

Sistemas Trifásicos

Oscar Perpiñán Lamigueiro

① Introducción

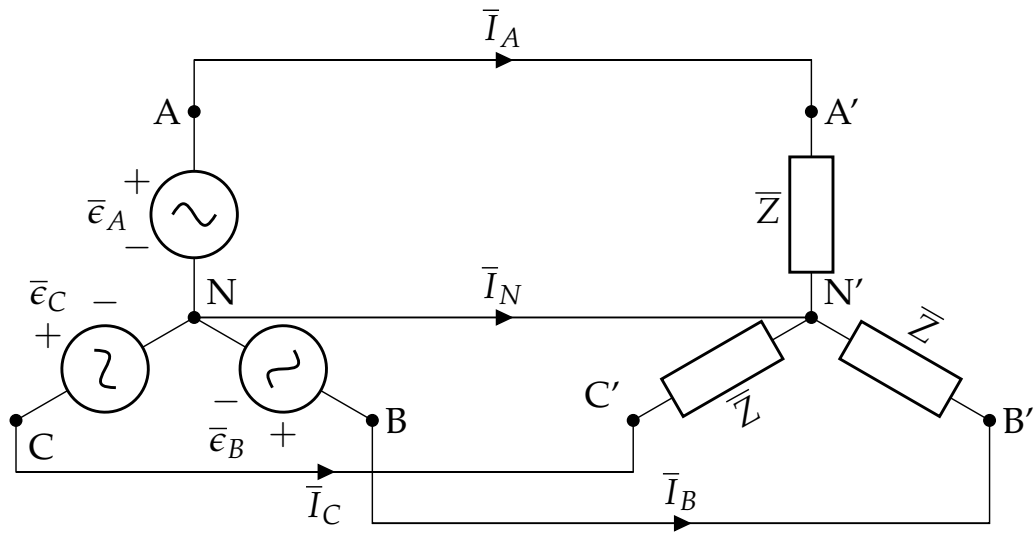
② Generadores

③ Receptores

④ Potencia en Sistemas Trifásicos

⑤ Compensación de Reactiva

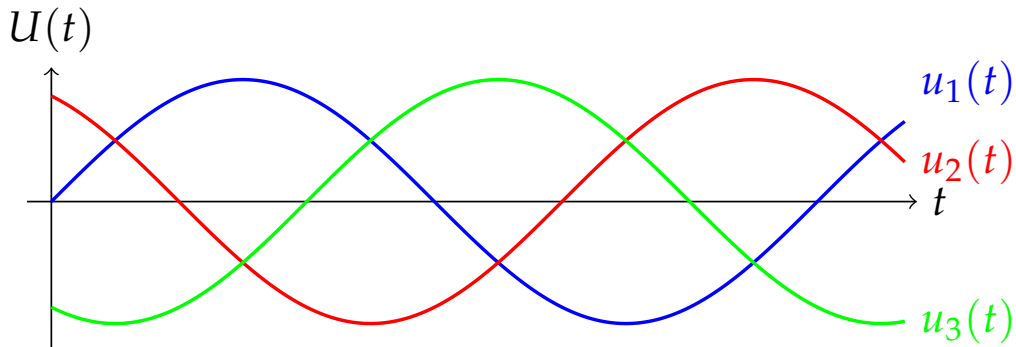
⑥ Medida de Potencia en Sistemas Trifásicos



Motivación de los sistemas trifásicos

- ▶ En un sistema trifásico la potencia instantánea es constante, evitando vibraciones y esfuerzos en las máquinas. (*La potencia instantánea de un sistema monofásico es pulsante.*)
- ▶ La masa de conductor necesaria en un sistema trifásico es un 25% inferior que en un monofásico para transportar la misma potencia.

Ondas Trifásicas

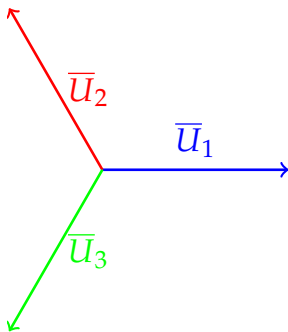


$$u_1(t) = U_0 \cos(\omega t)$$

$$u_2(t) = U_0 \cos(\omega t + 2\pi/3)$$

$$u_3(t) = U_0 \cos(\omega t - 2\pi/3)$$

Fasores de un sistema trifásico



$$\bar{U}_1 = U \angle 0$$

$$\bar{U}_2 = U \angle 2\pi/3$$

$$\bar{U}_3 = U \angle -2\pi/3$$

① Introducción

② Generadores

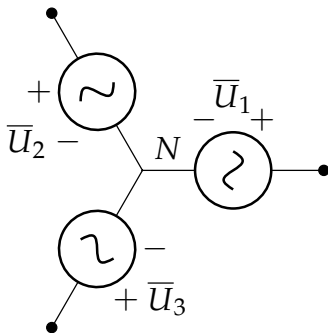
③ Receptores

④ Potencia en Sistemas Trifásicos

⑤ Compensación de Reactiva

⑥ Medida de Potencia en Sistemas Trifásicos

Conexión



$$u_1(t) = U_0 \cos(\omega t)$$

$$u_2(t) = U_0 \cos(\omega t + 2\pi/3)$$

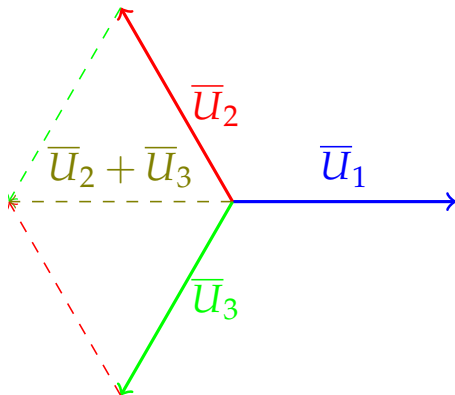
$$u_3(t) = U_0 \cos(\omega t - 2\pi/3)$$

$$\bar{u}_1 = U/0$$

$$\bar{u}_2 = U/2\pi/3$$

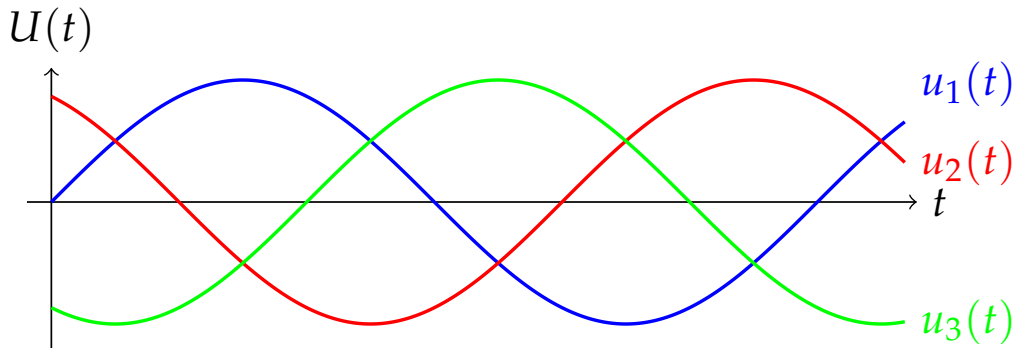
$$\bar{u}_3 = U/-2\pi/3$$

Las tensiones suman 0



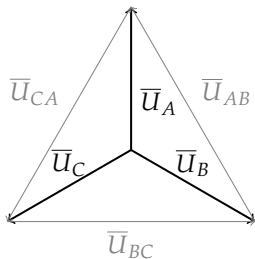
$$\overline{U}_1 + \overline{U}_2 + \overline{U}_3 = 0$$

