Probability and Bayes' Rule

Stephen G. Ware

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Narrative Intelligence LAB

THE UNIVERSITY of NEW ORLEANS

Story of AI

- Until now, virtually all problems in AI have been formulated as knowledge representation and search.
- Most problems have a clearly defined goal and iterative procedures that guaranteed finding it.
- Henceforth, most problems will rely on fuzzy values and fuzzily-defined goals.
- Most problems will be solved using statistical manipulations.

Probability

A **probability** is a number between 0 and 1 (inclusive) that indicates how likely some event is to occur or how likely it is that something is the case.

A probability of 0 indicates that it is impossible.

A probability of 1 indicates that it must happen.

A probability of 0.5 indicates a 50% chance.

Random Variables

Variables in probabilistic expressions are called **random variables**.

A random variable is similar to a variable in a constraint satisfaction problem in that it can have one of a pre-defined set of values.

A random variable is called “random” because, if its value is not known, it is often helpful to think of its value as being chosen at random.

Sample Space and Events

When speaking of probability, we refer to the set of all possible worlds as the **sample space**.

An **event** is a group of 1 or more possible worlds that all share some quality.

Rolling Dice: Sample Space

Sample space for rolling a single 6-sided die:

$D = 1$

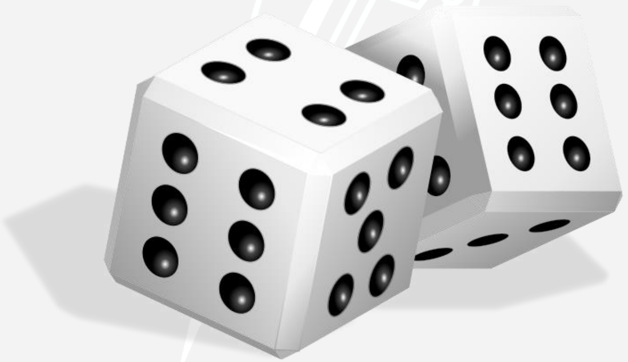
$D = 2$

$D = 3$

$D = 4$

$D = 5$

$D = 6$



Rolling Dice: Sample Space

The event of rolling a 1:

$D = 1$

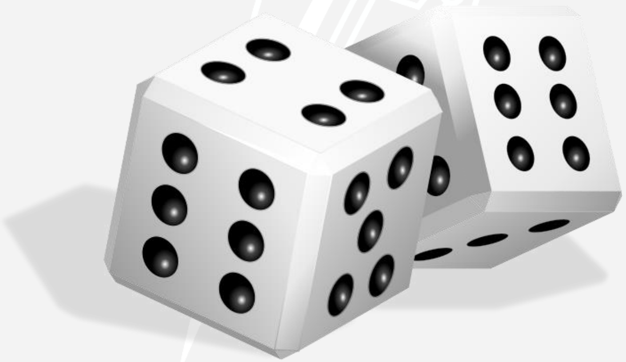
$D = 2$

$D = 3$

$D = 4$

$D = 5$

$D = 6$



Rolling Dice: Sample Space

The event of rolling a 1:

$D = 1$

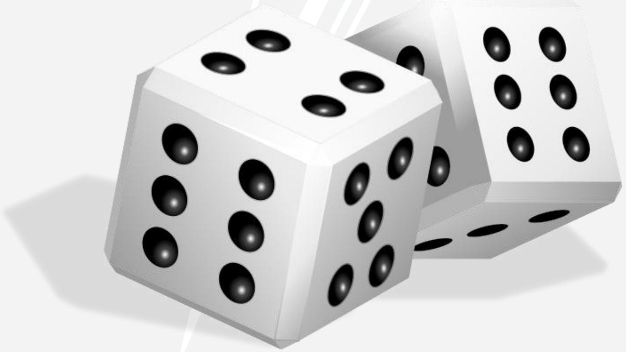
$D = 2$

$D = 3$

$D = 4$

$D = 5$

$D = 6$



Rolling Dice: Sample Space

The event of rolling a number higher than 3:

$D = 1$

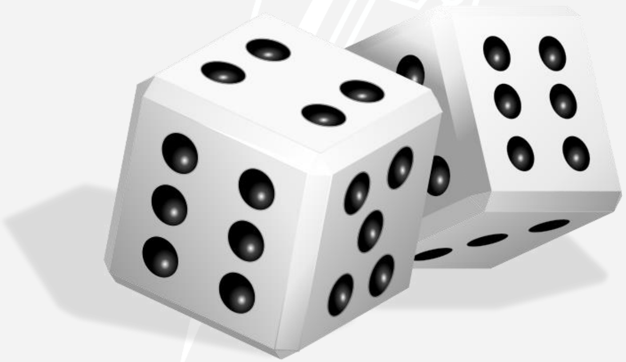
$D = 2$

$D = 3$

$D = 4$

$D = 5$

$D = 6$



Rolling Dice: Sample Space

The event of rolling a number higher than 3:

