微积分（一）下第 5 周第一次课作业答案与提示

复合函数微分法

1. 填空（均设 f 可微）：

1）设 $z=\arctan(xy)$，且 $y=e^x$，则 $\dfrac{\mathrm{d}z}{\mathrm{d}x}=\underline{y(1+x)/(1+x^2y^2)}$.

2）设 $u=f(x^2-y^2,e^{xy})$，则 $\dfrac{\partial u}{\partial x}=\underline{2xf_1'+ye^{xy}f_2'}$；$\dfrac{\partial u}{\partial y}=\underline{-2yf_1'+xe^{xy}f_2'}$.

3）设 $u=f(ax^2+by^2+cz^2)$，a，b，c 为常数，则 $\dfrac{\partial u}{\partial x}=\underline{2axf'}$.

4）设 $u=f\left(\dfrac{x}{y},\dfrac{y}{z}\right)$，则 $\dfrac{\partial u}{\partial x}=\underline{f_1'/y}$；$\dfrac{\partial u}{\partial y}=\underline{-xf_1'/y^2+f_2'/z}$；$\dfrac{\partial u}{\partial z}=\underline{-yf_2'/z^2}$.

5）设 $u=f(x,xy,xyz)$，则 $\dfrac{\partial u}{\partial x}=\underline{f_1'+yf_2'+yzf_3'}$；$\dfrac{\partial u}{\partial y}=\underline{xf_2'+xzf_3'}$；$\dfrac{\partial u}{\partial z}=\underline{xyf_3'}$.

2. 验证下列等式

1）设 $z=\dfrac{y}{f(x^2-y^2)}$，f 可微，则有 $\dfrac{1}{x}\dfrac{\partial z}{\partial x}+\dfrac{1}{y}\dfrac{\partial z}{\partial y}=\dfrac{z}{y^2}$.

2）设 $z=xy+xf(u)$，f 可微且 $u=\dfrac{y}{x}$，则有 $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=z+xy$. （略）

3. 设函数 f 二阶偏导数连续，求列出的二阶偏导数：

1）$u=f(x,\dfrac{x}{y})$，求 $\dfrac{\partial^2 u}{\partial x\partial y}$. 答案：$\dfrac{\partial^2 u}{\partial x\partial y}=-\dfrac{x}{y^2}f_{12}''-\dfrac{1}{y^2}f_2'-\dfrac{x}{y^3}f_{22}''$.

2）设 $z=\dfrac{1}{x}f(xy)+y\varphi(x+y)$，其中 f，φ 具有二阶连续导数，求 $\dfrac{\partial^2 z}{\partial x\partial y}$.

答案：$\dfrac{\partial^2 z}{\partial x\partial y}=yf''(xy)+\varphi'(x+y)+y\varphi''(x+y)$

3）$u=f(\sin x,\cos y,e^{x+y})$，求 $\dfrac{\partial^2 u}{\partial x\partial y}$.

答案：$\dfrac{\partial^2 u}{\partial x\partial y}=\cos x[-\sin y f_{12}''+e^{x+y}f_{13}'']+e^{x+y}f_3'+e^{x+y}[-\sin y f_{32}''+e^{x+y}f_{33}'']$.

4. 设 $u=f\left(\dfrac{y}{x}\right)+xg\left(\dfrac{y}{x}\right)$，其中 f,g 二次连续可微，证明 $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=0$.

（略）