ACTL3141 Assignment

Evan Xiong z5421149

Exploratory Data Analysis (EDA) on the Chilean Mortality Data

An analysis of the "Chilean Mortality" dataset has revealed several correlations in the data. The occurrence of annuitants dying is infrequent with approximately 92% of the population either remaining alive or censored during the investigation period spanning from 1st of January 2014 to 31st of December 2018. When we group annuitants by sex, it reveals an imbalance within the dataset as 57.2% of annuitants are female whereas 42.8% of annuitants are male.

The proportion of deaths by gender is illustrated in the proportional stacked bar graph seen in **Figure 1**. **Figure 1** indicates that there is a notable disparity in mortality rates between genders during the investigation period. The visualisation shows that 10.7% of males died and 6.0% of females died. The graph considers the number of deaths as proportions to accommodate the predominantly female dataset. This observation suggests that males are more susceptible to mortality, this is potentially attributed to lifestyle factors such as smoking or drug use. Additionally, a higher proportion of males are disabled compared to females, as 13.4% of male annuitants are disabled whereas 5.4% of female annuitants are disabled. Preexisting terminal medical conditions increase vulnerability to mortality, thereby contributing to higher mortality rates among disabled individuals.

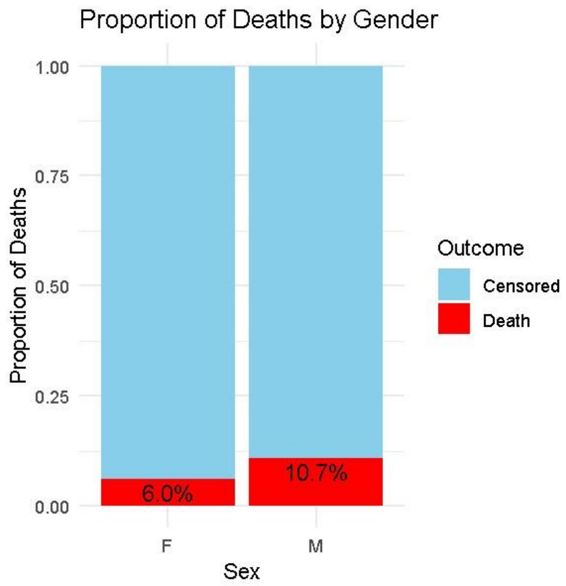


Figure 1: Proportion of Deaths by Gender

Figure 2 displays an overlapping histogram illustrating the distribution of the age of death by gender. A peak in the number of male deaths occurs between ages 70 and 72, while the number of female deaths peaks between ages 73 and 76. Moreover, the visualisation indicates a consistently higher number of male deaths across most ages, except at extreme ages near 100, where the male population in the dataset is significantly smaller. This observation reinforces the findings from **Figure 1**, highlighting the higher mortality rates among males compared to females, as the data indicates that more males are dying at an earlier age. The gradual decline in death frequency beyond this point reflects typical human mortality patterns. As reaching an old age such as 100 requires surviving to that age, thus, the likelihood of dying at such an old age diminishes.

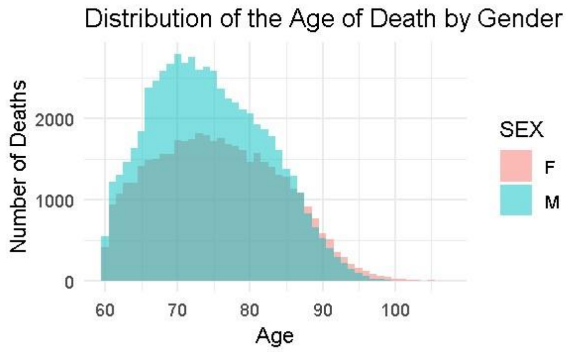


Figure 2: Distribution of the Age of Death by Gender

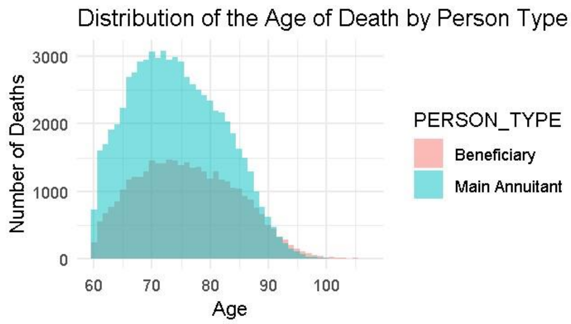


Figure 3: Distribution of the Age of Death by Person Type

In **Figure 3**, an overlapping histogram showcases the distribution of the age of death by person type. An elementary examination indicates that beneficiaries exhibit lower mortality rates compared to main annuitants, as evidenced by their consistently lower number of deaths and later peak in the number of deaths. This discrepancy arises from the fact that beneficiaries must survive longer than main annuitants to receive their benefits. Moreover, the disparity in the number of deaths is also influenced by the fact that 89.7% of beneficiaries are female whilst only 10.3% are male. As mentioned earlier, males generally experience lower mortality rates.