$D = 1$

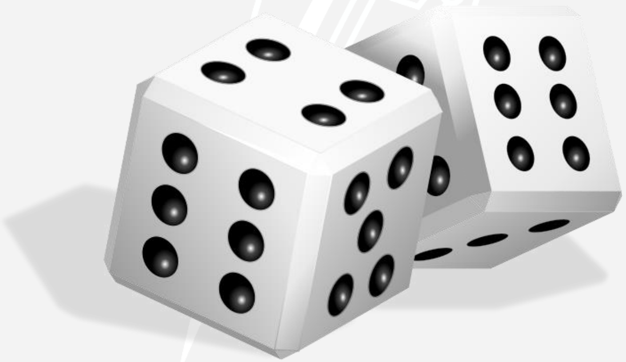
$D = 2$

$D = 3$

$D = 4$

$D = 5$

$D = 6$



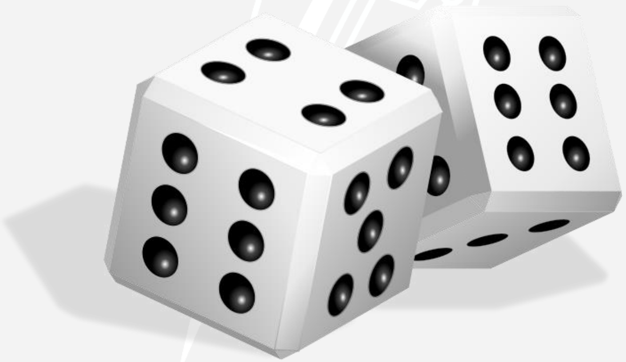
Rolling Dice: Sample Space

Sample space for rolling 2 6-sided dice:

$D_1 = 1 \wedge D_2 = 1$
$D_1 = 1 \wedge D_2 = 2$
$D_1 = 1 \wedge D_2 = 3$

…

$D_1 = 6 \wedge D_2 = 5$
$D_1 = 6 \wedge D_2 = 6$



Rolling Dice: Sample Space

The event of rolling doubles:

$D_1 = 1 \wedge D_2 = 1$

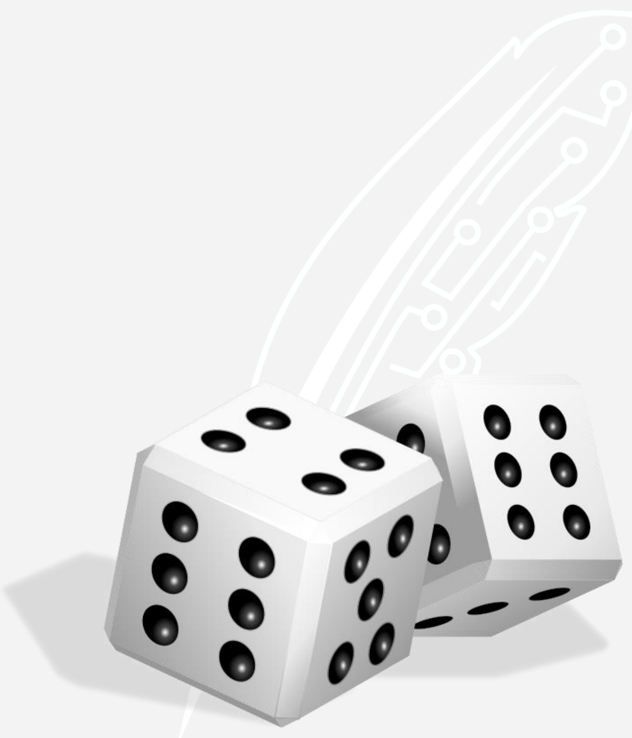
$D_1 = 1 \wedge D_2 = 2$

$D_1 = 1 \wedge D_2 = 3$

…

$D_1 = 6 \wedge D_2 = 5$

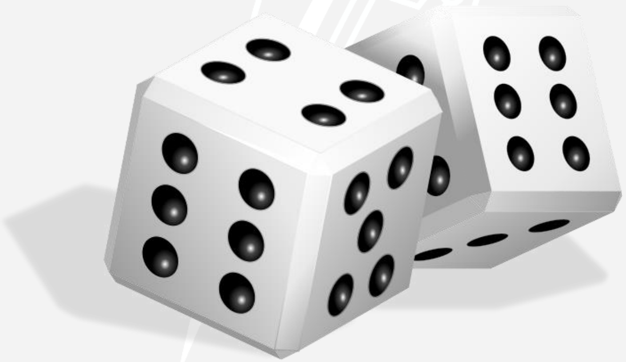
$D_1 = 6 \wedge D_2 = 6$



Prior Probability

The unconditional or **prior probability** of an event is the degree of belief that the event will happen in the absence of any other evidence.

