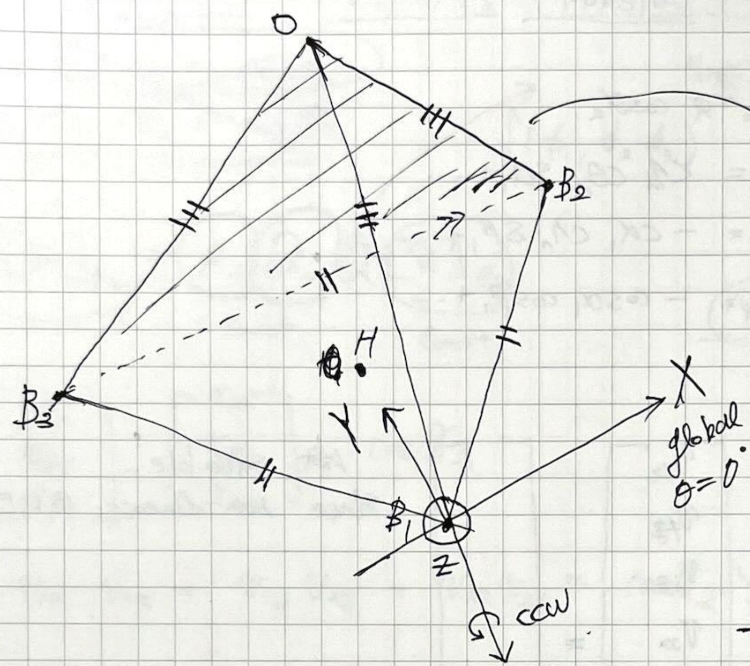
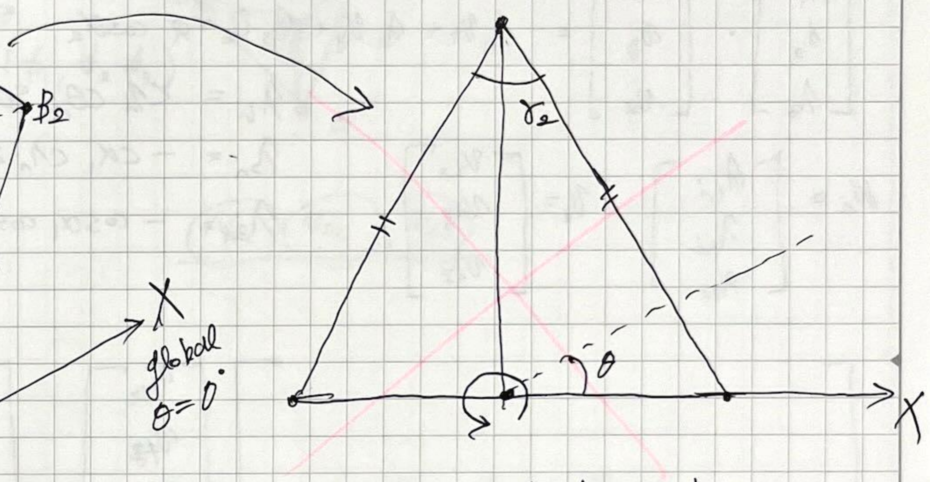




$\underline{v_i = R\, v_{i0}}$

R derived by infinitesimal displacement (applied user force)

$R_d,\ v_{10},\ v_{20},\ v_{30} \rightarrow \boxed{IK} \rightarrow \begin{bmatrix} x_d \\ y_d \\ \phi_d \end{bmatrix}$

