{U}_A = \frac{U}{\sqrt{3}} \angle 90^\circ$$

$$\bar{U}_B = \frac{U}{\sqrt{3}} \angle -30^\circ$$

$$\bar{U}_C = \frac{U}{\sqrt{3}} \angle -150^\circ$$

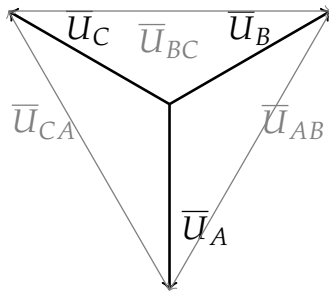


$$\bar{U}_{AB} = U \angle 120^\circ$$

$$\bar{U}_{BC} = U \angle 0^\circ$$

$$\bar{U}_{CA} = U \angle -120^\circ$$

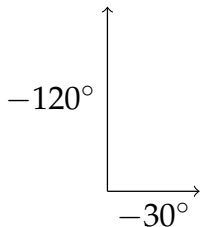
Secuencia de Fases Inversa (SFI)



$$\bar{U}_A = \frac{U}{\sqrt{3}} \angle -90^\circ$$

$$\bar{U}_B = \frac{U}{\sqrt{3}} \angle 30^\circ$$

$$\bar{U}_C = \frac{U}{\sqrt{3}} \angle 150^\circ$$



$$\bar{U}_{AB} = U \angle -120^\circ$$

$$\bar{U}_{BC} = U \angle 0^\circ$$

$$\bar{U}_{CA} = U \angle 120^\circ$$

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Tipos de Receptores

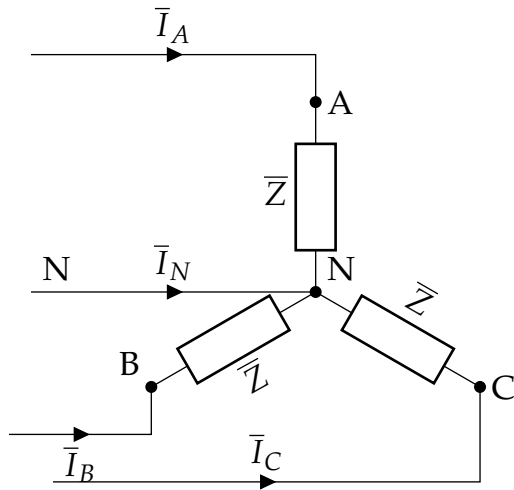
Conexión

- ▶ **Estrella** (punto común) Y
- ▶ **Triángulo** \triangle

Impedancias

- ▶ **Equilibrado** (las tres impedancias son idénticas en módulo **y** fase).
- ▶ **Desequilibrado**

Receptor en Estrella Equilibrado



$$\bar{I}_A = \frac{\bar{U}_A}{\bar{Z}} = \frac{U_f}{Z} \angle \pm 90^\circ - \theta$$

$$\bar{I}_B = \frac{\bar{U}_B}{\bar{Z}} = \frac{U_f}{Z} \angle \mp 30^\circ - \theta$$

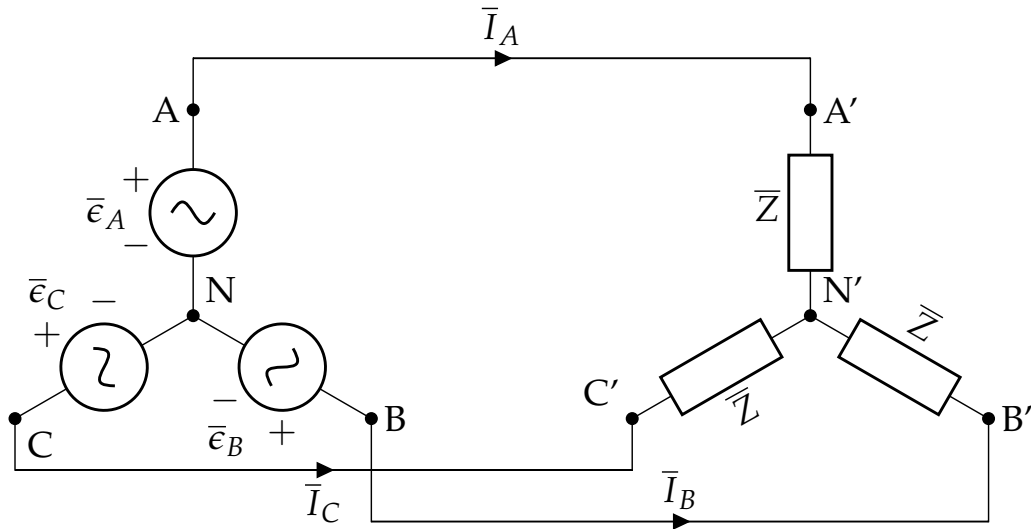
$$\bar{I}_C = \frac{\bar{U}_C}{\bar{Z}} = \frac{U_f}{Z} \angle \mp 150^\circ - \theta$$

$$|\bar{I}_A| = |\bar{I}_B| = |\bar{I}_C| = \frac{U_f}{Z}$$

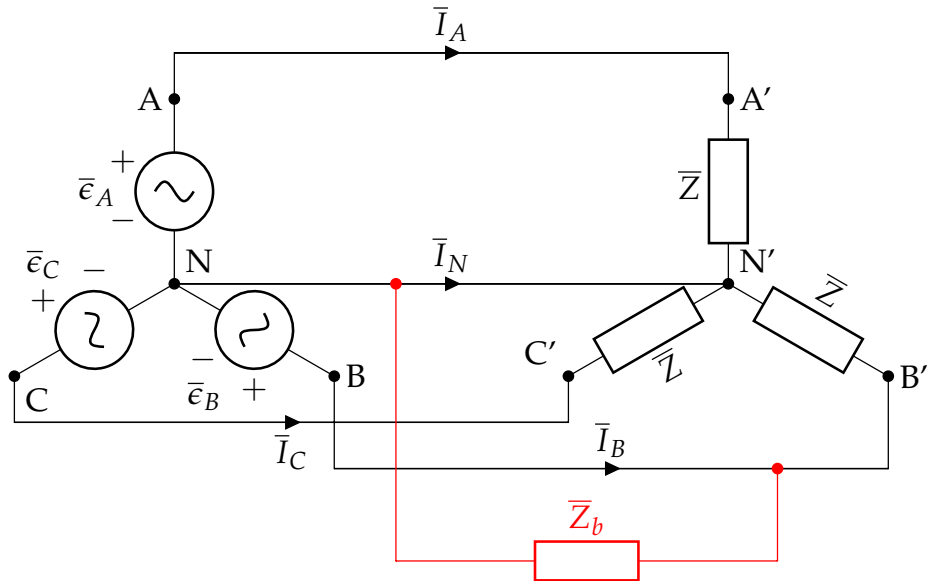
$$\bar{I}_A + \bar{I}_B + \bar{I}_C + \bar{I}_N = 0$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0 \rightarrow \boxed{\bar{I}_N = 0}$$