$$\underbrace{P}_{\text{“The probability of…”}}(doubles) =$$

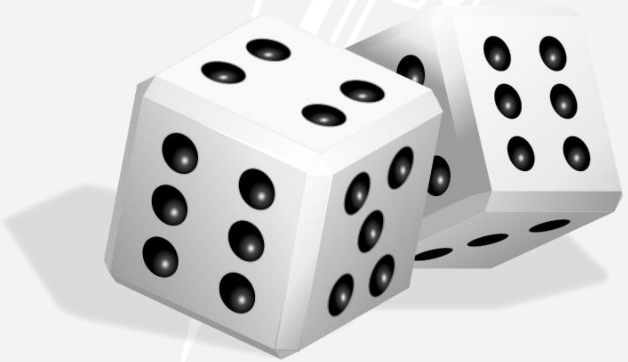


Prior Probability

The unconditional or **prior probability** of an event is the degree of belief that the event will happen in the absence of any other evidence.

$P(doubles) =$

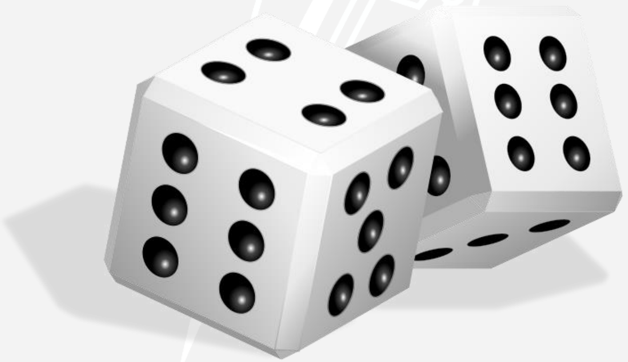
"What is the probability that doubles will be rolled?"



Prior Probability

The unconditional or **prior probability** of an event is the degree of belief that the event will happen in the absence of any other evidence.

$$P(doubles) = \frac{6}{36} = \frac{1}{6}$$

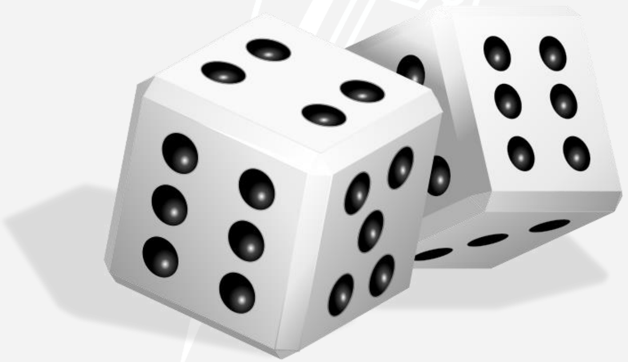


Prior Probability

The unconditional or **prior probability** of an event is the degree of belief that the event will happen in the absence of any other evidence.

$$P(doubles) = \frac{6}{36} = \frac{1}{6}$$

Total number of
possible worlds
i.e. 36 outcomes of rolling 2 dice

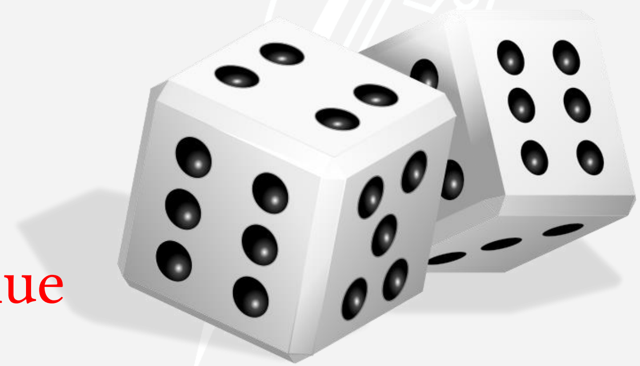


Prior Probability

The unconditional or **prior probability** of an event is the degree of belief that the event will happen in the absence of any other evidence.