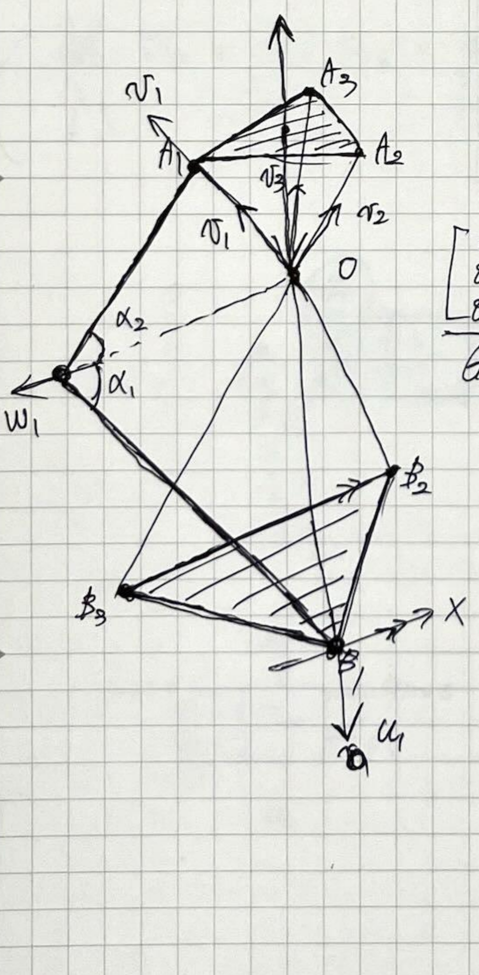
$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \rightarrow \boxed{FK} \rightarrow R_c,\ \begin{bmatrix} x_c \\ y_c \\ \phi_c \end{bmatrix}$

$\cancel{\begin{bmatrix} x_c \\ y_c \\ \phi_c \end{bmatrix} = f(R_c)}$

Euler angles

$\begin{bmatrix} \theta_{1c} \\ \theta_{2c} \\ \theta_{3c} \end{bmatrix} \rightarrow \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \rightarrow R_c \rightarrow \begin{bmatrix} x_c \\ y_c \\ \phi_c \end{bmatrix}$

Gosselin et al. (1994)



$w_i \cdot v_i = \cos\alpha_2$ $v_i \cdot v_j = \cos\gamma_2$ Cartesian?

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \cdot \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \lambda_1 v_x + \lambda_2 v_y + \lambda_3 v_z = \cos\alpha_2$$

$$\lambda_{1i} = c\eta_i \, c\theta_i \, s\alpha_1 + \cdots$$

$$\lambda_{2i} = -c\alpha_1 \, c\eta_i \, s\beta_1 + \cdots$$

$$\lambda_{3i} = -\cos\alpha_1 \cos\beta_1 + \cdots$$

$$w_i = \begin{bmatrix} \lambda_{1i} \\ \lambda_{2i} \\ \lambda_{3i} \end{bmatrix} \quad v_i = \begin{bmatrix} v_{ix} \\ v_{iy} \\ v_{iz} \end{bmatrix}$$

$$\begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{1z} \\ v_{2x} \\ v_{2y} \\ v_{2z} \\ v_{3x} \\ v_{3y} \\ v_{3z} \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

not solvable...
since non-linear S.O.E

Euler angles $\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \rightarrow R \begin{bmatrix} R_{11} & \cdots & R_{13} \\ \vdots & \ddots & \vdots \\ R_{31} & \cdots & R_{33} \end{bmatrix}$

$$\cancel{\begin{bmatrix} \alpha \\ \theta \\ \phi \end{bmatrix}}$$

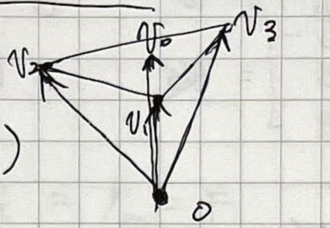
$$\cancel{\begin{bmatrix} \alpha \\ \theta \\ \phi \end{bmatrix}}$$

$\cancel{\cos\phi_2 = \cos\alpha_2}$ $\quad \phi_1, \phi_2, \phi_3 \quad$ $\cancel{u_i \cdot v_i = \cos\alpha_2}$

