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# Computer Viewing

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# Objectives

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- Introduce the mathematics of projection
- Introduce OpenGL viewing functions
- Look at alternate viewing APIs



# Computer Viewing

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- There are three aspects of the viewing process, all of which are implemented in the pipeline,
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix
  - Clipping
    - Setting the view volume



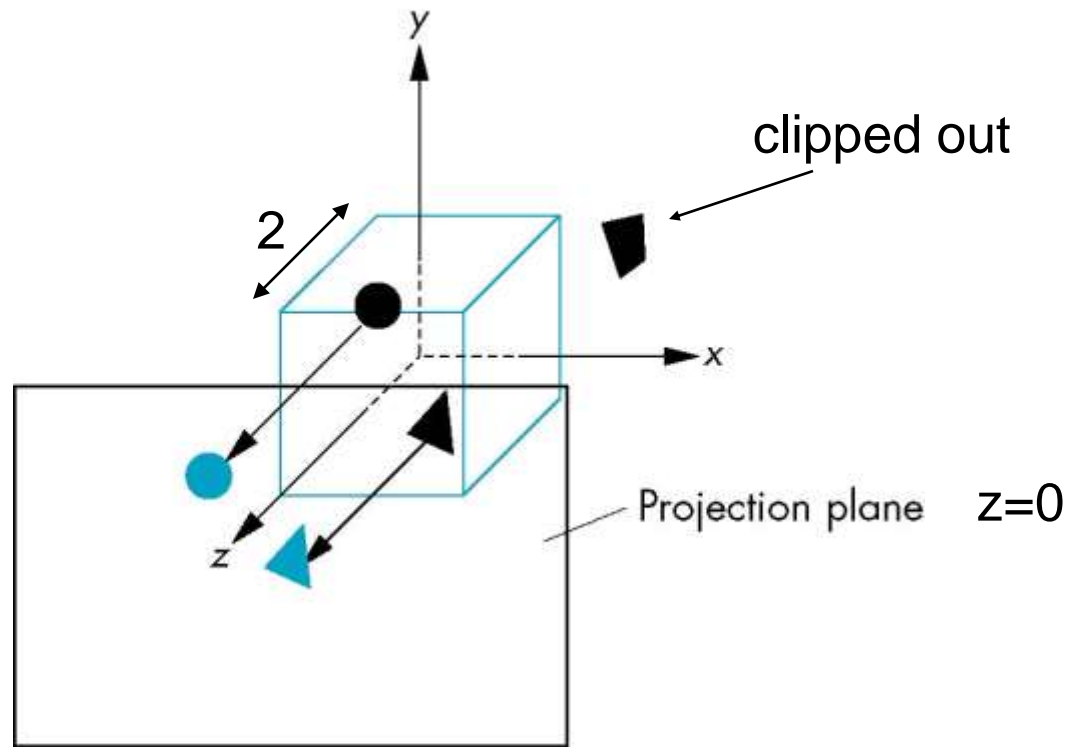
# The OpenGL Camera

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- In OpenGL, initially the object and camera frames are the same
  - Default model-view matrix is an identity
- The camera is located at origin and points in the negative z direction
- OpenGL also specifies a default view volume that is a cube with sides of length 2 centered at the origin
  - Default projection matrix is an identity



# Default Projection





# Moving the Camera Frame

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- If we want to visualize object with both positive and negative z values we can either
  - Move the camera in the positive z direction
    - Translate the camera frame
  - Move the objects in the negative z direction
    - Translate the world frame
- Both of these views are equivalent and are determined by the model-view matrix
  - Want a translation (`Translate(0.0, 0.0, -d);`)
    - $-d > 0$