$$P(doubles) = \frac{6}{36} = \frac{1}{6}$$

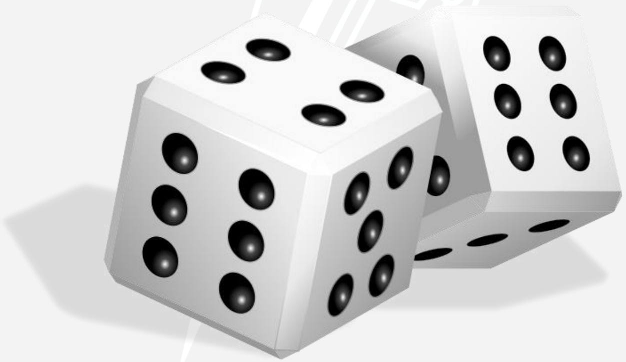
Number of possible worlds
in which the proposition is true
i.e. 6 worlds in which both die show same value



Posterior Probability

The conditional or **posterior probability** of an event is the degree of belief that the event will happen given some evidence.

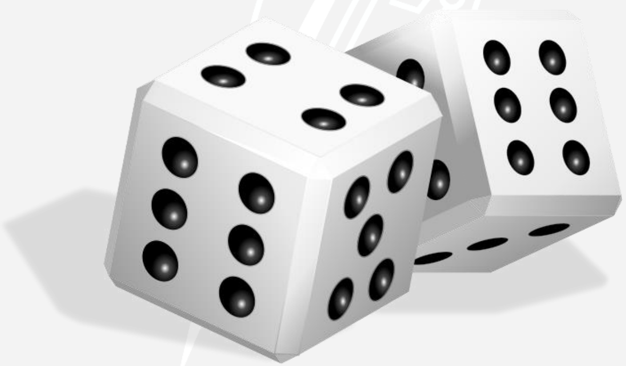
$$P(doubles \underbrace{|}_{\text{“given that…”}} D_1 = 4) =$$



Posterior Probability

The conditional or **posterior probability** of an event is the degree of belief that the event will happen given some evidence.

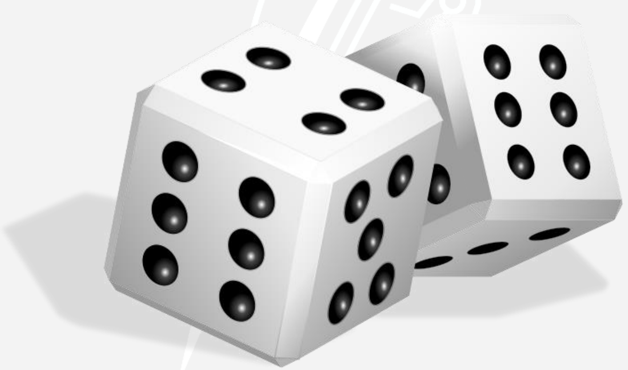
$$P(doubles|\underbrace{D_1 = 4}_{\text{evidence}}) =$$



Posterior Probability

The conditional or **posterior probability** of an event is the degree of belief that the event will happen given some evidence.

$$P(\underbrace{doubles}_{\text{event}}|D_1 = 4) =$$

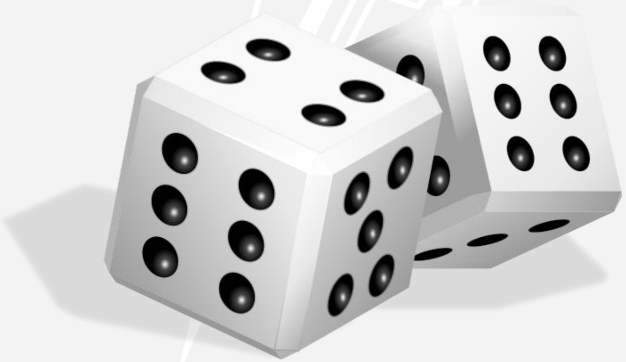


Posterior Probability

The conditional or **posterior probability** of an event is the degree of belief that the event will happen given some evidence.

$$P(doubles|D_1 = 4) =$$

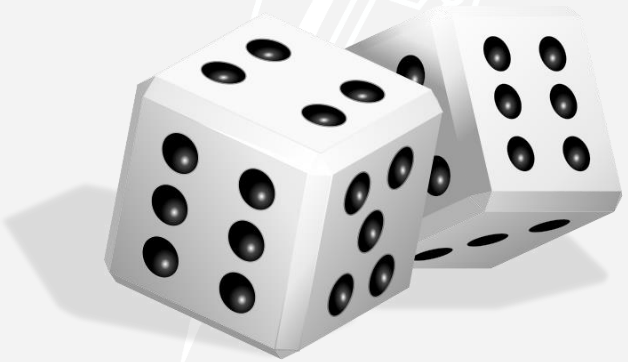
"What is the probability that doubles will be rolled given that the first die already shows a 4?"



