Kilonova Simulations

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1 Introduction

The purpose of this project is to create a simulation of a kilonova using Smooth-Particle-Hydrodynamics (SPH) methods. Using SPH is advantageous because each particle stores location and velocity data, conveniently handling particles moving quickly and spanning large scales without the complications of modifying mesh sizing as the problem scales with the merger. Additionally, we will provide the corresponding luminosity and characteristic timescale for the kilonova. Areas of interest include different cases such as circular orbit from differing mass, from equal mass merger and fly-by.

1.1 Background

Neutron Star(NS) mergers are some of the most energetic observable phenomena in the universe. The inspiral and merger of two neutron stars or a NS with a black hole is capable of producing a bright emission of electromagnetic radiation known as a kilonova. The concept of a thermal, electromagnetic emission being associated with a binary neutron star or neutron star with black hole collision was introduced in 1998 by Li and Paczyński [5]. They were later referred to as "kilonova" in 2010 by Metzger, et al who also demonstrated that the bright, supernova-like, emission is powered by r-process nucleosynthesis [6]. Meanwhile, the first detected kilonova was associated with GRB 130603B [10].

Kilonova events are also fascinating due to their multi-messenger potential, with GRB170817/GW170817 being the first such multi-messenger event [1]. Gravitational waves can be produced by the merger of two very massive, compact objects, such as neutron stars and black holes. Since Gw170817, searches have continued to look for more such multi-messenger events. In studying these systems, we can further narrow down the equation of state for neutron stars and better understand the dynamics under intense conditions. The equation of state for a neutron star still needs to be further constrained through observation and study. In this work, we will assume an equation of state.

For the purposes of a model, we differentiate between neutron stars ($1.18M_\odot \leq m_{ns} \leq 1.97M_\odot$) and black holes ($5M_\odot \leq m_{bh}$) by masses where $M_\odot$ is solar mass, m_{ns} is a neutron star's mass, and m_{bh} is the mass of a black hole[1]. We consider an upper limit to m_{bh}

[1]The jump between roughly 2 to 5 solar masses is known as a lower mass gap. Observations from the LIGO/Virgo/KAGRA collaboration has indicated that some compact binary collisions may form black holes within this range [4]. One example involving a mass gap object is GW190814 [2].

of $100M_\odot$.

Over the years, others have modeled kilonova after adopting a neutron star equation of state and adjusting what is included in their simulation for the given masses of two neutron stars. However, scaling in neutron star merger simulations is generally difficult due to order-of-magnitude changes in the size of the system before, during, and after the merger event.

2 Simulation

2.1 Setup

As two Neutron Stars spiral around each other, they form a small circular orbit[2]. Each orbit has a radii on the order of a few tens to a few hundreds of kilometers[3]. After the merger, the hot, densely packed material expands in a catastrophic explosion, growing to a size comparable to a solar system. This scaling issue poses a significant challenge to scientists simulating neutron star mergers.

To model the dynamics of the merger, we will assume spherical motion and adapt a relativistic model of the energy density. A value for central density 1665.3 (density at r = 0) was used.

Line element

$$ds^2 = -\alpha^2 dt^2 + \lambda_{ij}(dx^i + \beta^i dt)(dx^j \beta^j dt) \tag{1}$$

$$v^\mu = \frac{dx^\mu}{d\tau}\frac{d\tau}{dt} = \frac{U^\mu}{L} \tag{2}$$

Local energy density is

$$\rho = \rho_{rest} + u\rho_{rest}/c^2 = nm_o c^2(1 + u/c^2) \tag{3}$$

2.2 Computational methods

To handle the aforementioned scaling issue from section 1.1, we implement a Smooth-Particle-Hydrodynamics based approach to modeling the hydrodynamics of the merger rather than a mesh-based approach. As a preliminary test, we decided to model a star using SPH as a test-run for the merger simulation.

The first successful model of a Neutron Star merger using SPH methods was done more than two decades ago [10]. The advantages of using SPH as opposed to a mesh-based approach include ease when solving complex boundaries, although most modern merger simulations incorporate a coupled particle and mesh-based approach [9]. SPH

[2]Note that the orbits are not necessarily constrained to be circular. Some have highly eccentric, elliptical orbits that others have modeled over the years, for example, see Gold et al [3]. For our purposes and simplicity, we will for now assume a circular orbit.

[3]For scale reference, a neutron star radius is generally on the scale of about ten km.

provides a mesh-free framework for dealing with particle dynamics without the need for mesh scaling updates, greatly simplifying the scaling issue.

2.2.1 Simulating a Single Neutron Star: Pressure and Density

To simulate a single star with pressure and density, we use a Gaussian smoothing kernel in 3D and the density is calculated to be the mass times the gradient of the smoothing function, sampled from a calculated density result. Each particle has encoded a position and velocity and acceleration can be calculated using the known density and equation of state (EOS).

