



The University of New Mexico

Implementation III

Ed Angel

Professor Emeritus of Computer Science

University of New Mexico



The University of New Mexico

Objectives

- Survey Line Drawing Algorithms
 - DDA
 - Bresenham's Algorithm
- Aliasing and Antialiasing



Rasterization

- Rasterization (scan conversion)
 - Determine which pixels that are inside primitive specified by a set of vertices
 - Produces a set of fragments
 - Fragments have a location (pixel location) and other attributes such color and texture coordinates that are determined by interpolating values at vertices
- Pixel colors determined later using color, texture, and other vertex properties



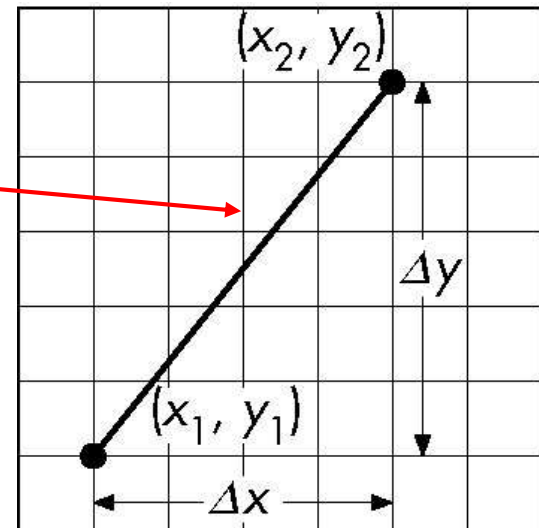
The University of New Mexico

Scan Conversion of Line Segments

- Start with line segment in window coordinates with integer values for endpoints
- Assume implementation has a **write_pixel** function

$$m = \frac{\Delta y}{\Delta x}$$

$$y = mx + h$$





DDA Algorithm

- Digital Differential Analyzer
 - DDA was a mechanical device for numerical solution of differential equations
 - Line $y=mx+ h$ satisfies differential equation
$$dy/dx = m = \Delta y/\Delta x = y_2-y_1/x_2-x_1$$

- Along scan line $\Delta x = 1$

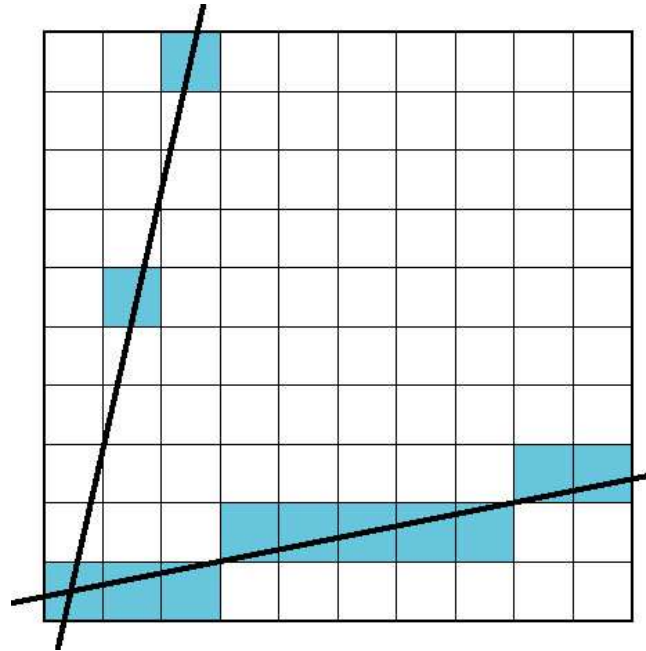
```
For (x=x1; x<=x2, ix++) {  
    y+=m;  
    write_pixel(x, round(y), line_color)  
}
```



