



# From 1,000,000 to Graham's Number

November 20, 2014 By Tim Urban

One to numbers post #2.

Last week, we started at 1 and slowly and steadily worked our way up to 1,000,000. We used dots. It was cute.

Well fun time's over. Today, shit gets real.

Before things get totally out of hand, let's start by working our way up the still-fathomable powers of 10 —

## Powers of 10

When we went from 1 to 1,000,000, we didn't need powers—we could just use a short string of digits to represent the numbers we were talking about. If we wanted to multiply a number by 10, we just added a zero.

But as you advance past a million, zeros start to become plentiful and you need a different notation. That's why we use powers. When people talk about exponential growth, they're referring to the craziness that can happen when you start using powers. For example:

If you multiply 9,845,625,675,438 by 8,372,745,993,275, the result is still smaller than  $8^{29}$ .

As we get bigger and bigger today, we'll stick with powers of 10, because when you start talking about really big numbers, what becomes relevant is the number of digits, not the digits themselves—i.e. every 70-digit number is somewhere between  $10^{69}$  and  $10^{70}$ , which is really all you need to know. So for at least the first part of this post, the powers of 10 can serve nicely as orders-of-magnitude "checkpoints".

Each time we up the power by one, we multiply the world we're in by ten, changing things significantly. Let's start off where we left off last time—

**$10^6$  (1 million – 1,000,000)** – The amount of dots in that [huge image](#) we finished up with last week. On my computer screen, that image was about 18cm x 450cm = .81 m<sup>2</sup> in area.

**$10^7$  (10 million)** – This brings us to a range that includes the number of steps it would take to walk around the Earth (40 million steps). If each of your steps around the Earth were represented by a dot like those from the grids in the last post, the dots would fill a 6m x 6m square.

**$10^8$  (100 million)** – Now we're at the number of books ever published in human history (130 million), and at the top of this range, the estimated number of words a human being speaks in a lifetime (860 million). Also in this range are the odds of winning the really big lotteries. A recent Mega Millions lottery had 1-in-175,711,536 odds of winning. To put those chances in perspective, that's about the number of seconds in six years. So it's like knowing a hedgehog will sneeze once and only once in the next six years and putting your hard-earned money down on one particular second—say, the 36th second of 2:52am on March 19th, 2017—and only winning if the one sneeze happens exactly at that second. Don't buy a Mega Millions ticket.

**$10^9$  (1 billion <sup>1</sup> – 1,000,000,000)** – Here we have the number of seconds in a century (about 3 billion), the number of living humans (7.125 billion), and to fit a billion dots, our dot image would cover two basketball courts.

**$10^{10}$  (10 billion)** – Now we're up to the years since the Big Bang (13.7 billion) and the number of seconds since Jesus Christ lived (60 billion).

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**10<sup>11</sup> (100 billion)** – This is about the number of stars in the Milky Way and the number of galaxies in the observable universe (100-400 billion)—so if a computer listed one observable galaxy every second since Christ, it wouldn't be anywhere close to finished currently.

**10<sup>12</sup> (1 trillion – 1,000,000,000,000)** – A million millions. The amount of pounds the scale would show if you put the whole human race on it (~1 trillion), the number of seconds humans have been around (~100,000 years = ~3 trillion seconds), and larger than both of those totals combined, the number of miles in one light year (6 trillion). A trillion is so big that you'd only need 4 trillion millimeters of ribbon

**9.8k** bow around the sun.  
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**10 trillion** – This is about as big as we can get for numbers we hear discussed in the real world, almost always related to nations and dollars—the US nominal GDP in 2013 was just under \$17 trillion, and its debt is currently just under \$18 trillion. Both of those are dwarfed by the number of cells in a human body (37 trillion).

9.8k

**100 trillion** – 100 trillion is about the number of letters in every published book in human history, as the number of bacteria in your body. <sup>2</sup> Also in this range is the total wealth of the world in 2013, which we discussed [at great length](#) in a previous post).

**10<sup>15</sup> (1 quadrillion)** – Okay goodbye normal words. People say the words million, billion, and trillion a lot. No one says quadrillion. It's really uncool to say the word quadrillion. <sup>3</sup> Most people opt for "a million billion" instead. Either way, there are about a quadrillion [ants on Earth](#). Comparing this to the bacteria fact, it's like you have 1/10th of the world's ants crawling around inside your body.

**10<sup>16</sup> (10 quadrillion)** – It's in this range that we get to the number of playing cards you'd have to accidentally knock off the table to cover the entire Earth (89 quadrillion). People would be mad at you.

**10<sup>17</sup> (100 quadrillion)** – The number of seconds since the Big Bang. Also the number of references to Kim Kardashian that entered my soundscape in the last week. Please stop.

**10<sup>18</sup> (1 quintillion)** – Also known as a billion billion, the word quintillion manages to be even less cool than a quadrillion. No one who has social skills ever says the word quintillion. Anyway, it's the number of cubic meters of water in all the Earth's oceans and the number of atoms in a grain of salt (1.2 quintillion). The number of grains of sand on every beach on Earth is about 7.5 quintillion—the same number of atoms in *six grains of salt*.

**10<sup>19</sup> (10 quintillion)** – The number of millimeters from here to the closest next star (38 quintillion millimeters).

**10<sup>20</sup> (100 quintillion)** – The number of meter-long steps it would take you to walk across the whole Milky Way. So many podcasts. And heard of a Planck volume? It's the smallest volume scientists talk about, so small you could fit 100 quintillion of them in a *proton*. More on Planck volumes later. Oh, and our dot image? By the time we get to 600 quintillion dots, the image would cover the surface of the Earth.

**10<sup>21</sup> (1 sextillion)** – Now we're even beyond the vocabulary of the weirdos. I don't think I've ever heard someone say "sextillion" out loud, and I hope to keep it that way.

