微积分（一）下第 5 周第二次课作业答案与提示

隐函数微分法

1. 填空: 设 $z = f(x,y)$ 由下列方程所确定，试求列出的偏导数.

1）设 $xyz = x+y+z$，则 $\frac{\partial z}{\partial x} = \underline{(1-yz)/(xy-1)}$.

2）设 $f(x+y, y+z, z+x) = 0$，f 可微，则 $\frac{\partial z}{\partial x} = -\frac{f_1'+f_3'}{f_2'+f_3'}$，$\frac{\partial z}{\partial y} = -\frac{f_1'+f_2'}{f_2'+f_3'}$.

3）设 $F(x, x+y, x+y+z) = 0$，F 可微，则 $\frac{\partial z}{\partial y} = -\frac{F_2'+F_3'}{F_3'}$.

4）设 $e^z - xyz = 0$，则 $\frac{\partial z}{\partial x} = \frac{z}{x(1-z)}$，$\frac{\partial^2 z}{\partial x^2} = \frac{z(z^2-2z+2)}{x^2(1-z)^3}$.

2. 设 $y = f(x,t)$，而 t 是由方程 $F(x,y,t) = 0$ 所确定的 x, y 的函数，试证：$\frac{dy}{dx} = \frac{f_x F_t - f_t F_x}{f_t F_y + F_t}$.　　提示：按由方程组确定的隐函数求导.

3. 设 $u = f(x,y,z)$ 具有连续偏导数，$z = z(x,y)$ 是由方程 $xe^x - ye^y - ze^z = 0$ 确定的二元函数，求 du.

答案：$du = f_x dx + f_y dy + f_z \frac{e^x(x+1)dx - e^y(y+1)dy}{e^z(z+1)}$.

4. 设 $z = z(x,y)$ 由方程 $z^3 - 2x\cos z + y = 0$ 所确定，求 $\frac{\partial^2 z}{\partial x \partial y}$.

答案：$z_{xy} = \frac{12z\cos x + 4x + 6z^2\sin z}{(3z^2 + 2x\sin z)^3}$.

5. 设 $\begin{cases} z = x^2 + y^2 \\ x^2 + 2y^2 + 3z^2 = 20 \end{cases}$，求 $\frac{dy}{dx}$，$\frac{dz}{dx}$.

答案：$\frac{dy}{dx} = \frac{-x(6z+1)}{2y(3z+1)}$，$\frac{dz}{dx} = \frac{x}{3z+1}$.

6. 设 $\begin{cases} u = f(ux, v+y) \\ v = g(u-x, v^2 y) \end{cases}$，$u$，$v$ 可微，求 $\frac{\partial u}{\partial x}$，$\frac{\partial v}{\partial x}$.

答案：$\frac{\partial u}{\partial x} = \frac{u f_1'(1-2vyg_2') - f_2'g_1'}{(1-xf_1')(1-2vyg_2') - f_2'g_1'}$，$\frac{\partial v}{\partial x} = \frac{g_1'(uf_1' + xf_1' - 1)}{(-xf_1')(1-2vyg_2') - f_2'g_1'}$.