The University of New Mexico

Problem

- DDA = for each x plot pixel at closest y
 - Problems for steep lines

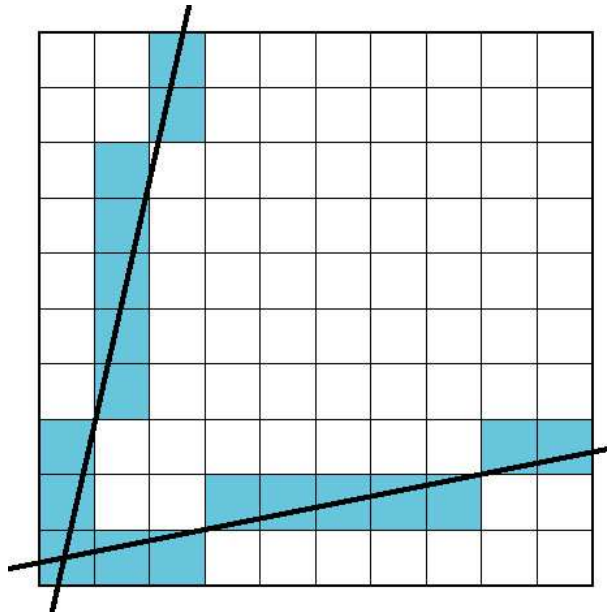




The University of New Mexico

Using Symmetry

- Use for $1 \geq m \geq 0$
- For $m > 1$, swap role of x and y
 - For each y , plot closest x





The University of New Mexico

Bresenham's Algorithm

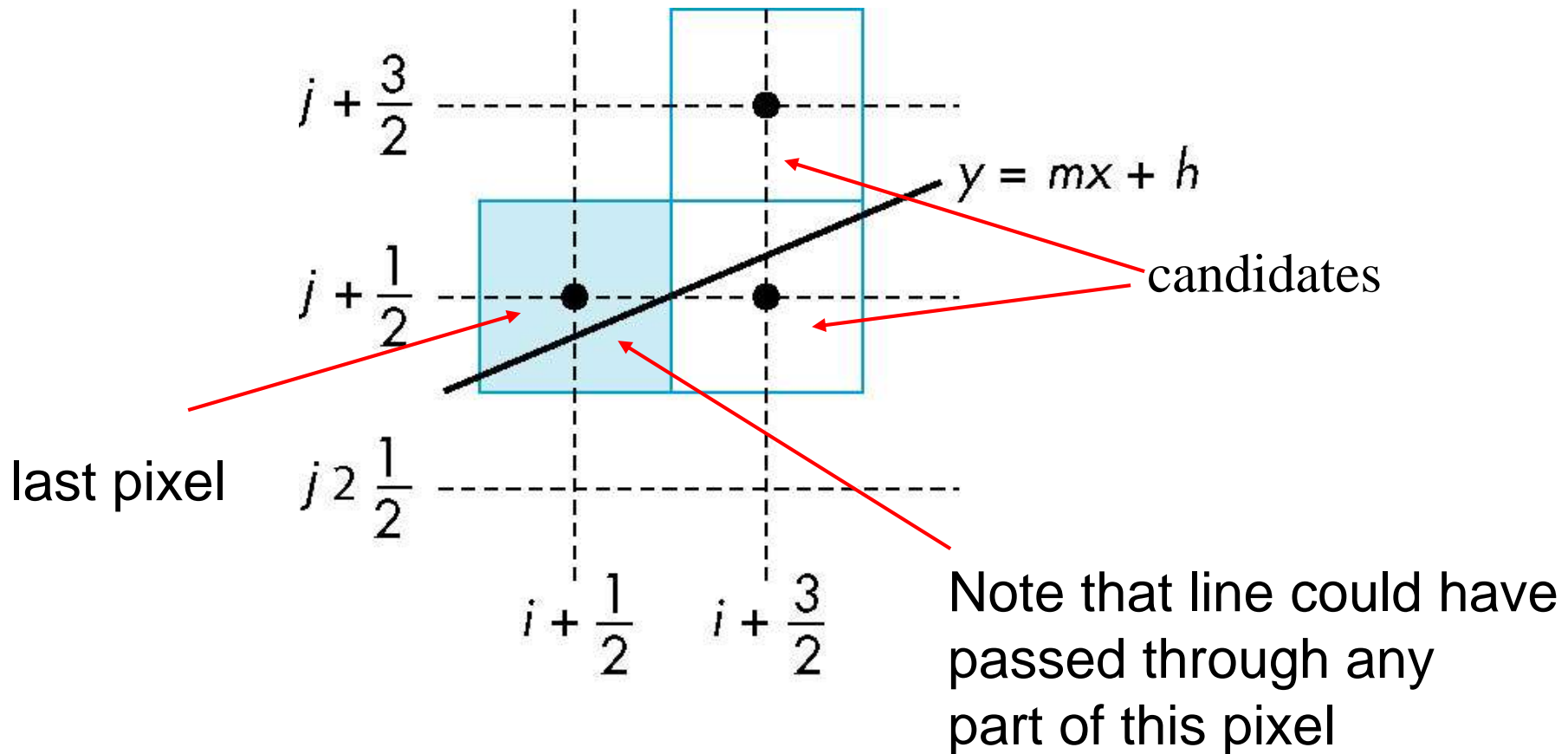
- DDA requires one floating point addition per step
- We can eliminate all fp through Bresenham's algorithm
- Consider only $1 \geq m \geq 0$
 - Other cases by symmetry
- Assume pixel centers are at half integers
- If we start at a pixel that has been written, there are only two candidates for the next pixel to be written into the frame buffer