Las tensiones suman 0



$$u_1(t) + u_2(t) + u_3(t) = 0$$

Tensiones de Fase y Línea



Tensiones de **Fase**: U_A, U_B, U_C

Tensiones de **Línea**: U_{AB}, U_{BC}, U_{CA}

$$\bar{U}_{AB} = \bar{U}_A - \bar{U}_B$$

$$\bar{U}_{BC} = \bar{U}_B - \bar{U}_C$$

$$\bar{U}_{CA} = \bar{U}_C - \bar{U}_A$$

$$\bar{U}_{AB} + \bar{U}_{BC} + \bar{U}_{CA} = 0$$

Tensiones de Fase y Línea

$$\bar{U}_A = U_f \angle \theta_f$$

$$\bar{U}_B = U_f \angle \theta_f - 120^\circ$$

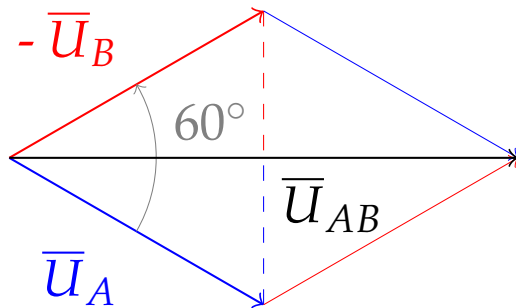
$$\bar{U}_{AB} = \bar{U}_A - \bar{U}_B =$$

$$= U_f \angle \theta_f - U_f \angle \theta_f - 120^\circ =$$

$$= U_f \angle \theta_f + U_f \angle \theta_f + 60^\circ$$

$$= 2 \cdot U_f \cdot \cos(30^\circ) \angle \theta_f + 30^\circ =$$

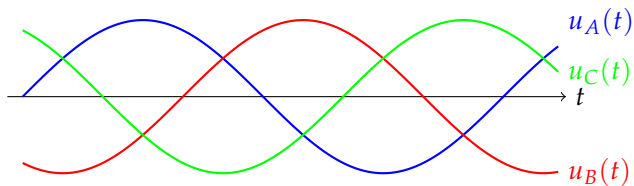
$$= \sqrt{3} U_f \angle \theta_f + 30^\circ$$



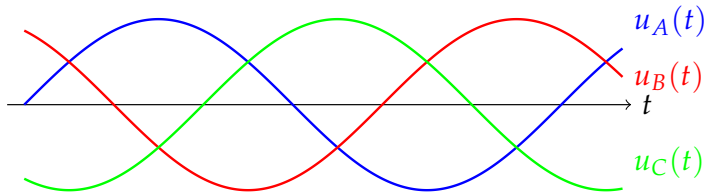
$$U = \sqrt{3} \cdot U_f$$
$$\theta_l = \theta_f + 30^\circ$$

Secuencia de Fases

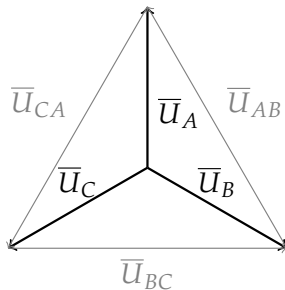
- Sentido en el que ocurren los máximos de cada fase.
- Secuencia de Fases Directa (**SFD**): ABC



- Secuencia de Fases Inversa (**SFI**): ACB



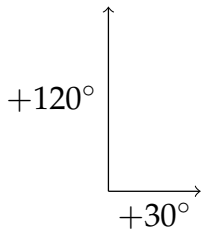
Secuencia de Fases Directa (SFD)



$$\bar{U}_A = \frac{U}{\sqrt{3}} \angle 90^\circ$$

$$\bar{U}_B = \frac{U}{\sqrt{3}} \angle -30^\circ$$

$$\bar{U}_C = \frac{U}{\sqrt{3}} \angle -150^\circ$$

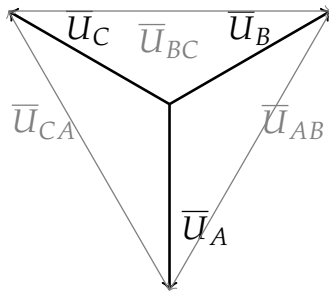


$$\bar{U}_{AB} = U \angle 120^\circ$$

$$\bar{U}_{BC} = U \angle 0^\circ$$

$$\bar{U}_{CA} = U \angle -120^\circ$$

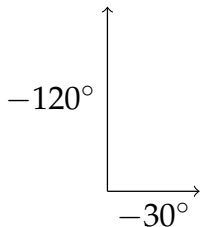
Secuencia de Fases Inversa (SFI)



$$\bar{U}_A = \frac{U}{\sqrt{3}} \angle -90^\circ$$

$$\bar{U}_B = \frac{U}{\sqrt{3}} \angle 30^\circ$$

$$\bar{U}_C = \frac{U}{\sqrt{3}} \angle 150^\circ$$



$$\bar{U}_{AB} = U \angle -120^\circ$$

$$\bar{U}_{BC} = U \angle 0^\circ$$

$$\bar{U}_{CA} = U \angle 120^\circ$$

① Introducción

② Generadores

③ Receptores

④ Potencia en Sistemas Trifásicos

⑤ Compensación de Reactiva

⑥ Medida de Potencia en Sistemas Trifásicos

Tipos de Receptores

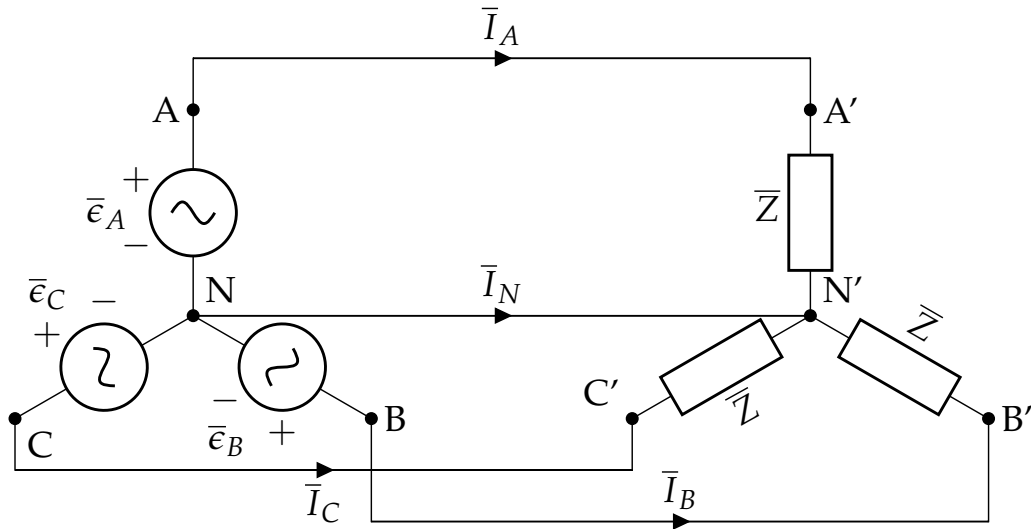
Conexión

- ▶ **Estrella** (punto común) Y
- ▶ **Triángulo** \triangle

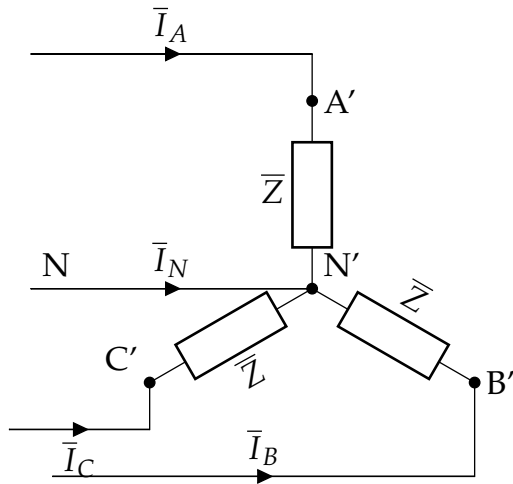
Impedancias

- ▶ **Equilibrado** (las tres impedancias son idénticas en módulo **y** fase).
- ▶ **Desequilibrado**

Receptor en Estrella Equilibrado



Receptor en Estrella Equilibrado



$$\bar{I}_A = \frac{\bar{U}_A}{\bar{Z}} = \frac{U_f}{Z} \angle \pm 90^\circ - \theta$$

$$\bar{I}_B = \frac{\bar{U}_B}{\bar{Z}} = \frac{U_f}{Z} \angle \mp 30^\circ - \theta$$