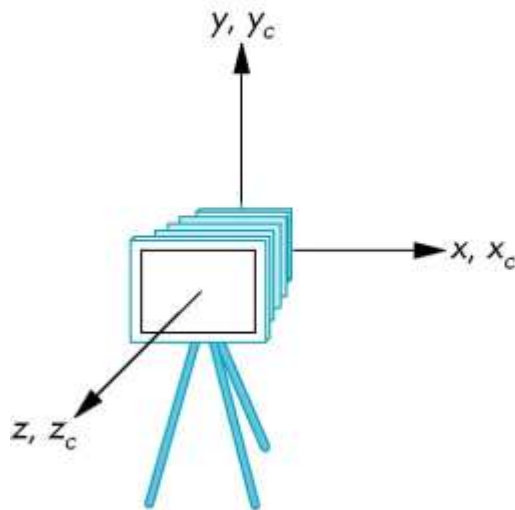
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# Moving Camera back from Origin

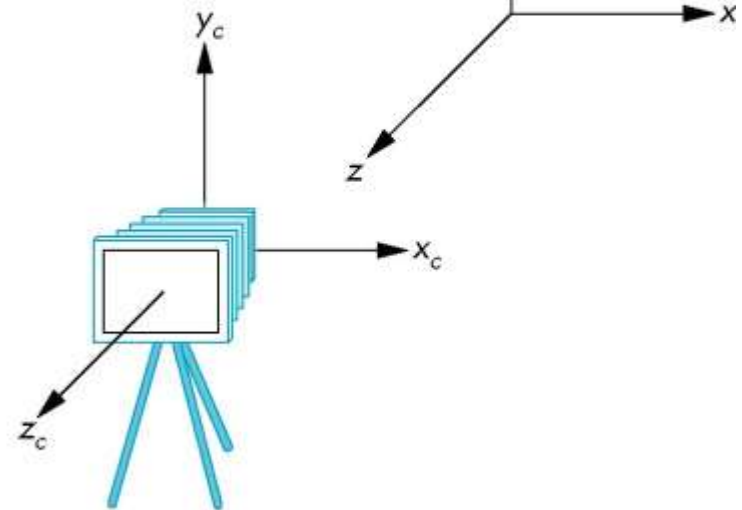
frames after translation by  $-d$

$$d > 0$$

default frames



(a)

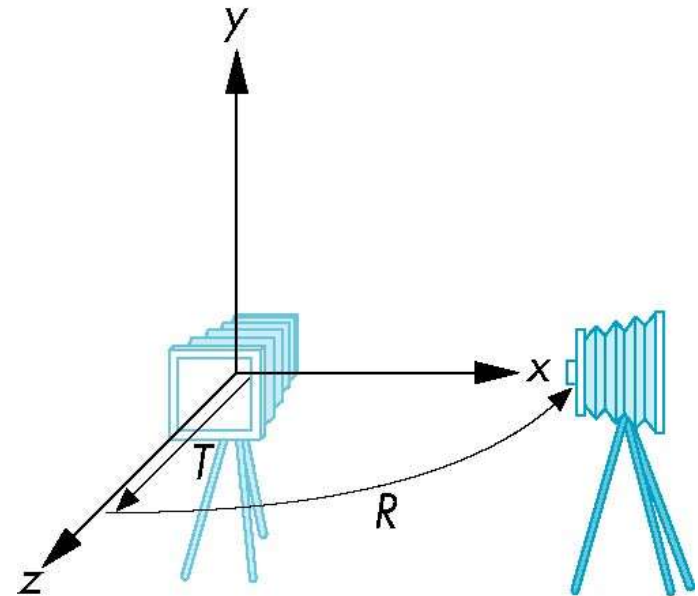


(b)



# Moving the Camera

- We can move the camera to any desired position by a sequence of rotations and translations
- Example: side view
  - Rotate the camera
  - Move it away from origin
  - Model-view matrix  $C = TR$







# OpenGL code

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- Remember that last transformation specified is first to be applied

```
// Using mat.h
```

```
mat4 t = Translate (0.0, 0.0, -d);  
mat4 ry = RotateY(90.0);  
mat4 m = t*ry;
```



# The LookAt Function

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- The GLU library contained the function `gluLookAt` to form the required modelview matrix through a simple interface
- Note the need for setting an up direction
- Replaced by `LookAt()` in `mat.h`
  - Can concatenate with modeling transformations
- Example: isometric view of cube aligned with axes

