An Explanation of Calibration-Free Pulse Oximetry

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1 Introduction

This is an attempt to clarify the ideas presented by Chugh and Kaur[1].

A valuble resource for understanding pulse oximetry in general is available at HowEquipmentWorks.com[2].

The EWB-Austin Instrumentation group is hoping to teach the world how to construct a practical pulse oximeter inexpensively to provide the medical benefits of pulse oximetry to parts of the world where it is not presently widespread. To do this, it must be understandable and inexpensive.

We have breadboarded simple equipment and analyzed signals with an Arudino Uno, constituting a very simple pulse oximeter.

Chugh and Kaur[1] have presented a theoretical approach to constructing a pulse oximeter that does not require calibration. If a pulse oximeter could be made withot calibration, this would be a large advantage for anyone in a developing country attempting to construct and use one correctly.

Chugh and Kaur[1] cite and build upon earlier works [3], which attempts to do hardware filtering and equalization of the signals. Both of these works cite a seminal work by Reddy et al. [4].

I believe all three of these papers have serious flaws. This paper is my attempt to explain this, mostly to myself. It is not ready to consider a critique of those papers.

I assert that [4], while valuable, is flawed or at least very confusing in its attempt to use "slope" of voltage against time, when clearly removing time from that analysis would be clearer. The means of deriving Figure 6 in that table is either incorrect or not explained.

Chugh and Kaur[1] cannot be correct as written, as it is clearly the case that differences in sensor sensitivities would affect their results, in my interpretation. Furthermore, that papers defines the quantity R in equations 1, 8, and 12, in ways which are mutually

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inconsistent (unless, very confusingly, they intent R to mean different quantities in these cases.

Note: Nitzan[5] is much clearer, giving a similar equation for $SaO2$ fundamental equation based on the standard R "ratio of ratios", roughly matching these papers.

2 Review of Chugh and Kaur

Chugh and Kaur[1] paper is concise, but not perfectly clear to us upon first reading. We here work through some of the math they present in order to verify it and to verify that we correctly understand it.

In section 1.i, the paper restates the Beer-Lambert law[6] as:

$$A = \ln \frac{I_o}{I_t} = \varepsilon \cdot C \cdot L \tag{1}$$

where A is the absorbance, ε is the wavelength-dependent extinction[1] coeffecint, C is the concentration of the absorbing material present in the path and L is the path length.

Using Wolframalpha.com to check this, we see that this formulation (with a change of variable names) is indeed just a restatement of the Beer-Lambert law as expressed by the Wikipedia article[6], although it seems that the computation of the absorbance from the incident and transmitted light via logarithm should be, according to Wikipedia, base 10:

$$A = \log_{10} \frac{I_o}{I_t} = \varepsilon \cdot C \cdot L \tag{2}$$

Chugh and Kaur state the multi-species version of the Beer-Lambert law, and then state the absorbance at the two distinct wavelenghts at which oxygenated hemoglobin and deoxygenated hemoglobin differ maximally, which they call RED and IR, in terms of the concentrations of two species (oxygenated (C_{hbo}) and deoxygenated (C_{hb})).

$$A_{RED} = (\varepsilon_{hbo(red)} \cdot C_{hbo} + \varepsilon_{hb(red)} \cdot C_{hb}) \cdot L \tag{3}$$

$$A_{IR} = (\varepsilon_{hbo(IR)} \cdot C_{hbo} + \varepsilon_{hb(IR)} \cdot C_{hb}) \cdot L \tag{4}$$

The then define the ration R as the ratio of the absorbance measured at these wavelengths:

$$R = \frac{A_{RED}}{A_{IR}} \tag{5}$$

[1]Note that Wikipedia[7] states the IUPAC discourages the term *e*xtinction coefficient in favor of *m*olar attunation coefficient.