**10<sup>23</sup> (100 sextillion)** – A rough estimate for the number of stars in the observable universe. You also had to deal with this number in high school—602 sextillion, or  $6.02 \times 10^{23}$ —is a mole, or Avogadro's Number, and the number of hydrogen atoms in a gram of hydrogen.

**10<sup>24</sup> (1 septillion)** – A trillion trillions. The Earth weighs about six septillion kilograms.

**10<sup>25</sup> (10 septillion)** – The number of drops of water in all the world's oceans.

**10<sup>27</sup> (1 octillion)** – If the Earth were hollow, it would take 1 octillion peas to pack it full. And I think we've heard just about enough from octillion.

Okay so now let's take a huge leap forward into a whole different territory—somewhere where the Earth's volume is too tiny and the Big Bang too recent to use in examples. In this new arena of number, only the observable universe—a sphere about 92 billion light years across—can handle the magnitude we're dealing with. <sup>4</sup>

**10<sup>80</sup>** – To get to 10<sup>80</sup>, you take trillion and you multiply it by a trillion, by a trillion, by a trillion, by a trillion, by a trillion, by a hundred million. No dot posters being sold for this number. So why did I stop here at this number? Because it's a common [estimate](#) for the number of atoms in the universe.

**10<sup>86</sup>** – And what if you wanted to pack the entire observable universe sphere with peas? You'd need 10<sup>86</sup> peas to make it happen.



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**10<sup>90</sup>** – This is how many medium size grains of sand (.5mm in diameter) it would take to pack the universe full.

## A Googol – $10^{100}$

The name googol came about when American mathematician Edward Kasner got cute one day in 1938 and asked his 9-year-old nephew Milton to come up with a name for  $10^{100}$ —1 with 100 zeros. Milton, being an inane 9-year-old, suggested “googol.” Kasner apparently decided this was a reasonable

**9.8k** Shares r, ran with it, and that was that. <sup>5</sup>

✓ big is a googol?

9.8k : number of grains of sand that could fit in the universe, times 10 billion. So picture the universe  
 9.8k icked with small grains of sand—for tens of billions of light years above the Earth, below it, in  
 9.8k f it, behind it, just sand. Endless sand. You could fly a plane for trillions of years in any direction  
 speed through it, and you'd never get to the end of the sand. Lots and lots and lots of sand.

Imagine that you stop the plane at some point, reach out the window, and grab one grain of sand. Look at it under a powerful microscope—and what you see is that it's actually not a single grain, but 10 billion microscopic grains wrapped in a membrane, all of which together is the size of a normal grain of sand. If that were the case for every single grain of sand in this hypothetical—if each were actually a bundle of 10 billion tinier grains—the *total* number of those microscopic grains would be a googol.

We're running out of room here on both the small and big end of things to fit these numbers into the physical world, but three more for you:

**$10^{113}$**  – The number of hydrogen atoms it would take to pack the universe full of them.

**$10^{122}$**  – The number of protons you could fit in the universe.

**10<sup>185</sup>** – Back to the Planck volume (the smallest volume I’ve ever heard discussed in science). How many of these smallest things could you fit in the very biggest thing, the observable universe? 10<sup>185</sup>. Without being able to go smaller or bigger on either end, we’ve reached the largest number where the physical world can be used to visualize it.

# A Googolplex – $10^{\text{googol}}$

After popularizing the newly-named *googol*, Krasner could barely keep his pants on with this adorable new schtick and asked his nephew to coin *another* term. He could barely finish the question before Milton opened his un-nuanced mouth and declared the number *googolplex*, which he, in typical Milton form, described as “one, followed by writing zeroes until you get tired.”<sup>6</sup> At this, Krasner showed some uncharacteristic restraint, ignoring Milton and giving the number an actual definition:  $10^{\text{googol}}$  or 1 with a googol zeros written after it. With its full written-out exponent, a googolplex looks like this:

[illegible]

So a googol is 1 with just 100 zeros after it, which is a number 10 billion times bigger than the grains of sand that would fill the universe. Can you possibly imagine what kind of number is produced when you put a *googol* zeros after the 1?

There's no possible way to wrap your head around that number—the best we can do is try to understand how long it would take to *write* the number. What I wrote above is just the *exponent*—actually writing a googolplex *out* involves writing a googol zeros. First, let's figure out *where* we'd write these zeros.

As we've discussed, filling the universe with sand only gets you a ten billionth of the way to a googol, so what we'd have to do is fill the universe to the brim with sand, get a very tiny pen, and write 10 billion zeros *on each grain of sand*. If you did this and then looked at a completed grain under a microscope, you'd see it covered with 10 billion microscopic zeros. If you did that on every single grain of sand filling the universe, you'd have successfully written down the number googolplex.

And just how long would it take to do that?

Well I just tested how fast a human can reasonably write zeros, and I wrote 36 zeros in 10 seconds.<sup>7</sup> At that rate, if from the age of 5 to the age of 85, all I did for 16 hours a day, every single day, was write zeros at that rate, I'd finish *one half of a grain of sand in my lifetime*. You'd need to dedicate two full human lives to finish one grain of sand. About 107 billion human beings have ever lived in the history of the species. If every single human dedicated every waking moment of their lives to writing zeros on grains of sand, as a species we'd have by now filled a cube with a side of 1.7m—about the height of a human—with completed sand grains. That's *it*.

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Now to get a glimpse at how big the actual *number* is—as the Numberphilers [explain](#), the total possible quantum states that could occur in the space occupied by a human (i.e. every possible arrangement of atoms that could happen in that space) is *far* less than a googolplex. What this means is that if there were a universe with a volume of a googolplex cubic meters (an extraordinarily large space), random probability suggests that there would be *exact copies of you* in that universe. Why? Because every possible arrangement of matter in a human-sized space would likely occur many, many times in a space that vast, meaning everything that could possibly exist would exist—including you. Including you with *cat whiskers* but normal otherwise. Including you but a one-foot tall version. Including you exactly how *9.8k Shares* except instead of a pinky finger on your left hand you have Napoleon's penis there as your fifth What I'm saying isn't science fiction—it's the reality of a space that large.