Survival Analysis
Parametric Approaches

The cumulative hazard depicted in **Appendix A.1** reveals an exponential relationship with time, suggesting a survival function of the form $S(t) = e^{Bc^x}$. This form of the survival function poses challenges for using parametric approaches to conduct survival analysis, in which only the Weibull distribution bear a vague resemblance to the mortality of the annuitants.

Non-Parametric/Semi-Parametric Approaches
Kaplan-Meier (KM)

Before performing Kaplan-Meier estimation, it essential to establish the assumptions of this estimation. Firstly, we assume lives are independent and the censoring is non-informative. The non-informative censoring is generally a reasonable assumption, as in the mortality dataset annuitants are seen to be censored at the end of the investigation period. However, the assumption of independent live maybe less realistic as it's common for individuals to have multiple pension accounts. When creating the survival objects, annuitant ages reached up to roughly 130 years old. This is unrealistic as the oldest person recorded Jeanne Calment was aged 122. Due to the scarcity of data and the absence of verification for these supercentenarians, they have been excluded from the dataset.

Grouping annuitants by sex, disability status, and type of annuitant has allowed us to assess the significance of these covariates. Kaplan-Meier estimates, as shown in **Appendix A.2**, provide insights into the impact of each covariate. Comparing survival functions in **Figure A.2.1** reveals that male annuitants exhibit higher mortality rates than female annuitants. This is evident from the curve representing females being consistently above that of males, indicating a lower probability of survival for males each year compared to females. Moreover, applying the same reasoning to the survival functions in **Figure A.2.2** and **Figure A.2.3**, we can deduce that healthy annuitants and beneficiaries experience lower mortality rates compared to disabled annuitants and main annuitants respectively. To support our graphical findings and determine whether the differences in the survival functions are significant we can conduct numerical analysis using the Log-rank test. The results of the Log-rank tests are presented in **Figure 4**.

Subgroups	Test Statistic	P-Value
Male vs Female	8697	$p \approx 0$
Healthy vs Disabled	21220	$p \approx 0$
Main Annuitant vs Beneficiary	4067	$p \approx 0$

Figure 4: Log-rank Test Results by Subgroups

The Log-rank test assesses whether there exists a significant disparity in the hazard rates among two different populations across $\forall\ t \leq \tau$. The Log-rank test achieves this by evaluating the validity of the null hypothesis $H_0\colon h_1(t) = h_2(t)$, where $h_1(t)$ is the hazard rate of the first subgroup and $h_2(t)$ is the hazard rate of the second subgroup. In the case where we consider the population of male and female annuitants, the Log-rank test at the 5% significance level yields a test statistic of 8697 as shown in **Figure 4**. This corresponds to a p-value approximately equal to 0. With the p-value being less than 0.05, we reject the null hypothesis, indicating that the hazard rates of the two subgroups are different. Consequently, this implies that the survival functions of the two populations also vary as $S(t) = \mathrm{e}^{-H(t)}$, where $H(t)$ is the cumulative hazard rates and equals $\int_0^t h(t)$. Conducting the Log-rank test on the populations of healthy and disabled annuitants, as well as the populations of main annuitants and beneficiaries, further reinforces our graphical observations in which the survival rates of these subgroups differ.