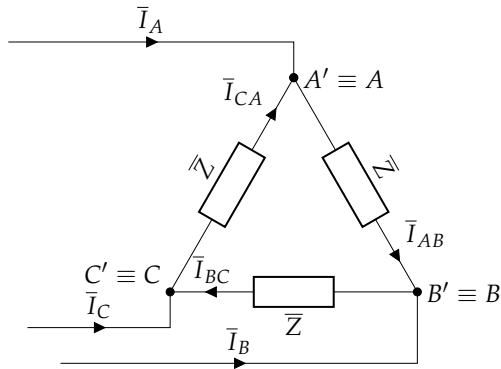
Receptor en Estrella Equilibrado



Receptor en Estrella con Carga Monofásica



Receptor en Triángulo Equilibrado



$$\bar{I}_{AB} = \frac{\bar{U}_{AB}}{\bar{Z}} = \frac{U}{Z} \angle \pm 120^\circ - \theta$$

$$\bar{I}_{BC} = \frac{\bar{U}_{BC}}{\bar{Z}} = \frac{U}{Z} \angle 0 - \theta$$

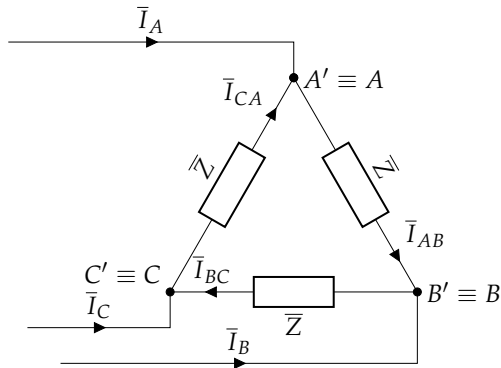
$$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}} = \frac{U}{Z} \angle \mp 120^\circ - \theta$$

$$\bar{I}_{AB} + \bar{I}_{BC} + \bar{I}_{CA} = 0$$

Corriente de Fase:

$$I_f = |\bar{I}_{AB}| = |\bar{I}_{BC}| = |\bar{I}_{CA}| = \frac{U}{Z}$$

Receptor en Triángulo Equilibrado



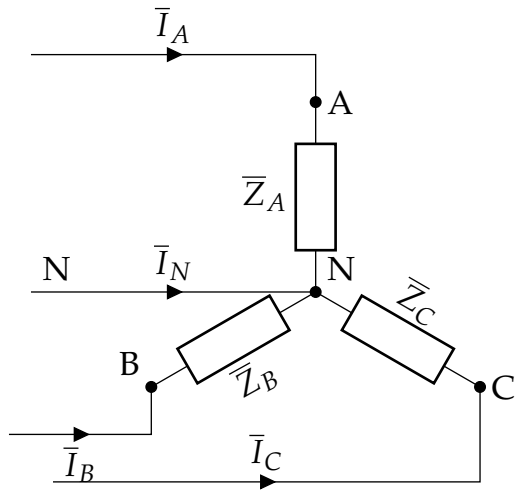
$$\begin{aligned}\bar{I}_A &= \bar{I}_{AB} - \bar{I}_{CA} = \sqrt{3} \cdot \frac{U}{Z} \angle \pm 90^\circ - \theta \\ \bar{I}_B &= \bar{I}_{BC} - \bar{I}_{AB} = \sqrt{3} \cdot \frac{U}{Z} \angle \mp 30^\circ - \theta \\ \bar{I}_C &= \bar{I}_{CA} - \bar{I}_{BC} = \sqrt{3} \cdot \frac{U}{Z} \angle \mp 150^\circ - \theta\end{aligned}$$

Corriente de Línea:

$$I = |\bar{I}_A| = |\bar{I}_B| = |\bar{I}_C| = \sqrt{3} \cdot \frac{U}{Z}$$

$$I = \sqrt{3} \cdot I_f$$

Receptor en Estrella Desequilibrado con Neutro



$$\bar{I}_A = \frac{\bar{U}_A}{\bar{Z}_A}$$

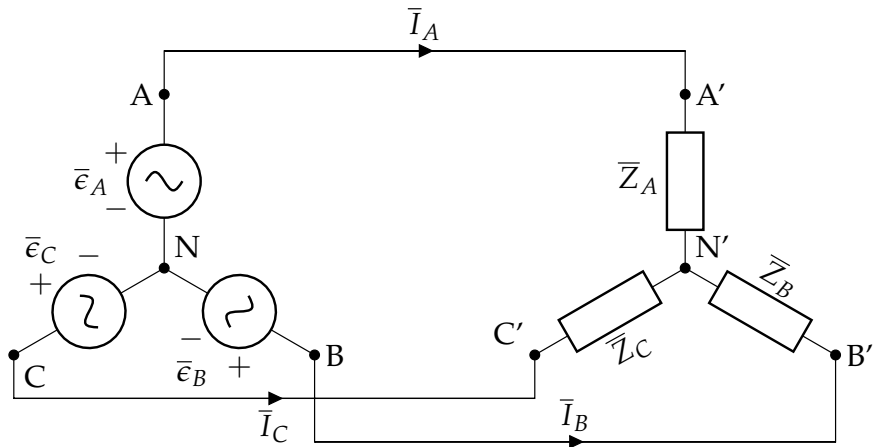
$$\bar{I}_B = \frac{\bar{U}_B}{\bar{Z}_B}$$

$$\bar{I}_C = \frac{\bar{U}_C}{\bar{Z}_C}$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C + \bar{I}_N = 0$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C \neq 0 \rightarrow \boxed{\bar{I}_N \neq 0}$$

Receptor en Estrella Desequilibrado sin Neutro



$$\bar{U}_N \neq \bar{U}_{N'}$$

Método del desplazamiento del neutro

Ecuaciones del receptor:

$$\bar{U}_{A'N'} = \bar{I}_A \cdot \bar{Z}_A$$

$$\bar{U}_{B'N'} = \bar{I}_B \cdot \bar{Z}_B$$

$$\bar{U}_{C'N'} = \bar{I}_C \cdot \bar{Z}_C$$

Ecuación del nudo N' :

$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0$$

Método del desplazamiento del neutro

Relacionamos las tensiones en el receptor con las tensiones del generador:

$$\bar{U}_{A'N'} = \bar{U}_{AN} - \bar{U}_{N'N}$$

$$\bar{U}_{B'N'} = \bar{U}_{BN} - \bar{U}_{N'N}$$

$$\bar{U}_{C'N'} = \bar{U}_{CN} - \bar{U}_{N'N}$$

Despejamos las corrientes teniendo en cuenta estas relaciones:

$$\bar{I}_A = \frac{\bar{U}_{AN} - \bar{U}_{N'N}}{\bar{Z}_A}$$

$$\bar{I}_B = \frac{\bar{U}_{BN} - \bar{U}_{N'N}}{\bar{Z}_B}$$

$$\bar{I}_C = \frac{\bar{U}_{CN} - \bar{U}_{N'N}}{\bar{Z}_C}$$

Método del desplazamiento del neutro

Finalmente, usando la ecuación del nudo N' despejamos la tensión $U_{N'N}$ (tensión de desplazamiento del neutro)*:

$$\bar{U}_{N'N} = \frac{\bar{U}_{AN} \cdot \bar{Y}_A + \bar{U}_{BN} \cdot \bar{Y}_B + \bar{U}_{CN} \cdot \bar{Y}_C}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C}$$

Una vez calculada esta tensión $\bar{U}_{N'N}$ se pueden calcular las corrientes de línea:

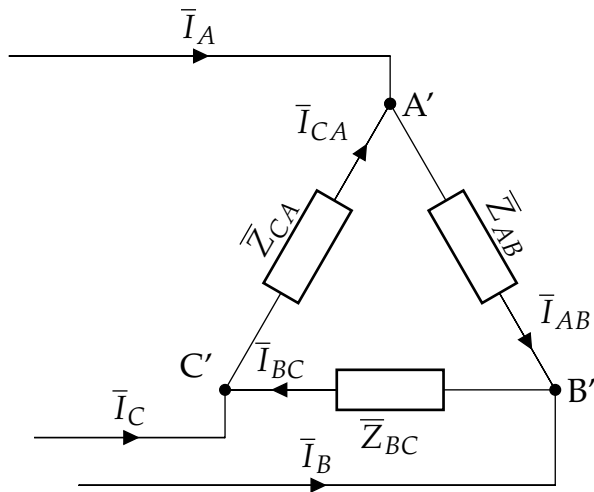
$$\bar{I}_A = (\bar{U}_{AN} - \bar{U}_{N'N}) \cdot \bar{Y}_A$$

$$\bar{I}_B = (\bar{U}_{BN} - \bar{U}_{N'N}) \cdot \bar{Y}_B$$

$$\bar{I}_C = (\bar{U}_{CN} - \bar{U}_{N'N}) \cdot \bar{Y}_C$$

*Se puede llegar a este mismo resultado aplicando el teorema de Millman.

