EE239AS
HW3
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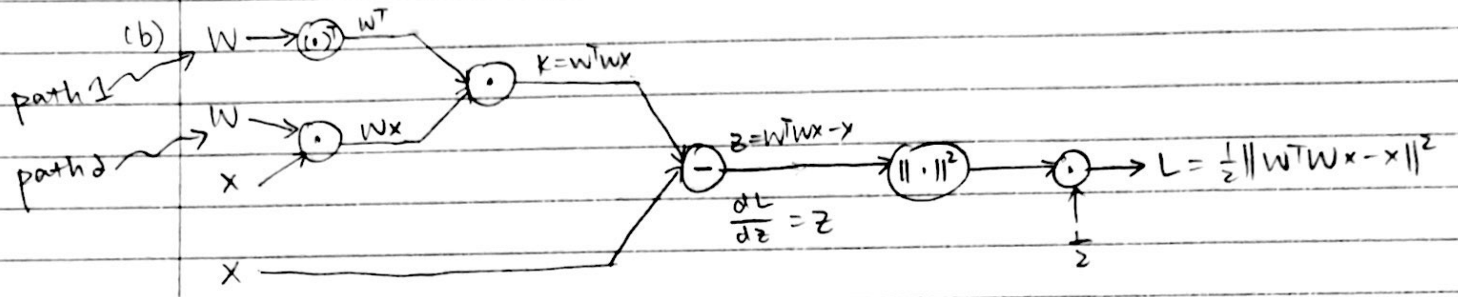
Problem 1 (a) Since Wx is a lower dimension representation of x, W^TWx wants to reconstruct a $\hat{x}$ as similar as x. By minimizing $L = \frac{1}{2}\|W^TWx - x\|^2$, if this loss goes to zero, then W will be an eigenvector of x and $W^TW = I$, then Wx could preserve the most important information about x with reduced dimension.

(b)



(c) These two paths should be summed up. Mathematically,

$$\frac{dL}{dW} = \underbrace{\left(\frac{dL}{dW^T}\right)^T}_{\text{path 1}} + \underbrace{\frac{d(Wx)}{dW} \cdot \frac{dL}{d(Wx)}}_{\text{path 2}}$$

(d) Let $z = W^TWx - x$ $\Rightarrow \frac{dL}{dz} = \frac{1}{2} \cdot (2z) = z$

Let $K = W^TW$

$$\Rightarrow \frac{dL}{dK} = \frac{dL}{dz}$$

path 1: $\frac{dL}{dW^T} = \frac{dK}{dW^T} \cdot \frac{dL}{dK} = \frac{dL}{dK}(Wx)^T = z(Wx)^T$

$\because \frac{dL}{dW^T} = \left(\frac{dL}{dW}\right)^T$ when L is a scalar $\Rightarrow \frac{dL}{dW} = \left(\frac{dL}{dW^T}\right)^T = \left(z(Wx)^T\right)^T$

$= Wxz^T$

path 2: $\frac{dL}{d(Wx)} = \frac{dK}{d(Wx)} \frac{dL}{dK} = (W^T)^T \frac{dL}{dK} = Wz$

$\frac{dL}{dW} = \frac{d(Wx)}{dW} \frac{dL}{d(Wx)} = \frac{dL}{d(Wx)} x^T = Wzx^T$

$\Rightarrow$ To sum up, $\frac{dL}{dW} = Wxz^T + Wzx^T$

$$= Wx(W^TWx - x)^T + W(W^TWx - x)x^T$$
$$= Wx[(W^TW - I)x]^T + W[(W^TW - I)x]x^T$$
$$= Wxx^T(W^TW - I)^T + W(W^TW - I)xx^T$$