Cox Regression

Conducting a Cox Regression on the Chilean Mortality dataset enhances our Kaplan-Meier analysis by allowing us to quantify the effect of each covariate. Before proceeding with the Cox regression, it's essential to address the assumptions underlying the regression model. Cox Regression is a proportional hazard model. In this model, the relative risk $\frac{\lambda(t;Z_1)}{\lambda(t;Z_2)}$ [1] remains constant $\forall\, t$. Statistical tests based on the scaled Schoenfeld residuals are utilised to assess the independence between the residuals and time. The results of these tests seen in **Figure A.3.1** have yielded p-values approximately equal to 0 for each of the covariates. This indicates that the tests are statistically significant for each of the covariates, this implies that the assumption of proportional hazard cannot be upheld. As the covariates change with time, which leads to a situation where the relative risk does not remain constant $\forall\, t$. Additionally, it's reasonable to consider the presence of multicollinearity in the regression model given the correlation between the covariates; sex, disability status and type of annuitant. Specifically, there appears to be a correlation between sex and the type of annuitant, as the beneficiaries are predominately female in the dataset. Thus, the coefficients for these covariates coincide with each other. Consequently, the results obtained from Cox Regression may not accurately reflect Chilean mortality as the regression analysis assumes proportional hazard and the absence of multicollinearity. To validate our findings, a confirmatory test involving the examination of the Cox-Snell residuals can be done. The residual plot showcased in **Figure A.3.2** reveals a lack of linearity in the residuals. This deviation from a straight line suggests that the residuals are not exponentially distributed, indicating a poor fit of the Cox Regression model to the data.

The Cox Regression results presented in **Figure A.4.1** provide insight into the coefficient values for each covariate in our model. The covariates of the regression model serve as indicators, in which Z_{i1}[2] $= 1$ if the annuitant is healthy, $Z_{i2} = 1$ if the annuitant is male and $Z_{i3} = 1$ if the type of annuitant is a main annuitant. The coefficient value for Z_{i1} is -1.091442. The negative value suggests a negative correlation with the hazard rate, indicating that healthy annuitants experience lower mortality rates. The coefficient value for Z_{i2} is 0.561295, which indicates a positive correlation with the hazard rate. This implies that male annuitants experience higher mortality rates. The coefficients value for Z_{i3} is -0.153251. From this, we can infer that annuitants who are main annuitants experience lower mortality rates. This contradicts our earlier findings from the KM analysis, where we deduce that beneficiaries experience lower mortality rates. The discrepancy arises from the multicollinearity in the model as the "SEX" covariate captures much of what the "PERSON_TYPE" covariate entails. An elementary analysis of the deviance table depicted in **Figure A.4.2** allows us to determine whether there was significant improvement in the model when increasing model complexity via the addition of a covariate. The low p-values for each covariates indicates that all our covariates are statistically significant and contribute heavily to improving the regression's fit.

Justification of Life Table

Figure 4.5 plots the initial population l_x against the age x from the 5 Chilean life tables used for pricing and reserving life annuities. According to the relationship $l_x = l_0$[3] $\times S(x)$ we can see that the initial population l_x is a scaled version of the survival function $S(x)$. From this we can assess whether our survival analysis supports the current use of 5 life tables. Upon examining the l_x plots, we consistently observe higher values for females compared to males, indicating that females experience lower mortality rates compared to males as according to the previously established relationship, a larger l_x corresponds to a higher $S(x)$. This is consistent with our survival analysis findings. Similarly, the l_x plots for non-disabled annuitants are consistently higher than the l_x plots for disabled annuitants, which suggest that healthy annuitants experience lower mortality rates

[1] $\lambda(t;Z_1)$ is the hazard rate of an individual with risk factor Z_1 and $\lambda(t;Z_2)$ is the hazard rate of an individual with risk factor Z_2.
[2] Z_i is the risk factors for the i[th] annuitant
[3] l_0 is a constant that represents the initial population at age 0.

compared to disabled annuitants, this thereby aligns with our findings from our survival analysis. Comparing the l_x plots for female beneficiaries and female main annuitant reveals a discrepancy as female beneficiaries appear to experience higher mortality rates. This observation contradicts our EDA and survival analysis which suggests that beneficiaries generally experience lower mortality rates. This inconsistency arises from the fact that beneficiaries are predominantly female. When performing survival analysis, we disregarded gender when comparing the mortality of beneficiaries and main annuitants, this leaves us unable to capture the relationship between female beneficiaries and female annuitant. Therefore, it is completely plausible that female beneficiaries experience higher mortality rates compared to female main annuitants.

Graduation

Figure 5 illustrates the graduation of the crude rates $\hat{q}_x$[4] using four different approaches. A visual examination of the curves reveals a significant disparity at extreme ages. The parametric methods of Gompertz and Makeham tend to overestimate mortality at extreme ages, indicating their limitations in capturing mortality trends accurately in this age range. When employing a natural cubic spline to graduate, we observe rates that better align with the crudes rate at older ages compared to the parametric approaches of Gompertz and Makeham. However, upon comparison with the graduated rates obtained using a smoothing spline, it becomes evident that the natural cubic spline approach captures more noise, this is particularly noticeable at age 98. This suggests that the graduation achieved using a smoothing spline seems the most appropriate. Similar to survival analysis, numerical analysis in the form of various statistical tests are conducted to confirm that smoothing splines are the most suitable approach for graduating the crude rates. The results of these statistical tests are illustrated in **Appendix A.6**.