Receptor en Triángulo Desequilibrado



$$\bar{I}_{AB} = \frac{\bar{U}_{AB}}{\bar{Z}_{AB}}$$

$$\bar{I}_{BC} = \frac{\bar{U}_{BC}}{\bar{Z}_{BC}}$$

$$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}_{CA}}$$

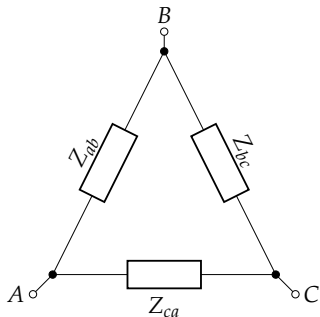
$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA}$$

$$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB}$$

$$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC}$$

Transformación de receptores

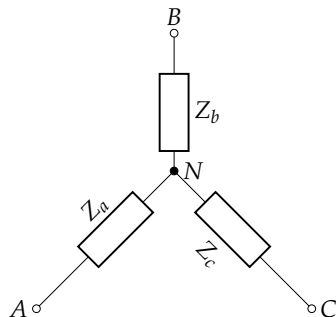
Triángulo a Estrella



$$\bar{Z}_a = \frac{\bar{Z}_{ab} \cdot \bar{Z}_{ca}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}}$$

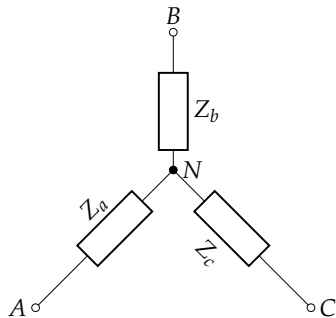
$$\bar{Z}_b = \frac{\bar{Z}_{bc} \cdot \bar{Z}_{ca}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}}$$

$$\bar{Z}_c = \frac{\bar{Z}_{ca} \cdot \bar{Z}_{ab}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}}$$



Transformación de receptores

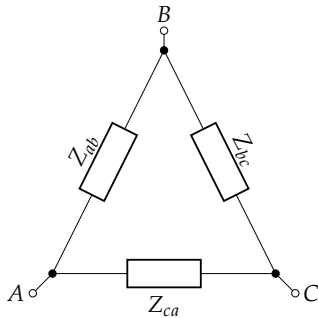
Estrella a Triángulo



$$\bar{Y}_{ab} = \frac{\bar{Y}_a \bar{Y}_b}{\bar{Y}_a + \bar{Y}_b + \bar{Y}_c}$$

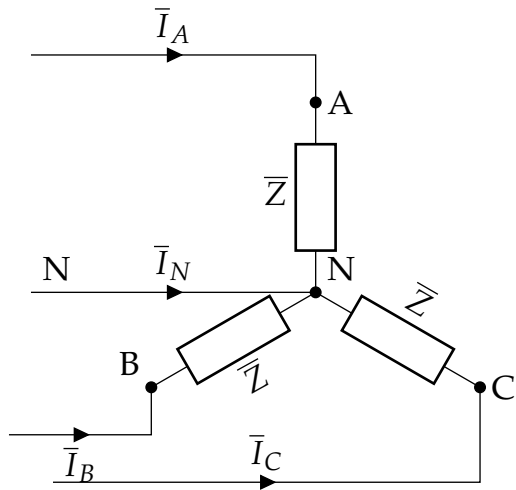
$$\bar{Y}_{bc} = \frac{\bar{Y}_b \bar{Y}_c}{\bar{Y}_a + \bar{Y}_b + \bar{Y}_c}$$

$$\bar{Y}_{ca} = \frac{\bar{Y}_c \bar{Y}_a}{\bar{Y}_a + \bar{Y}_b + \bar{Y}_c}$$



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Receptor en Estrella Equilibrado



$$P = 3 \cdot P_Z = 3 \cdot U_Z I_Z \cos(\theta)$$

$$Q = 3 \cdot Q_Z = 3 \cdot U_Z I_Z \sin(\theta)$$

$$I_Z = I$$

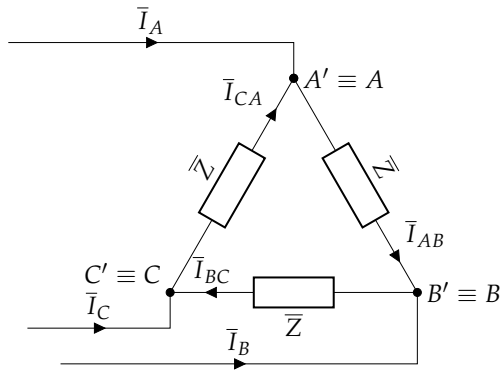
$$U_Z = U_F$$

$$P = 3U_F I \cos(\theta) = \sqrt{3}UI \cos(\theta)$$

$$Q = 3U_F I \sin(\theta) = \sqrt{3}UI \sin(\theta)$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{3}UI$$

Receptor en Triángulo Equilibrado



$$P = 3 \cdot P_Z = 3 \cdot U_Z I_Z \cos(\theta)$$

$$Q = 3 \cdot Q_Z = 3 \cdot U_Z I_Z \sin(\theta)$$

$$I_Z = I_F$$

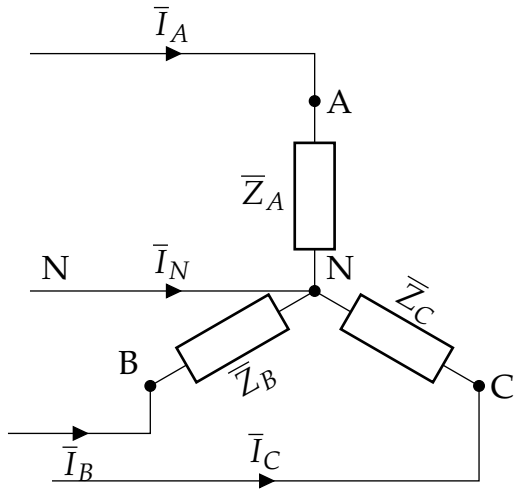
$$U_Z = U$$

$$P = 3UI_F \cos(\theta) = \sqrt{3}UI \cos(\theta)$$

$$Q = 3UI_F \sin(\theta) = \sqrt{3}UI \sin(\theta)$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{3}UI$$