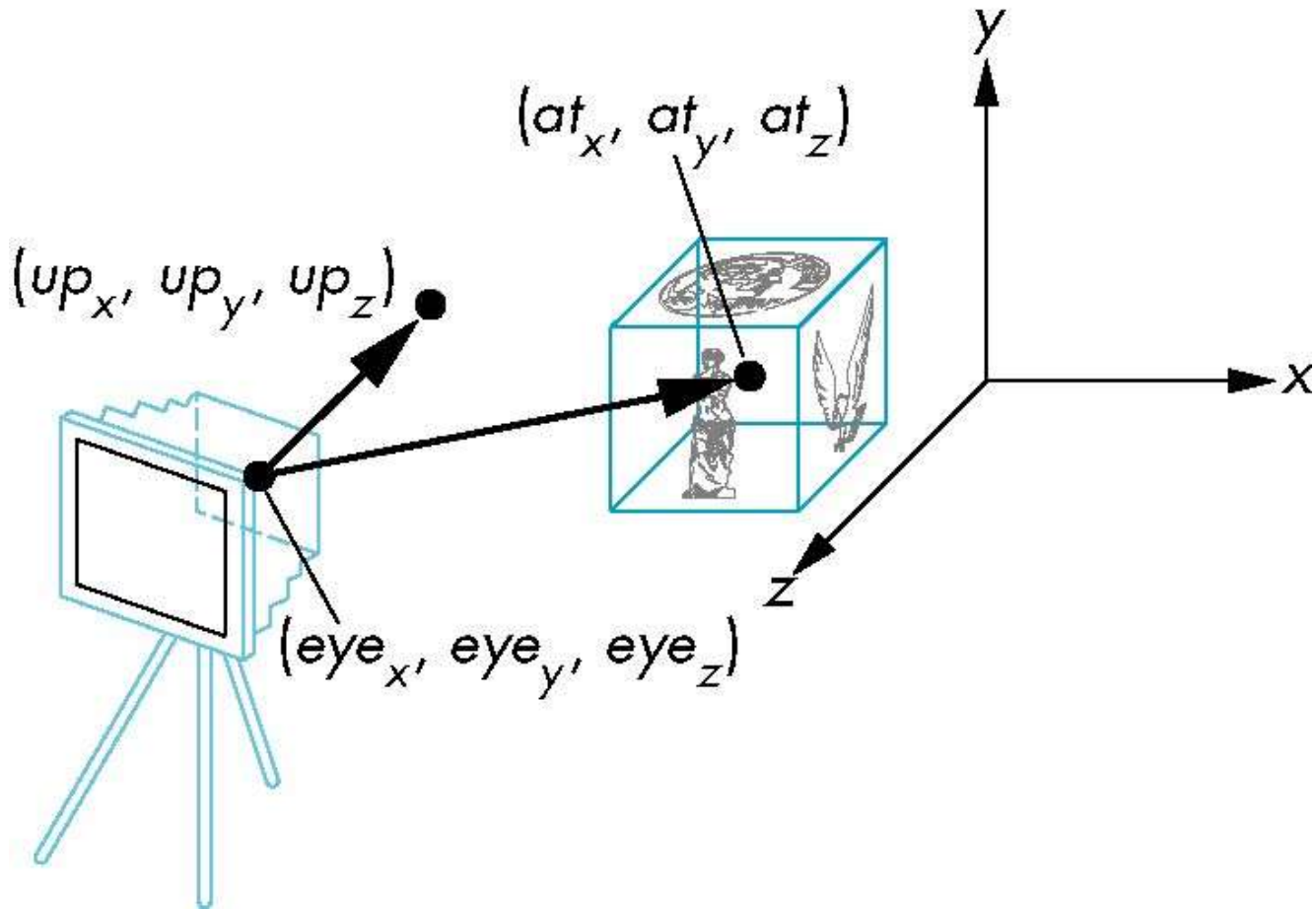
```
mat4 mv = LookAt(vec4 eye, vec4 at, vec4 up);
```