The University of New Mexico

Candidate Pixels

$$1 \geq m \geq 0$$





The University of New Mexico

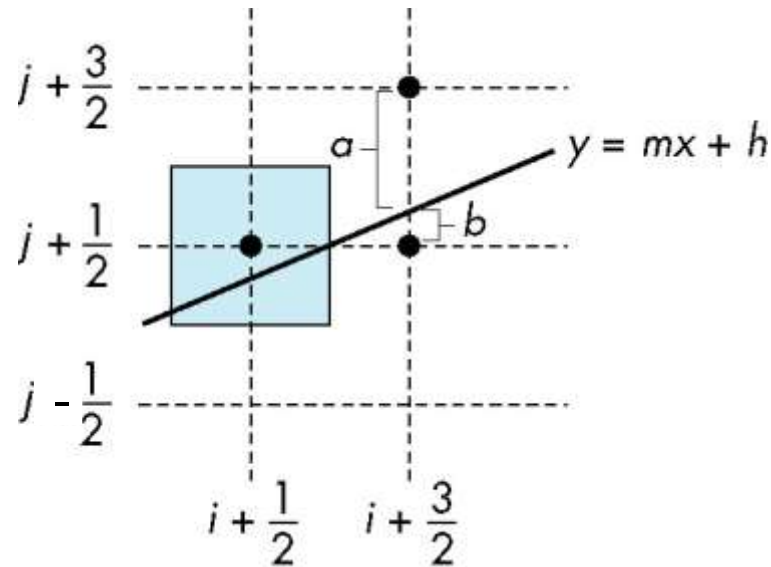
Decision Variable

$$d = \Delta x(b-a)$$

d is an integer

$d > 0$ use upper pixel

$d < 0$ use lower pixel





Incremental Form

- More efficient if we look at d_k , the value of the decision variable at $x = k$

$$d_{k+1} = d_k - 2\Delta y, \quad \text{if } d_k < 0$$

$$d_{k+1} = d_k - 2(\Delta y - \Delta x), \quad \text{otherwise}$$

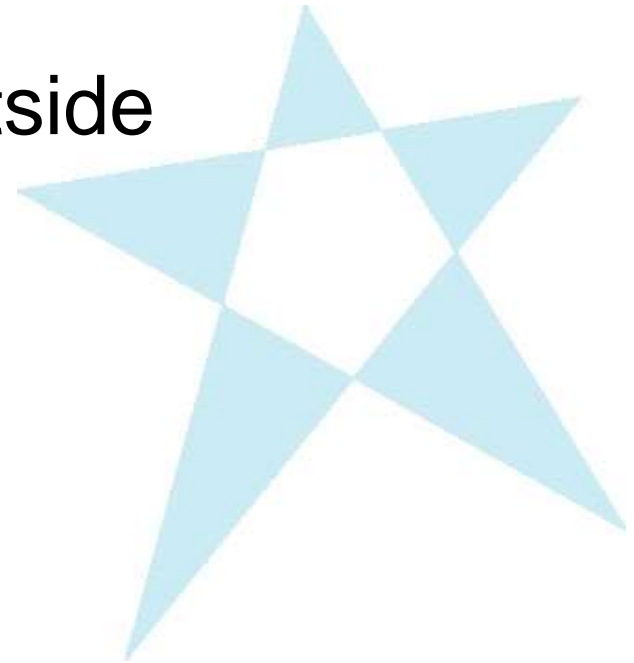
- For each x , we need do only an integer addition and a test
- Single instruction on graphics chips