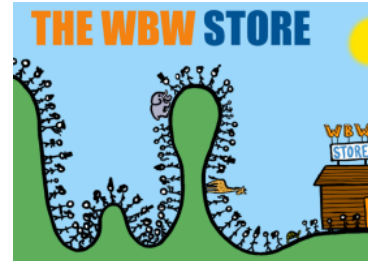
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## Graham's Number

How sometimes you go through life, and you're lost but you don't even know it, and then one day a right person comes along and you realize what you had been looking for this whole time?

How I feel about Graham's number.

Huge numbers have always both tantalized me and given me nightmares, and until I learned about Graham's number, I thought the biggest numbers a human could ever conceive of were things like "A googolplex to the googolplexth power," which would blow my mind when I thought about it. But when I learned about Graham's number, I realized that not only had I not scratched the surface of a truly huge number, I had been *incapable of doing so*—I didn't have the tools. And now that I've gained those tools (and you will too today), a googolplex to the googolplexth power sounds like a kid saying "100 plus 100!" when asked to say the biggest number he could think of.

Before we dive in, why is Graham's number even a number people talk about?

I'm not gonna really explain this because the explanation is really boring and confusing—here's the official problem Ronald Graham (a living American mathematician) was working on when he came up with it:

*Connect each pair of geometric vertices of an  $n$ -dimensional hypercube to obtain a complete graph on  $2^n$  vertices. Color each of the edges of this graph either red or blue. What is the smallest value of  $n$  for which every such coloring contains at least one single-colored complete subgraph on four coplanar vertices?*

I told you it was boring and confusing. Anyway, there's no single answer to the problem, but Graham's proof includes a lower and upper bound, and Graham's number was one version of an upper bound for  $n$  that Graham came up with.

He came up with the number in 1977, and it gained recognition when a colleague [wrote about it](#) in *Scientific American* and called it "a bound so vast that it holds the record for the largest number ever used in a serious mathematical proof." The number ended up in the *Guinness Book of World Records in 1980* for the same reason, and though it has today been surpassed, it's still renowned for being the biggest number most people ever hear about. That's why Graham's number is a thing—it's not just an arbitrarily huge number, it's actually relevant in the world of math.

So anyway, I said above that I had been limited in the kind of number I could even imagine because I lacked the tools—so what are the tools we need to do this?

It's actually one key tool: the hyperoperation sequence.

The hyperoperation sequence is a series of mathematical operations (e.g. addition, multiplication, etc.), where each operation in the sequence is an iteration up from the previous operation. You'll understand in a second. Let's start with the first and simplest operation: counting.

### Operation Level 0 – Counting

If I have 3 and I want to go up from there, I go 3, 4, 5, 6, 7, and so on until I get where I want to be. Not a high-powered operation.

### Operation Level 1 – Addition

Addition is an iteration up from counting, which we can call "iterated counting"—so instead of doing 3, 4, 5, 6, 7, I can just say  $3 + 4$  and skip straight to 7. Addition being "iterated counting" means that addition is like a counting *shortcut*—a way to bundle all the counting steps into one, more concise step.

### Operation Level 2 – Multiplication

One level up, multiplication is iterated addition—an addition shortcut. Instead of saying  $3 + 3 + 3 + 3$ , multiplication allows us to bundle all of those addition steps into one higher-operation step and say  $3 \times 4$ . Multiplication is a more powerful operation than addition and you can create way bigger numbers with it. If I add two eight-digit numbers together, I'll end up with either an eight or nine-digit number. But if I *multiply* two eight-digit numbers together, I end up with either a 15 or 16-digit number—much bigger.

### Operation Level 3 – Exponentiation (↑)

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g up another level, exponentiation is iterated multiplication. Instead of saying  $3 \times 3 \times 3 \times 3$ , exponentiation allows me to bundle that string into the more concise  $3^4$ .

9.8k

he thing is, this is where most people stop. In the real world, exponentiation is the highest operation we tend to ever use in the hyperoperation sequence. And when I was envisioning my huge

9.8k

plex<sup>googolplex</sup> number, I was doing the very best I could using the highest level I knew—exponentiation. On Level 3, the way to go as huge as possible is to make the base number massive *and* exponent number massive. Once I had done that, I had maxed out.

g to breaking through the ceiling to the *really* big numbers is understanding that you can go up more levels of operations—you can keep iterating up infinitely. *That's* the way numbers get truly huge.

And to do this, we need a different kind of notation. So far, we've worked with a different symbol on each level (+, x, and a superscript)—but we don't want to have to remember a ton of different symbols if we're gonna be working with a bunch of different operations levels. So we'll use Knuth's up-arrow notation, which is one symbol that can be used on any level.

Knuth's up-arrow notation starts on Operation Level 3, replacing exponentiation with a single up arrow: ↑. So to use up-arrow notation, instead of saying  $3^4$ , we say  $3 \uparrow 4$ , but they mean the same thing.

$$3 \uparrow 4 = 81$$

$$2 \uparrow 3 = 8$$

$$5 \uparrow 5 = 3,125$$

$$1 \uparrow 38 = 1$$

Got it? Good.

Now let's move up a level and start seeing the insane power of the hyperoperation sequence:

### Operation Level 4 – Tetration (↑↑)

Tetration is iterated exponentiation. Before we can understand how to bundle a string of exponentiation the way exponentiation bundles a string of multiplication, we need to understand what a "string of exponentiation" even is.