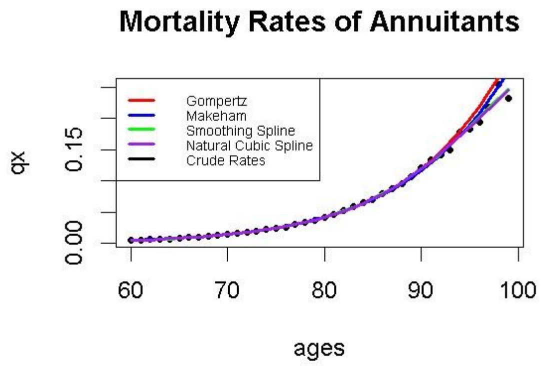


Figure 5: Comparison of the Graduated Rates

Smoothness tests are omitted from consideration as certain graduation methods inherently guarantee smoothness to a predetermined degree. To evaluate the graduation's adherence to data, we can consider six statistical tests.

To obtain an overall goodness of fit of the graduation, we can consider the Chi-Square Test of Fit. The test looks at the $\sum_{all\ ages\ x} z_x^2$[5], if the sum of the squared deviations is too large, the graduation fits the data poorly; a sign of overgraduation. An analysis of the test's results at the 5% significance level in **Figure A.6.1** shows that the smoothing spline approach has passed the test, this indicates the model has an appropriate fit. Unlike the smoothing spline approach, when performing the Chi-Square Test of Fit on the Gompertz, Makeham and Natural Cubic Spline models, we can see that the p-value are all lower than 0.05. This means that the mortality experience does not conform to Gompertz law, Makeham law or to a Natural Cubic Spline. Further analysis is needed to determine why the fit is poor for these methods and whether the graduation is biased. To complement the Chi-Square Test of Fit, the Standardised Deviations Test serves as another valuable tool. This test assesses the normality of the standardised deviations Z_x, providing insights into whether the graduated rates exhibit overgraduation or undergraduation. The test results depicted in **Figure A.6.2** shows that the test statistic for the Gompertz approach is 12.834, this yields a p-value approximately equal to 0. Such a low p-value suggests that the deviations do not conform to a standard normal distribution. Additionally, the high test statistic indicates that the graduated rates are overgraduated. Conversely,

[4] The methodology behind the calculations of the crudes rates can be found in appendix A.6.
[5] z_x^2 refers to the standardised deviation at age x.