The University of New Mexico

Polygon Scan Conversion

- Scan Conversion = Fill
- How to tell inside from outside
 - Convex easy
 - Nonsimple difficult
 - Odd even test
 - Count edge crossings
 - Winding number



odd-even fill



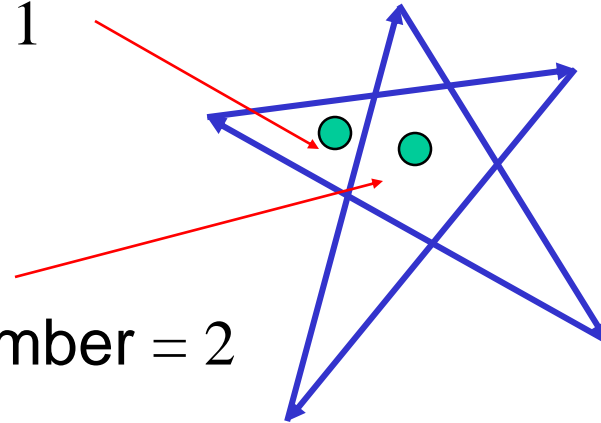
The University of New Mexico

Winding Number

- Count clockwise encirclements of point

winding number = 1

winding number = 2



- Alternate definition of inside: inside if winding number $\neq 0$



The University of New Mexico

Filling in the Frame Buffer

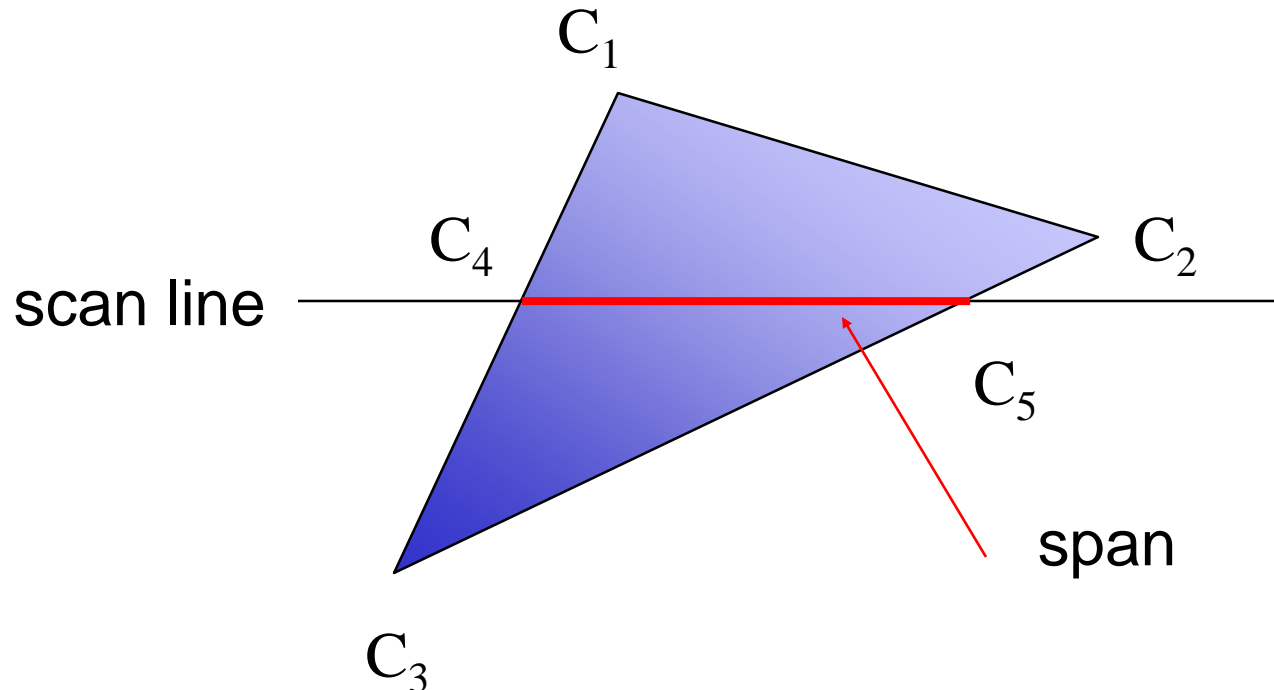
- Fill at end of pipeline
 - Convex Polygons only
 - Nonconvex polygons assumed to have been tessellated
 - Shades (colors) have been computed for vertices (Gouraud shading)
 - Combine with z-buffer algorithm
 - March across scan lines interpolating shades
 - Incremental work small