Posterior Probability

The conditional or **posterior probability** of an event is the degree of belief that the event will happen given some evidence.

$$P(doubles|D_1 = 4) = \frac{1}{6}$$

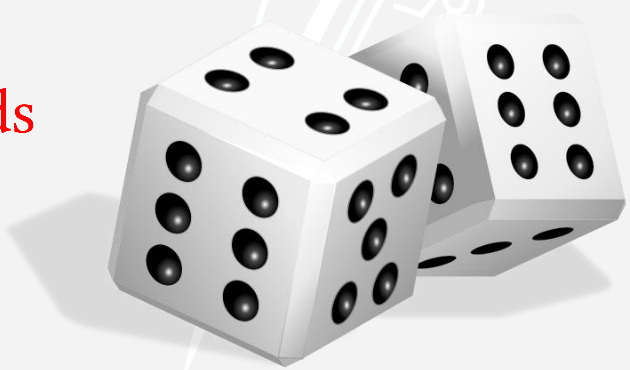


Posterior Probability

The conditional or **posterior probability** of an event is the degree of belief that the event will happen given some evidence.

$$P(doubles|D_1 = 4) = \frac{1}{6}$$

Total number of possible worlds
in which the evidence holds
i.e. 6 worlds where the
first die shows a 6

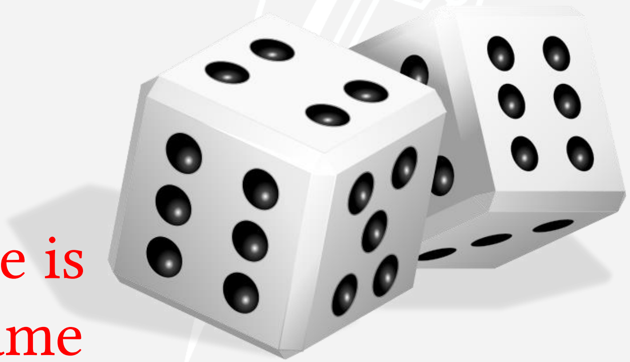


Posterior Probability

The conditional or **posterior probability** of an event is the degree of belief that the event will happen given some evidence.

$$P(doubles|D_1 = 4) = \frac{1}{6}$$

Total number of worlds in
which the proposition is true
i.e. Of the 6 possible worlds, there is
only 1 where the die show the same



Posterior Probability

Posterior probabilities are defined in terms of prior probabilities.

$$P(event|evidence) = \frac{P(event \wedge evidence)}{P(evidence)}$$

Posterior Probability

Posterior probabilities are defined in terms of prior probabilities.

$$P(doubles|D_1 = 4) = \frac{P(doubles \wedge D_1 = 4)}{P(D_1 = 4)}$$

Posterior Probability

Posterior probabilities are defined in terms of prior probabilities.

$$P(doubles|D_1 = 4) = \frac{P(doubles \wedge D_1 = 4)}{6}$$

Posterior Probability

Posterior probabilities are defined in terms of prior probabilities.

$$P(doubles|D_1 = 4) = \frac{1}{6}$$

Probability Distribution

A **probability distribution** is a table of all possible values for a random variable and their associated probabilities, which must sum to 1.

Variable	Value	Probability
$Cavity$	$true$	0.2
	$false$	0.8

20% of people who visit the dentist's office have a cavity, and the other 80% do not.

Probability Distribution

A **probability distribution** is a table of all possible values for a random variable and their associated probabilities, which must sum to 1.

Variable	Value	Probability
Weather	*sunny*	0.60
	rainy	0.10
	cloudy	0.29
	snowy	0.01

Joint Distribution

We often want to talk about multiple variables at the same time. When we combine multiple variables into a single table, we call this a joint distribution.

Let's consider both the presence of a cavity and the weather at the same time.

Joint Distribution

A **joint probability distribution** is a table of all possible values for two or more random variables and their associated probabilities. All cells in the table must sum to 1.

Probability Notation

We use an italic normal font P to indicate the probability of some specific event. This indicates a single number.

We us a non-italic bold font $\mathbf{P}$ to indicate the probability distribution of one or more variables. This indicates a table of numbers.

Probability Notation

Random variable names are capitalized.

Values are written in all lowercase.

When a random variable name appears by itself, it means "all possible values for this variable."

We can indicate a specific value for a variable by using "=" or by simply writing the value.

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

Indicates a table of values

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

Name of a variable

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

No "=" means all possible values for this variable

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

Variable with no "=", so all possible values for this variable also

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

		Variable	
		Cavity	
		Value	
Variable	Value	*true*	*false*
Weather	*sunny*	0.120	0.480
	rainy	0.020	0.080
	cloudy	0.058	0.232
	snowy	0.002	0.008

This is the full joint probability distribution for the variables *Weather* and *Cavity*.

