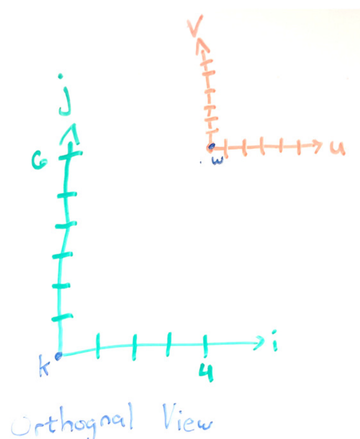
Edward Thomas Garcia
CSCI 5631-001 Fall 2016
Homework 4

1) The composite transformation matrix $M=TRT^{-1}$ is applied to set of vertices. Which of the component transformations is the first one applied? What does this imply about how you should build your composite transformations?

The order can be described with parenthesis: $M=(T * (R * (T^{-1})))$ In code, the above transformation should be processed from right-to-left as follows:

$M = T^{-1}$
M *= R
M *= T

2) Imagine you have two right handed frames, one with representation in terms of the vectors [i j k] and another with corresponding vectors [u v w]. For every unit we move in the [i j k] frame we move 2 units in the [u v w] frame and the origin of the [u v w] frame can be represented at [4i 6j 2k]. Derive the transform $M_{[ijk]<-[uvw]}$ such that, when applied to any point P in [u v w] we get its representation in [i j k]. Show how you accomplished this.



Step 1
Find the matrix that would translate a point from [u v w] to [i j k].

$$T = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 2
Every 2 units in [u v w] count as 1 unit in [i j k]. Hence to convert from [u v w] to [i j k], we must divide by 2.

$$S = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 3: $M_{[ijk]<-[uvw]}$ = TS = $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 & 4 \\ 0 & 0.5 & 0 & 6 \\ 0 & 0 & 0.5 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Note: that scaling is done first because if translating was done first, then the subsequent scaling would amplify the translation.

3) How can you tell if a homogenous transformation matrix is a pure rotation? A rotation about X? Y? Z? Why?

A pure rotation matrix does not translate, scale, sheer, or do any other transformations except rotate. Hence it would take the form:

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ **for rotation about**

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ **for rotation about Y**

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ **for rotation about Z**

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4) How can you tell if a homogenous matrix is a pure translation?

A pure translation matrix does not rotate, scale, sheer, or do any other transformations except translate. Hence it would take the form:

$$T = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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5) How can you tell if a homogenous matrix is a rigid body transformation?

This one is harder to tell just looking at a 4x4 matrix of values. But a rigid body transformation preserves the angles and lengths of the object being transformed. This can be found out if the matrix is “special orthogonal”. This can be found by the following tests on the top left 3x3 matrix:

- **Is each row vector (1x3) a unit vector?**
- **Is each column vector (3x1) a unit vector?**
- **Is each vector perpendicular to each other?**