Receptor en Estrella Desequilibrado

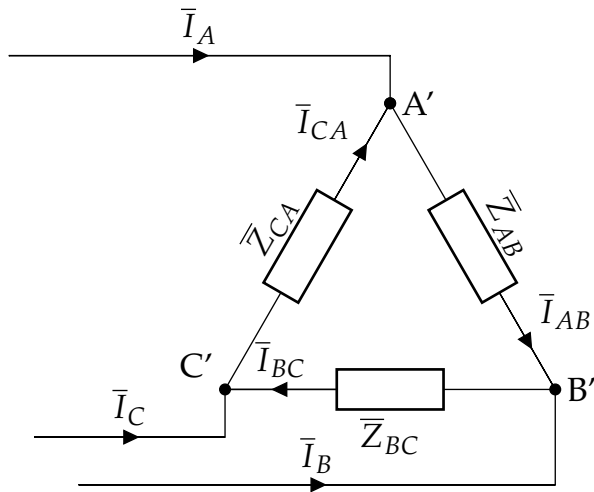


$$P = P_A + P_B + P_C$$

$$Q = Q_A + Q_B + Q_C$$

$$\bar{S} = P + jQ$$

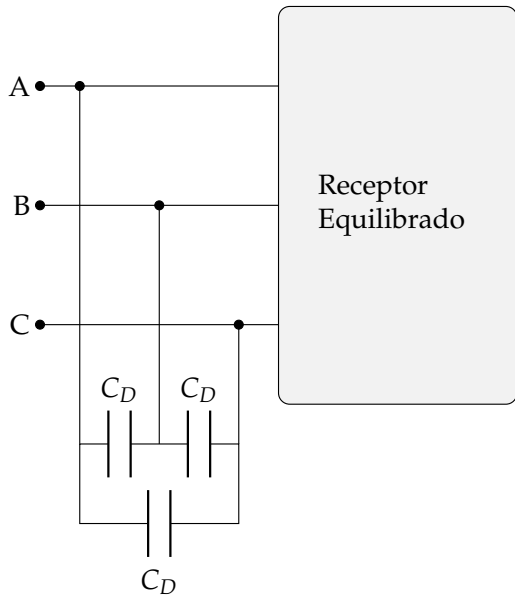
Receptor en Triángulo Desequilibrado



$$P = P_{AB} + P_{BC} + P_{CA}$$
$$Q = Q_{AB} + Q_{BC} + Q_{CA}$$
$$\bar{S} = P + jQ$$

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Conexión en Triángulo



$$Q = P \tan \theta$$

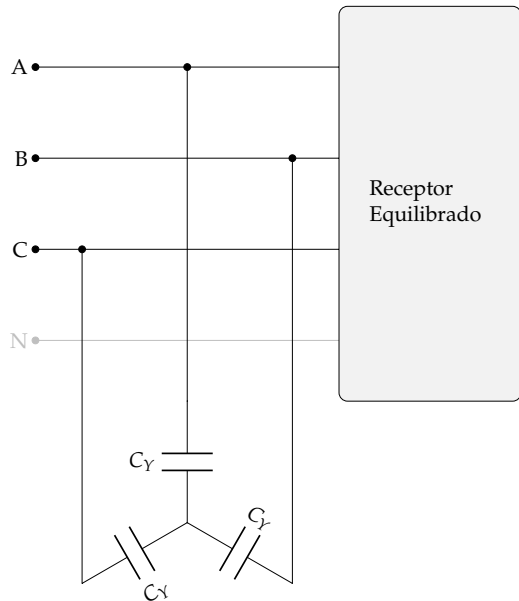
$$Q' = P \tan \theta' =$$

$$= Q - Q_c$$

$$Q_c = 3 \cdot \omega C_{\Delta} \cdot U^2$$

$$C_{\Delta} = \frac{P(\tan \theta - \tan \theta')}{3\omega U^2}$$

Conexión en Estrella



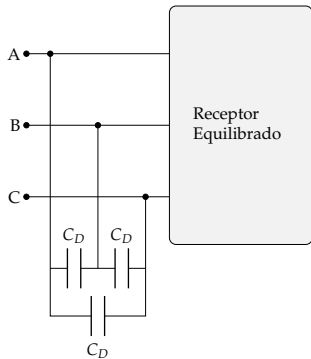
$$Q = P \tan \theta$$

$$Q' = P \tan \theta' =$$
$$= Q - Q_c$$

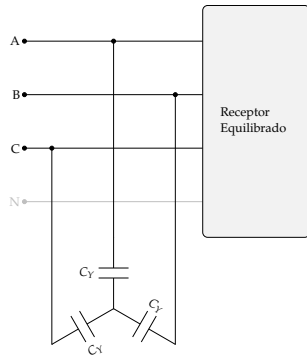
$$Q_c = 3 \cdot \omega C_Y \cdot U_f^2$$

$$C_Y = \frac{P(\tan \theta - \tan \theta')}{\omega U^2}$$

Comparación Estrella-Triángulo



$$C_{\Delta} = \frac{P(\tan \theta - \tan \theta')}{3\omega U^2}$$

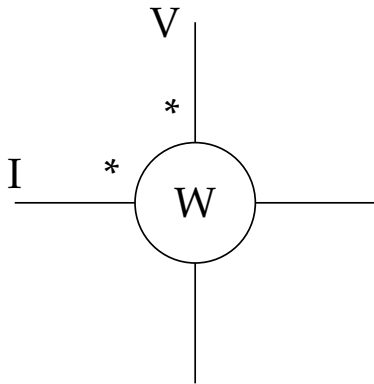


$$C_Y = \frac{P(\tan \theta - \tan \theta')}{\omega U^2}$$

Dado que $C_Y = 3 \cdot C_{\Delta}$ la **configuración recomendada** es **triángulo**.

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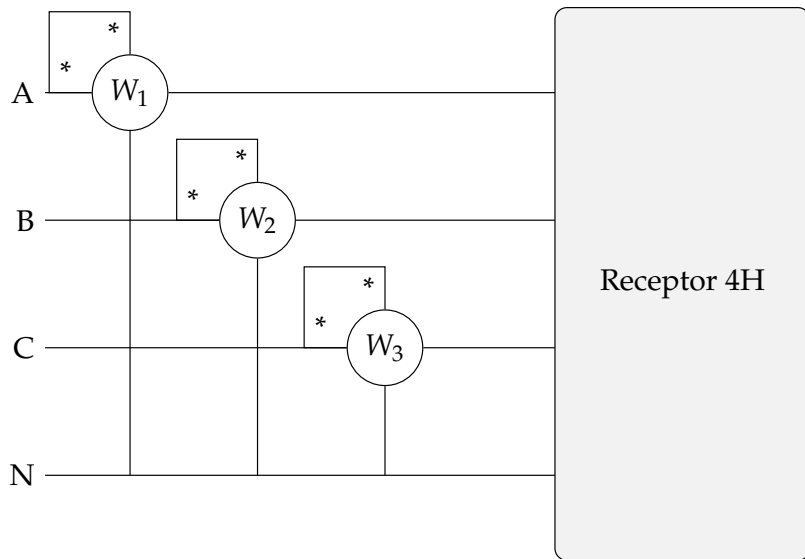
Recordatorio: vatímetro