the results of the Standardised Deviation Test on the other graduation approaches indicate that the deviations of the Makeham, Natural Cubic Splines and Smoothing Splines approaches are normally distributed. However, an additional concern arises, particularly regarding the test statistics for the Makeham and Natural Cubic Spline graduation approaches, as their test statistics are very small. The test statistics suggest that the deviations are clustered up together near $Z_x \approx 0$. This is a sign of undergraduation which suggests the model has captured an excessive amount of noise in the data. A confirmatory test using Q-Q plots to the test normality of the deviations can be found in the appendix. To evaluate the overall bias of the graduation approaches, we employ the Signs Test and Cumulative Deviation Test. The Signs Test determines whether the graduated rates are consistently overestimating or underestimating the crude rates. The presence of bias, where the graduated rates are on average too high or low, indicates a poor fit to the data as slights shift in the graduated rates could enhance the model's fit significantly. The Signs Test achieves this by checking that roughly half of the graduated rates are above the crude rates and half are below the crude rates. **Figure A.6.3** details the results of the Signs Test. Since the p-value for all the graduation approaches are above the 5% significance level, we cannot sufficiently reject the null hypothesis H_0. Thus, there's insufficient evidence to suggest an imbalance in positive and negative biases. The Cumulative Deviation Test serves to confirm the findings obtained from the Signs Test. The test assesses the sum of the deviations, which are then standardised. Specifically, it calculates $\frac{\sum_{ages}(d_x - E_x^c \dot{\mu}_{x+\frac{1}{2}})}{\sqrt{\sum_{ages} E_x^c \dot{\mu}_{x+\frac{1}{2}}}}$, where d_x refers to actual number of deaths and $E_x^c \dot{\mu}_{x+\frac{1}{2}}$ refers to the expected number of deaths. If the accumulated deviations are too high or too low, it suggests that the graduated rates are biased. The results of these tests are presented in **Figure A.6.4**. The p-value for all the graduation approaches are above the 5% significance level, indicating insufficient evidence to reject the null hypothesis. This reinforces the notion of a balance in positive and negative biases across the graduation approaches. A weakness in these tests is their inability to detect a large positive or negative cumulative deviation over part of the whole of the age range. As positive signs at some ages can be cancelled by negative signs at other ages. Therefore, the Grouping of Signs Test is employed such that we can determine whether there are local biases in the graduated rates. The Grouping of Signs Test assesses whether the observed number of runs in each graduation approach would be expected if the biases were arranged randomly. If the number of runs falls below the critical threshold, the rates are overgraduated, meaning they maybe consistently too high or too low over certain parts of the curve. The result of these tests depicted in **Figure A.6.5** shows that the test statistic for the Gompertz approach is 4, this yields a p-value approximately equal to 0. This suggests that the graduated rates have likely overestimated or underestimated the crude rates across most of the curve, indicating a poor fit and overgraduation. For the other graduation approaches the p-values surpassed the 5% significance level. Therefore, there is insufficient evidence to reject the null hypothesis, indicating that the number of runs present in the graduation is not a significant concern. The serial correlation test evaluates whether correlation exists between the sequence of standardised deviations and its lagged sequence. In this test, we examine serial correlation at lag 1, as autocorrelation tends to diminish quickly due to limited available information. An elementary analysis of the autocorrelation plots depicted in **Figure A.6.6** reveals that at lag 1 only the graduated rates obtained from the smoothing spline fall within the 95% confidence intervals on the serial correlation values. This suggests that there is insufficient evidence to indicate correlation between the sequence of standardised deviations and its lagged sequence. Considering all these adherence tests collectively, we can conclude that the smoothing spline is the most appropriate model for graduating the crude rates. This determination is because the smoothing spline model is the only one to pass all the adherence tests successfully.

Comparison with the Sex-Specific Chilean Life Tables

Figure 6 presents a comparison between the initial mortality rates q_x derived from the Chilean Life Tables and the mortality rates obtained through the process of graduation using smoothing splines.

The visualisation indicates that our graduated unisex mortality rates are consistently underestimating the mortality rates for males and overestimating the mortality rates for females. This observation suggests that graduating unisex mortality rates may not accurately capture the mortality experience.