Using straightforward algebra, R becomes independent of the path length L:

$$R = \frac{(\varepsilon_{hbo(red)} \cdot C_{hbo} + \varepsilon_{hb(red)} \cdot C_{hb})}{(\varepsilon_{hbo(IR)} \cdot C_{hbo} + \varepsilon_{hb(IR)} \cdot C_{hb})} \tag{6}$$

Using straightforward algebra, they rearrange this:

$$C_{hb} = C_{hbo} \frac{R \cdot \varepsilon_{hbo(IR)} - \varepsilon_{hbo(red)}}{\varepsilon_{hb(red)} - R \cdot \varepsilon_{hb(IR)}} \tag{7}$$

Note: Chugh and Kaur use slightly different variable names than used at Wikipedia. Possibly in this paper we will change the names yet again to make them more conformant to that style.

They then define SpO_2 (periphereal oxygen saturation) [8, 9]:

$$SpO_2 = \frac{C_{hbo}}{C_{hbo} + C_{hb}} \tag{8}$$

Then the substitute 7 into 8 and simplify:

$$SpO_2 = \frac{100(\varepsilon_{hbo(red)} - R \cdot \varepsilon_{hb(IR)})}{(\varepsilon_{hb(red)} - \varepsilon_{hbo(red)}) + R \cdot (\varepsilon_{hbo(IR)} - \varepsilon_{hb(IR)})} \tag{9}$$

The then claim that R can be computed by taking the base-10 log of that AC component of the RED and IR signals.

I believe they paper suggests this to be the the difference between the the maximum of the time-varying signal and the minimum of the time-varying signal.

(Only an electrical engineer would understand the term "AC component" to mean the time-varying signal. I believe a better terminology is the "pulsative signal". This is the part of the signal remaining when the unvarying signal is removed. In terms of the FFT, the very low frequencies of the signal may be removed (those lower than the lowest human pulse, which is about 0.5 Hz).)

However, the absolute difference between the greatest and least transmittance (and inversely absorbance) is highly dependent on the machinery and physical body measured.

Define the "Pulsative Absorbance" at frequency λ.

$$P_\lambda = \frac{\text{max of moving average of absorbance}}{\text{min of moving average of absorbance}} \tag{10}$$

Where the absorbance is computed by the definition:

$$A_\lambda = \log_{10} \frac{I_{\lambda o}}{I_{\lambda t}} \tag{11}$$

If we make the reasonable assumption that I_t is proportional to our our measured signal, then we can substitute and remove the dependence ont he intensity of the transmitted light:

$$P_\lambda = \frac{\max A_\lambda}{\min A_\lambda} \tag{12}$$

So we can create the ratio of the blood-full absorbance to the ratio of the blood-empty absorbance:

$$P_\lambda = \frac{\max A_\lambda}{\min A_\lambda} \tag{13}$$

Can it be that they mean the ratio of the pulsative part of the signal to the non-pulsative part of the signal? This is the change in the absorbance.

3 Finding Peak-to-Peak Values: A novel algorithm

Using this approach, a fundamental need it so measure the peak-to-peak difference of the time-varying part of the signal. This can be accomplished by keeping a running maximum and a running minimum.

On October 28th, I had coded an approach to this that works. I was not able to find a good algorithm for the Aduino.

Additional, I believe I have discovered a new algorithm that will be effective for the Arduino, with a constant running time and extremely good performance with a constant amount of space.

I have not yet coded this algorithm. This is my attempt to record my thoughts.

3.1 DROPBORINGLIN: A fast constant space algorithm for the running maximum of a sampled signal

Douglas [10] provides a good algorithm *MAXLIST* for computing a running maximum. This algorithm builds upon his observation that one may keep an ordered list of potential maxima.

In order to produce a constant space algorithm with good error characteristics, we define *DROPBORINGLIN* whose fundamental insight is that we can define a ring buffer of constant size to keep the samples. If the signal is not monotone decreasing, the observation *MAXLIST* observation tells us that only a small number of samples need be stored, as they tend to be higher than, and therefore obviate, older samples.

However, if the signal is monotone decreasing, to be perfect we might have to keep every sample, which cannot be done in small space.

Therefore we observe that the error produced by dropping an individual sample varies, and is easily computed. If we ignore the fact that we are dealing with digital signal processing series which is probably a sampling of a smooth waveform, we can compute this area as two right triangles.

