Hierarchical Modeling I

Ed Angel

Professor Emeritus of Computer Science,

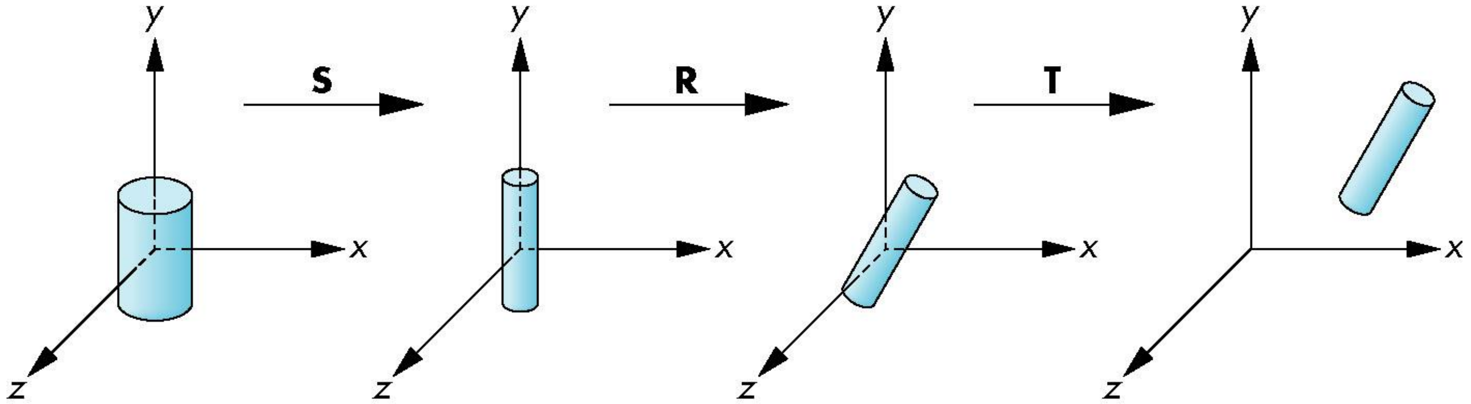
University of New Mexico

Objectives

- Examine the limitations of linear modeling
    - Symbols and instances
- Introduce hierarchical models
    - Articulated models
    - Robots
- Introduce Tree and DAG models

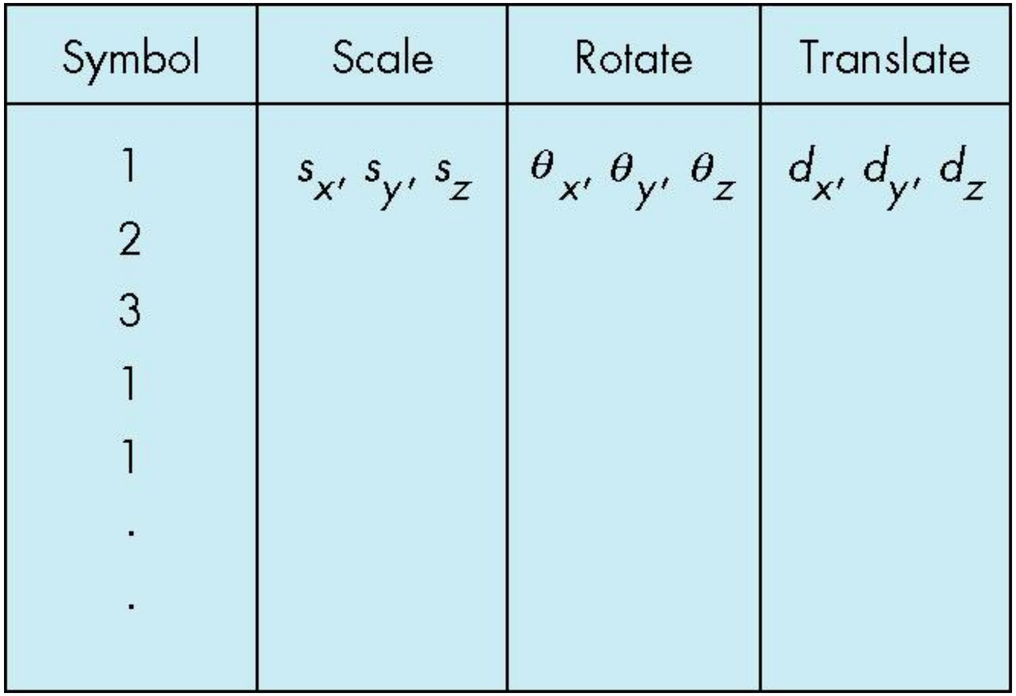
Instance Transformation

- Start with a prototype object (a *symbol*)
- Each appearance of the object in the model is an *instance*
    - Must scale, orient, position
    - Defines instance transformation



