Evaluating Program Semantics Reasoning with Type Inference in System *F*

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NEURAL INFORMATION PROCESSING SYSTEMS

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Test-time compute: let's think step by step

- Chain-of-Thought prompting [Wei+22] enables *reasoning* in LLMs.
- CoT decomposes multi-step problems into intermediate (reasoning) steps.
- CoT becomes a *training paradigm* [Ope24; Dee+25; Mue+25].

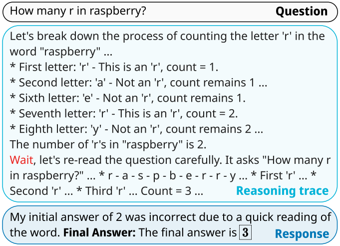

Figure 1: TTC example [Mue+25].

- Pre-trained knowledge: rules
- In-context knowledge + prompt: assumptions/hypotheses
- CoT: reasoning
  - step 1: lemma 1
  - step 2: lemma 2
  - …
- Response: a proposition

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The evaluation gap

Reasoning LLMs are evaluated on math and coding:

- Mathematical reasoning
  - AIME 2024: high school math competition
  - MATH-500: math problem solving
- Code generation
  - LiveCodeBench [Jai+25]: LeetCode, online coding contests
  - SWE-Bench [Jim+24]: patch generation

What is missing in the idea of *reasoning* for code?

- *program-centric* deductive system
- $\implies$ *reason* about structural logic behind programs

Propositions as types

- Types in the programming $\cong$ propositions in the logic [Wad15].
- A function type $A \rightarrow B \cong$ the proposition $A \implies B$ [Cur34].
- Type inference is natural deduction.

```haskell
not :: Bool -> Bool
(.) :: (b -> c) -> (a -> b) -> a -> c
span :: (a -> Bool) -> [a] -> ([a], [a])

break p =  span (not . p)
-- complete the following type signature for `break`
break :: (a -> Bool) -> [a] -> ([a],[a])
```

Listing 1: Example task for the `break` function

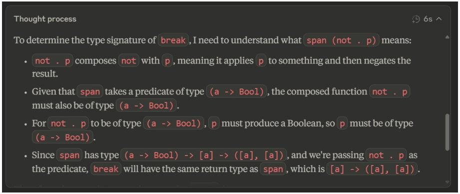

Figure 2: Claude 3.7' extended thinking mode on the task in Listing 1, ✓.

But wait, are LLMs really reasoning about *program semantics*?

Input transformations leads to significant performance drop:

- Semantic-preserving code transformations [AD22; Yan+22; Liu+23].
- Perturbations on math problems [Mir+24; Jia+24; Gul+24].

```haskell
f2 :: (t1 -> T1) -> [t1] -> ([t1], [t1])
f3 :: T1 -> T1
f4 :: (t1 -> t2) -> (t3 -> t1) -> t3->t2

f1 p = f2 (f3 `f4` p)
-- complete the following type signature for `f1`
f1 :: (t1 -> T1) -> [t1] -> ([t1], [t1])
```

Listing 2: The task in 1 after alpha-rewrite

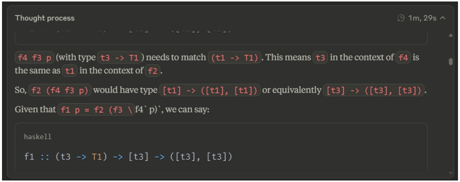

Figure 3: Claude 3.7' extended thinking mode on the task in Listing 2, ✓.