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

		Variable	
		Cavity	
		Value	
Variable	Value	*true*	*false*
Weather	*sunny*	0.120	0.480
	rainy	0.020	0.080
	cloudy	0.058	0.232
	snowy	0.002	0.008

$$P(Weather = sunny \wedge Cavity = true) = ?$$

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

		Variable	
		$Cavity$	
		Value	
Variable	Value	$true$	$false$
$Weather$	$sunny$	0.120	0.480
	$rainy$	0.020	0.080
	$cloudy$	0.058	0.232
	$snowy$	0.002	0.008

$P(Weather = sunny \wedge Cavity = true) = ?$

Indicates a specific number

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

		Variable	
		Cavity	
		Value	
Variable	Value	*true*	*false*
Weather	*sunny*	0.120	0.480
	rainy	0.020	0.080
	cloudy	0.058	0.232
	snowy	0.002	0.008

$$P(\underbrace{Weather = sunny}_{\text{Indicates a value for this variable}} \wedge Cavity = true) = ?$$

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

		Variable	
		Cavity	
		Value	
Variable	Value	*true*	*false*
Weather	*sunny*	0.120	0.480
	rainy	0.020	0.080
	cloudy	0.058	0.232
	snowy	0.002	0.008

$$P(Weather = sunny \wedge \underbrace{Cavity = true}_{\text{Indicates a value for this variable}}) = ?$$

Probability Notation

$\mathbf{P}(Weather, Cavity)$=

		Variable	
		Cavity	
		Value	
Variable	**Value**	*true*	*false*
Weather	*sunny*	0.120	0.480
	rainy	0.020	0.080
	cloudy	0.058	0.232
	snowy	0.002	0.008

$$P(Weather = sunny \wedge Cavity = true) = 0.120$$

Probability Notation

$\mathbf{P}(Weather, Cavity) =$

		Variable	
		Cavity	
		Value	
Variable	Value	$true$	$false$
Weather	$sunny$	0.120	0.480
	$rainy$	0.020	0.080
	$cloudy$	0.058	0.232
	$snowy$	0.002	0.008

$P(sunny \wedge true) = 0.120$

Equivalent, but abbreviated

Probability Notation

$\mathbf{P}(sunny, Cavity) =$

Probability Notation

$\mathbf{P}(sunny, Cavity) =$

Indicates a table of values

Probability Notation

$\mathbf{P}(sunny, Cavity) =$

Only show cases where
$Weather = sunny$

Probability Notation

$\mathbf{P}(sunny, Cavity) =$

This is the name of a value, so only show cases where $Weather = sunny$

Probability Notation

$\mathbf{P}(sunny, Cavity) =$

This is the name of a variable, and there is no "=", so show all values for this variable

Probability Notation

$\mathbf{P}(sunny, Cavity) =$

		Variable	
		Cavity	
		Value	
Variable	Value	*true*	*false*
Weather	*sunny*	0.2	0.8

Notice how the cells sum to 1

Probability Notation

$\mathbf{P}(sunny, Cavity) =$

		Variable	
		Cavity	
		Value	
Variable	Value	*true*	*false*
Weather	*sunny*	0.2	0.8

This is a joint probability distribution (because it involves more than one variable), but we don't call it the "full" join distribution, because we are only showing one possible value for *Weather*.

Dentistry Domain

Consider an AI system designed to diagnose whether or not a patient visiting the dentist has a cavity.

We will consider 3 Boolean random variables:

- *toothache* indicates whether or not the patient reports having a toothache
- *catch* indicates whether or not the dentist's metal hook catches in the patient's tooth
- *cavity* indicates whether or not the patient has a cavity

Dentistry Domain

The dentist collects information (i.e. the values of *toothache* and *catch*) and inputs that information into the system.

The system reports a probability for the value of *cavity* based on the input. In other words, the system tells the dentist how likely it is that the patient has a cavity.

Dentistry Full Joint Distribution

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

$$P(cavity) = ?$$

What is the prior probability of having a cavity?

In other words, when a new patient walks in and we don't know anything about him or her, what is the likelihood that the patient has a cavity?

Dentistry Full Joint Distribution

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$P(cavity) = ?$$

What is the prior probability of having a cavity?

We only consider worlds in which $cavity$ is true.

Dentistry Full Joint Distribution

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$P(cavity) = ?$$

What is the prior probability of having a cavity?

We consider worlds in which both $toothache$ and $\neg toothache$ are true, and also worlds in which both $catch$ and $\neg catch$ is true.