Symbol-Instance Table

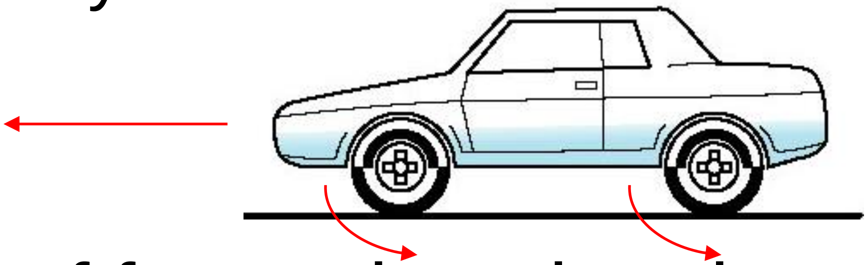
Can store a model by assigning a number to each symbol and storing the parameters for the instance transformation



Symbol	Scale	Rotate	Translate
1	s_x, s_y, s_z	$\theta_x, \theta_y, \theta_z$	d_x, d_y, d_z
2			
3			
1			
1			
.			
.			

Relationships in Car Model

- Symbol-instance table does not show relationships between parts of model
- Consider model of car
  - Chassis + 4 identical wheels
  - Two symbols



- Rate of forward motion determined by rotational speed of wheels

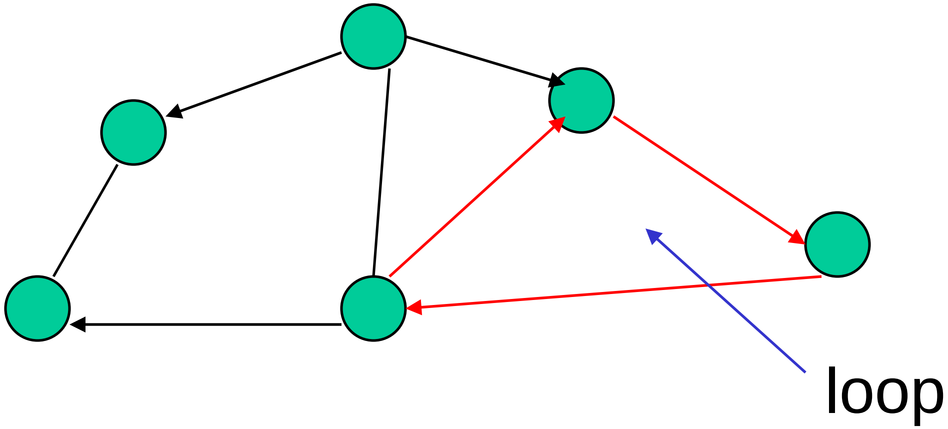
Structure Through Function Calls

```
car(speed)
{
    chassis()
    wheel(right_front);
    wheel(left_front);
    wheel(right_rear);
    wheel(left_rear);
}
```

- Fails to show relationships well
- Look at problem using a graph

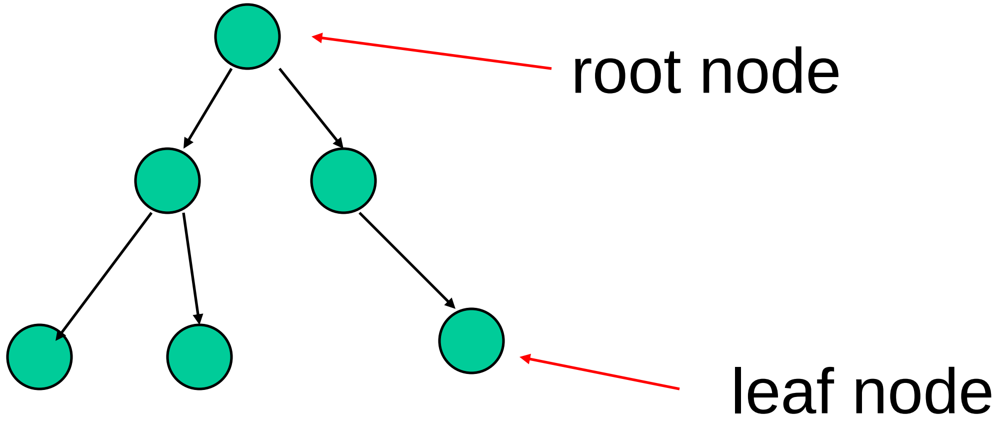
Graphs

- Set of *nodes* and *edges (links)*
- Edge connects a pair of nodes
  - Directed or undirected
- *Cycle*: directed path that is a loop



