# An Explanation of Calibration-Free Pulse Oximetry

Robert L. Read *

*Founder, Public Invention, an educational non-profit.*

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## 1 Introduction

This is an attempt to clarify the ideas presented by Chugh and Kaur[1].

A valuble resource for understanding pulse oximetry in general is available at HowEquipmentWorks.com[2].

The EWB-Austin Instrumentation group is hoping to teach the world how to construct a practical pulse oximeter inexpensively to provide the medical benefits of pulse oximetry to parts of the world where it is not presently widespread. To do this, it must be understandable and inexpensive.

We have breadboarded simple equipment and analyzed signals with an Arudino Uno, constituting a very simple pulse oximeter.

Chugh and Kaur[1] have presented a theoretical approach to constructing a pulse oximeter that does not require calibration. If a pulse oximeter could be made withot calibration, this would be a large advantage for anyone in a developing country attempting to construct and use one correctly.

Chugh and Kaur[1] cite and build upon earlier works [3], which attempts to do hardware filtering and equalization of the signals. Both of these works cite a seminal work by Reddy et al. [4].

I believe all three of these papers have serious flaws. This paper is my attempt to explain this, mostly to myself. It is not ready to consider a critique of those papers.

I assert that [4], while valuable, is flawed or at least very confusing in its attempt to use "slope" of voltage against time, when clearly removing time from that analysis would be clearer. The means of deriving Figure 6 in that table is either incorrect or not explained.

Chugh and Kaur[1] cannot be correct as written, as it is clearly the case that differences in sensor sensitivities would affect their results, in my interpretation. Furthermore, that papers defines the quantity $R$ in equations 1, 8, and 12, in ways which are mutually

*read.robert@gmail.com

inconsistent (unless, very confusingly, they intent R to mean different quantities in these cases.

Note: Nitzan[**?**] is much clearer, giving a similar equation for $SaO2$ fundamental equation based on the standard $R$ "ratio of ratios", roughly matching these papers.

## 2 Review of Chugh and Kaur

Chugh and Kaur[1] paper is concise, but not perfectly clear to us upon first reading. We here work through some of the math they present in order to verify it and to verify that we correctly understand it.

In section 1.$i$, the paper restates the Beer-Lambert law[5] as:

$$A = \ln \frac{I_o}{I_t} = \varepsilon \cdot C \cdot L \tag{1}$$

where $A$ is the absorbance, $\varepsilon$ is the wavelength-dependent extinction[1] coeffecint, $C$ is the concentration of the absorbing material present in the path and $L$ is the path length.

Using Wolframalpha.com to check this, we see that this formulation (with a change of variable names) is indeed just a restatement of the Beer-Lambert law as expressed by the Wikipedia article[5], although it seems that the computation of the absorbance from the incident and transmitted light via logarithm should be, according to Wikipedia, base 10:

$$A = \log_{10} \frac{I_o}{I_t} = \varepsilon \cdot C \cdot L \tag{2}$$

Chugh and Kaur state the multi-species version of the Beer-Lambert law, and then state the absorbance at the two distinct wavelenghts at which oxygenated hemoglobin and deoxygenated hemoglobin differ maximally, which they call $RED$ and $IR$, in terms of the concentrations of two species (oxygenated ($C_{hbo}$) and deoxygenated ($C_{hb}$)).

$$A_{RED} = (\varepsilon_{hbo(red)} \cdot C_{hbo} + \varepsilon_{hb(red)} \cdot C_{hb}) \cdot L \tag{3}$$

$$A_{IR} = (\varepsilon_{hbo(IR)} \cdot C_{hbo} + \varepsilon_{hb(IR)} \cdot C_{hb}) \cdot L \tag{4}$$

The then define the ration $R$ as the ratio of the absorbance measured at these wavelengths:

$$R = \frac{A_{RED}}{A_{IR}} \tag{5}$$

[1] Note that Wikipedia[6] states the IUPAC discourages the term *e*xtinction coefficient in favor of *m*olar attunation coefficient.

Using straightforward algebra, R becomes independent of the path length $L$:

$$R = \frac{(\varepsilon_{hbo(red)} \cdot C_{hbo} + \varepsilon_{hb(red)} \cdot C_{hb})}{(\varepsilon_{hbo(IR)} \cdot C_{hbo} + \varepsilon_{hb(IR)} \cdot C_{hb})} \tag{6}$$

Using straightforward algebra, they rearrange this:

$$C_{hb} = C_{hbo} \frac{R \cdot \varepsilon_{hbo(IR)} - \varepsilon_{hbo(red)}}{\varepsilon_{hb(red)} - R \cdot \varepsilon_{hb(IR)}} \tag{7}$$

Note: Chugh and Kaur use slightly different variable names than used at Wikipedia. Possibly in this paper we will change the names yet again to make them more conformant to that style.

They then define $SpO_2$ (periphereal oxygen saturation) [7, 8]:

$$SpO_2 = \frac{C_{hbo}}{C_{hbo} + C_{hb}} \tag{8}$$

Then the substitute 7 into 8 and simplify:

$$SpO_2 = \frac{100(\varepsilon_{hbo(red)} - R \cdot \varepsilon_{hb(IR)})}{(\varepsilon_{hb(red)} - \varepsilon_{hbo(red)}) + R \cdot (\varepsilon_{hbo(IR)} - \varepsilon_{hb(IR)})} \tag{9}$$

The then claim that R can be computed by taking the base-10 log of that AC component of the $RED$ and $IR$ signals.

I believe they paper suggests this to be the the difference between the the maximum of the time-varying signal and the minimum of the time-varying signal.

(Only an electrical engineer would understand the term "AC component" to mean the time-varying signal. I believe a better terminology is the "pulsative signal". This is the part of the signal remaining when the unvarying signal is removed. In terms of the FFT, the very low frequencies of the signal may be removed (those lower than the lowest human pulse, which is about 0.5 Hz).)

However, the absolute difference between the greatest and least transmittance (and inversely absorbance) is highly dependent on the machinery and physical body measured.

Define the "Pulsative Absorbance" at frequency $\lambda$.

$$P_\lambda = \frac{\text{max of moving average of absorbance}}{\text{min of moving average of absorbance}} \tag{10}$$

Where the absorbance is computed by the definition:

$$A_\lambda = \log_{10} \frac{I_{\lambda o}}{I_{\lambda t}} \tag{11}$$

If we make the reasonable assumption that $I_t$ is proportional to our our measured signal, then we can substitute and remove the dependence ont he intensity of the transmitted light:

$$P_\lambda = \frac{\max A_\lambda}{\min A_\lambda} \tag{12}$$

So we can create the ratio of the blood-full absorbance to the ratio of the blood-empty absorbance:

$$P_\lambda = \frac{\max A_\lambda}{\min A_\lambda} \tag{13}$$

Can it be that they mean the ratio of the pulsative part of the signal to the non-pulsative part of the signal? This is the change in the absorbance.

## References


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