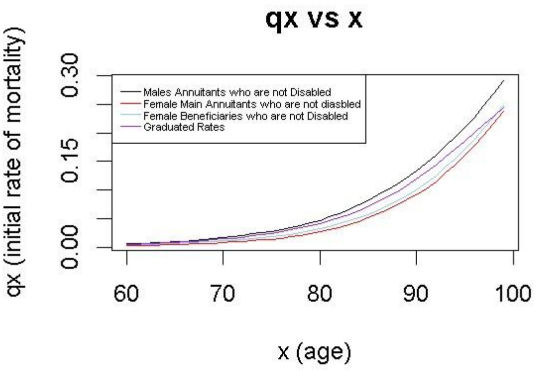


Figure 6: Comparison of the Graduated Rates with the Life Table Mortality

Ethical Implications of Unisex Annuity Pricing

When pricing annuity products, it's crucial to consider socioeconomic variables like gender due to their significant correlation with mortality rates. Failing to account for these variables could lead to annuity products being either overpriced or underpriced. In the context of Dobrin's ethical framework, the stakeholders involved in this ethical dilemma are the insurer who offer these products and the policyholders who purchase these products from the insurer. Policyholders seek to lower the prices of the annuity products, while insurers aim to boost their annual profits. Implementing gender as a rating factor in insurance introduces inherent unfairness because gender is not a factor under a policyholder's control, unlike risk-reducing actions such as installing window locks. Females who experience higher mortality rates would have to pay relatively higher to receive the same benefit as males. Another rising concern emerges with the growing population of sexual minorities who may not identify strictly as male or female. In this context, insurers using gender as a rating factor may not be prioritising the diverse needs of all customers. This concern can potentially be addressed through the application of data science techniques, such as risk profiling based on facial recognition from selfies. However, this approach raises concerns regarding privacy invasion. Moreover, the complexity of risk profiling algorithms may lead to a lack of transparency between the policyholder and insurer. The possible courses of action involve implementing gender as a rating factor or not. When evaluating the possible courses of action, we can analyse them through the perspectives of utilitarianism and deontology. From a utilitarian standpoint, using gender a rating factor may not be the most favourable choice. In Chile, significant gender disparities exist, with females earning on average 22% less than males and a lower percentage of females participating in the labour force compared to males. Implementing gender as a rating factor would disadvantage roughly 50.37% of the population as females would be required to pay relatively higher to receive the same benefit as males. This is unfair, as fewer females may have the financial means to afford annuity products thereby hindering their ability to receive adequate pension benefits. From a deontological perspective, using gender as a rating factor may also be not the most favourable choice as gender is not within a policyholder's control. Therefore, it would be morally wrong to penalise females for a factor out their control. On the other hand, implementing gender as a rating factor is a favourable choice for insurers, as our survival analysis indicates that gender is a significant variable that affects mortality. Insurance companies would suffer huge losses if unisex life tables were used to price their annuities products as the life tables would consistently overestimate the mortality rates for females which means the insurer would have to pay more benefits. Additionally, when underwriters assess policyholders, gender specific diseases like breast cancer and prostate cancer are considered. Disregarding these gender specific diseases results in poor estimations of the mortality rate as cancer on average reduces a person's life expectancy by 30%. Considering all these factors, despite its unfairness, it is imperative to implement gender as a rating factor. The dilemma faced by insurers, where they must ensure sustainability while potentially further disadvantaging the female population in Chile, evokes feelings of guilt and shame. To address concerns regarding privacy and the growing population of sexual minorities, the Chilean government could consider implementing legislation similar to "The Anti-Discrimination Act 1977" in New South Wales. This may provide reassurance to policyholders.

Appendix
A.1 Cumulative Hazard Over Time

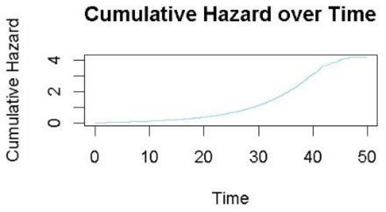


Figure A.1: Cumulative Hazard Over Time

A.2 KM Estimates of the Survival Functions

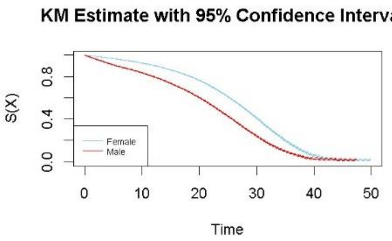


Figure A.2.1 KM Estimates by Gender

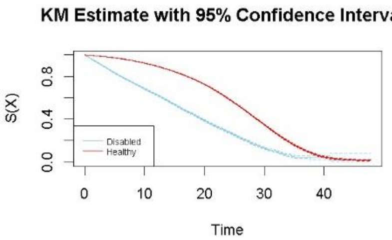


Figure A.2.2 KM Estimates by Health

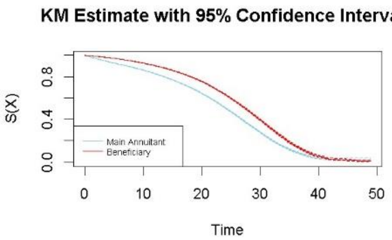


Figure A.2.3 KM Estimates by Person Type

A.3 Statistical Tests and Graphical Diagnostics based on the scaled Schoenfeld Residuals

	chisq	df	p
HEALTH	2222	1	<2e-16
SEX	410	1	<2e-16
PERSON_TYPE	433	1	<2e-16
GLOBAL	2295	3	<2e-16