So far, all we've done with exponentiation is one computation—a base number and a power it's raised to. But what if we put two of these computations together, like:

$$2^{2^2}$$

We get a *power tower*. Power towers are incredibly powerful, because they start at the top and work their way down. So  $2^{2^2} = 2^{(2^2)} = 2^4 = 16$ . Nothing that impressive yet, but check out:

$$3^{3^{3^3}}$$

Using parentheses to emphasize the top down order:  $3^{3^{3^3}} = 3^{3^{(3^3)}} = 3^{3^{27}} = 3^{(3^{27})} = 3^{7,625,597,484,987} =$  a 3.6 trillion-*digit* number

Remember, a googol and its universe-filling microscopic mini-sand is only a 100-digit number. So all it takes is a power tower of 3s stacked 4 high to *dwarf* a googol, as well as  $10^{185}$ , the number of Planck volumes to fill the universe and our physical world maximum. It's not as big as a googolplex, but we can take care of that easily by just adding one more 3 to the stack:

$3^{3^{3^3}} = 3^{(3^{3^{3^3}})} = 3^{(3.6 \text{ trillion-digit number})} =$  way bigger than a googolplex, which is  $10^{(100\text{-digit number})}$ . As for a googolplex itself, power towers allow us to immediately humiliate it by writing it as:

$10^{10^{100}}$  or, more typically,  $10^{10^{10^2}}$ . So you can imagine what kind of number you get when you start making *tal* power towers. Tetration is intense.

Now those towers are Level 3, exponential strings, the same way  $3 \times 3 \times 3 \times 3$  is a Level 2, multiplication string. We use Level 3 to bundle that Level 2 string into  $3^4$ , or  $3 \uparrow 4$ . So how do we use Level 4 to bundle an exponential string? Double arrows.

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$3^{3^{3^3}}$  is the same as saying  $3 \uparrow (3 \uparrow (3 \uparrow 3))$ . We bundle those 4 one-arrow 3s into  $3 \uparrow \uparrow 4$ .

Likewise,  $3 \uparrow \uparrow 5 = 3 \uparrow (3 \uparrow (3 \uparrow (3 \uparrow 3))) = 3^{3^{3^{3^3}}}$

$4 \uparrow \uparrow 7 = 4 \uparrow (4 \uparrow (4 \uparrow (4 \uparrow (4 \uparrow (4 \uparrow 4))))))$  = a power tower of 4s 7 high.

Here's the general rule:

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## Tetration

(Operation Level 4)

$a \uparrow \uparrow b$  means a **power tower** of  $a$ 's,  $b$  high

$$a \uparrow \uparrow b = \underbrace{a \uparrow (a \uparrow (\dots \uparrow a))}_{b \text{ copies of } a} = \underbrace{a^{\dots^a}}_{b \text{ copies of } a}$$

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We're about to move up another level, and this is about to become more complex, so before we move on, make sure you *really* understand Level 4 and what  $\uparrow \uparrow$  means—just remember that  $a \uparrow \uparrow b$  is a power tower of  $a$ 's,  $b$  high.

## Operation Level 5 – Pentation ( $\uparrow \uparrow \uparrow$ )

Pentation, or iterated tetration, bundles double arrow strings together into a single operation.

The pattern we've seen is each new level bundles a string of the previous level together by using a  $b$  term as the *length* of the string. For example:

**Multiplication** bundles together a string of **addition**:

$$a \times b = \underbrace{a + a + \dots + a}_{b \text{ copies of } a}$$

**Exponentiation** bundles together a string of **multiplication**:

$$a \uparrow b \text{ or } a^b = \underbrace{a \times a \times \dots \times a}_{b \text{ copies of } a}$$

**Tetration** bundles together a string of **exponentiation**:

$$a \uparrow \uparrow b = \underbrace{a \uparrow (a \uparrow (\dots \uparrow a))}_{b \text{ copies of } a} = \underbrace{a^{\dots^a}}_{b \text{ copies of } a}$$

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In each case,  $a$  is the base number and  $b$  is the length of the string being bundled.

So what does pentation bundle together? How can you have a *string of power towers*?

The answer is what I call a "power tower feeding frenzy". Here's how it works:

You have a string of power towers standing next to each other, in a particular order, all using the same base number. The thing that differs between them is the *height* of each tower. The first tower's height is the same number as the base number. You process that tower down to its full expanded outcome, and that outcome becomes *the height* of the next tower. You then process *that* tower, and the outcome becomes the *height* of the *next* tower. And so on. Each tower's outcome "feeds" into the next tower and becomes its height—hence the feeding frenzy. Here's why this happens:

$3 \uparrow \uparrow \uparrow 4$  means a string of  $(3 \uparrow \uparrow 3)$  operations, 4 long. So:

$$3 \uparrow \uparrow \uparrow 4 = 3 \uparrow \uparrow (3 \uparrow \uparrow (3 \uparrow \uparrow 3))$$

Remember, when you see  $\uparrow \uparrow$  it means a single power tower that's  $b$  high, so:

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$$3 \uparrow\uparrow\uparrow 4 = 3 \uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow\uparrow 3)) = 3 \uparrow\uparrow (3 \uparrow\uparrow 3^{3^3})$$

Now, you might remember from before that  $3^3 = 3^{27} = 7,625,597,484,987$ . So:

$$3 \uparrow\uparrow\uparrow 4 = 3 \uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow\uparrow 3)) = 3 \uparrow\uparrow (3 \uparrow\uparrow 3^{3^3}) = 3 \uparrow\uparrow (3 \uparrow\uparrow 7,625,597,484,987)$$

So the first tower of height 3 processed down into 7 trillion-ish. Now the next parentheses we're dealing with is  $(3 \uparrow\uparrow 7,625,597,484,987)$ , where the outcome of the first tower is the *height* of this second tower. And how high would that tower of 7 trillion-ish 3s be?