As a test, we modeled a $3M_{\odot}$ star using 100, 1,000, and 4,000 particles and polytropic index 1. In Figure 2 we have a model of the single neutron star at various particle counts with Gaussian smoothing function [7] and smoothing length 1. The Newton-Raphson Method was used to determine the initial number density at r = 0 and the simulation converged after 5 iterations. Density is shown in the table at the bottom. Initial Pressure at the center can be calculated from number density:

$$P_0 = \frac{363.44 n_i^{2.54}}{\rho_s} \tag{4}$$

$$M_0 = (4\pi G^3 \rho_s)^{-0.5} \tag{5}$$

$$R = GM_0 \tag{6}$$

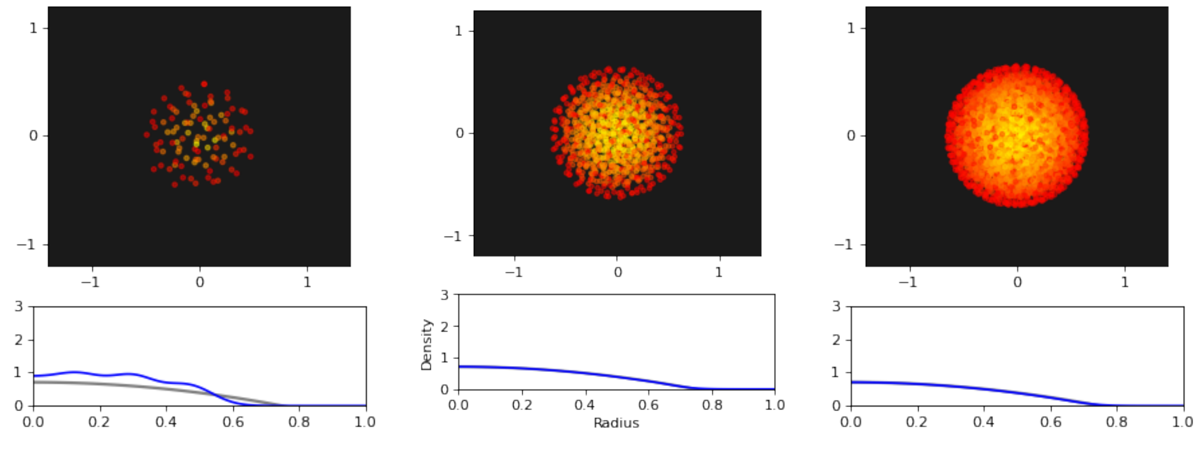

Figure 1: *A simple star model using SPH. Images show the star modeled with 100 points, 1,000 points, and 4,000 points.*

Euler's method is used to update the mass and pressure of each point. A plot of the convergence can be found in Figure 2.

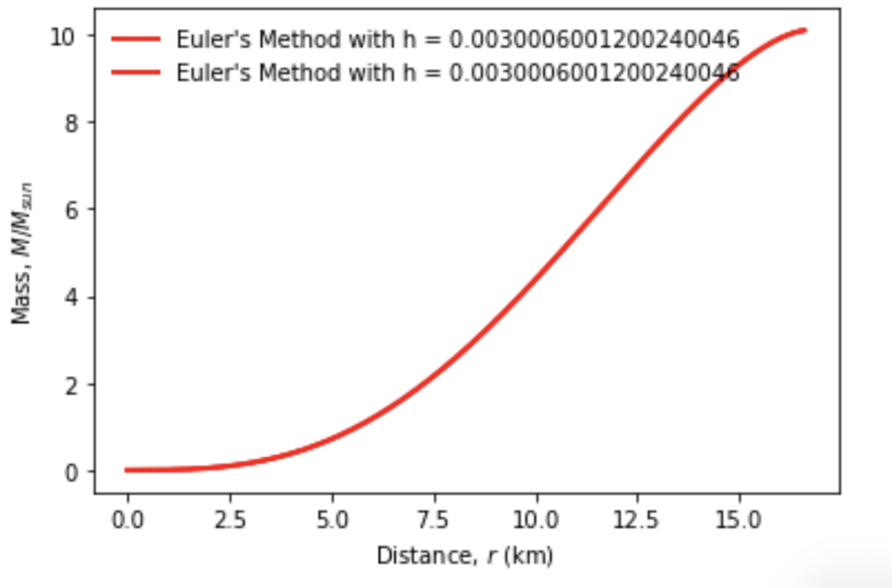

Figure 2: *Mass convergence with Euler's method. Initial density,* $\rho_s = 1665.3 MeV/fm3$*, total mass* $= 10M_\odot$*, radius of the neutron star* $= 16.55$ *km, and* $P < 9e-05$*, found after 916 runs.*

2.2.2 Simulating a Single Neutron Star: Time Iteration

To test out our ability to model a Neutron star using SPH, we implemented a basic SPH simulation to model a single star as outlined in [7].