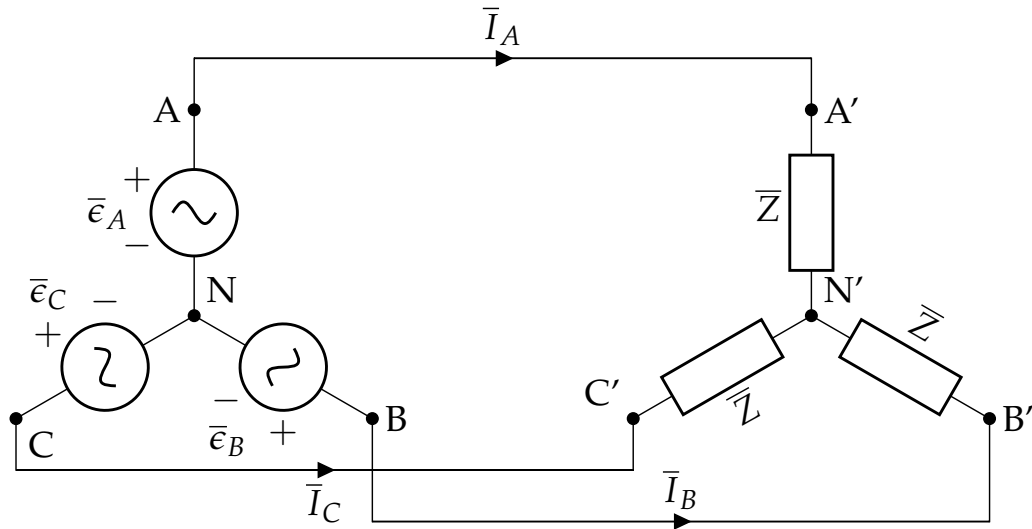
$$\bar{I}_C = \frac{\bar{U}_C}{\bar{Z}} = \frac{U_f}{Z} \angle \mp 150^\circ - \theta$$

$$|\bar{I}_A| = |\bar{I}_B| = |\bar{I}_C| = \frac{U_f}{Z}$$

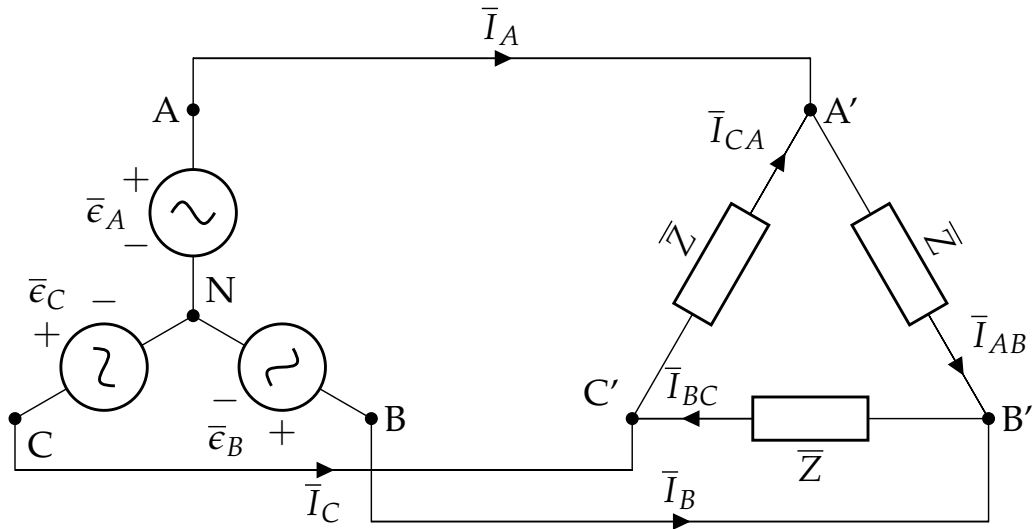
$$\bar{I}_A + \bar{I}_B + \bar{I}_C + \bar{I}_N = 0$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C = 0 \rightarrow \boxed{\bar{I}_N = 0}$$

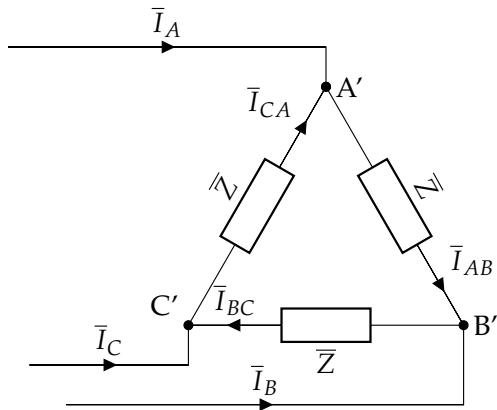
Receptor en Estrella Equilibrado



Receptor en Triángulo Equilibrado



Receptor en Triángulo Equilibrado



$$\bar{I}_{AB} = \frac{\bar{U}_{AB}}{\bar{Z}} = \frac{U}{Z} \angle \pm 120^\circ - \theta$$

$$\bar{I}_{BC} = \frac{\bar{U}_{BC}}{\bar{Z}} = \frac{U}{Z} \angle 0 - \theta$$

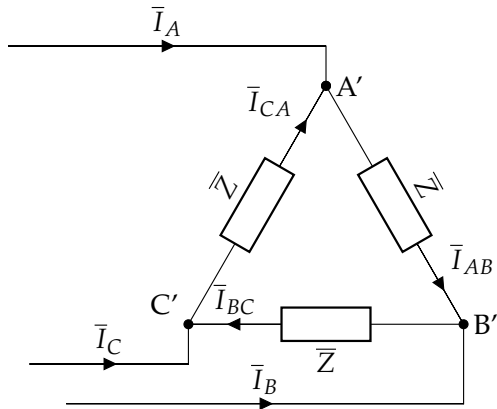
$$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}} = \frac{U}{Z} \angle \mp 120^\circ - \theta$$

$$\bar{I}_{AB} + \bar{I}_{BC} + \bar{I}_{CA} = 0$$

Corriente de Fase:

$$I_f = |\bar{I}_{AB}| = |\bar{I}_{BC}| = |\bar{I}_{CA}| = \frac{U}{Z}$$

Receptor en Triángulo Equilibrado



$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA} = \sqrt{3} \cdot \frac{U}{Z} \angle \pm 90^\circ - \theta$$

$$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB} = \sqrt{3} \cdot \frac{U}{Z} \angle \mp 30^\circ - \theta$$

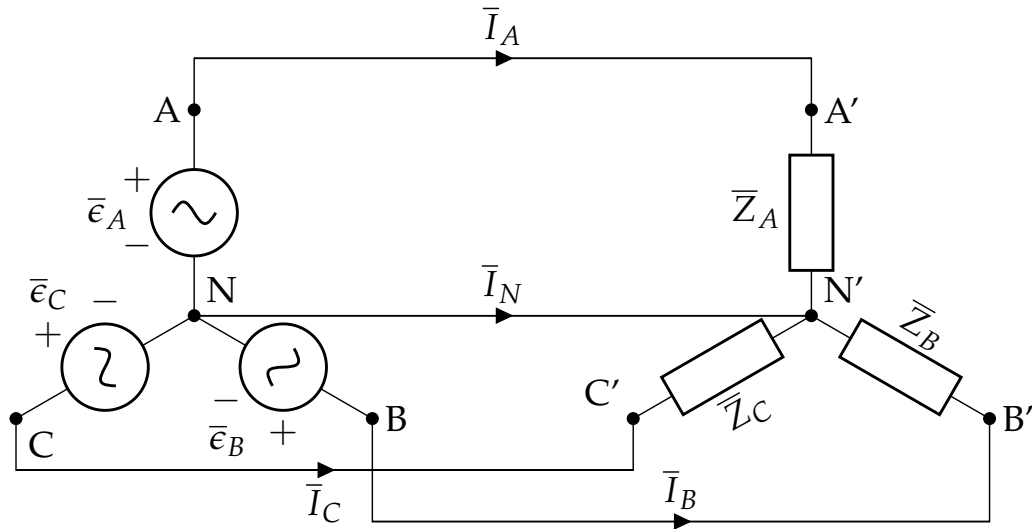
$$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC} = \sqrt{3} \cdot \frac{U}{Z} \angle \mp 150^\circ - \theta$$

Corriente de Línea:

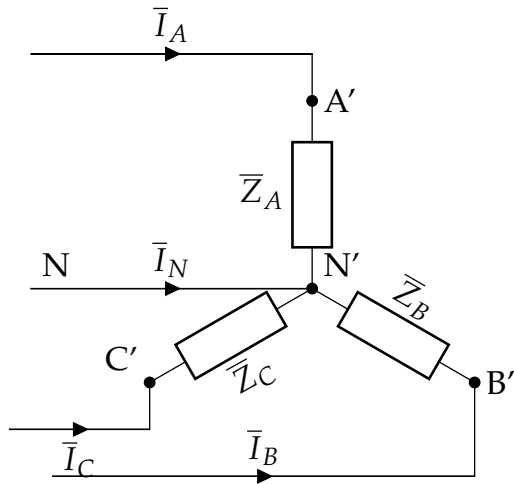
$$I = |\bar{I}_A| = |\bar{I}_B| = |\bar{I}_C| = \sqrt{3} \cdot \frac{U}{Z}$$

$$I = \sqrt{3} \cdot I_f$$

Receptor en Estrella Desequilibrado con Neutro



Receptor en Estrella Desequilibrado con Neutro



$$\bar{I}_A = \frac{\bar{U}_A}{\bar{Z}_A}$$

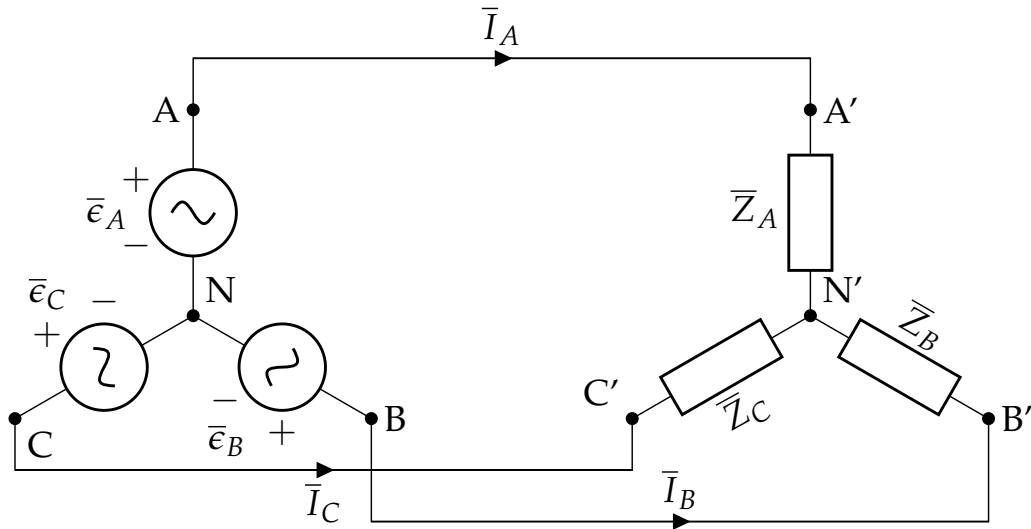
$$\bar{I}_B = \frac{\bar{U}_B}{\bar{Z}_B}$$

$$\bar{I}_C = \frac{\bar{U}_C}{\bar{Z}_C}$$

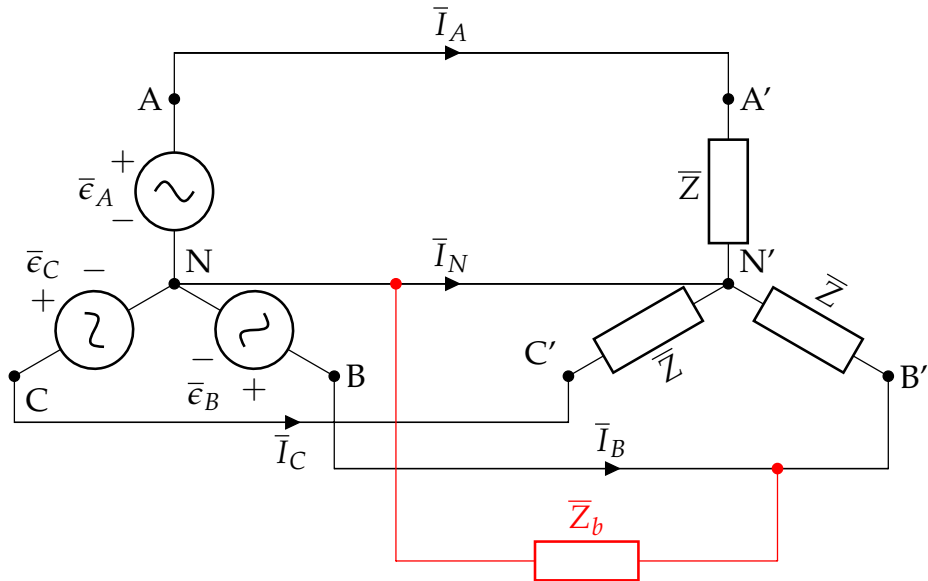
$$\bar{I}_A + \bar{I}_B + \bar{I}_C + \bar{I}_N = 0$$

$$\bar{I}_A + \bar{I}_B + \bar{I}_C \neq 0 \rightarrow \boxed{\bar{I}_N \neq 0}$$

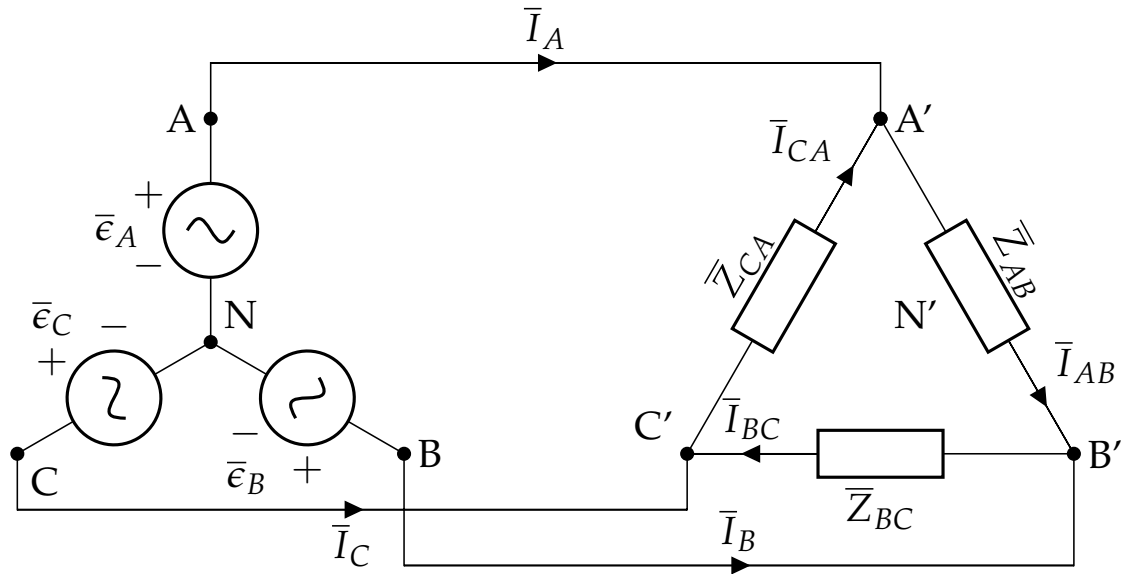
Receptor en Estrella Desequilibrado sin Neutro



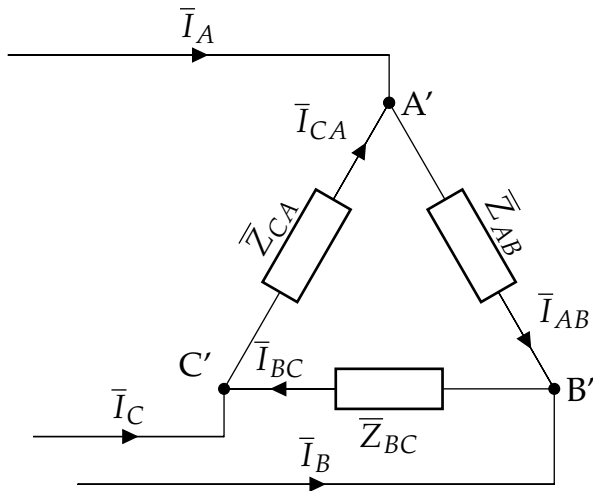
Receptor en Estrella con Carga Monofásica