The University of New Mexico

Using Interpolation

C_1 C_2 C_3 specified by `glColor` or by vertex shading
 C_4 determined by interpolating between C_1 and C_3
 C_5 determined by interpolating between C_2 and C_3
interpolate between C_4 and C_5 along span





Flood Fill

- Fill can be done recursively if we know a seed point located inside (WHITE)
- Scan convert edges into buffer in edge/inside color (BLACK)

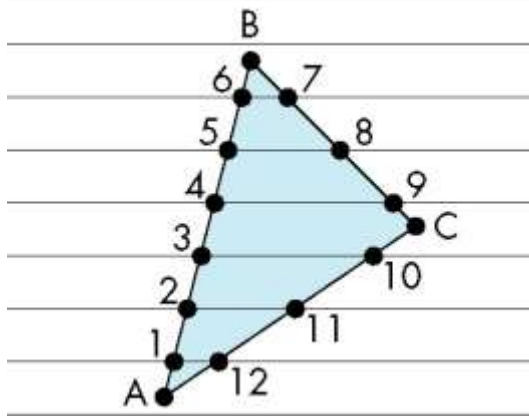
```
flood_fill(int x, int y) {  
    if(read_pixel(x,y) == WHITE) {  
        write_pixel(x,y,BLACK);  
        flood_fill(x-1, y);  
        flood_fill(x+1, y);  
        flood_fill(x, y+1);  
        flood_fill(x, y-1);  
    }  
}
```



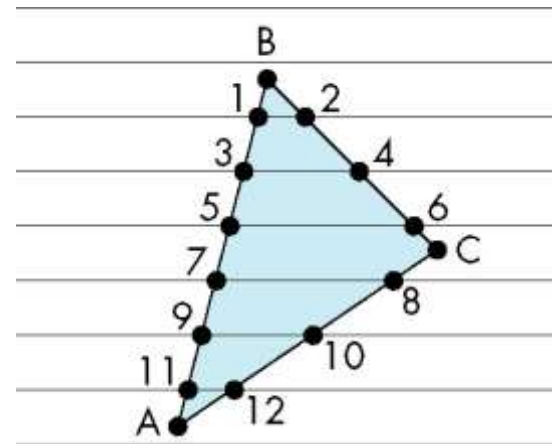

The University of New Mexico

Scan Line Fill

- Can also fill by maintaining a data structure of all intersections of polygons with scan lines
 - Sort by scan line
 - Fill each span