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each 3 is two centimeters high, which is about how big my written 3's are, the tower would rise 150 million kilometers high, which would touch the sun. Even if we used tiny, typed 2mm 3's, our tower would reach the moon and back to the Earth and back to the moon *forty* times before finishing. If you wrote those tiny 3's on the ground instead, the tower would wrap around the earth 400 times. Let's call this tower the "sun tower," because it stretches all the way to the sun. So what we have is:

9.8k

$$= 3 \uparrow\uparrow (3 \uparrow\uparrow (3 \uparrow\uparrow 3)) = 3 \uparrow\uparrow (3 \uparrow\uparrow 3^{3^3}) = 3 \uparrow\uparrow (3 \uparrow\uparrow 7,625,597,484,987) = 3 \uparrow\uparrow (\text{sun tower})$$

That  $3 \uparrow\uparrow (\text{sun tower})$  operation is a power tower of 3's whose height is the number you get when you multiply out the entire sun tower (and this final tower we're building won't even come close to fitting in the observable universe). And we don't get to our final value of  $3 \uparrow\uparrow\uparrow 4$  until we multiply *out* this final tower.

So using  $\uparrow\uparrow\uparrow$ , or pentation, creates a power tower feeding frenzy, where as you go, each tower's *height* begins to become incomprehensible, let alone the actual final value. Written generally:

## Pentation

(Operation Level 5)

$a \uparrow\uparrow\uparrow b$  means a **power tower feeding frenzy** with  $b$  towers of  $a$ 's

$$a \uparrow\uparrow\uparrow b = \underbrace{a \uparrow\uparrow (a \uparrow\uparrow (\dots \uparrow\uparrow a))}_{b \text{ copies of } a}$$

Each parentheses is a **power tower**, which, when multiplied all the way out, becomes the "b position" in the power tower of the next parentheses outward — i.e. the *height* of that power tower.

$$a \uparrow\uparrow\uparrow b = \left. \begin{array}{c} a^{\dots a} \\ a^{\dots a} \\ \vdots \\ a \end{array} \right\} b-1 \text{ towers}$$

**Power tower feeding frenzy**  
The first tower at the bottom has a height of  $a$ . Once multiplied out, that becomes the *height* of the next tower, which then, when multiplied out, feeds into the next tower as *its* height, and so on.

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We're gonna go up one more level—

## Operation Level 6 – Hexation ( $\uparrow\uparrow\uparrow\uparrow$ )

So on Level 4, we're dealing with a string of Level 3 exponents—a power tower. On Level 5, we're dealing with a string of Level 4 power towers—a power tower feeding frenzy. On Level 6, aka hexation or iterated pentation, we're dealing with a string of power tower feeding frenzies—what we'll call a "power tower feeding frenzy psycho festival." Here's the basic idea:

A power tower feeding frenzy happens. The final number the frenzy produces becomes *the number of towers in the next feeding frenzy*. Then that frenzy happens and produces an even more ridiculous number, which then becomes *the number of towers* for the *next* frenzy. And so on.

$3 \uparrow\uparrow\uparrow\uparrow 4$  is a power tower feeding frenzy psycho festival, during which there are 3 total  $\uparrow\uparrow\uparrow$  feeding frenzies, each one dictating the number of towers in the next one. So:

$$3 \uparrow\uparrow\uparrow\uparrow 4 = 3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow\uparrow (3 \uparrow\uparrow\uparrow 3))$$

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Now remember from before that  $3 \uparrow \uparrow 3$  is what turns into the sun tower. So:

$$3 \uparrow \uparrow \uparrow 4 = 3 \uparrow \uparrow (3 \uparrow \uparrow (3 \uparrow \uparrow 3)) = 3 \uparrow \uparrow (3 \uparrow \uparrow (\text{sun tower}))$$

Since  $\uparrow \uparrow \uparrow$  means a power tower feeding frenzy, what we have here with  $3 \uparrow \uparrow \uparrow$  (sun tower) is a feeding frenzy with a multiplied-out-sun-tower number of *towers*. When *that* feeding *finally* finishes, the outcome becomes the number of towers in the final feeding frenzy. The psycho festival ends when that final feeding frenzy produces it's final number. Here's hexation explained generally:

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9.8k

## Hexation

(Operation Level 6)

$a \uparrow \uparrow \uparrow b$  means  
a power tower feeding frenzy psycho festival  
with  $b$  towers of  $a$ 's

$$a \uparrow \uparrow \uparrow b = \underbrace{a \uparrow \uparrow (a \uparrow \uparrow (\dots \uparrow \uparrow a))}_{b \text{ copies of } a}$$

Each parenthesis is a **power tower feeding frenzy**, which, when multiplied *all* the way out, becomes the "b" position in the power tower feeding frenzy of the next parentheses outward—i.e. the *number of towers* of that power tower feeding frenzy.

$$a \uparrow \uparrow \uparrow b = \underbrace{\left\{ \begin{array}{c} a \\ \left\{ \begin{array}{c} a^a \\ \vdots \\ a^a \end{array} \right\} \\ a \end{array} \right\}}_{b-1 \text{ feeding frenzies}} \dots \left\{ \begin{array}{c} a \\ \left\{ \begin{array}{c} a^a \\ \vdots \\ a^a \end{array} \right\} \\ a \end{array} \right\} \right\} a-1$$

**A power tower feeding frenzy**

**Power tower feeding frenzy psycho festival**  
The first feeding frenzy (on the far right) starts with an  $a$  number of towers that feed into each other, producing an ultimately huge total. That total becomes the *number of towers* in the next feeding frenzy, and so on. A group of feeding frenzies is a feeding frenzy psycho festival, and results from using four arrows, or hexation.

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And that's how the hyperoperation sequence works. You can keep increasing the arrows, and each arrow you add *dramatically* explodes the scope you're dealing with. So far, we've gone through the first seven operations in the sequence, including the first four arrow levels:

$\uparrow$  = power

$\uparrow \uparrow$  = power tower

$\uparrow \uparrow \uparrow$  = power tower feeding frenzy

$\uparrow \uparrow \uparrow \uparrow$  = power tower feeding frenzy psycho festival

So now that we have the toolkit, **let's go through Graham's number:**

Graham's number is going to be equal to a term called  $g_{64}$ . We'll get there. First, we need to start back with a number called  $g_1$ , and then we'll work our way up. So what's  $g_1$ ?

$$g_1 = 3 \uparrow \uparrow \uparrow 3$$

Hexation. You get it. Kind of. So let's go through it.