Vatímetro: equipo de medida de 4 terminales (1 par para tensión, 1 par para corriente)

$$W = \Re(\bar{U} \cdot \bar{I}^*)$$

Sistema de 4 Hilos



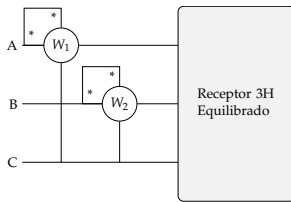
$$W_1 = \Re(\bar{U}_A \cdot \bar{I}_A^*) = P_A$$

$$W_2 = \Re(\bar{U}_B \cdot \bar{I}_B^*) = P_B$$

$$W_3 = \Re(\bar{U}_C \cdot \bar{I}_C^*) = P_C$$

$$P = W_1 + W_2 + W_3$$

Sistema de 3 Hilos Equilibrado



SFD

$$W_1 = UI \cos(\theta - 30^\circ)$$

$$W_2 = UI \cos(\theta + 30^\circ)$$

$$P = W_1 + W_2$$

$$Q = \sqrt{3}(W_1 - W_2)$$

$$\tan \theta = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2}$$

SFI

$$W_1 = UI \cos(\theta + 30^\circ)$$

$$W_2 = UI \cos(\theta - 30^\circ)$$

$$P = W_1 + W_2$$

$$Q = \sqrt{3}(W_2 - W_1)$$

$$\tan \theta = \sqrt{3} \frac{W_2 - W_1}{W_1 + W_2}$$

Otras conexiones: 3H SFD

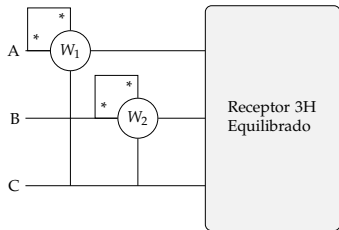
$$(ABC) :: A \triangleright B \triangleright C \implies \{AB, BC, CA\}$$

$$W_1 = UI \cos(\theta - 30^\circ)$$

$$P = W_1 + W_2$$

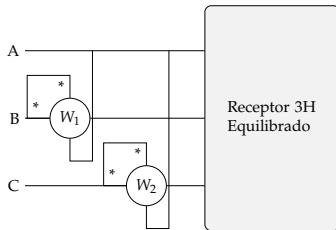
$$W_2 = UI \cos(\theta + 30^\circ)$$

$$Q = \sqrt{3}(W_1 - W_2)$$



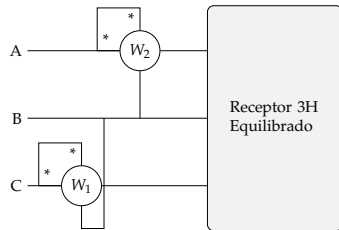
$W_1 : AC \notin SFD$

$W_2 : BC \in SFD$



$W_1 : BA \notin SFD$

$W_2 : CA \in SFD$



$W_1 : CB \notin SFD$

$W_2 : AB \in SFD$

Otras conexiones: 3H SFI

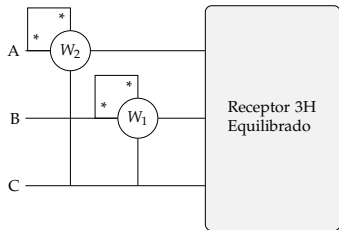
$$(ACB) :: A \triangleright C \triangleright B \implies \{AC, CB, BA\}$$

$$W_1 = UI \cos(\theta - 30^\circ)$$

$$P = W_1 + W_2$$

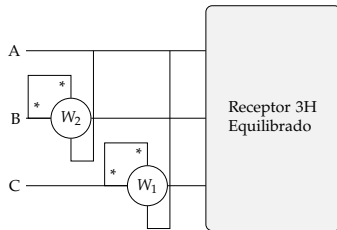
$$W_2 = UI \cos(\theta + 30^\circ)$$

$$Q = \sqrt{3}(W_1 - W_2)$$



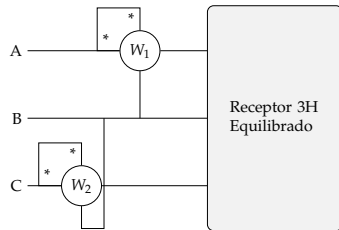
$W_1 : BC \notin SFI$

$W_2 : AC \in SFI$



$W_1 : CA \notin SFI$

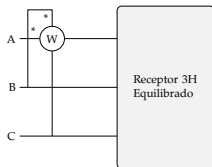
$W_2 : BA \in SFI$



$W_1 : AB \notin SFI$

$W_2 : CB \in SFI$

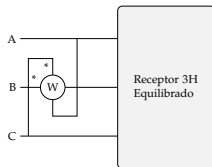
Conexiones para medida de reactiva



$$W = \Re(\bar{U}_{BC} \cdot \bar{I}_A^*)$$

$BC \in SFD$

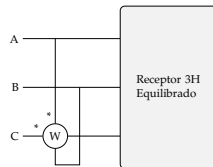
$BC \notin SFI$



$$W = \Re(\bar{U}_{CA} \cdot \bar{I}_B^*)$$

$CA \in SFD$

$CA \notin SFI$



$$W = \Re(\bar{U}_{AB} \cdot \bar{I}_C^*)$$

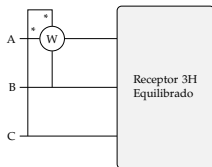
$AB \in SFD$

$AB \notin SFI$

$$SFD \rightarrow \boxed{W = \frac{Q}{\sqrt{3}}}$$

$$SFI \rightarrow \boxed{W = -\frac{Q}{\sqrt{3}}}$$

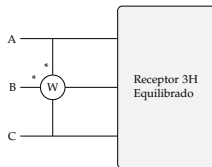
Conexiones para medida de reactiva



$$W = \Re(\bar{U}_{CB} \cdot \bar{I}_A^*)$$

$CB \notin SFD$

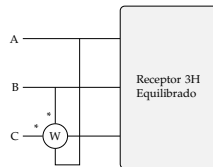
$CB \in SFI$



$$W = \Re(\bar{U}_{AC} \cdot \bar{I}_B^*)$$

$AC \notin SFD$

$AC \in SFI$



$$W = \Re(\bar{U}_{BA} \cdot \bar{I}_C^*)$$

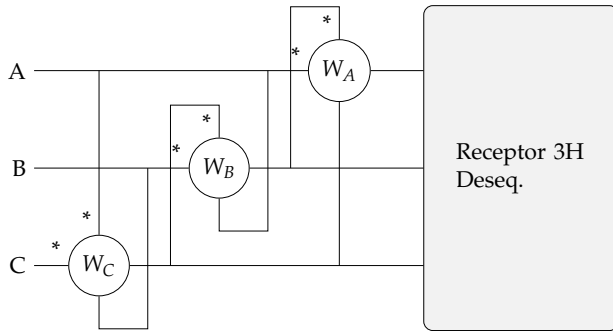
$BA \notin SFD$

$BA \in SFI$

$$SFD \rightarrow \boxed{W = -\frac{Q}{\sqrt{3}}}$$

$$SFI \rightarrow \boxed{W = \frac{Q}{\sqrt{3}}}$$

Medida de la reactiva con receptor desequilibrado



$$W_A = \Re(\bar{U}_{BC} \cdot \bar{I}_A^*)$$

$$W_B = \Re(\bar{U}_{CA} \cdot \bar{I}_B^*)$$

$$W_C = \Re(\bar{U}_{AB} \cdot \bar{I}_C^*)$$

$$\bar{U}_{AB} = \pm\sqrt{3} \cdot \bar{U}_C \cdot e^{j\pi/2}$$

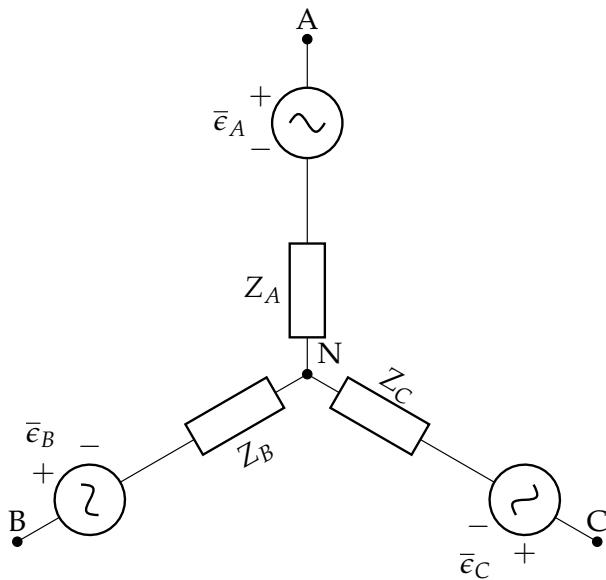
$$\bar{U}_{BC} = \pm\sqrt{3} \cdot \bar{U}_A \cdot e^{j\pi/2}$$