Dentistry Full Joint Distribution

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

$$P(cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$$

When a new, unknown patient walks in, there is a 20% chance that he or she has a cavity.

Naïve Solution

One (bad) solution would be to simply report a 20% chance of having a cavity all the time.

However, we can do better if we consider some additional information.

Dentistry Full Joint Distribution

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$P(cavity|toothache) = ?$$

What is the posterior probability of the patient having a cavity given that the patient has a toothache?

Dentistry Full Joint Distribution

	toothache	
	catch	*¬catch*
cavity	0.108	0.012
¬cavity	0.016	0.064

$$P(cavity|toothache) = ?$$

We can disregard all worlds in which $\neg toothache$.

Dentistry Full Joint Distribution

	toothache
cavity	0.108+0.012
$\neg$*cavity*	0.016+0.064

$$P(cavity|toothache) = ?$$

We need to consider both *catch* and $\neg$*catch*, so we sum those columns and combine them into 1.

Dentistry Full Joint Distribution

	toothache
cavity	0.12
$\neg$*cavity*	0.08

$$P(cavity|toothache) = ?$$

We need to consider both *catch* and $\neg$*catch*, so we sum those columns and combine them into 1.

Dentistry Full Joint Distribution

	toothache
cavity	0.12
$\neg$*cavity*	0.08

$$P(cavity|toothache) = ?$$

The cells in the table no longer sum to 1!

We need to **normalize** the table.

Dentistry Full Joint Distribution

	toothache
cavity	0.12α
$\neg$*cavity*	0.08α

$$P(cavity|toothache) = ?$$

Normalization means multiplying each cell in the table by a single number which will cause all the cells to sum to 1 again.

$$0.12\alpha + 0.08\alpha = 1$$

Dentistry Full Joint Distribution

	toothache
cavity	0.12α
$\neg$*cavity*	0.08α

$$P(cavity|toothache) = ?$$

Normalization means multiplying each cell in the table by a single number which will cause all the cells to sum to 1 again.

$$\alpha = 5$$

Dentistry Full Joint Distribution

	toothache
cavity	0.6
$\neg$*cavity*	0.4

$$P(cavity|toothache) = ?$$

Now that we have calculated $\mathbf{P}(Cavity|toothache)$, we can look up the value we want in the table.

Dentistry Full Joint Distribution

	$toothache$
$cavity$	0.6
$\neg cavity$	0.4

$$P(cavity|toothache) = 0.6$$

Now that we have calculated $\mathbf{P}(Cavity|toothache)$, we can look up the value we want in the table.

If the patient reports having a toothache, there is a 60% chance that he or she has a cavity.

Causality and Diagnosis

When we speak of cause and effect, we speak of two events that often occur together (the cause first, followed by the effect).

First order logic is well suited to reasoning in the **causal direction**. Logical deduction asks, "Given some cause, what will be the effect?"

Logic is poorly suited to reasoning in the opposite, or **diagnostic direction**. "Give the effect, what was the cause?"

Bayes' Rule

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

$$P(b|a) = \frac{P(a \wedge b)}{P(a)}$$

This is the definition of a posterior probability, written twice.

Bayes' Rule

$$P(a|b)P(b) = P(a \wedge b)$$

$$P(b|a)P(a) = P(a \wedge b)$$

This is an equivalent form of those same definitions, often called the **product rule**.

Note that the right hand side of both equations is the same.

Bayes' Rule

$$P(b|a)P(a) = P(a|b)P(b)$$

Combine both equations into one.

Bayes' Rule

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)}$$

Divide by $P(a)$.

Bayes' Rule

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)}$$

This formula is **Bayes Rule**, named for its discoverer, Rev. Thomas Bayes. It was further developed into the form above by Pierre-Simon Laplace.

Bayes' Rule

$$P(cause|effect) = \frac{P(effect|cause)P(cause)}{P(effect)}$$

This formula is **Bayes Rule**, named for its discoverer, Rev. Thomas Bayes. It was further developed into the form above by Pierre-Simon Laplace.

It has proven very helpful in diagnostic reasoning.

Bayes' Rule

$$P(cause|effect) = \frac{P(effect|cause)P(cause)}{P(effect)}$$

We have observed some effect, and given something which might be the cause, we want to know how likely it is that the cause caused the effect.

To calculate this, we need three things.

Bayes' Rule

$$P(cause|effect) = \frac{P(effect|cause)P(cause)}{P(effect)}$$

We need to know the prior probability of the effect.

In other words, we need to know how likely the effect is to happen regardless of other factors.

Bayes' Rule

$$P(cause|effect) = \frac{P(effect|cause)P(cause)}{P(effect)}$$

We need to know the prior probability of the cause.