Receptor en Triángulo Desequilibrado



Receptor en Triángulo Desequilibrado



$$\bar{I}_{AB} = \frac{\bar{U}_{AB}}{\bar{Z}_{AB}}$$

$$\bar{I}_{BC} = \frac{\bar{U}_{BC}}{\bar{Z}_{BC}}$$

$$\bar{I}_{CA} = \frac{\bar{U}_{CA}}{\bar{Z}_{CA}}$$

$$\bar{I}_A = \bar{I}_{AB} - \bar{I}_{CA}$$

$$\bar{I}_B = \bar{I}_{BC} - \bar{I}_{AB}$$

$$\bar{I}_C = \bar{I}_{CA} - \bar{I}_{BC}$$

- ① Introducción
- ② Generadores
- ③ Receptores
- ④ Potencia en Sistemas Trifásicos**
- ⑤ Compensación de Reactiva
- ⑥ Medida de Potencia en Sistemas Trifásicos

Potencia Instantánea en Sistemas Equilibrados

$$u_A(t) = \sqrt{2}U_f \cos(\omega t + 90^\circ)$$

$$u_B(t) = \sqrt{2}U_f \cos(\omega t - 30^\circ)$$

$$u_C(t) = \sqrt{2}U_f \cos(\omega t - 150^\circ)$$

$$i_A(t) = \sqrt{2}I_f \cos(\omega t + 90^\circ - \theta)$$

$$i_B(t) = \sqrt{2}I_f \cos(\omega t - 30^\circ - \theta)$$

$$i_C(t) = \sqrt{2}I_f \cos(\omega t - 150^\circ - \theta)$$

$$p_A(t) = u_A(t) \cdot i_A(t)$$

$$p_B(t) = u_C(t) \cdot i_B(t)$$

$$p_C(t) = u_C(t) \cdot i_C(t)$$

$$p(t) = p_A(t) + p_B(t) + p_C(t)$$

Potencia Instantánea en Sistemas Equilibrados

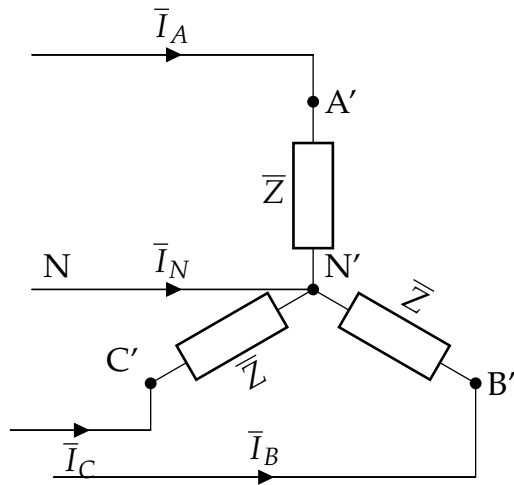
$$\begin{aligned} p(t) = & \sqrt{2}U_f \cos(\omega t + 90^\circ) \cdot \sqrt{2}I_f \cos(\omega t + 90^\circ - \theta) + \\ & + \sqrt{2}U_f \cos(\omega t - 30^\circ) \cdot \sqrt{2}I_f \cos(\omega t - 30^\circ - \theta) + \\ & + \sqrt{2}U_f \cos(\omega t - 150^\circ) \cdot \sqrt{2}I_f \cos(\omega t - 150^\circ - \theta) \end{aligned}$$

$$\cos(\alpha) \cdot \cos(\beta) = \frac{1}{2} \cdot (\cos(\alpha + \beta) + \cos(\alpha - \beta))$$

$$\begin{aligned} p(t) = & U_f I_f [\cos(2\omega t + 180^\circ - \theta) + \cos(\theta)] + \\ & + U_f I_f [\cos(2\omega t - 60^\circ - \theta) + \cos(\theta)] + \\ & + U_f I_f [\cos(2\omega t - 300^\circ - \theta) + \cos(\theta)] \end{aligned}$$

$$p(t) = 3 \cdot U_f \cdot I_f \cdot \cos(\theta) = \sqrt{3} \cdot U \cdot I \cdot \cos(\theta)$$

Receptor en Estrella Equilibrado



$$P = 3 \cdot P_Z = 3 \cdot U_Z I_Z \cos(\theta)$$

$$Q = 3 \cdot Q_Z = 3 \cdot U_Z I_Z \sin(\theta)$$

$$I_Z = I$$

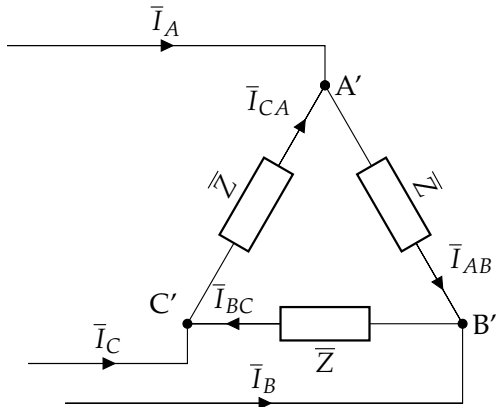
$$U_Z = U_F$$

$$P = 3U_F I \cos(\theta) = \sqrt{3}UI \cos(\theta)$$

$$Q = 3U_F I \sin(\theta) = \sqrt{3}UI \sin(\theta)$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{3}UI$$

Receptor en Triángulo Equilibrado



$$P = 3 \cdot P_Z = 3 \cdot U_Z I_Z \cos(\theta)$$

$$Q = 3 \cdot Q_Z = 3 \cdot U_Z I_Z \sin(\theta)$$

$$I_Z = I_F$$

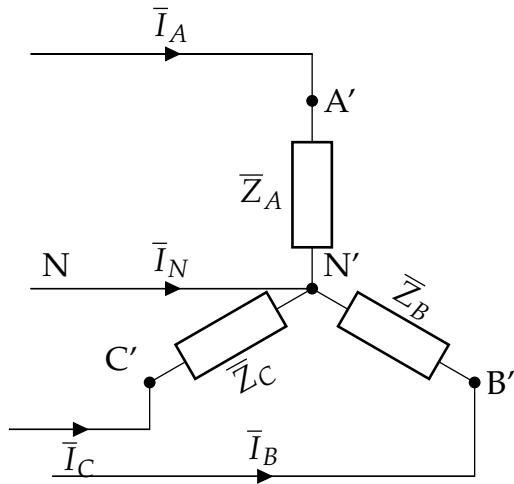
$$U_Z = U$$

$$P = 3UI_F \cos(\theta) = \sqrt{3}UI \cos(\theta)$$

$$Q = 3UI_F \sin(\theta) = \sqrt{3}UI \sin(\theta)$$

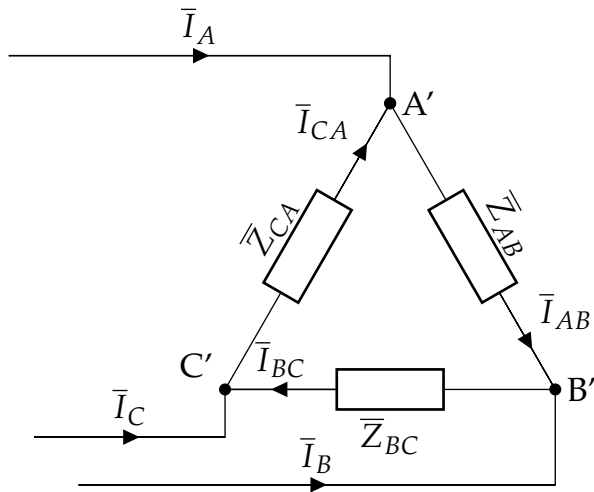
$$S = \sqrt{P^2 + Q^2} = \sqrt{3}UI$$

Receptor en Estrella Desequilibrado



$$P = P_A + P_B + P_C$$
$$Q = Q_A + Q_B + Q_C$$
$$\bar{S} = P + jQ$$

Receptor en Triángulo Desequilibrado



$$P = P_{AB} + P_{BC} + P_{CA}$$

$$Q = Q_{AB} + Q_{BC} + Q_{CA}$$

$$\bar{S} = P + jQ$$

Comparativa Monofásica-Trifásica

Comparemos un sistema monofásico y un sistema trifásico (3H) que transmiten la **misma potencia activa** y funcionan a la **misma tensión entre líneas**.