$$\bar{U}_{CA} = \pm\sqrt{3} \cdot \bar{U}_B \cdot e^{j\pi/2}$$

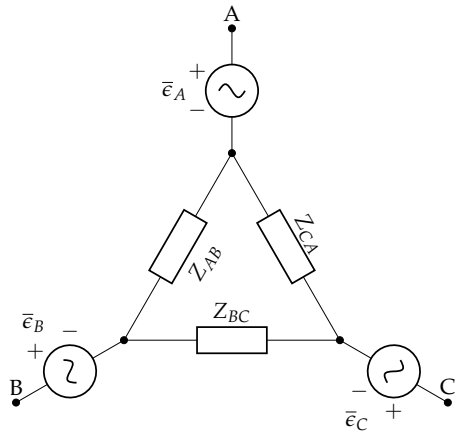
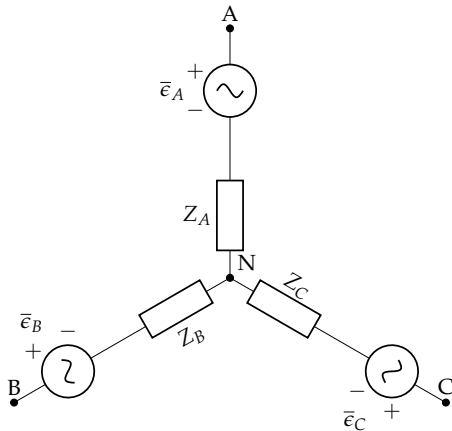
$$W_A + W_B + W_C = \pm Q / \sqrt{3}$$

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- ⑥ **Conversión de Fuentes Reales**
- ⑦ Estudio generalizado de los sistemas trifásicos

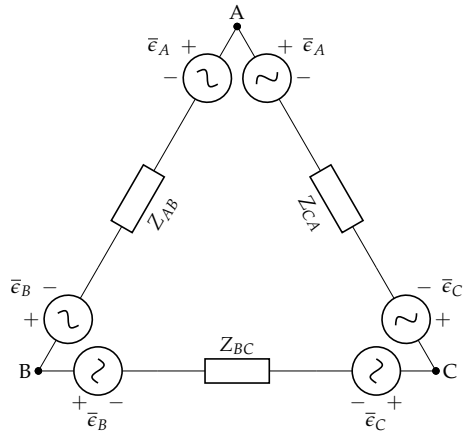
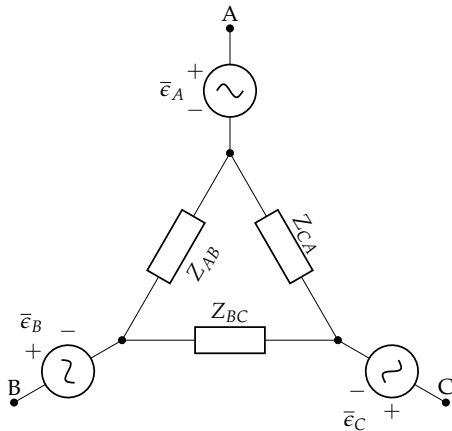
Estrella a Triángulo



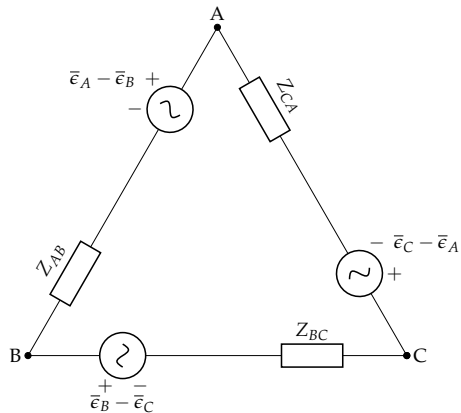
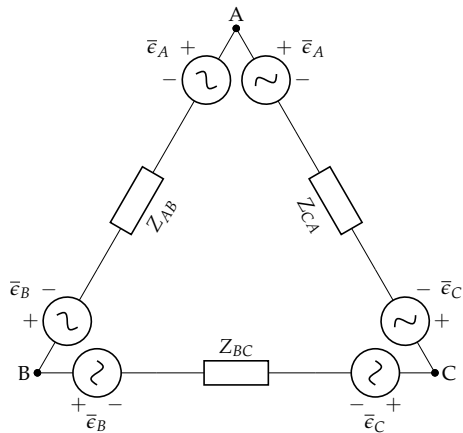
Transformamos impedancia



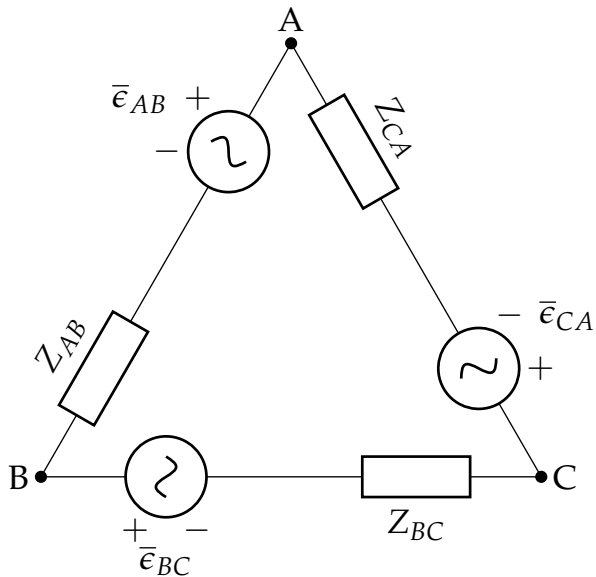
Aplicamos movilidad de fuentes



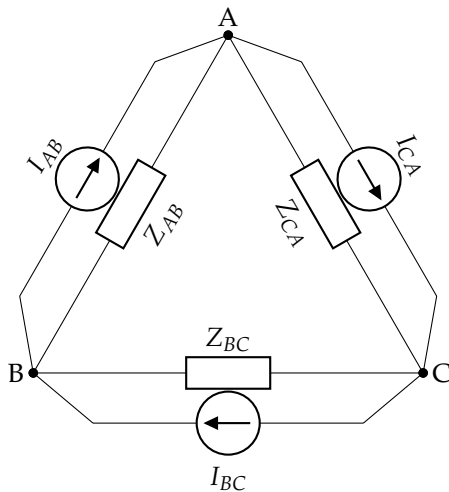
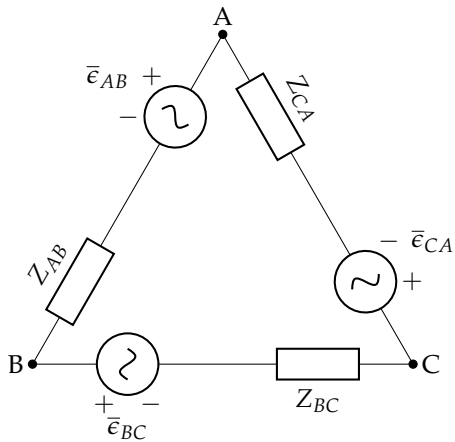
Asociamos fuentes