Figure A.3.1: Statistical Test based on the Scaled Schoenfeld Residuals

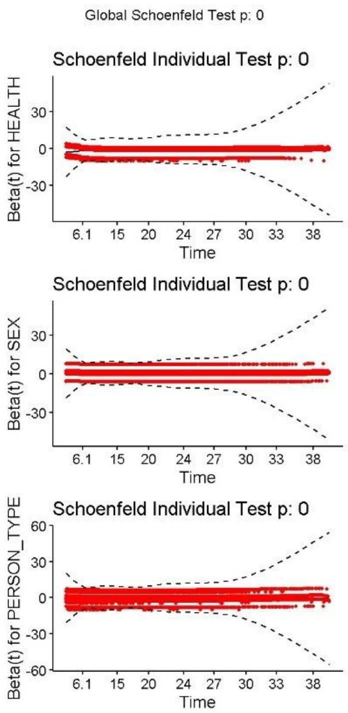


Figure A.3.2: Graphical Diagnostics based on the Scaled Schoenfeld Residuals

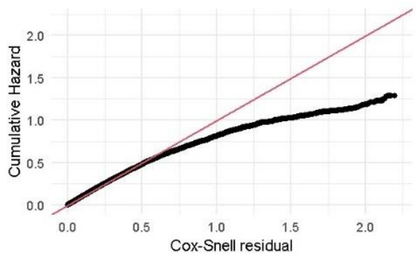


Figure A.3.3: Cox-Snell Residual Plot

A.4 Cox Regression Output

```
Call:
coxph(formula = cens_annuitant ~ HEALTH + SEX + PERSON_TYPE,
    data = annuitant_data, method = "breslow")

  n= 1291830, number of events= 103819

                               coef exp(coef)  se(coef)       z Pr(>|z|)
HEALTHHealthy             -1.091442  0.335732  0.009152 -119.25   <2e-16 ***
SEXM                       0.561295  1.752941  0.008674   64.71   <2e-16 ***
PERSON_TYPEMain Annuitant -0.153251  0.857914  0.009322  -16.44   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

                          exp(coef) exp(-coef) lower .95 upper .95
HEALTHHealthy                0.3357     2.9786    0.3298    0.3418
SEXM                         1.7529     0.5705    1.7234    1.7830
PERSON_TYPEMain Annuitant    0.8579     1.1656    0.8424    0.8737

Concordance= 0.623  (se = 0.001 )
Likelihood ratio test= 20381  on 3 df,   p=<2e-16
Wald test            = 24508  on 3 df,   p=<2e-16
Score (logrank) test = 26920  on 3 df,   p=<2e-16
```