From our initial conditions we can calculate pressure and density values for a Neutron star and incorporate them into the model. We use time iteration and update the particle motion during the simulation. Using our known pressure and density values we can calculate the acceleration for each particle, and the position and velocity (stored values) receive a kick of

$$v+ = a * dt/2 \tag{7}$$

$$x+ = v * dt/2 \tag{8}$$

during each time step. In our model, the mass converged to a value that was high for a neutron star ($10M_\odot$) so the code will need to be modified to correct for this issue. However, the preliminary data gives a procedure for simulating the density of large, compact objects. In the next checkpoint, we will modify the code to better match true Neutron Star conditions.

2.2.3 Dynamical System (Merger)

The binary orbital speed can be estimated as $\sqrt{GM/(R_A + R_B)}$ which for an equal mass merger is:

$$v_{orbital}/c = \sqrt{C} = 0.39(C/0.15)^{1/5} \tag{9}$$

$$v_r/c = 0.39(C/0.15)^{1/2} \tag{10}$$

For an equal mass merger, the dynamics is primarily dominated by orbital motion [8].

2.3 Quantities to inspect

In addition to the SPH binary neutron star modeling, we plan to provide deliverables for the system. At this time, the primary deliverable will be the peak luminosity. That is,

$$L_{peak} \approx 1.3 * 10^{30} \frac{erg}{sec} \left(\frac{\epsilon_d}{0.5}\right)^{0.5} \left(\frac{M_{ej}}{0.01 M_{\odot}}\right)^{0.35} \left(\frac{\nu}{0.01c}\right)^{0.65} \left(\frac{\kappa}{10cm^2g^{-1}}\right)^{-0.65} \tag{11}$$

[11].

Wherein we assume the ejecta and expansion is homogeneous and write the ejecta mass to be,

$$M_{ej} = \frac{4\pi}{3}\rho_d \nu t \tag{12}$$

.

Here, κ is the opacity, ρ_d is the density, ν is the expansion velocity, and ϵ_d is the energy deposition.

Similarly, we can write,

$$t_{peak} \approx \left(\frac{3\kappa M_{ej}}{4\pi c\nu}\right)^{0.5} \tag{13}$$

for the characteristic timescale [11].

Both L_{peak} (eq. 11) and t_{peak} (eq. 13) will be provided as deliverables of our work.

3 Conclusion

Kilonovae are the electromagnetic counterparts to binary neutron star mergers or neutron star/ black hole mergers. They are highly energetic and fascinating astrophysical events.

We plan to model the binary neutron star merger through SPH methods and provide deliverables such as the peak luminosity and characteristic timescale. We plan to provide a simulation for circular orbits of various mass, equal mass, and a fly-by. We hope to provide a compelling model of these events.

References

[1] Benjamin P Abbott. "Multi-messenger observations of a binary neutron star merger". In: (2017).

[2] Richard Abbott et al. "GW190814: gravitational waves from the coalescence of a 23 solar mass black hole with a 2.6 solar mass compact object". In: *The Astrophysical Journal Letters* 896.2 (2020), p. L44.

[3] Roman Gold et al. "Eccentric binary neutron star mergers". In: *Physical Review D* 86.12 (2012), p. 121501.

[4] Anuradha Gupta et al. "Black holes in the low-mass gap: Implications for gravitational-wave observations". In: *Physical Review D* 101.10 (2020), p. 103036.

[5] Li-Xin Li and Bohdan Paczyński. "Transient events from neutron star mergers". In: *The Astrophysical Journal* 507.1 (1998), p. L59.

[6] BD Metzger et al. "Electromagnetic counterparts of compact object mergers powered by the radioactive decay of r-process nuclei". In: *Monthly Notices of the Royal Astronomical Society* 406.4 (2010), pp. 2650–2662.

[7] Philip Mocz. *Create your own smoothed-particle hydrodynamics simulation (with python)*. Sept. 2020. URL: `https://philip-mocz.medium.com/create-your-own-smoothed-particle-hydrodynamics-simulation-with-python-76e1cec505f1`.

[8] David Radice, Sebastiano Bernuzzi, and Albino Perego. "The Dynamics of Binary Neutron Star Mergers and GW170817". In: *Annual Review of Nuclear and Particle Science* 70.1 (Oct. 2020), pp. 95–119. DOI: `10.1146/annurev-nucl-013120-114541`. URL: `https://doi.org/10.1146%5C%2Fannurev-nucl-013120-114541`.

[9] Stephan Rosswog and P Diener. "SPHINCS_BSSN: a general relativistic smooth particle hydrodynamics code for dynamical spacetimes". In: *Classical and Quantum Gravity* 38.11 (2021), p. 115002.

[10] NR Tanvir et al. "A 'kilonova'associated with the short-duration γ-ray burst GRB 130603B". In: *Nature* 500.7464 (2013), pp. 547–549.

[11] WeiKang Zheng et al. "Gamma-Ray Burst in the Swift/Fermi Era and Beyond". In: *Advances in Astronomy* 2018 (2018), pp. 1–1.