vertex order generated
by vertex list

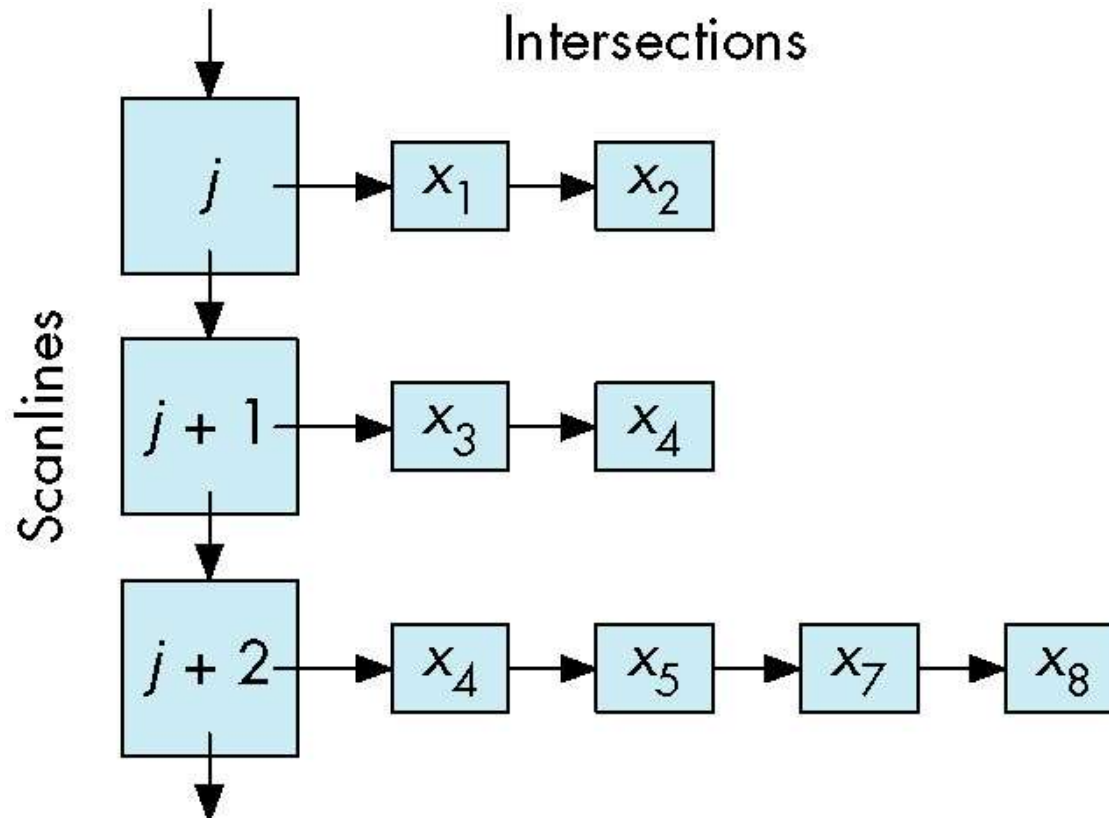


desired order



The University of New Mexico

Data Structure

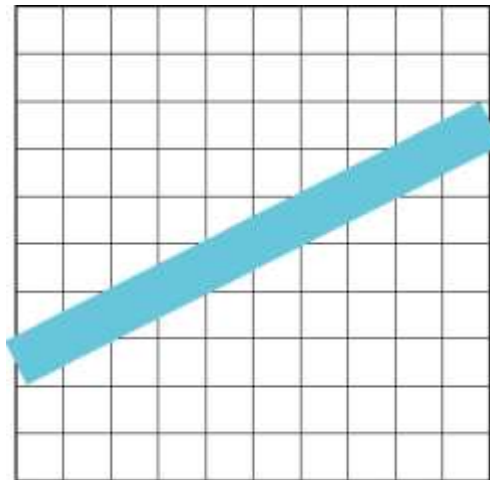




The University of New Mexico

Aliasing

- Ideal rasterized line should be 1 pixel wide



- Choosing best y for each x (or visa versa) produces aliased raster lines

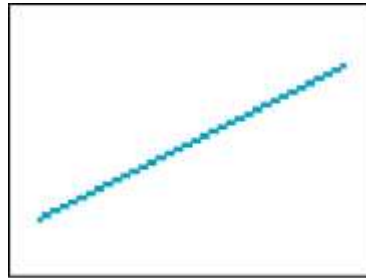


The University of New Mexico

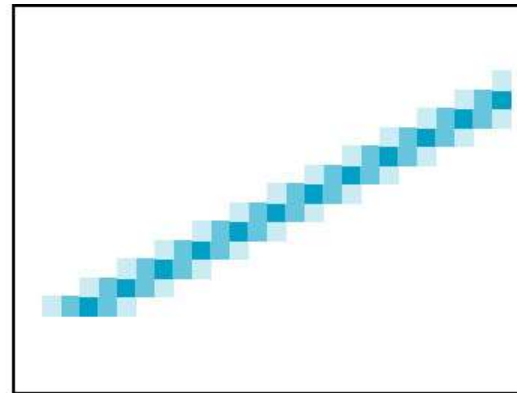
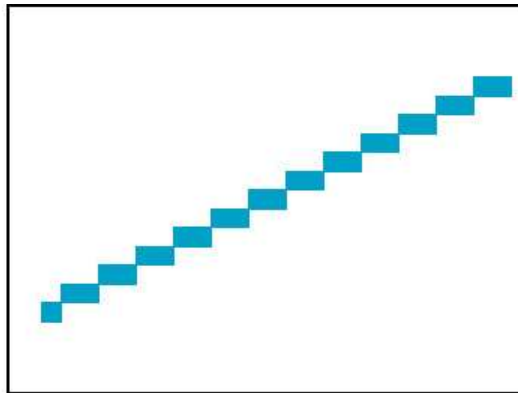