Tree

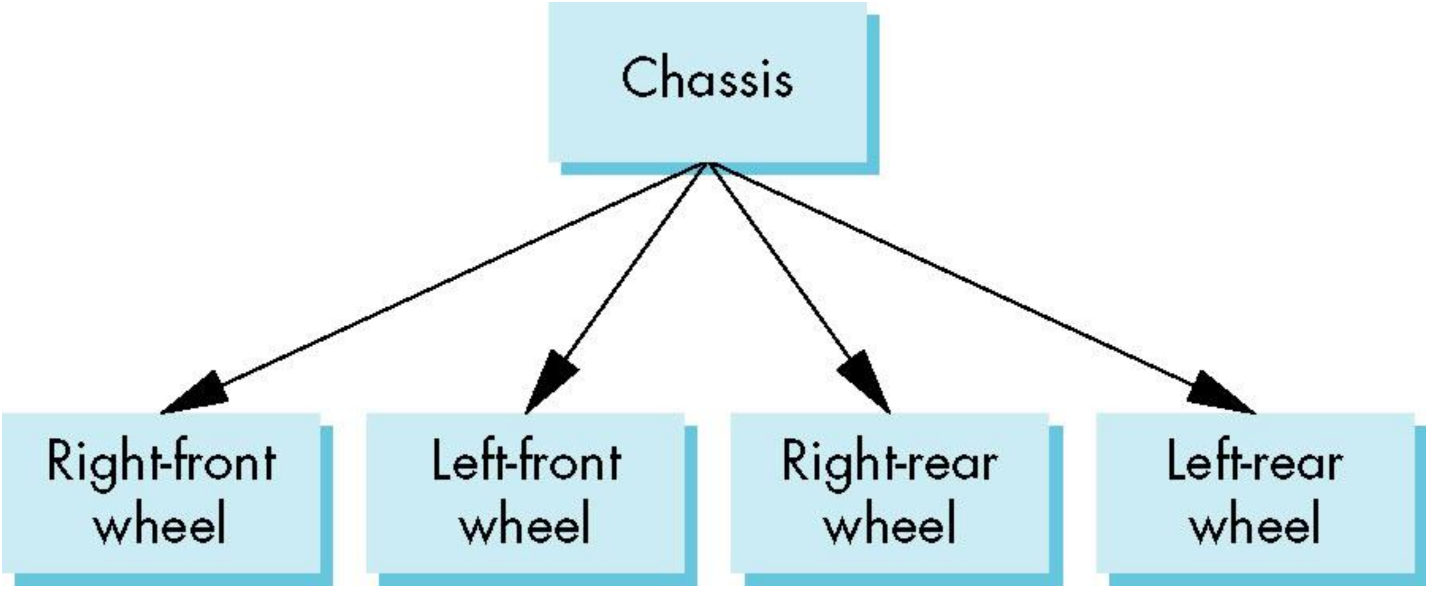
- Graph in which each node (except the root) has exactly one parent node
  - May have multiple children
  - Leaf or terminal node: no children