$$UI_1 \cos \theta = P_1 = P_3 = \sqrt{3}UI_3 \cos \theta \rightarrow \boxed{I_1 = \sqrt{3}I_3}$$

Las **pérdidas en la línea** deben ser **iguales** para salvar la **misma distancia**:

$$2R_1I_1^2 = P_{1l} = P_{3l} = 3R_3I_3^2$$

Sustituyendo la relación de corrientes y teniendo en cuenta la relación entre resistencia y sección:

$$2 \cdot R_1 \cdot 3I_3^2 = 3 \cdot R_3I_3^2 \rightarrow R_1 = \frac{1}{2}R_3 \rightarrow \boxed{S_1 = 2 \cdot S_3}$$

Finalmente, la relación entre masas de conductor es:

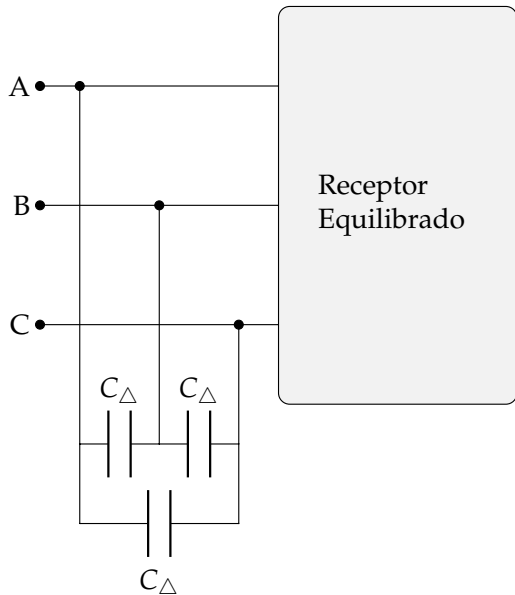
$$\frac{m_3}{m_1} = \frac{3 \cdot S_3}{2 \cdot S_1} = \boxed{\frac{3}{4}}$$

- ① Introducción
- ② Generadores
- ③ Receptores
- ④ Potencia en Sistemas Trifásicos
- ⑤ Compensación de Reactiva**
- ⑥ Medida de Potencia en Sistemas Trifásicos

Objetivo

- ▶ Sea un receptor **equilibrado inductivo** del que conocemos P , Q y, por tanto, su factor de potencia $\cos \theta$.
- ▶ Para reducir la potencia reactiva del sistema debemos instalar un **banco de condensadores** que suministrarán una potencia reactiva Q_c .
- ▶ Como **resultado**, la potencia reactiva y el factor de potencia del sistema serán $Q' = Q - Q_c$ y $\cos \theta' > \cos \theta$.
- ▶ En trifásica existen dos posibilidades:
 - ▶ Conexión en triángulo: C_{Δ}
 - ▶ Conexión en estrella: C_Y .

Conexión en Triángulo



$$Q = P \tan \theta$$

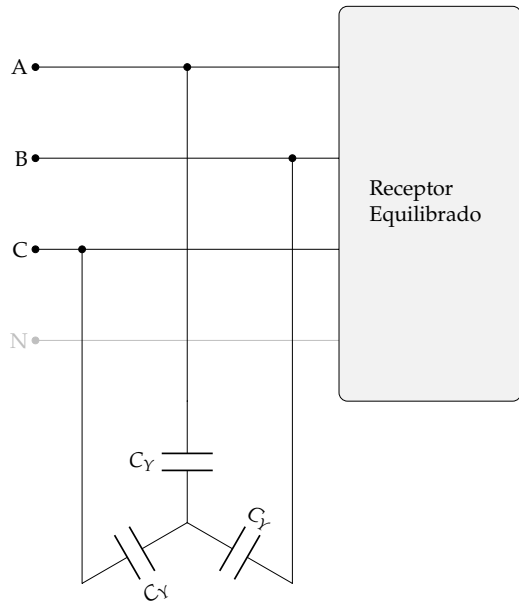
$$Q' = P \tan \theta' =$$

$$= Q - Q_c$$

$$Q_c = 3 \cdot \omega C_{\Delta} \cdot U^2$$

$$C_{\Delta} = \frac{P(\tan \theta - \tan \theta')}{3\omega U^2}$$

Conexión en Estrella



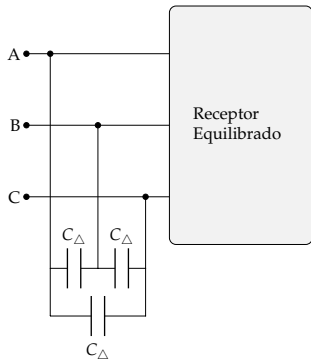
$$Q = P \tan \theta$$

$$Q' = P \tan \theta' =$$
$$= Q - Q_c$$

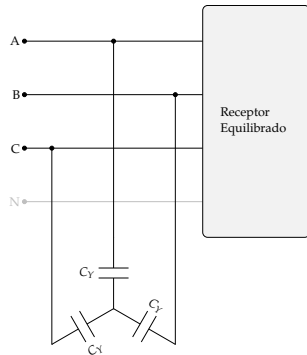
$$Q_c = 3 \cdot \omega C_Y \cdot U_f^2$$

$$C_Y = \frac{P(\tan \theta - \tan \theta')}{\omega U^2}$$

Comparación Estrella-Triángulo



$$C_{\Delta} = \frac{P(\tan \theta - \tan \theta')}{3\omega U^2}$$

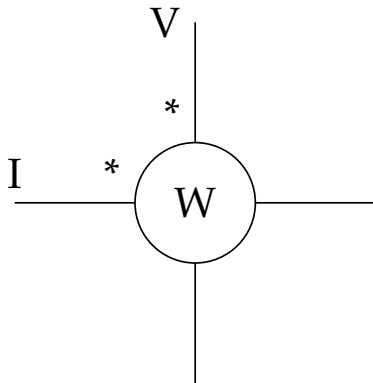


$$C_Y = \frac{P(\tan \theta - \tan \theta')}{\omega U^2}$$

Dado que $C_Y = 3 \cdot C_{\Delta}$ la **configuración recomendada** es **triángulo**.

- ① Introducción
- ② Generadores
- ③ Receptores
- ④ Potencia en Sistemas Trifásicos
- ⑤ Compensación de Reactiva
- ⑥ Medida de Potencia en Sistemas Trifásicos

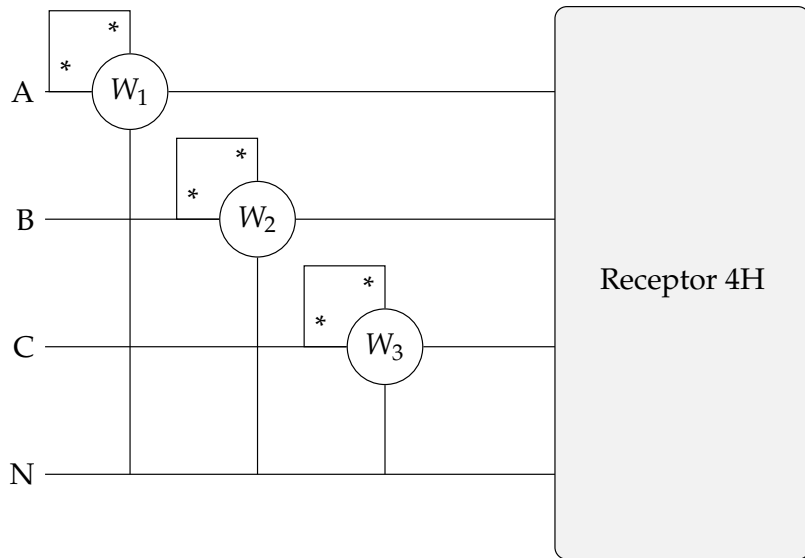
Recordatorio: vatímetro



Vatímetro: equipo de medida de 4 terminales (1 par para tensión, 1 par para corriente)