However, we can do even better if we assume that our time series is coming from a samples series of limited frequency, and that therefore if the window slides over a sample in time the maximum should not abruptly change, but is better appxoimated by a linear interpolation.

Each of the samples which is not either the newest or the oldest can then be thought of as giving us information about how far away the signal is from a linear interpolation between the preceding and succeeding samples. Some samples are more surpsising than others, in that they are further away from this linear interpolation. We propose that the best way to compute the maximum in limited space is to drop the sample that is the "most boring". Since this is easily computed for each sample, we have a simple algorithm that operates in constant space. When the signal is monotone decreasing, the algorithm may be erroneous, but will only be erroneous by an amount which we can can bound and minimize.

This algorithm is particularly well-suited to implementation on an Arduino, where memory is limited.

If we examine 1 and imagine that there is a time series which is sampled at points A, B, C, D, and E. The colored triangles represent the "error" that occurs if we compute the window ending between A and E, assuming that signal does not rise after that. It is visually apparent that B is a rather boring point compared to D in that the error that is produced by droppoing point B is much smaller than the error produced by dropping point D.

This observation is the heart of the algorithm: if the signal never rises so that you run out of space, it is better to drop the boring points.

A good data structure for this algorithm is a ring buffer containing R "samples", which contian both a value and a time, and a "client data field" which contains the boringness. By the observation of $MAXLIST$ we observe that it behooves us to maintain the invariant that the elemets of this buffer are ordered with increasing time and decreasing value.

The fundamental operation of a running algorithm is the addition of a sample more recent than any in our ring, which we call simple $INPUT$.

The algorithm $INPUT$ takes a current ring and a new sample (v, t). It returns an approximation to the maximum of the signal in a window of duration d.

1. If $t - d$ is greater than the penultimate sample, drop the last sample from the ring.

2. Remove all elements from the ring which have a value less than v.

3. If the ring is full, drop the most boring element, computed from the head.

4. Add (v, t) to the head of the ring.

5. If the ring contains 3 elements, assign a boringness to the element next to the head.

6. Return the linear interpolation of $(t-d)$ based on the last sample and the penultimate sample.

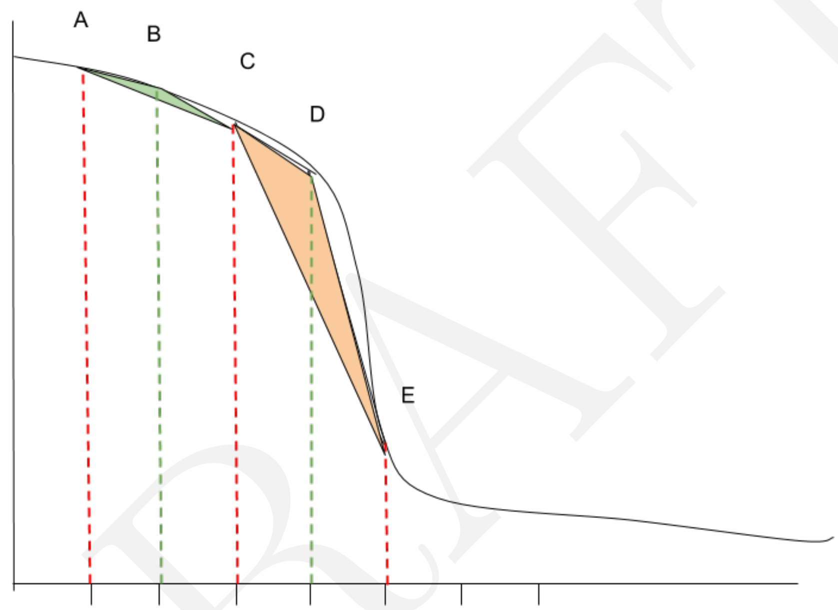

Figure 1: A decreasing sampled signal.

The function $BORING$ assigns a real number to an element in the ring B if it has a successor C and predecssor A in the ring. It takes the index of the element in the ring and the ring as its argument. A small number is considered "more boring". This is the area of the triangle

1.

2.

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