① $\cancel{u_i \cdot v_i = \cos\alpha_2}$

② $\cancel{v_i \cdot v_0 = \cos\gamma_2}$

$f(\phi_1, \phi_2, \phi_3)$



$$[v_i]_{P_0} = \underbrace{P_i Q_i}_{\text{const.}} Q \underbrace{[v_i]_{P_2}}_{\text{const.}}$$

v_{1c} v_{2c} v_{3c}

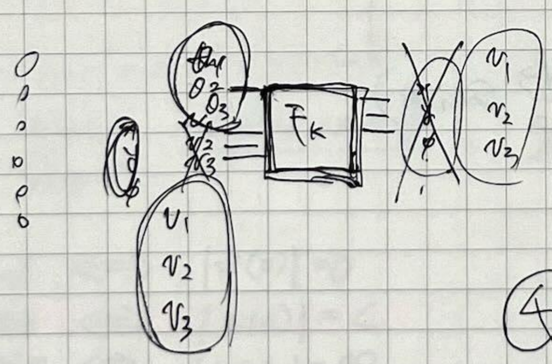
$$\begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{1z} \end{bmatrix} \cdot \begin{bmatrix} v_{2x} \\ v_{2y} \\ v_{2z} \end{bmatrix} = \cos\gamma_2$$

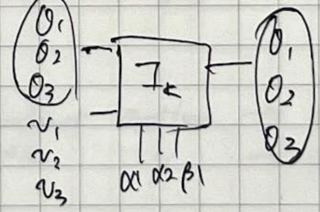
$$\left(\begin{array}{l} v_{1x} \cdot v_{2x} + v_{1y} v_{2y} + v_{1z} v_{2z} = \cos\gamma_2 \\ \qquad \qquad '' \\ \qquad \qquad '' \end{array} \right)$$

6 system of equations.
→ 9 variable?

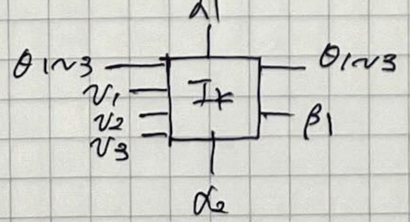
$$\left(\begin{array}{l} \cancel{u_{11}} \cdot v_{1x} + \cancel{u_{12}} \cdot v_{1y} + \cancel{u_{13}} \cdot v_{1z} = \cos\alpha_2 \\ \lambda_1 \qquad \lambda_2 \; '' \qquad \lambda_3 \\ \qquad \qquad '' \end{array} \right)$$

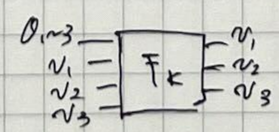
x, y, ψ → v_1, v_2, v_3





④ ①



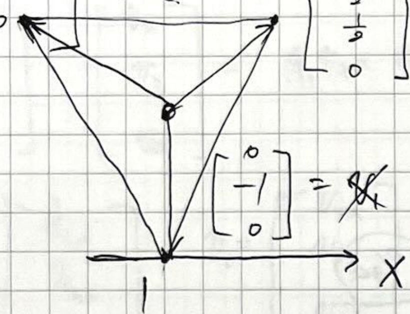


$\begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} = \cancel{v_3}$

$\cancel{\beta_2 = 90^\circ}$

$\begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} = \cancel{v_2}$

$\theta = \begin{bmatrix} \pi/2 \\ \pi/2 \\ \pi/2 \end{bmatrix}$



$\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \cancel{v_1}$

$\cancel{\beta_2 = 90^\circ}$

$\cancel{\alpha_2 = 0^\circ}$

$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 45^\circ$

$\theta_1 = \theta_2 = \theta_3 = 90^\circ$

$v_1 = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad v_2 = \begin{bmatrix} \frac{\sqrt{3}}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ 1/\sqrt{2} \end{bmatrix} \quad v_3 = \begin{bmatrix} -\frac{\sqrt{3}}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$\theta \to 95^\circ$

$\frac{1}{\sqrt{2}} = \frac{2}{\sqrt{3}} \sin\frac{\gamma}{2}$

$\frac{\sqrt{6}}{4} \cancel{\frac{\sqrt{3}}{2\sqrt{2}}} = \sin\frac{\gamma}{2}$

$\sin\beta = \frac{2}{\sqrt{3}} \sin\frac{\gamma}{2}$

$\gamma = 2\,\mathrm{asin}\left(\frac{\sqrt{3}}{2} \sin\beta\right)$