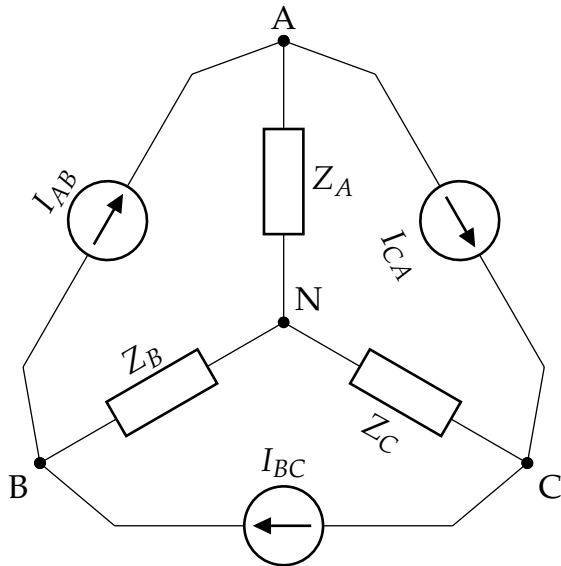
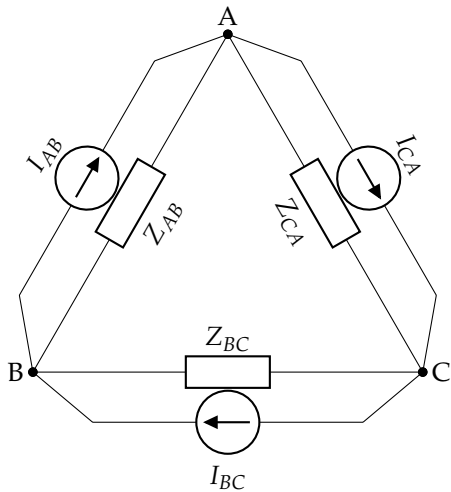
Triângulo a Estrela



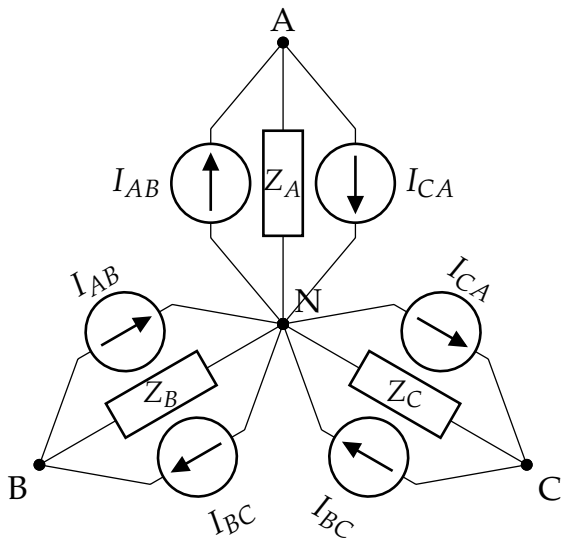
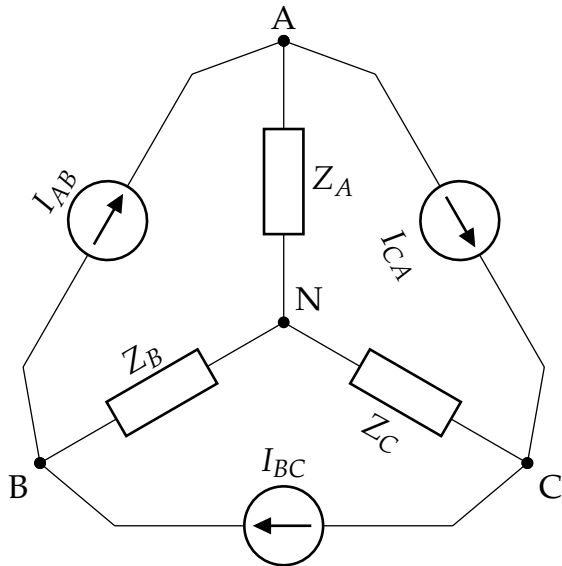
Transformamos fuentes



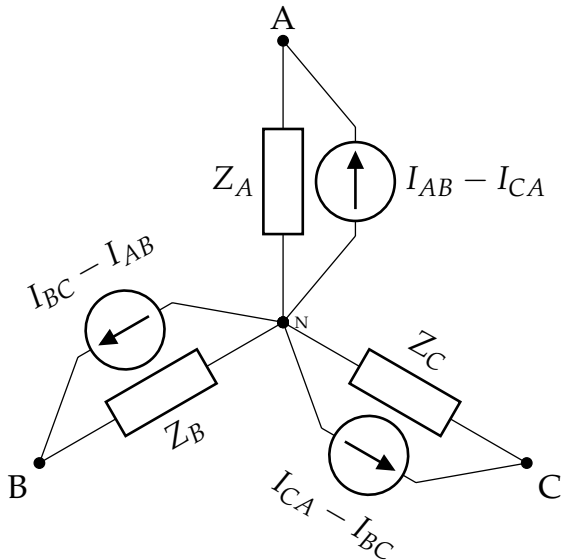
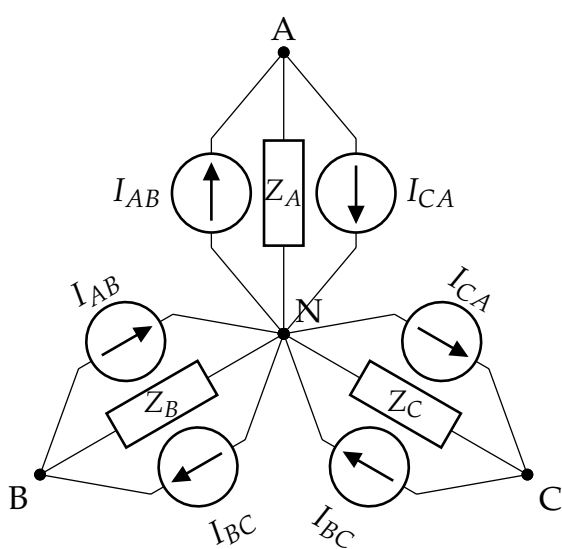
Transformamos impedancias



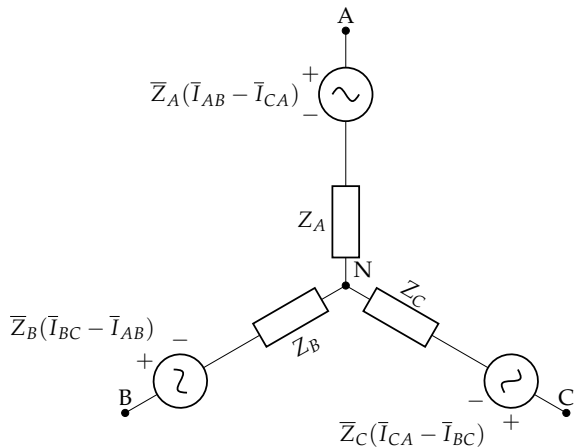
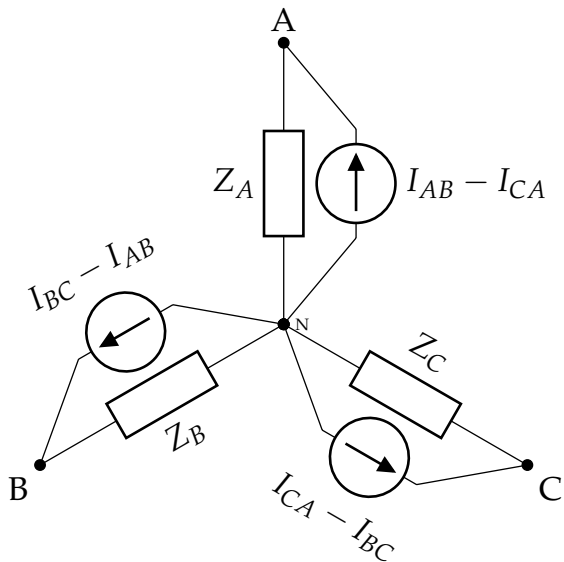
Aplicamos movilidad de fuentes