root node

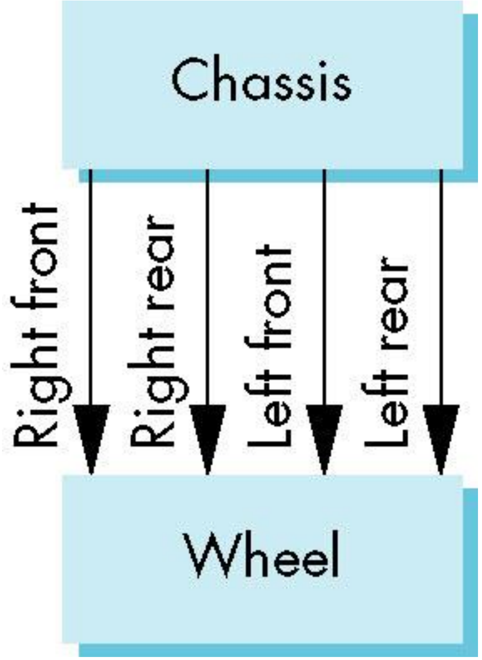
leaf node

Tree Model of Car



DAG Model

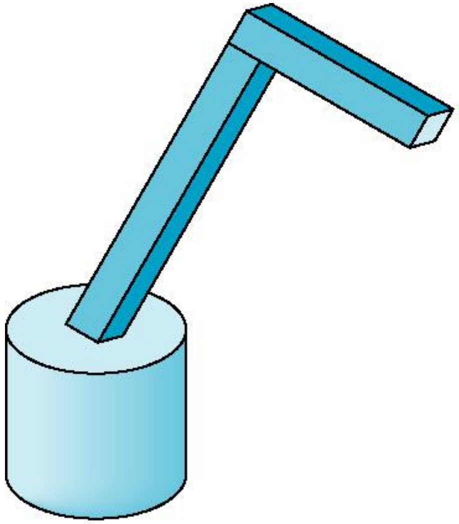
- If we use the fact that all the wheels are identical, we get a *directed acyclic graph*
  - Not much different than dealing with a tree



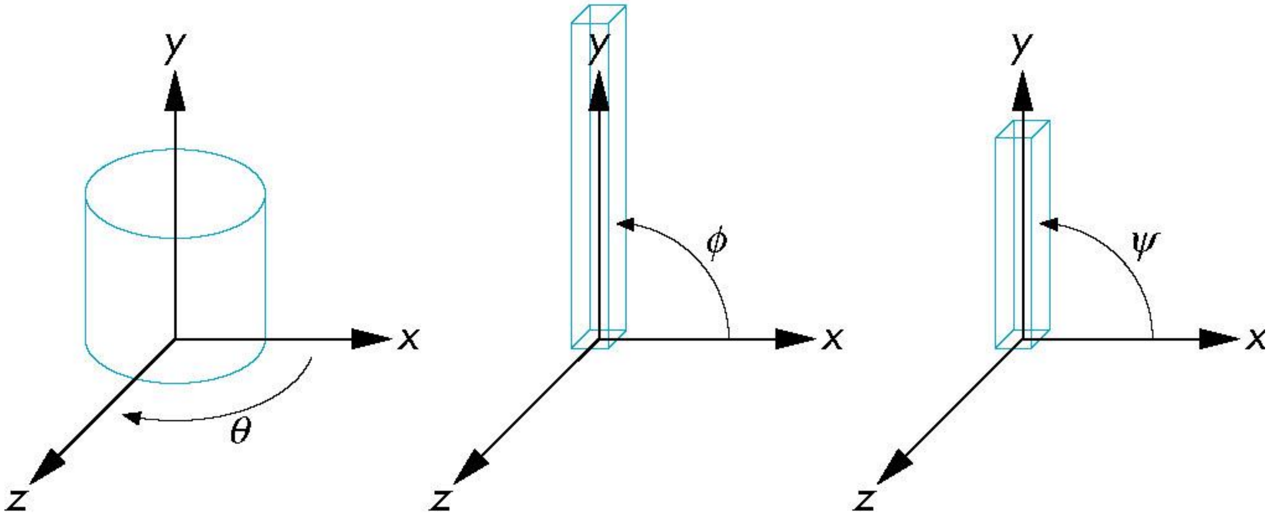
Modeling with Trees

- Must decide what information to place in nodes and what to put in edges
- Nodes
  - What to draw
  - Pointers to children
- Edges
  - May have information on incremental changes to transformation matrices (can also store in nodes)

Robot Arm



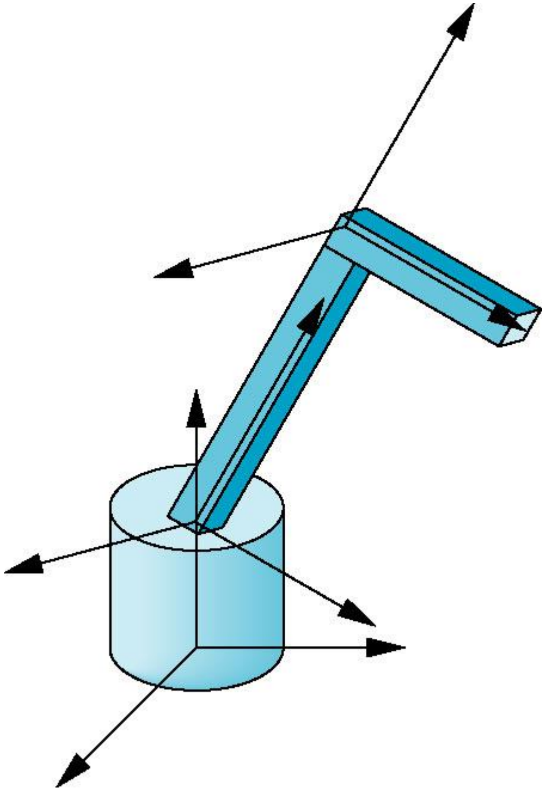
robot arm



parts in their own
coodinate systems

Articulated Models

- Robot arm is an example of an *articulated model*
  - Parts connected at joints
  - Can specify state of model by giving all joint angles



