Repaso de trigonometría y números complejos

1. Trigonometría

Ecuación fundamental

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

Cuadratura

$$\sin(\theta + \pi/2) = \cos(\theta) \qquad \sin(\theta - \pi/2) = -\cos(\theta)$$
$$\cos(\theta + \pi/2) = -\sin(\theta) \qquad \cos(\theta - \pi/2) = \sin(\theta)$$

Suma y resta de ángulos

$$\begin{aligned} \cos(\alpha - \beta) &= \cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta) \\ \cos(\alpha + \beta) &= \cos(\alpha) \cdot \cos(\beta) - \sin(\alpha) \cdot \sin(\beta) \\ \sin(\alpha - \beta) &= \sin(\alpha) \cdot \cos(\beta) - \cos(\alpha) \cdot \sin(\beta) \\ \sin(\alpha + \beta) &= \sin(\alpha) \cdot \cos(\beta) + \cos(\alpha) \cdot \sin(\beta) \end{aligned}$$

Productos y ángulo doble

$$\cos(\alpha) \cdot \cos(\beta) = \frac{1}{2} \cdot [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$
$$\cos(2\alpha) = 2 \cdot \cos^2(\alpha) - 1$$
$$\cos(2\alpha) = 1 - 2 \cdot \sin^2(\alpha)$$
$$\sin(2\alpha) = 2 \cdot \sin(\alpha) \cdot \cos(\alpha)$$

Derivadas e integrales

$$\frac{\mathrm{d}\sin(\omega t + \theta)}{\mathrm{d}t} = \omega \cdot \cos(\omega t + \theta)$$

$$\frac{\mathrm{d}\cos(\omega t + \theta)}{\mathrm{d}t} = -\omega \cdot \sin(\omega t + \theta)$$

$$\int \sin(\omega t + \theta) \, \mathrm{d}t = -\frac{1}{\omega} \cdot \cos(\omega t + \theta) + k$$

$$\int \cos(\omega t + \theta) \, \mathrm{d}t = \frac{1}{\omega} \cdot \sin(\omega t + \theta) + k$$

Aprovechando las relaciones de cuadratura, podemos comprobar que las derivadas adelantan $\pi/2$:

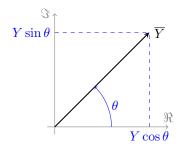
$$\frac{\mathrm{d}\sin(\omega t + \theta)}{\mathrm{d}t} = \omega \cdot \sin(\omega t + \theta + \pi/2)$$
$$\frac{\mathrm{d}\cos(\omega t + \theta)}{\mathrm{d}t} = \omega \cdot \cos(\omega t + \theta + \pi/2)$$

Y las integrales <u>retrasan</u> $\pi/2$:

$$\int \sin(\omega t + \theta) dt = \frac{1}{\omega} \cdot \sin(\omega t + \theta - \pi/2) + k$$
$$\int \cos(\omega t + \theta) dt = \frac{1}{\omega} \cdot \cos(\omega t + \theta - \pi/2) + k$$

2. Números complejos

Definición



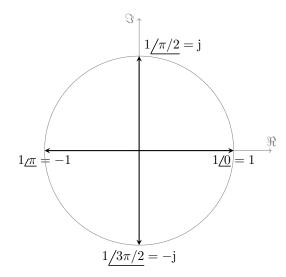
$$\overline{Y} = Y \cdot e^{j\theta}$$
 $\overline{Y} = Y \cdot [\cos(\theta) + j \cdot \sin(\theta)]$
= $Y/\underline{\theta}$ = $a_Y + j b_Y$

$$\begin{aligned} |\overline{Y}| &= \sqrt{a_Y^2 + b_Y^2} \\ &= \sqrt{Y^2 \cos^2(\theta) + Y^2 \sin^2(\theta)} \\ &= Y \end{aligned}$$

Fórmula de Euler

$$e^{j\theta} = \cos(\theta) + j \cdot \sin(\theta)$$

Complejo unitario



$$1/0 = e^{j0} = 1$$

$$1/\pi/2 = e^{j\pi/2} = j$$

$$1/\pi = e^{j\pi} = -1$$

$$1/3\pi/2 = e^{j3\pi/2} = -j$$

$$j^{2} = e^{j \pi/2} \cdot e^{j \pi/2}$$
 $= e^{j \pi}$
 $= -1$
 $1/j = 1/e^{j \pi/2}$
 $= e^{-j \pi/2}$
 $= e^{j 3\pi/2}$
 $= -j$

Operaciones

$$\begin{split} \overline{Y} + \overline{Z} &= \left[Y \cos(\theta_Y) + Z \cos(\theta_Z) \right] + j \cdot \left[Y \sin(\theta_Y) + Z \sin(\theta_Z) \right] \\ \overline{Y} - \overline{Z} &= \left[Y \cos(\theta_Y) - Z \cos(\theta_Z) \right] + j \cdot \left[Y \sin(\theta_Y) - Z \sin(\theta_Z) \right] \end{split}$$

$$\overline{Y} \cdot \overline{Z} = (Y \cdot Z) \cdot e^{\theta_Y + \theta_Z}$$
$$= (Y \cdot Z) / \theta_Y + \theta_Z$$
$$\overline{Y}^2 = Y^2 / 2\theta_Y$$

$$\frac{\overline{Y}}{\overline{Z}} = \frac{Y}{Z} \cdot e^{\theta_Y - \theta_Z}$$
$$= \frac{Y}{Z} / \theta_Y - \theta_Z$$

Conjugado

$$\overline{Y}^* = Y \cdot e^{-j\theta}$$

$$= Y / -\theta$$

$$= Y \cdot [\cos(\theta) - j \cdot \sin(\theta)]$$

$$= a_Y - j b_Y$$

$$\overline{Y}\cdot\overline{Y}^*=|Y|^2$$