Asociamos fuentes

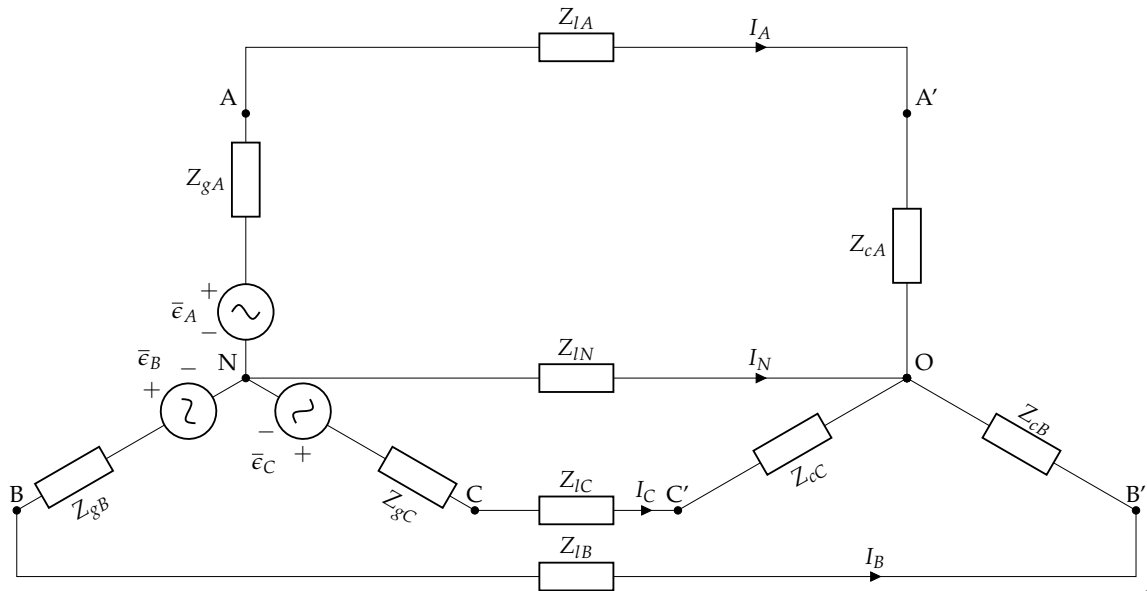


Transformamos fuentes

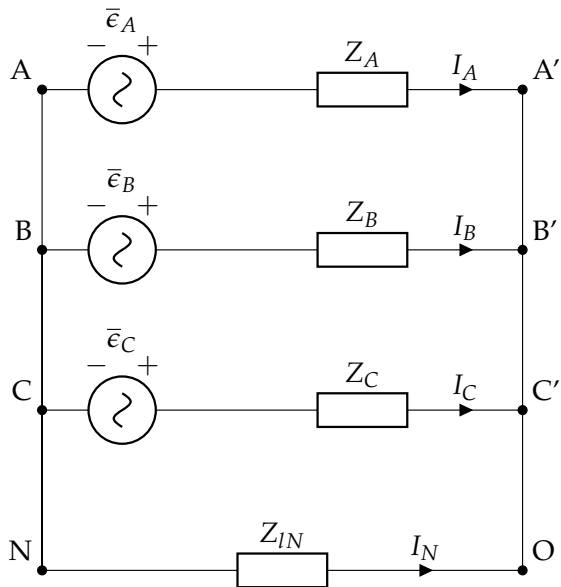


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Planteamiento del sistema



Agrupamos impedancias de generador, línea y receptor

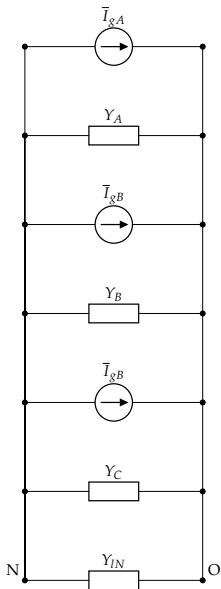


$$\bar{Z}_A = \bar{Z}_{gA} + \bar{Z}_{lA} + \bar{Z}_{cA}$$

$$\bar{Z}_B = \bar{Z}_{gB} + \bar{Z}_{lB} + \bar{Z}_{cB}$$

$$\bar{Z}_C = \bar{Z}_{gC} + \bar{Z}_{lC} + \bar{Z}_{cC}$$

Conversión de fuentes



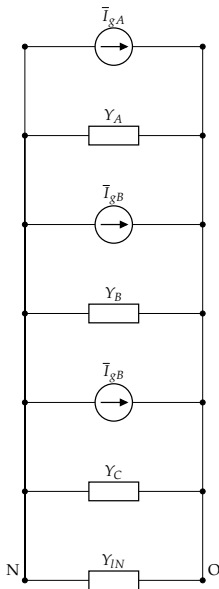
$$\bar{I}_{gA} = \bar{\epsilon}_A \cdot \bar{Y}_A$$

$$\bar{I}_{gB} = \bar{\epsilon}_B \cdot \bar{Y}_B$$

$$\bar{I}_{gC} = \bar{\epsilon}_C \cdot \bar{Y}_C$$

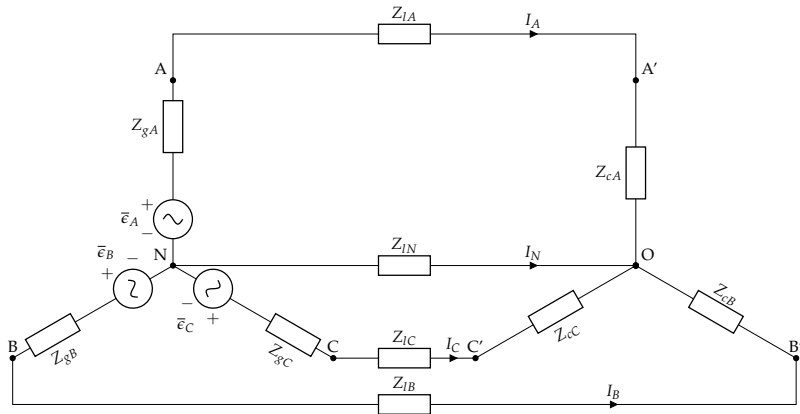
$$\bar{U}_{ON} = \frac{\bar{I}_{gA} + \bar{I}_{gB} + \bar{I}_{gC}}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C + \bar{Y}_{IN}}$$

Tensión de desplazamiento del neutro



$$\bar{U}_{ON} = \frac{\bar{e}_{gA} \cdot \bar{Y}_A + \bar{e}_{gB} \cdot \bar{Y}_B + \bar{e}_{gC} \cdot \bar{Y}_C}{\bar{Y}_A + \bar{Y}_B + \bar{Y}_C + \bar{Y}_{IN}}$$

Cálculo de corrientes



$$\bar{I}_A = \frac{\bar{e}_A - \bar{U}_{ON}}{\bar{Z}_{gA} + \bar{Z}_{lA} + \bar{Z}_{cA}}$$

$$\bar{I}_B = \frac{\bar{e}_B - \bar{U}_{ON}}{\bar{Z}_{gB} + \bar{Z}_{lB} + \bar{Z}_{cB}}$$

$$\bar{I}_C = \frac{\bar{e}_C - \bar{U}_{ON}}{\bar{Z}_{gC} + \bar{Z}_{lC} + \bar{Z}_{cC}}$$

$$\bar{I}_N = -\bar{I}_A - \bar{I}_B - \bar{I}_C$$