Figure A.4.1: Cox Regression Output

```
Analysis of Deviance Table
 Cox model: response is cens_annuitant
Terms added sequentially (first to last)

              loglik    Chisq Df Pr(>|Chi|)
NULL        -1218271
HEALTH      -1210879 14783.82  1  < 2.2e-16 ***
SEX         -1208217  5325.21  1  < 2.2e-16 ***
PERSON_TYPE -1208081   271.62  1  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure A.4.2: Deviance Table

A.5 2020 Chilean Life Tables

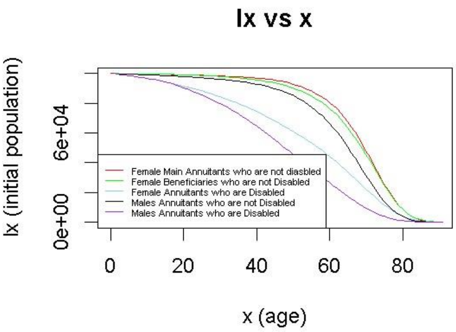


Figure A.5: 2020 Chilean Life Tables

A.6 Statistical Analysis of Graduation

The crude rates $\hat{q}_x = \frac{d_x}{E_x}$ were calculated by finding d_x and E_x first. d_x and E_x are both computed in R via the use of various for loops.

To use these statistical tests, we assume that:

- Lives are independent
- There is no heterogeneity in each age
- The expected deaths should be greater than 5 for each age such that we can use the normal approximation

The graduated μ_x obtained from Gompertz and Makeham law were transformed to q_x such that better comparisons with the Chilean life tables can be made.
The knots for the natural cubic splines were calculated by using a weighted averages of the knots used in the Australian life tables.
The spar parameter for the smoothing spline is 0.6, this value was obtained from trial and error.

Chi-Square Test of Fit

Method	Test Statistic	P-Value
Gompertz	83.62415	$p = 2.823816e-05$
Makeham	53.61765	$p = 0.03789372$
Natural Cubic Spline	49.71409	$p = 0.04001462$
Smoothing Spline	24.87979	$p = 0.8069883$

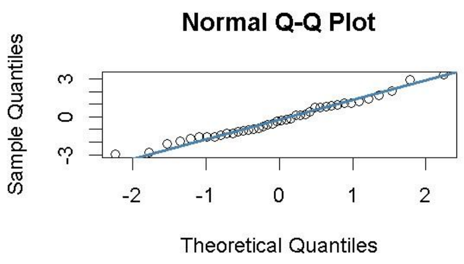
Figure A.6.1 Chi-Square Test of Fit Results

Standardised Deviation Test

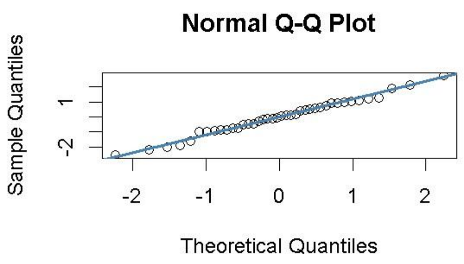
Method	Test Statistic	P-Value
Gompertz	12.834	$p = 0.005009$
Makeham	0.51696	$p = 0.9151$
Natural Cubic Spline	1.0052	$p = 0.8$
Smoothing Spline	3.8017	$p = 0.2837$

Figure A.6.2 Standardised Deviation Test Results

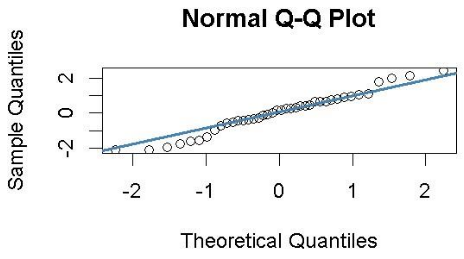
Q-Q Plots
Deviations Under Gompertz Approach



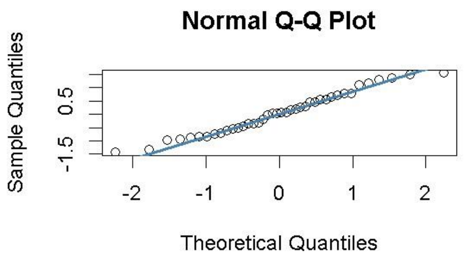
Deviations Under Makeham Approach



Deviations Under Natural Cubic Spline Approach



Deviations Under Smoothing Spline Approach



Signs Test

Method	Test Statistic	P-Value
Gompertz	17	$p = 0.4296$
Makeham	20	$p = 1$
Natural Cubic Spline	22	$p = 0.6358$
Smoothing Spline	22	$p = 0.6358$

Figure A.6.3 Signs Test Results

Cumulative Deviation Test

Method	Test Statistic	P-Value
Gompertz	0.4070625	$p = 0.6839621$
Makeham	0.2492856	$p = 0.8031399$
Natural Cubic Spline	0.1687758	$p = 0.865973$
Smoothing Spline	0.3863985	$p = 0.6992016$

Figure A.6.4 Cumulative Deviation Test Results

Grouping of Signs Test

Method	Test Statistic	P-Value
Gompertz	4	$p = 0.0001130225$
Makeham	8	$p = 0.1029516$
Natural Cubic Spline	8	$p = 0.1064289$
Smoothing Spline	8	$p = 0.1064289$

Figure A.6.5 Grouping of Signs Test Results

Serial Correlation Test

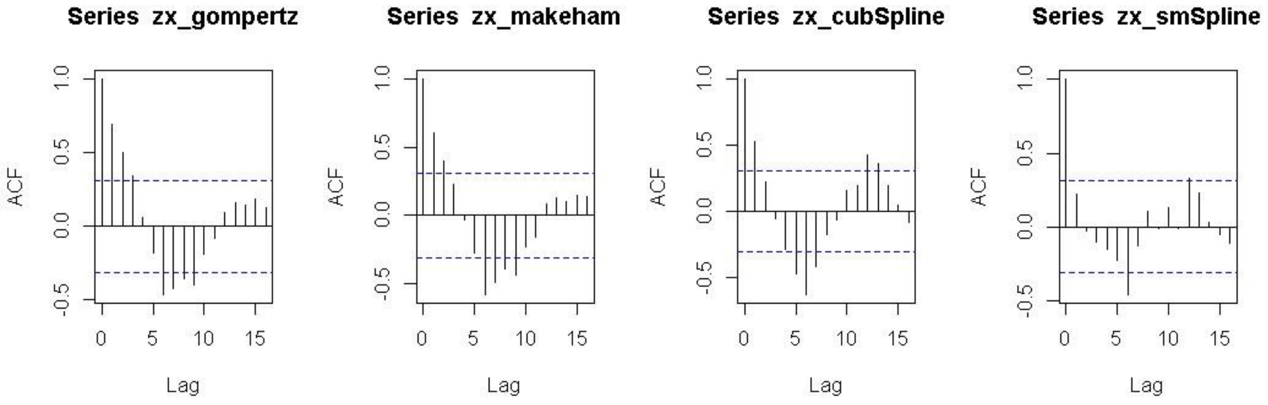


Figure A.6.6 Serial Correlation Test Results

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Generative AI usage

- Generative AI was used to help me understand what is wrong with my code.
- "Why is my code wrong?"