Since there are four arrows, it looks like we have a power tower feeding frenzy psycho festival on our hands. Here's how it looks visually:



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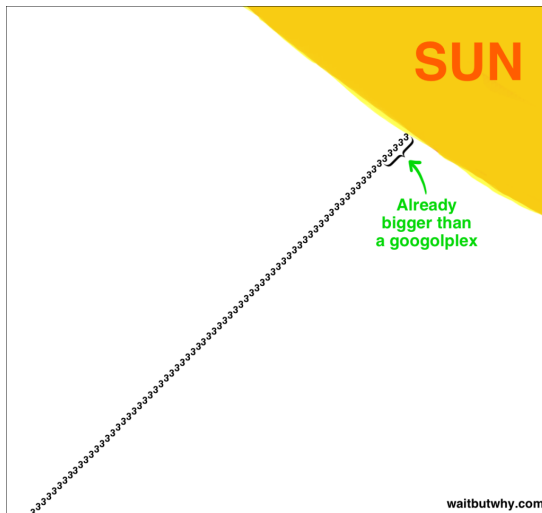


*And that's only the top few centimeters of the sun tower.*

9.8k  
Shares

9.8k

9.8k



ve get a meter down, the number is truly *far, far, far* bigger than we could ever fathom. And that's *a meter down*.

**The tower goes down 150 million kilometers.**

Let's call the final outcome of this multiplied-out sun tower INSANITY in all caps. We can't comprehend even a few centimeters multiplied out, so 150 million km is gonna be called INSANITY and we'll just live with it.

So back to where we were:

$$g_1 = 3 \uparrow \uparrow \uparrow \uparrow 3 = 3 \uparrow \uparrow \uparrow (3 \uparrow \uparrow \uparrow 3) = 3 \uparrow \uparrow \uparrow (\text{sun tower})$$

And now we can replace the sun tower with the final number that it produces:

$$g_1 = 3 \uparrow \uparrow \uparrow \uparrow 3 = 3 \uparrow \uparrow \uparrow (3 \uparrow \uparrow \uparrow 3) = 3 \uparrow \uparrow \uparrow (\text{sun tower}) = 3 \uparrow \uparrow \uparrow \text{INSANITY}$$

Alright, we're ready for the second of our two feeding frenzies. And here's the thing about this second feeding frenzy—

So you know how upset I just got about this whole INSANITY thing?

*That was the outcome of a feeding frenzy with only two towers.* The first little one multiplied out and fed into the second one and the outcome was INSANITY.

Now for this second feeding frenzy...

There are an INSANITY number of *towers*.

We'll move on in a minute, and I'll stop doing these dramatic one sentence paragraphs, I promise—but just absorb that for a second. INSANITY was so big there was no way to talk about it. Planck volumes in the universe is a joke. A googolplex is laughable. It's too big to be part of my life. And that's the number of *towers* in the second feeding frenzy.

$$g_1 = 3 \uparrow \uparrow \uparrow \uparrow 3 = 3 \uparrow \uparrow \uparrow \text{INSANITY} = \left. \begin{array}{c} 3^{3^{\dots^3}} \\ 3^{3^{\dots^3}} \\ \vdots \\ 3 \end{array} \right\} \text{INSANITY number of towers}$$

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So we have an INSANITY number of towers, each one being multiplied alllllllll the way down to determine the *height* of the next one, until somehow, somewhere, at some point in a future universe, we multiply our *final* tower of this second feeding frenzy out...and *that* number—let's call it NO I CAN'T EVEN—is the final outcome of the  $3 \uparrow \uparrow \uparrow 3$  power tower feeding frenzy psycho festival.

That number—NO I CAN'T EVEN—is  $g_1$ .

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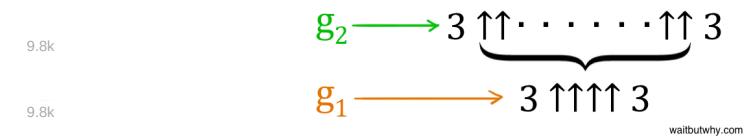
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So  $g_1$  is  $3 \uparrow \uparrow \uparrow 3$ , aka NO I CAN'T EVEN.

9.8k Shares xt step is we need to get to  $g_2$ . Here's how we get there:



losely at that drawing until you realize how not okay it is. Then let's continue.

h. We spent all day *clawing* our way up from one arrow to four, *coping* with the hardships each new operation level presented us with, absorbing the outrageous effect of adding each new arrow in. We went slowly and steadily and we ended up at NO I CAN'T EVEN.

Then Graham decides that for  $g_2$ , he'll just do the same thing as he did in  $g_1$ , **except instead of four arrows, there would be NO I CAN'T EVEN arrows.**

Arrows. The *entire*  $g_1$  now feeds into  $g_2$  as its number of *arrows*.

Just going to a *fifth* arrow would have made my head explode, but the number of arrows in  $g_2$  isn't five—it's *far, far more* than the number of Planck volumes that could fit in the universe, *far, far more* than a googolplex, and *far, far more* than INSANITY. **And that's the number of arrows.** That's the level of *operation*  $g_2$  uses. Graham's number iterates on the concept of iterations. It bundles the hyperoperation sequence *itself*.

Of course, we won't even pretend to do anything with that information other than laugh at it, stare at it, and be aroused by it. There's nothing we could possibly say about  $g_2$ , so we won't.

And how about  $g_3$ ?