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# gluLookAt

`LookAt(eye, at, up)`





# Other Viewing APIs

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- The LookAt function is only one possible API for positioning the camera
- Others include
  - View reference point, view plane normal, view up (PHIGS, GKS-3D)
  - Yaw, pitch, roll
  - Elevation, azimuth, twist
  - Direction angles



# Projections and Normalization

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- The default projection in the eye (camera) frame is orthogonal
- For points within the default view volume

$$x_p = x$$

$$y_p = y$$

$$z_p = 0$$

- Most graphics systems use *view normalization*
  - All other views are converted to the default view by transformations that determine the projection matrix
  - Allows use of the same pipeline for all views



# Homogeneous Coordinate Representation

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default orthographic projection

$$x_p = x$$

$$y_p = y$$

$$z_p = 0$$

$$w_p = 1$$

$$\mathbf{p}_p = \mathbf{M}\mathbf{p}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

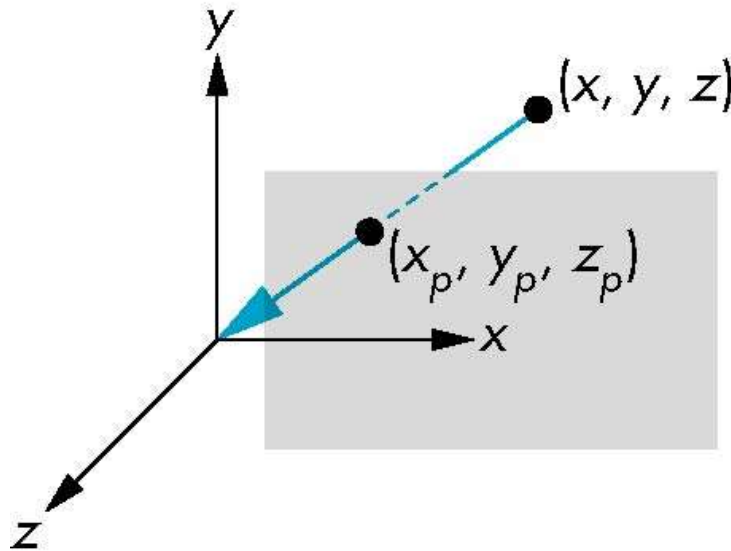
In practice, we can let  $\mathbf{M} = \mathbf{I}$  and set the  $z$  term to zero later



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# Simple Perspective

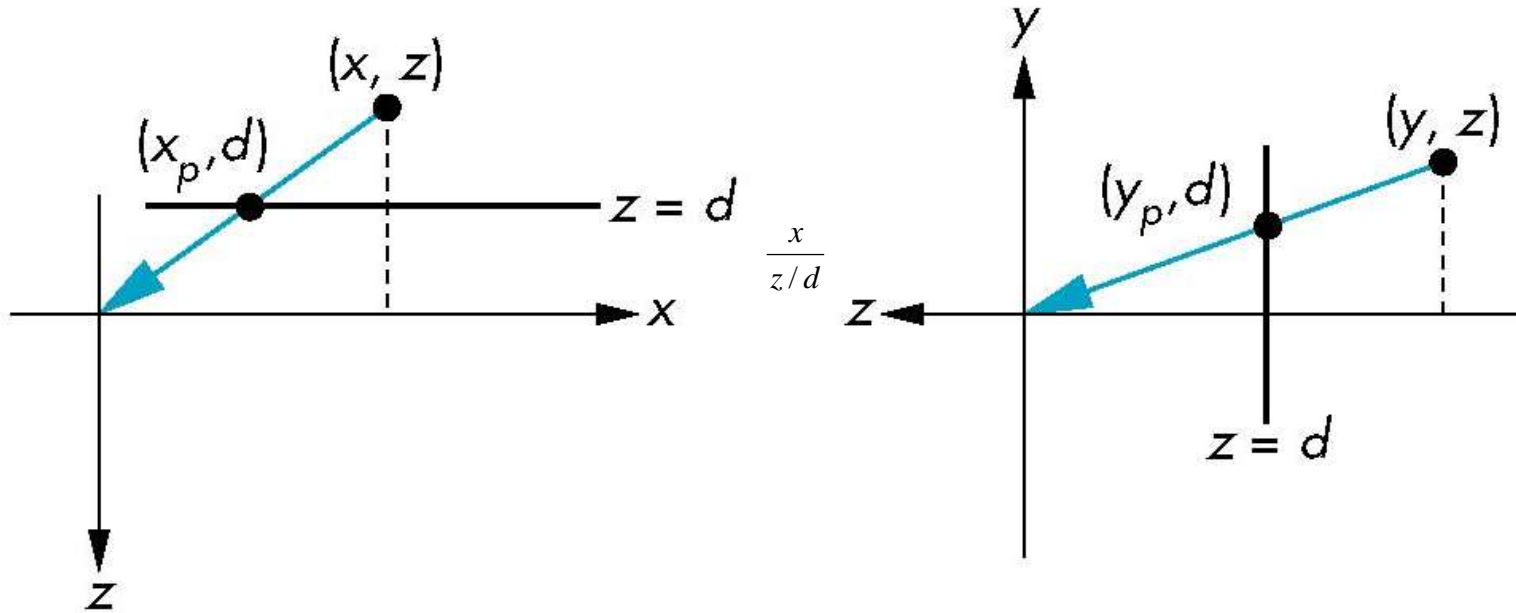
- Center of projection at the origin
- Projection plane  $z = d$ ,  $d < 0$