$$W = \Re(\bar{U} \cdot \bar{I}^*)$$

Sistema de 4 Hilos



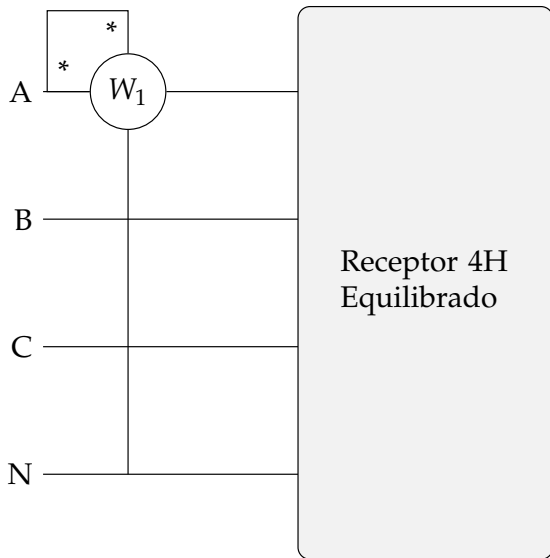
$$W_1 = \Re(\bar{U}_A \cdot \bar{I}_A^*) = P_A$$

$$W_2 = \Re(\bar{U}_B \cdot \bar{I}_B^*) = P_B$$

$$W_3 = \Re(\bar{U}_C \cdot \bar{I}_C^*) = P_C$$

$$P = W_1 + W_2 + W_3$$

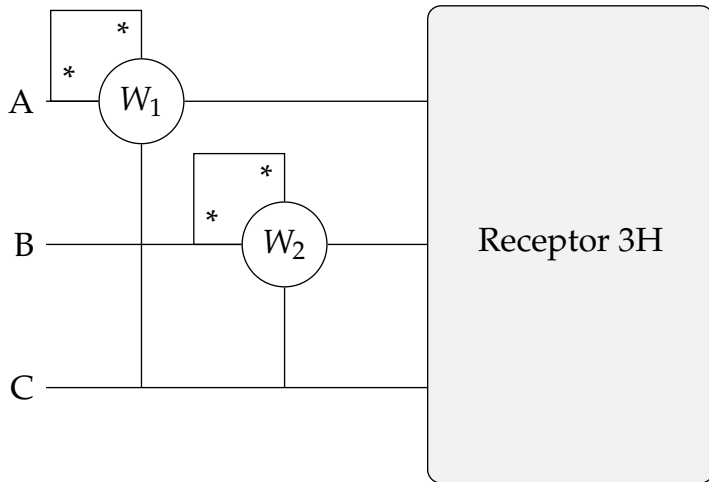
Sistema de 4 Hilos Equilibrado



$$P_A = P_B = P_C$$

$$P = 3 \cdot W_1$$

Sistema de 3 Hilos



Montaje de Aron

$$W_1 = \Re(\bar{U}_{AC} \cdot \bar{I}_A^*)$$

$$W_2 = \Re(\bar{U}_{BC} \cdot \bar{I}_B^*)$$

$$W_1 + W_2 = ?$$

Sistema de 3 Hilos

Desarrollamos las dos expresiones usando corrientes de fase y obviando el operador \Re :

$$\overline{U}_{AC} \cdot \bar{I}_A^* = \overline{U}_{AC} \cdot (\bar{I}_{AB}^* - \bar{I}_{CA}^*)$$

$$\overline{U}_{BC} \cdot \bar{I}_B^* = \overline{U}_{BC} \cdot (\bar{I}_{BC}^* - \bar{I}_{AB}^*)$$

Sumamos las dos expresiones:

$$\overline{U}_{AC} \cdot \bar{I}_A^* + \overline{U}_{BC} \cdot \bar{I}_B^* = \overline{U}_{AC} \cdot \bar{I}_{AB}^* - \overline{U}_{AC} \cdot \bar{I}_{CA}^* + \overline{U}_{BC} \cdot \bar{I}_{BC}^* - \overline{U}_{BC} \cdot \bar{I}_{AB}^*$$

Y agrupamos, teniendo en cuenta que $\overline{U}_{AB} + \overline{U}_{BC} + \overline{U}_{CA} = 0$:

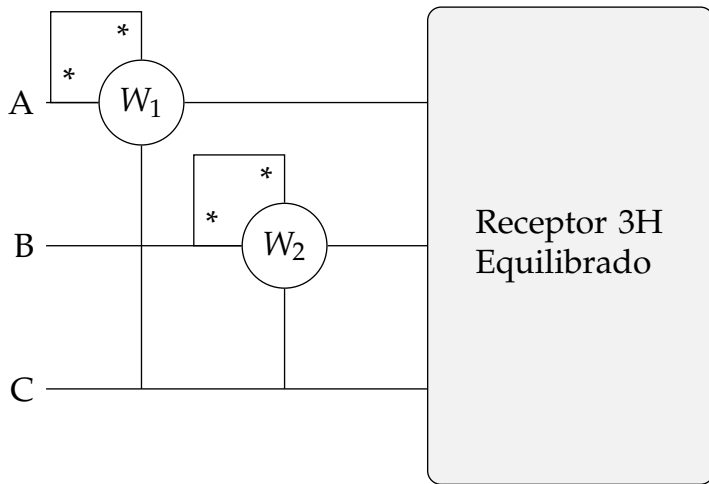
$$\overline{U}_{CA} \cdot \bar{I}_{CA}^* + \overline{U}_{BC} \cdot \bar{I}_{BC}^* + (\overline{U}_{AC} \cdot \bar{I}_{AB}^* - \overline{U}_{BC} \cdot \bar{I}_{AB}^*) = \bar{S}_{CA} + \bar{S}_{BC} + \bar{S}_{AB}$$

Extrayendo la parte real de este resultado obtenemos:

$$\Re(\bar{S}_{AB} + \bar{S}_{BC} + \bar{S}_{CA}) = P \rightarrow \boxed{W_1 + W_2 = P}$$

Sistema de 3 Hilos Equilibrado

Cuando el sistema es equilibrado, las lecturas individuales de los vatímetros tienen significado.



$$W_1 = \Re(\bar{U}_{AC} \cdot \bar{I}_A^*) = ?$$

$$W_2 = \Re(\bar{U}_{BC} \cdot \bar{I}_B^*) = ?$$

Sistema de 3 Hilos Equilibrado

Supongamos SFD:

$$\bar{U}_{AC} = -\bar{U}_{CA} = U/\underline{-120^\circ + 180^\circ} = U/\underline{60^\circ}$$

$$\bar{I}_A = I/\underline{90^\circ - \theta}$$

$$\bar{U}_{AC} \cdot \bar{I}_A^* = UI/\underline{\theta - 30^\circ} \rightarrow \boxed{W_1 = UI \cos(\theta - 30^\circ)}$$

$$\bar{U}_{BC} = U/\underline{0}$$

$$\bar{I}_B = I/\underline{-30^\circ - \theta}$$

$$\bar{U}_{BC} \cdot \bar{I}_B^* = UI/\underline{\theta + 30^\circ} \rightarrow \boxed{W_2 = UI \cos(\theta + 30^\circ)}$$

Sistema de 3 Hilos Equilibrado

Desarrollamos los dos cosenos:

$$\cos(30^\circ - \theta) = \cos 30^\circ \cos \theta + \sin 30^\circ \sin \theta$$

$$\cos(30^\circ + \theta) = \cos 30^\circ \cos \theta - \sin 30^\circ \sin \theta$$

