
1) Let matrix A be: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Let vector w be: $\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$. What is the result of the operation Aw ?

$$Aw = \begin{bmatrix} (1)(-2) + (0)(2) + (0)(0) \\ (0)(-2) + (1)(2) + (0)(0) \\ (0)(-2) + (0)(2) + (1)(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$

2) Let matrix B be: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. What is the result of the operation Bw ?

$$Bw = \begin{bmatrix} (1)(-2) + (2)(2) + (3)(0) \\ (4)(-2) + (5)(2) + (6)(0) \\ (7)(-2) + (8)(2) + (9)(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

3) Let matrix C be: $\begin{bmatrix} 1 & 4 & 3 \\ 4 & 5 & 1 \\ 7 & 2 & 2 \end{bmatrix}$. What is the result of the operation AC ? CA ?

$$AC = \begin{bmatrix} (1)(1) + (0)(4) + (0)(7) & (1)(4) + (0)(5) + (0)(2) & (1)(3) + (0)(1) + (0)(2) \\ (0)(1) + (1)(4) + (0)(7) & (0)(4) + (1)(5) + (0)(2) & (0)(3) + (1)(1) + (0)(2) \\ (0)(1) + (0)(4) + (1)(7) & (0)(4) + (0)(5) + (1)(2) & (0)(3) + (0)(1) + (1)(2) \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 5 & 1 \\ 7 & 2 & 2 \end{bmatrix}$$

$$CA = \begin{bmatrix} (1)(1) + (4)(0) + (3)(0) & (1)(0) + (4)(1) + (3)(0) & (1)(0) + (4)(0) + (3)(1) \\ (4)(1) + (5)(0) + (1)(0) & (4)(0) + (5)(1) + (1)(0) & (4)(0) + (5)(0) + (1)(1) \\ (7)(1) + (2)(0) + (2)(0) & (7)(0) + (2)(1) + (2)(0) & (7)(0) + (2)(0) + (2)(1) \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 5 & 1 \\ 7 & 2 & 2 \end{bmatrix}$$

4) Let vector v be: $\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$. What is the length of w ? v ? Normalize w . What is the new length? Show your work.

$$\text{Length of } w = \|w\| = \sqrt{-2^2 + 2^2 + 0^2} = \sqrt{8} \approx 2.828$$

$$\text{Length of } v = \|v\| = \sqrt{2^2 + 0^2 + 3^2} = \sqrt{13} \approx 3.606$$

$$\text{Normalized } w = \hat{w} = \frac{w}{\|w\|} = \begin{bmatrix} -2/2.828 \\ 2/2.828 \\ 0/2.828 \end{bmatrix} = \begin{bmatrix} -0.707 \\ 0.707 \\ 0 \end{bmatrix}$$

$$\text{Length of } \hat{w} = \|\hat{w}\| = \sqrt{-0.707^2 + 0.707^2 + 0^2} = 1$$

5) What is the angle between w and v ? What is the cross product $w \times v$? Show your work, and do your best to draw the second operation.

$$w \cdot v = (-2)(2) + (2)(0) + (0)(3) = -4$$

$$\theta = \cos^{-1} \frac{w \cdot v}{\|w\| \|v\|} = \cos^{-1} \frac{-4}{(2.828)(3.606)} = \cos^{-1} -0.392 = 113.079^\circ \text{ or } 1.974 \text{ rad}$$

$$w \times v = \begin{bmatrix} (2)(3) - (0)(0) \\ (0)(2) - (-2)(3) \\ (-2)(0) - (2)(2) \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ -4 \end{bmatrix}$$

