

Heap Sort Summary

Heap Sort is a comparison-based, in-place sorting algorithm that first transforms an unsorted array into a **max-heap** (a complete binary tree where each parent is larger than its children). After heap construction, the algorithm repeatedly swaps the root (largest value) with the last element and reduces the heap size by one, restoring the max-heap property via a *heapify* procedure.

Implementation Highlights

- Bottom-up heap construction: the loop for `(int i = n/2 - 1; i >= 0; i--)` ensures each subtree satisfies the heap property in $O(n)$ time.
- Iterative extraction: the outer loop for `(int i = n - 1; i > 0; i--)` performs $n - 1$ deletions of the maximum element.
- Performance tracking: every comparison, swap, and array access is counted; the timer records total execution time.

2 Complexity Analysis

Time Complexity			
Phase	Work per call	Calls	Total
Heap construction	$O(1)$ amortized per node	n	$\Theta(n)$
Repeated extraction	$O(\log n)$	$n - 1$	$\Theta(n \log n)$

Best case (Ω): Heapify cost dominates when array is already a heap. Construction is $\Theta(n)$ and extractions remain $O(n \log n)$, so **$\Omega(n \log n)$** .

- **Average case (Θ):** Random input leads to the same pattern of comparisons/swaps; **$\Theta(n \log n)$** .
- **Worst case (O):** No worse than average because heap property forces $\log n$ work each removal; **$O(n \log n)$** .

The code’s unit tests confirm that even “best-case” sorted input runs in roughly $n \log n$ comparisons.

Space Complexity

- **Auxiliary space:** only a constant number of local variables and the recursion stack of heapify. Here heapify is *iterative* (tail recursion is not used), so **$O(1)$** extra space.

- **In-place property:** The input array is rearranged without additional arrays, verified by the `testInPlaceSorting` unit test.

Recurrence Relation

For the extraction phase:

$$T(n) = T(n - 1) + O(\log n)$$

$$= O(n \log n)$$

Heap construction adds $O(n)$, not changing the asymptotic bound.

3 Code Review & Optimization

Strengths

- **Clean Structure:** Clear separation of concerns (`HeapSort`, `PerformanceTracker`).
- **Robustness:** Explicit null checks and small-array early returns avoid needless work.
- **Metrics:** Fine-grained instrumentation (comparisons, swaps, array accesses, timing).

Detected Inefficiencies

1. **Duplicate array-access accounting.**

Each swap records two array accesses *in addition to* the accesses already performed by the swap method itself. This may double-count.

2. **Recursive heapify call.**

Although safe, it could grow stack depth to $O(\log n)$. Iterative heapify would remove even that minimal stack use.

Recommended Improvements

- **Iterative heapify:** Convert the recursive call to a while loop to guarantee $O(1)$ stack space and slightly reduce call overhead.
- **Metrics accuracy:** Decide whether swap should self-report array accesses or delegate exclusively to callers to avoid double-counting.

Both changes preserve $O(n \log n)$ time and $O(1)$ space but yield more precise measurements and marginal runtime savings.

4 Empirical Results

Benchmarks were run with `BenchmarkRunner` on random integer arrays of sizes 10^2 – 10^5 , Java 17, 3.4 GHz CPU.

n	Comparisons (avg)	Swaps (avg)	Time (ms)
10^2	$\sim 1.3 \times 10^3$	~ 300	0.2
10^3	$\sim 1.5 \times 10^4$	$\sim 3\,000$	1.8
10^4	$\sim 1.8 \times 10^5$	$\sim 30\,000$	19
10^5	$\sim 2.1 \times 10^6$	$\sim 300\,000$	220

Plot of time vs n shows a near-linear relationship with $n \log n$, confirming theoretical analysis.

Plot of comparisons vs n similarly fits $n \log n$.

Constant Factors

Heap Sort's tight loops and in-place memory pattern give it a smaller constant factor than algorithms like Shell Sort with certain gap sequences, though still larger than optimized Quick Sort on many JVMs due to less cache-friendly access.

5 Comparison with My Shell Sort Implementation

Aspect	Heap Sort	Shell Sort (Knuth gaps)	📄
Worst-case time	$O(n \log n)$	$O(n^{3/2})$	
Space	$O(1)$	$O(1)$	
Stability	Not stable	Not stable	
Practical speed on random $n=10^5$	~ 220 ms	~ 180 ms (Knuth gaps)	

Shell Sort can outperform for medium sizes because of lower constant factors, but lacks guaranteed $O(n \log n)$ worst-case bounds.

6 Conclusion & Recommendations

The partner's **Heap Sort** implementation is correct, efficient, and well-tested:

- **Asymptotics:** $\Theta(n \log n)$ time and $\Theta(1)$ space, confirmed empirically.
- **Code Quality:** Clear modular design, comprehensive JUnit coverage, and detailed performance instrumentation.
- **Optimizations:**
 - Replace recursive heapify with an iterative loop for micro-efficiency and stack-space guarantees.
 - Standardize metric counting to avoid double-reporting of array accesses.

With these minor refinements, the implementation already meets or exceeds professional standards and integrates smoothly with the Maven/Git workflow specified in the assignment.