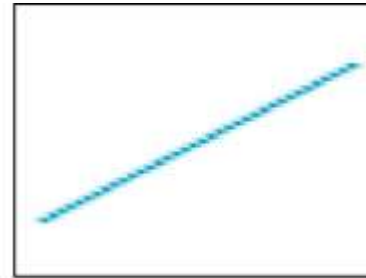
Antialiasing by Area Averaging

- Color multiple pixels for each x depending on coverage by ideal line

original



antialiased



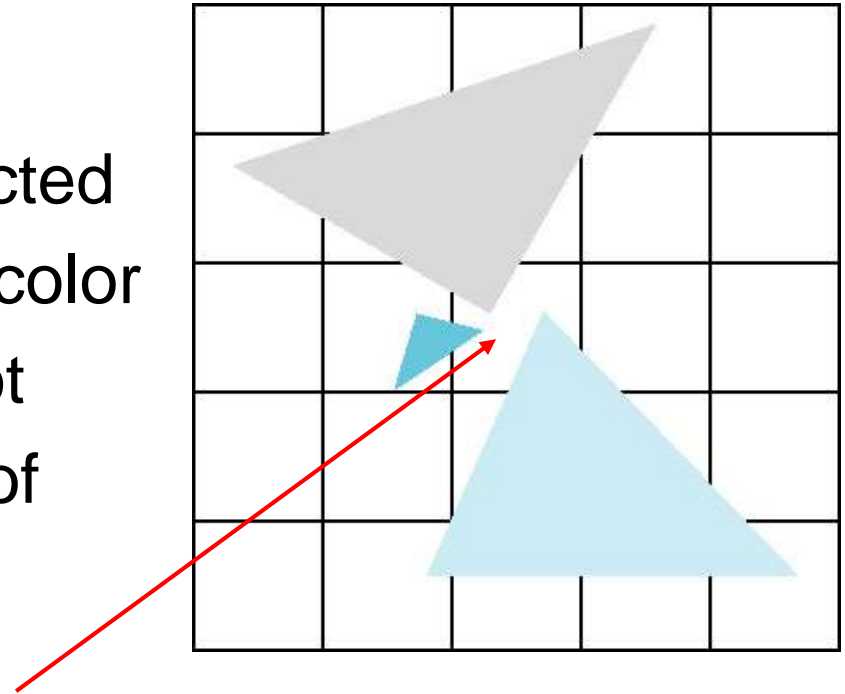
magnified



The University of New Mexico

Polygon Aliasing

- Aliasing problems can be serious for polygons
 - Jaggedness of edges
 - Small polygons neglected
 - Need compositing so color of one polygon does not totally determine color of pixel



All three polygons should contribute to color