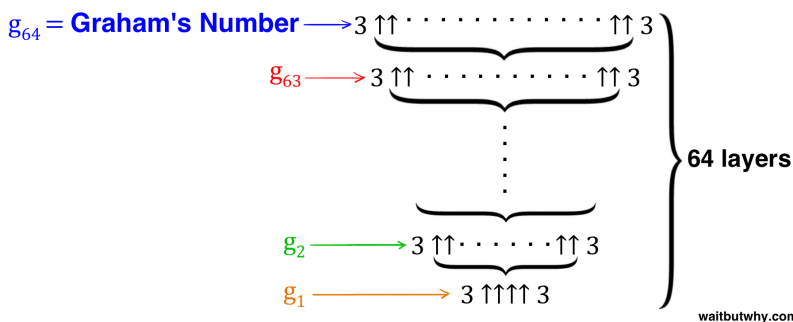
You guessed it—once the laughable  $g_2$  is all multiplied out, that becomes the number of *arrows* in  $g_3$ .

And then this happens again for  $g_4$ . And again for  $g_5$ . And again and again and again, all the way up to  $g_{64}$ .

$g_{64}$  is Graham's number.

All together, it looks like this:

## Graham's Number



So there you go. A new thing to have nightmares about.

P.S. Writing this post made me *much* less likely to pick “infinity” as my answer to [this week's dinner table question](#). Imagine living a *Graham's number amount of years*.<sup>8</sup> Even if hypothetically, conditions stayed the same in the universe, in the solar system, and on Earth forever, there is no way the human brain is built to withstand spans of time like that. I'm horrified thinking about it. I think it would be the

gravest of grave errors to punch infinity into the calculator—and this is from someone who’s openly terrified of death. Weirdly, thinking about Graham’s number has actually made me feel a little bit calmer about death, because it’s a reminder that *I don’t actually want to live forever*—I *do* want to die at some point, because remaining conscious for eternity is even scarier. Yes, death comes way, *way* too quickly, but the thought “I do want to die at some point” is a very novel concept to me and actually makes me more relaxed than usual about our mortality.

P.P.S If you must, another Wait But Why [post](#) on large numbers.

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- Daniel Arseneault • 19 days ago

yeah, if you thought g64 was bad just imagine g65... or even worse, gg1... or WAY worse, ggg.....gg1 with a g1 amount of g's, what i like to call gh1 (the h stands for hyper, as this is iterated g's. you can see where im going.). gh2 would be a g2 amount of g's, and gh(g1) is what i like to call ghh1. and so on... until we get to u1 (the u now stands for Ultimate), which would be a gh1 amount of h's after the g. then u64 would be the Ultimately Huge Number.

^

 | 

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- lj307 • 2 months ago

This was fun. Fun as fuck.

^

 | 

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**GuccizBud** • 2 months ago

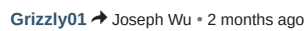
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9.8k

9.8k

**Joseph Wu** • 4 months ago

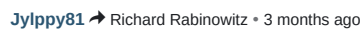
3 ^ | v • Reply • Share ›



1 ^ | v • Reply • Share ›



^ | v • Reply • Share »



7 ^ | v • Reply • Share ›



^ | v • Reply • Share »



11 ^ | v • Reply • Share ›



1 ^ | v • Reply • Share ›



1 ^ | v • Reply • Share ›



i did not like this. not at all. i hated this. this did things to my brain that shouldn't be allowed  
so, please tell me if i got this right:

did i get it

4 ^ | v • Reply • Share ›



13/17



^ | v • Reply • Share >



**Chris H** → Todd French • 5 months ago

Did you try crying about it?

^ | v • Reply • Share >

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**Edward Unger** • a year ago

There is a semi-facetious "law" that says almost every number is a very very large number. Reading this post about Graham's Number makes that clear. I would like to formulate another law, modestly named after myself.

"Almost every arithmetical operation is useless." I would like to have the option of living forever, though.

^ | v • Reply • Share >

9.8k

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**Eric Zhan** • a year ago

Why stop? Why not make a new operation,  $\uparrow$ , that would be the number of arrows? For example,  $3\uparrow 3$  would be  $3(\text{arrow})(\text{arrow})(\text{arrow})3$ . Going like this, we could construct a number way larger than Graham's number.

3 ^ | v • Reply • Share >

9.8k

**City Boy** • a year ago

Has anyone mentioned this:

<https://www.scottaaronson.c...>

I have been trying to figure out which gives bigger numbers faster.

Any comment?

1 ^ | v • Reply • Share >



**Rickard Edman** • a year ago

Let's go crazy; graham's number subscripted with graham's number...

1 ^ | v • Reply • Share >



**cst1992** → Rickard Edman • a year ago

You mean g subscripted with Graham's number?

^ | v • Reply • Share >



**Statalyzer** • a year ago

It would make more sense for power-towers to start on the bottom and not the top, even though this makes the numbers smaller.

^ | v • Reply • Share >



**Richard Connor** • a year ago

Imagine the universe IS big enough to contain Graham's number, as it may very well be. Imagine our universe is finite, but could quite comfortably accommodate a Graham's number of Graham's numbers.

2 ^ | v • Reply • Share >



**Luiz Henrique** • a year ago

Good explanation... but I think that  $g_1$  is enough to have nightmares and  $g_2$  can blow one's mind LOL

^ | v • Reply • Share >



**Lord Fnord** • 2 years ago

"18cm x 450cm = .81 m<sup>2</sup> in area."

Uh, no. And you don't even need to think to know that this is wrong: it would mean your computer screen was almost a square yard. Like, hunh?

The number you're looking for is .081 square metres.

^ | v • Reply • Share >



**BarnZarn** → Lord Fnord • 2 months ago

I'm pretty sure my computer screen is not 4.5 meters long

^ | v • Reply • Share >



**Lappro** → Lord Fnord • 2 years ago

No the article is correct.  $m^2 = m * m$ .

18cm = 0.18m

450cm = 4.5m

So we get  $0.18m * 4.5m = 0.81 m^2$

g ^ | v • Reply • Share >



**Anonymous Person** • 2 years ago

What's Graham's Number? Its  $G_{64}$ . What is this? Its more than googolplexianth<sup>googolplexianth</sup>. Googolplexianth times! Or using knoth's arrow, it is  $G_{64} \gg \text{googolplexianth} \uparrow \text{googolplexianth}$ !