# Perspective Equations

Consider top and side views



$$x_p = \frac{x}{z/d}$$

$$y_p = \frac{y}{z/d}$$

$$z_p = d$$



# Homogeneous Coordinate Form

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consider  $\mathbf{q} = \mathbf{M}\mathbf{p}$  where  $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$



# Perspective Division

- However  $w \neq 1$ , so we must divide by  $w$  to return from homogeneous coordinates
- This *perspective division* yields

$$x_p = \frac{x}{z/d} \quad y_p = \frac{y}{z/d} \quad z_p = d$$

the desired perspective equations

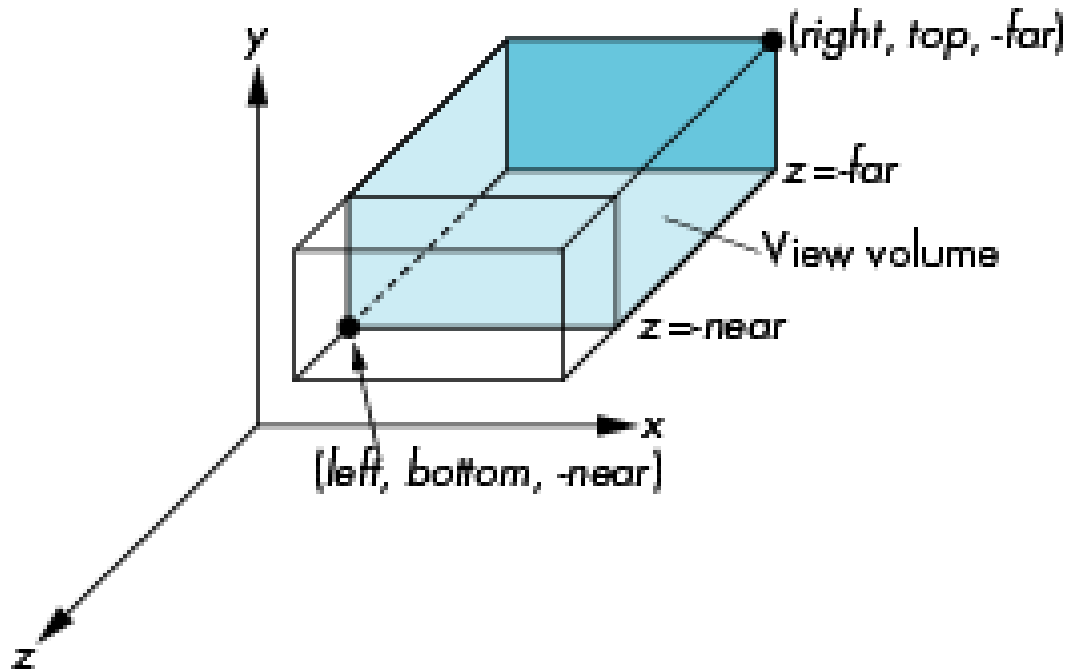
- We will consider the corresponding clipping volume with `mat.h` functions that are equivalent to deprecated OpenGL functions



