

An Explanation of Calibration-Free Pulse Oximetry

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April 5, 2018

1 Introduction

This is an attempt to clarify the ideas presented by Chugh and Kaur[1].

A valuable resource for understanding pulse oximetry in general is available at HowEquipmentWorks.com[2].

The EWB-Austin Instrumentation group is hoping to teach the world how to construct a practical pulse oximeter inexpensively to provide the medical benefits of pulse oximetry to parts of the world where it is not presently widespread. To do this, it must be understandable and inexpensive.

We have breadboarded simple equipment and analyzed signals with an Arudino Uno, constituting a very simple pulse oximeter.

Chugh and Kaur[1] have presented a theoretical approach to constructing a pulse oximeter that does not require calibration. If a pulse oximeter could be made without calibration, this would be a large advantage for anyone in a developing country attempting to construct and use one correctly.

2 Review of Chugh and Kaur

Chugh and Kaur[1] paper is concise, but not perfectly clear to us upon first reading. We here work through some of the math they present in order to verify it and to verify that we correctly understand it.

In section 1.i, the paper restates the Beer-Lambert law[3] as:

$$A = \ln \frac{I_o}{I_t} = \varepsilon \cdot C \cdot L \quad (1)$$

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where A is the absorbance, ε is the wavelength-dependent extinction¹ coefficient, C is the concentration of the absorbing material present in the path and L is the path length.

Using Wolframalpha.com to check this, we see that this formulation (with a change of variable names) is indeed just a restatement of the Beer-Lambert law as expressed by the Wikipedia article[3], although it seems that the computation of the absorbance from the incident and transmitted light via logarithm should be, according to Wikipedia, base 10:

$$A = \log_{10} \frac{I_o}{I_t} = \varepsilon \cdot C \cdot L \quad (2)$$

Chugh and Kaur state the multi-species version of the Beer-Lambert law, and then state the absorbance at the two distinct wavelengths at which oxygenated hemoglobin and deoxygenated hemoglobin differ maximally, which they call RED and IR , in terms of the concentrations of two species (oxygenated (C_{hbo}) and deoxygenated (C_{hb})).

$$A_{RED} = (\varepsilon_{hbo(red)} \cdot C_{hbo} + \varepsilon_{hb(red)} \cdot C_{hb}) \cdot L \quad (3)$$

$$A_{IR} = (\varepsilon_{hbo(IR)} \cdot C_{hbo} + \varepsilon_{hb(IR)} \cdot C_{hb}) \cdot L \quad (4)$$

They then define the ration R as the ratio of the absorbance measured at these wavelengths:

$$R = \frac{A_{RED}}{A_{IR}} \quad (5)$$

Using straightforward algebra, R becomes independent of the path length L :

$$R = \frac{(\varepsilon_{hbo(red)} \cdot C_{hbo} + \varepsilon_{hb(red)} \cdot C_{hb})}{(\varepsilon_{hbo(IR)} \cdot C_{hbo} + \varepsilon_{hb(IR)} \cdot C_{hb})} \quad (6)$$

Using straightforward algebra, they rearrange this:

$$C_{hb} = C_{hbo} \frac{R \cdot \varepsilon_{hbo(IR)} - \varepsilon_{hbo(red)}}{\varepsilon_{hb(red)} - R \cdot \varepsilon_{hb(IR)}} \quad (7)$$

Note: Chugh and Kaur use slightly different variable names than used at Wikipedia. Possibly in this paper we will change the names yet again to make them more conformant to that style.

They then define SpO_2 (periphereal oxygen saturation) [5, 6]:

$$SpO_2 = \frac{C_{hbo}}{C_{hbo} + C_{hb}} \quad (8)$$

¹Note that Wikipedia[4] states the IUPAC discourages the term extinction coefficient in favor of molar attenuation coefficient.

Then the substitute 7 into 8 and simplify:

$$SpO_2 = \frac{100(\varepsilon_{hbo(red)} - R \cdot \varepsilon_{hb(IR)})}{(\varepsilon_{hb(red)} - \varepsilon_{hbo(red)}) + R \cdot (\varepsilon_{hbo(IR)} - \varepsilon_{hb(IR)})} \quad (9)$$

The then claim that R can be computed by taking the base-10 log of that AC component of the *RED* and *IR* signals.

I believe they paper suggests this to be the the difference between the the maximum of the time-varying signal and the minimum of the time-varying signal.

(Only an electrical engineer would understand the term “AC component” to mean the time-varying signal. I believe a better terminology is the “pulsative signal”. This is the part of the signal remaining when the unvarying signal is removed. In terms of the FFT, the very low frequencies of the signal may be removed (those lower than the lowest human pulse, which is about 0.5 Hz).)

However, the absolute difference between the greatest and least transmittance (and inversely absorbance) is highly dependent on the machinery and physical body measured.

Define the “Pulsative Absorbance” at frequency λ .

$$P_\lambda = \frac{\max \text{ of moving average of absorbance}}{\min \text{ of moving average of absorbance}} \quad (10)$$

Where the absorbance is computed by the definition:

$$A_\lambda = \log_{10} \frac{I_{\lambda o}}{I_{\lambda t}} \quad (11)$$

If we make the reasonable assumption that I_t is proportional to our our measured signal, then we can substitute and remove the dependence ont he intensity of the transmitted light:

$$P_\lambda = \frac{\max A_\lambda}{\min A_\lambda} \quad (12)$$

Hello:

$$P_\lambda = \frac{\max A_\lambda}{\min A_\lambda} \quad (13)$$

Can it be that they mean the ratio of the pulsative part of the signal to the non-pulsative part of the signal? This is the change in the absorbance.

References

- [1] S. Chugh and J. Kaur. Low cost calibration free pulse oximeter. In *2015 Annual IEEE India Conference (INDICON)*, pages 1–5, Dec 2015.
- [2] How pulse oximeters work explained simply. https://www.howequipmentworks.com/pulse_oximeter/. Accessed: 2018-04-05.

- [3] Wikipedia contributors. Beerlambert law — wikipedia, the free encyclopedia, 2017. [Online; accessed 5-April-2018].
- [4] Wikipedia contributors. Molar attenuation coefficient — wikipedia, the free encyclopedia, 2018. [Online; accessed 5-April-2018].
- [5] Wikipedia contributors. Pulse oximetry — wikipedia, the free encyclopedia, 2018. [Online; accessed 5-April-2018].
- [6] Wikipedia contributors. Oxygen saturation (medicine) — wikipedia, the free encyclopedia, 2018. [Online; accessed 5-April-2018].