Remember: googol, 1 followed by 100 zeros. Googol plex, 1 followed by a googol zeros! Googolplexian, 1 followed by a googolplex of zeros, Googolplexianth, 1 followed by a Googolplexian of zeros!

^ | v • Reply • Share >

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**Nobody** → Anonymous Person • 2 years ago

Not a good analogy. This works even for g1. G2 is so big no analogy that doesn't copy its creation can describe it. Just stop attempting to analogise how big it is using the real world since it is too small.

9.8k  
Shares

I did find one good analogy. Take a random person.

You: Did you hear about the Graham's number?

Them: No.

You: Okay, think of the largest number you can describe (infinity doesn't count).

Them: OK.

You: It's bigger.

16 ^ | v • Reply • Share >

9.8k

9.8k

**Ondřej Vitík** • 2 years ago

It is incomprehensible and nightmare-invoking. But that's not a reason for wanting to die.

Your reason is simply you are used to people having to die. Or maybe used to boredom. Or maybe used to experiencing 1 year as one year, although old people experience it as half a year and God G64 times differently than you. And with SAI we will become something comparable to God, at least in some senses.

^ | v • Reply • Share >



**Enderman1323** • 2 years ago

Some loser stole this document

<https://urbantimes.co/2014/...>

^ | v • Reply • Share >



**Enderman1323** → Enderman1323 • 2 years ago

As in the link is to the stolen document, Tim Urban's is the original

^ | v • Reply • Share >



**Nic Morales** • 2 years ago

But what about G65?

2 ^ | v • Reply • Share >



**Lord Fnord** → Nic Morales • 2 years ago

A lot of people have good feelings about the number 0.5 and don't want to insult it. They might refer to somebody like you, Nic, as a 1/G65-wit.

2 ^ | v • Reply • Share >



**Nic Morales** → Lord Fnord • 2 years ago

Woha, someone feels smart today!

2 ^ | v • Reply • Share >



**Anonymous Person** → Nic Morales • 2 years ago

were only talking about grahams number because it is used for serious mathmatecal proof

If we were talking numbers with no purpose, then here, go to G (down in subscript)g64. G grahams numberethed

1 ^ | v • Reply • Share >



**Doom\_Jasper** • 2 years ago

When you're talking about the  $10^{80}$ , if thats the number of atoms in the universe, than how can the number of peas that can fit ( $10^{86}$ ) be bigger? sorry if this is a stupid question

^ | v • Reply • Share >



**Mark** → Doom\_Jasper • 2 years ago

Yeah it's the number of atoms that ARE in the universe, not that could fit. The average density is about six protons per cubic meter I've read (which means it's probably much lower outside stars and black holes). The number of peas per square meter is a bit more than 10 million, so the number match pretty well ( $10^6$  more peas can fit than there are atoms).

^ | v • Reply • Share >



**Sinis** → Doom\_Jasper • 2 years ago

Peas are very concentrated volume of atoms. Space is mostly empty, therefore no atoms to fill it up.

1 ^ | v • Reply • Share >



**Taira Schug** → Sinis • 2 years ago

Space is not empty, there's virtual particles which come in and out of existence. This is one of the properties of space. Look into Hawking Radiation. This goes into how Black Holes actually



## From 1,000,000 to Graham's Number - Wait But Why

1 ^ | v • Reply • Share ›

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I think even the number of all possible arrangements of the position, momentum, spin of every atom in the whole universe would still be way way smaller than Graham's number, let alone the number of possible arrangement of atoms in our brain, which is probably smaller than  $g_1$ . So it would be impossible for us to live graham's number years, even with advanced biotechnology, because our brains can't even hold that much memory.

^ | v • Reply • Share x

The creepiest thing about that is that even is somehow minds could hold that much information, the number of different "events" that could happen, just probabilistically, is way smaller than the Graham's number, so you would be living every possible "thing", in every possible order, with every possible variation, over, and over, and over again, "almost infinitely" many times. (well, asuming the universe somehow resets, like a Big Bounce thing... otherwise you just live in the heat death of the universe, with complete darkness and no events happening)

1 ^ | v • Reply • Share ›



You can forget everything and still be alive, can't you?

^ | v • Reply • Share ›

So really, if you just keep adding subscripts below other subscripts, couldn't you "hexate" downwards? Instead of a building a tower of superscriptions you dig a giant hole of subscripted G's? Like  $3\downarrow\downarrow\downarrow\downarrow\downarrow$  etc...

2 ^ | v • Reply • Share ›

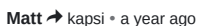


Subscripts aren't an operation, they are just "names" for the different versions of the same variable. In the case of "g"s, they indicate the place of the number in the sequence (e.g. "g26" is the 26th number of the sequence).

^ | v • Reply • Share ›

It's pretty big

2 ^ | v • Reply • Share ›



Correct.

^ | v • Reply • Share ›

Just dropping by to inform you that in a paraller universe I have \$g\_{(sub)}(g\_{(sub)}googolplex), and I'm not even the richest person there.

^ | v • Reply • Share x



Inflation is a bitch, huh?

^ | v • Reply • Share ›

And if I paraphrase Carl Sagan, Graham's number is no closer to infinity than the number one is.


15 ^ | v • Reply • Share ›

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 asdf — Username checks out.


7 comments • 2 years ago




 **keyce** — You are assuming that we will have to recreate what we sense by will, which is not the plan. The plan is to send scans of what our brain ...

337 comments • 2 years ago



 **Tim L.** — Probably a little anal/OCD too, not just a typical weirdo

 **Josh Bambrick** — Curiously, similar thoughts have often occurred to me. However, I typically consider questions where what you have to gain is much ...

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And still almost all numbers are larger than Graham's number  
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**Mark Henry**  
plus 1  
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 **Emily Hanby**  
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