TITLE

NAME:

Example

- 1. Let $y_1 = 6$, and for each $n \in \mathbb{N}$ define $y_{n+1} = \frac{2y_n 6}{3}$.
 - (a) Use induction to prove that the sequence satisfies $y_n > -6$ for all $n \in \mathbb{N}$.

Proof. We use induction on n.

Base Case: n = 1

$$y_{n+1} = y_2 = \frac{2y_1 - 6}{3} = 2 > -6$$

We prove the base case.

Inductive Step: Let $k \in \mathbb{N}, k \geq n$, suppose $y_k > -6$,

$$y_{k+1} = \frac{2y_k - 6}{3}$$

$$> \frac{2(-6) - 6}{3}$$

$$> \frac{-18}{3}$$

$$> -6$$

This proves the inductive step.

Conclude by Induction, the result holds.

(b) Use another induction argument to show the sequence $(y_1, y_2, y_3, ...)$ is decreasing.

Proof. We use strong induction on n.

Base Case: n = 1 and n = 2

$$n = 1 : y_1 = 6$$

 $n = 2 : y_2 = 2 < y_1$

the base case holds.

Induction Step: Let $k \in \mathbb{N}, k \geq 2$ and suppose $y_n < y_{n-1} \forall n \leq k$, then we have

$$y_{k+1} = \frac{2y_k - 6}{3}, y_k = \frac{2y_{k-1} - 6}{3}$$

$$\implies y_{k+1} - y_k = \frac{(2y_k - 6) - (2y_{k-1} - 6)}{3}$$

$$y_k < y(k-1) \implies (2y_k - 6) < (2y_{k-1} - 6)$$

$$\implies y_{k+1} < y_k$$

The inductive step holds. Prove by strong induction, the statement holds.