Relationships in Robot Arm

- Base rotates independently
  - Single angle determines position
- Lower arm attached to base
  - Its position depends on rotation of base
  - Must also translate relative to base and rotate about connecting joint
- Upper arm attached to lower arm
  - Its position depends on both base and lower arm
  - Must translate relative to lower arm and rotate about joint connecting to lower arm

Required Matrices

- Rotation of base: $\mathbf{R}_b$
  - Apply $\mathbf{M} = \mathbf{R}_b$ to base
- Translate lower arm <u>relative</u> to base: $\mathbf{T}_{lu}$
- Rotate lower arm around joint: $\mathbf{R}_{lu}$
  - Apply $\mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu}$ to lower arm
- Translate upper arm <u>relative</u> to upper arm: $\mathbf{T}_{uu}$
- Rotate upper arm around joint: $\mathbf{R}_{uu}$
  - Apply $\mathbf{M} = \mathbf{R}_b \mathbf{T}_{lu} \mathbf{R}_{lu} \mathbf{T}_{uu} \mathbf{R}_{uu}$ to upper arm

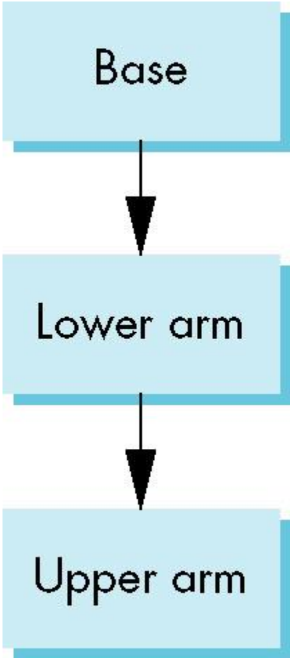
OpenGL Code for Robot

```
mat4 ctm;
robot_arm()
{
    ctm = RotateY(theta);
    base();
    ctm *= Translate(0.0, h1, 0.0);
    ctm *= RotateZ(phi);
    lower_arm();
    ctm *= Translate(0.0, h2, 0.0);
    ctm *= RotateZ(psi);
    upper_arm();
}
```

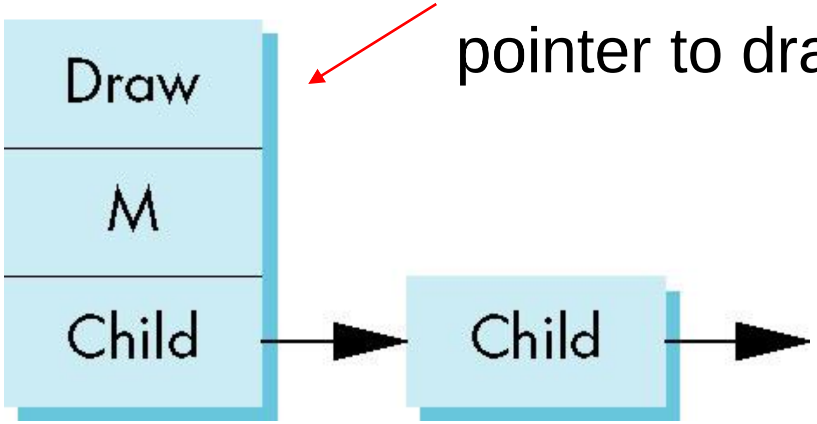
Tree Model of Robot

- Note code shows relationships between parts of model
  - Can change “look” of parts easily without altering relationships
- Simple example of tree model
- Want a general node structure

for nodes



Possible Node Structure



Code for drawing part or pointer to drawing function

linked list of pointers to children

matrix relating node to parent

Generalizations

- Need to deal with multiple children
  - How do we represent a more general tree?
  - How do we traverse such a data structure?
- Animation
  - How to use dynamically?
  - Can we create and delete nodes during execution?