Si sumamos obtenemos la potencia activa:

$$W_1 + W_2 = \sqrt{3}UI \cos \theta = P$$

Si restamos obtenemos la potencia reactiva (salvo un factor):

$$W_1 - W_2 = UI \sin \theta = \frac{Q}{\sqrt{3}}$$

Por tanto, también podemos calcular el ángulo del receptor:

$$\tan \theta = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2}$$

Sistema de 3 Hilos Equilibrado

Repetimos el desarrollo con SFI:

$$\bar{U}_{AC} = -\bar{U}_{CA} = U/\underline{120^\circ + 180^\circ} = U/\underline{-60^\circ}$$

$$\bar{I}_A = I/\underline{-90^\circ - \theta}$$

$$\bar{U}_{AC} \cdot \bar{I}_A^* = UI/\underline{\theta + 30^\circ} \rightarrow \boxed{W_1 = UI \cos(\theta + 30^\circ)}$$

$$\bar{U}_{BC} = U/\underline{0}$$

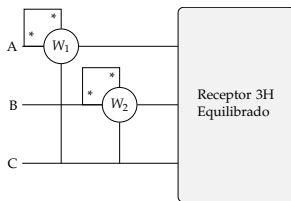
$$\bar{I}_B = I/\underline{30^\circ - \theta}$$

$$\bar{U}_{BC} \cdot \bar{I}_B^* = UI/\underline{\theta - 30^\circ} \rightarrow \boxed{W_2 = UI \cos(\theta - 30^\circ)}$$

$$\boxed{W_1 + W_2 = \sqrt{3}UI \cos \theta = P}$$

$$\boxed{W_1 - W_2 = -UI \sin \theta = -\frac{Q}{\sqrt{3}}}$$

Sistema de 3 Hilos Equilibrado



SFD

$$W_1 = UI \cos(\theta - 30^\circ)$$

$$W_2 = UI \cos(\theta + 30^\circ)$$

$$P = W_1 + W_2$$

$$Q = \sqrt{3}(W_1 - W_2)$$

$$\tan \theta = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2}$$

SFI

$$W_1 = UI \cos(\theta + 30^\circ)$$

$$W_2 = UI \cos(\theta - 30^\circ)$$

$$P = W_1 + W_2$$

$$Q = \sqrt{3}(W_2 - W_1)$$

$$\tan \theta = \sqrt{3} \frac{W_2 - W_1}{W_1 + W_2}$$

Otras conexiones: 3H SFD

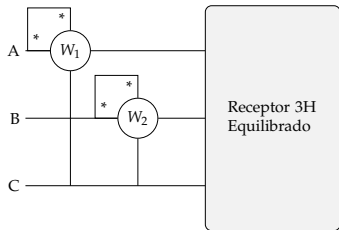
$$(ABC) :: A \triangleright B \triangleright C \implies \{AB, BC, CA\}$$

$$W_1 = UI \cos(\theta - 30^\circ)$$

$$P = W_1 + W_2$$

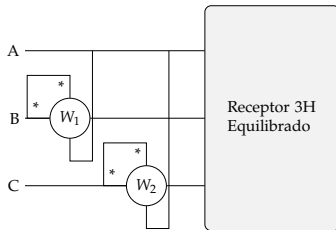
$$W_2 = UI \cos(\theta + 30^\circ)$$

$$Q = \sqrt{3}(W_1 - W_2)$$



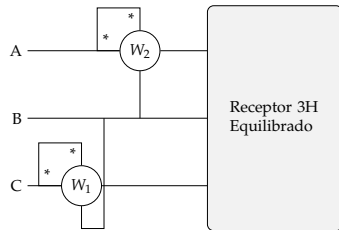
$W_1 : AC \notin SFD$

$W_2 : BC \in SFD$



$W_1 : BA \notin SFD$

$W_2 : CA \in SFD$



$W_1 : CB \notin SFD$

$W_2 : AB \in SFD$

Otras conexiones: 3H SFI

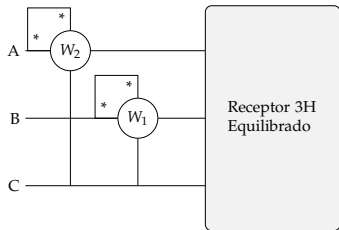
$$(ACB) :: A \triangleright C \triangleright B \implies \{AC, CB, BA\}$$

$$W_1 = UI \cos(\theta - 30^\circ)$$

$$P = W_1 + W_2$$

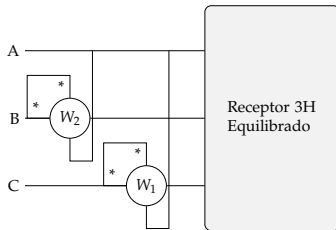
$$W_2 = UI \cos(\theta + 30^\circ)$$

$$Q = \sqrt{3}(W_1 - W_2)$$



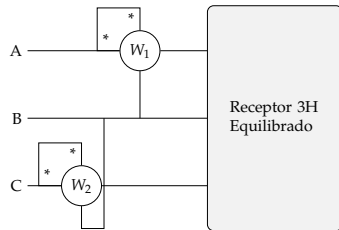
$W_1 : BC \notin SFI$

$W_2 : AC \in SFI$



$W_1 : CA \notin SFI$

$W_2 : BA \in SFI$



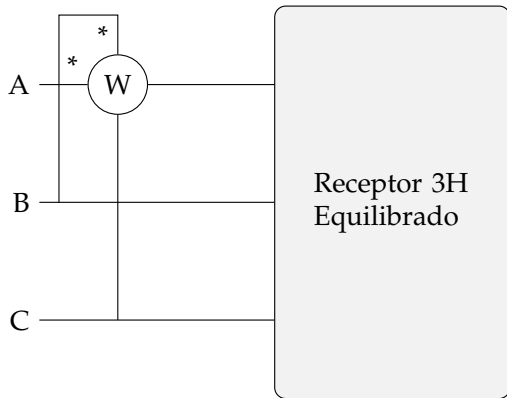
$W_1 : AB \notin SFI$

$W_2 : CB \in SFI$

Medida de Reactiva con un Vatímetro

Cuando el sistema está equilibrado, es posible medir la potencia reactiva con un único vatímetro.

Supongamos **SFD**:



$$W = \Re(\bar{U}_{BC} \cdot \bar{I}_A^*)$$

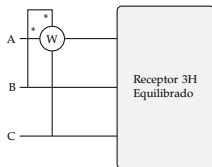
$$\bar{U}_{BC} = U \angle 0$$

$$\bar{I}_A = I \angle 90^\circ - \theta$$

$$\begin{aligned} W &= \Re(U I \angle \theta - 90^\circ) = \\ &= UI \sin(\theta) \end{aligned}$$

$$W = \frac{Q}{\sqrt{3}}$$

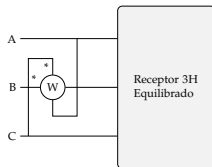
Conexiones para medida de reactiva



$$W = \Re(\bar{U}_{BC} \cdot \bar{I}_A^*)$$

$BC \in SFD$

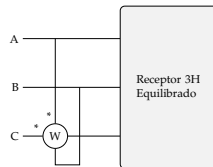
$BC \notin SFI$



$$W = \Re(\bar{U}_{CA} \cdot \bar{I}_B^*)$$

$CA \in SFD$

$CA \notin SFI$



$$W = \Re(\bar{U}_{AB} \cdot \bar{I}_C^*)$$

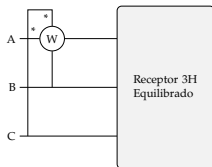
$AB \in SFD$

$AB \notin SFI$

$$SFD \rightarrow \boxed{W = \frac{Q}{\sqrt{3}}}$$

$$SFI \rightarrow \boxed{W = -\frac{Q}{\sqrt{3}}}$$

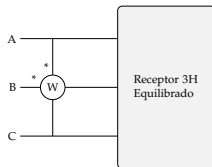
Conexiones para medida de reactiva



$$W = \Re(\bar{U}_{CB} \cdot \bar{I}_A^*)$$

$CB \notin SFD$

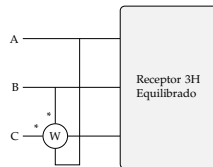
$CB \in SFI$



$$W = \Re(\bar{U}_{AC} \cdot \bar{I}_B^*)$$

$AC \notin SFD$

$AC \in SFI$



$$W = \Re(\bar{U}_{BA} \cdot \bar{I}_C^*)$$

$BA \notin SFD$

$BA \in SFI$

$$SFD \rightarrow \boxed{W = -\frac{Q}{\sqrt{3}}}$$

$$SFI \rightarrow \boxed{W = \frac{Q}{\sqrt{3}}}$$