In other words, we need to know how likely the cause is to happen regardless of other factors.

Bayes' Rule

$$P(cause|effect) = \frac{P(effect|cause)P(cause)}{P(effect)}$$

We need to know the posterior probability of the effect given the cause.

In other words, we need to know how often the cause causes the effect.

Bayes' Rule in Dentistry Domain

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$P(cavity|ache \wedge catch) = \frac{P(ache \wedge catch|cavity)P(cavity)}{P(ache \wedge catch)}$$

Bayes' Rule in Dentistry Domain

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$P(cavity|ache \land catch) = \frac{P(ache \land catch|cavity)P(cavity)}{P(ache \land catch)}$$

Find $P(ache \land catch)$

NIL

Bayes' Rule in Dentistry Domain

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$P(cavity|ache \wedge catch) = \frac{P(ache \wedge catch|cavity)P(cavity)}{0.124}$$

Find $P(cavity)$

Bayes' Rule in Dentistry Domain

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

$$P(cavity|ache \land catch) = \frac{P(ache \land catch|cavity)0.2}{0.124}$$

Find $P(ache \land catch|cavity)$

Bayes' Rule in Dentistry Domain

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108α	0.012α	0.072α	0.008α

$$P(cavity|ache \wedge catch) = \frac{P(ache \wedge catch|cavity)0.2}{0.124}$$

Find $P(ache \wedge catch|cavity)$

Bayes' Rule in Dentistry Domain

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.54	0.06	0.36	0.04

$$P(cavity|ache \wedge catch) = \frac{P(ache \wedge catch|cavity)0.2}{0.124}$$

Find $P(ache \wedge catch|cavity)$

Bayes' Rule in Dentistry Domain

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.54	0.06	0.36	0.04

$$P(cavity|ache \land catch) = \frac{0.54 \cdot 0.2}{0.124} \approx 0.871$$

When a patient complains of a toothache and the hook catches in the tooth, there is an 87% chance that the patient has a cavity.

Independence

Sometimes, the value of one variables does not influence the value of another.

Consider $\mathbf{P}(Cavity, Weather)$. The weather outside and the state of a patient's tooth are not related, so we can reason about them separately.

Absolute Independence

We say that random variables a and b are **absolutely independent** when $P(a|b) = P(a)$.

Equivalently, $P(a \wedge b) = P(a)P(b)$.

In other words, the fact that we know the values of b does not give us any extra information about the value of a (and vice versa).

Absolute Independence

If we assume the weather and the presence of a cavity have no influence on one another:

$$\mathbf{P}(Cavity|Weather) = \mathbf{P}(Cavity)$$

In other words, we can't improve our cavity diagnosis system by having the dentist input the weather. The chances of having a cavity are the same in all kinds of weather.

Conditional Independence

Let us assume than cavities cause toothaches and that cavities cause hooks to catch in teeth.

Can we say that $Toothache$ and $Catch$ are absolutely independent?

Toothaches do not cause the hook to catch, nor does the hook catching cause a toothache. However, that is not what the question is asking.

Conditional Independence

Let us assume than cavities cause toothaches and that cavities cause hooks to catch in teeth.

Can we say that $Toothache$ and $Catch$ are absolutely independent?

No, they are not absolutely independent. If one has a toothache, then there is a higher chance one has a cavity, which means there is a higher chances that the hook will catch.

Conditional Independence

Let us assume than cavities cause toothaches and that cavities cause hooks to catch in teeth.

Can we say that $Toothache$ and $Catch$ are absolutely independent?

However, because these two variables do not directly influence one another, we can say that they are independent given the presence of absence of a cavity!

Conditional Independence

We say that two random variables a and b are **conditionally independent** of one another given some third variable z when

$$P(a \wedge b|z) = P(a|z)P(b|z).$$

Conditional Independence

$$\mathbf{P}(Toothache, Catch|Cavity) = \mathbf{P}(Toothache|Cavity)\mathbf{P}(Catch|Cavity)$$

In other words, once we have ruled out the influence of $Cavity$, $Toothache$ and $Catch$ are independent of one another.

Conditional Independence

$$\mathbf{P}(Toothache, Catch|Cavity) = \mathbf{P}(Toothache|Cavity)\mathbf{P}(Catch|Cavity)$$

If we only consider worlds where $Cavity$ is true, then $Toothache$ and $Catch$ are absolutely independent. Likewise, if we only consider worlds where $Cavity$ is false, then $Toothache$ and $Catch$ are absolutely independent.

Thus, we say they are conditionally independent.

Independence

Independence assumptions reduce the size of the table needed to represent the full joint probability distribution.

Conditional independence assumptions are often more easily available than absolute independence assumptions.