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# OpenGL Orthogonal Viewing

`Ortho(left, right, bottom, top, near, far)`



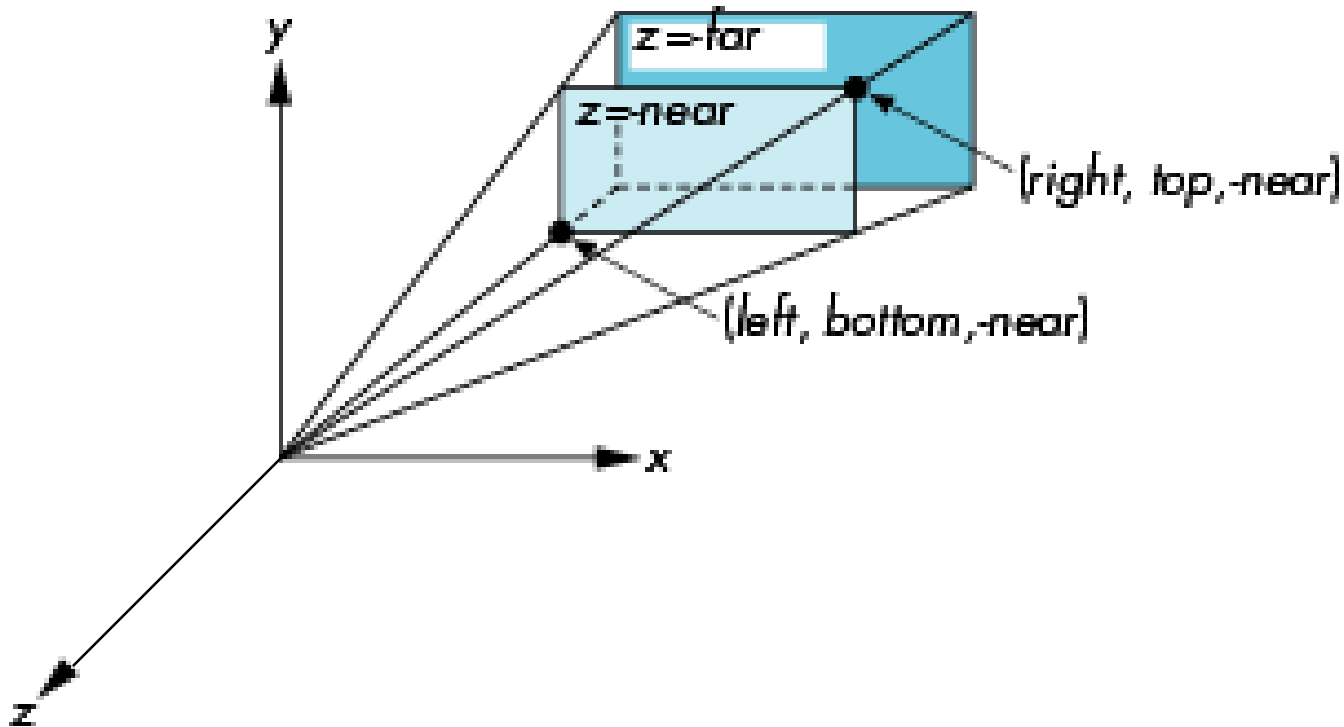
`near` and `far` measured from camera



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# OpenGL Perspective

**Frustum(left, right, bottom, top, near, far)**





# Using Field of View

- With **Frustum** it is often difficult to get the desired view
- **Perspective (fovy, aspect, near, far)** often provides a better interface