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Benchmark construction

We use the Haskell Prelude [Jon03] for the following reasons:

1. A formal deductive type system: System F (and System $F_{<}$ [1]).
2. The type signature is concise and is decoupled from the function body.
3. Type equivalence is formally verifiable (w/ signature only).
4. Haskell is the most popular language that meets these conditions.

```haskell
map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = f x : map f xs
```

Listing 3: Parametric polymorphic

```haskell
(==) :: Eq a => a -> a -> Bool
x == y = not (x /= y)
```

Listing 4: Ad-hoc polymorphism

[1] for bounded quantification

Removing natural language from tasks

We design three *alpha-rewrite* operators to remove NL from the tasks:

- NL-Type: `Int`, `Char`, `Bool`, `Eq`, `Ord` ... → `T1`, `T2`, `T3`, `T4`, `T5` ...
- Type-Var: `a`, `b`, `c`, `d`, `e` ... → `t1`, `t2`, `t3`, `t4`, `t5` ...
- Binding: `map`, `not`, `foldl`, `(+)` ... → `f1`, `f2`, `f3`, `f4` ...

Implementation details:

- Operators have type `Task -> Either Task Error`.
- Operators are commutative and associative under composition [Kle65].
- Transformed tasks are *alpha-equivalent* to the original tasks.

Construction pipeline and statistics

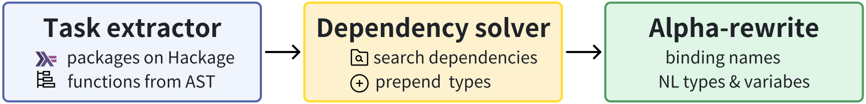

Figure 4: Pipeline to construct TF-Bench.

In total, TF-Bench has 188 tasks,

- 26.6% are monomorphic functions,
- 32.4% are parametric polymorphisms,
- and 41.0% are ad-hoc polymorphisms.

TF-Bench$_{\text{pure}}$ is the NL-free version of TF-Bench.

Evaluation methodology

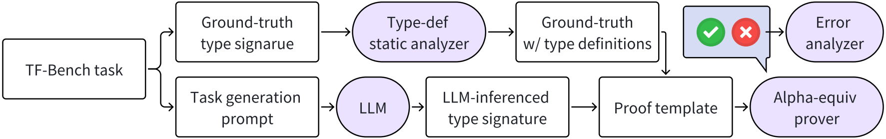

Figure 5: Pipeline to evaluate LLMs on TF-Bench.

1. We use the ground-truth signature to add definitions for new types.
2. We prompt the LLMs with tasks to generated a type signature.
3. We construct a proof using the ground-truth and generated signatures.
4. If the proof compiles, the two signatures are equivalent.
5. Otherwise we analyze what the error is.

Research questions

We ask the following questions around *program semantics reasoning*:

1. What is the performance gap of LLMs on TF-Bench and TF-Bench$_{\text{pure}}$?
2. How effective is TTC of models after reinforcement learning?
3. Can fine-tuning on math/code improve reasoning?

RQ1. The performance gap

Model	Version	TTC	Acc	Acc_{pure}	RS
Claude-3.5-sonnet	2024-06-20	✗	85.46	48.97	57.3
Claude-3.7-sonnet	2025-02-19	✓	<u>90.42</u>	**55.85**	<u>61.77</u>
GPT-O3-mini	2025-01-31	✓	**90.43**	48.40	53.52
GPT-O3	2025-04-16	✓	81.91	<u>52.66</u>	**64.29**
DeepSeek-V3	2025-03-25	✗	83.51	43.62	52.23
DeepSeek-R1	2025-01-20	✓	86.70	44.15	50.92
Qwen3	30B-A3B	✓	81.38	40.43	49.68
	32B	✓	87.94	43.09	49.00
	235B-A22B-FP8	✓	85.11	44.15	51.87

Table 1: Main evaluation results. $\text{RS}(m) = {}^{\text{Acc}_{\text{pure}}(m)}/_{\text{Acc}(m)}$.

RQ2. The effectiveness of TTC

Model	TTC	Acc	Acc_{pure}	RE
Qwen3-235B-FP8	✗	80.49	35.64	1.37
	✓	86.70	44.15	
Claude-3.7-sonnet	✗	87.77	46.81	3.41
	✓	90.42	55.85	
Gemini-2.5-flash	✗	78.19	30.32	3.90
	✓	83.51	51.06	

Table 2: Reasoning effectiveness of top LLMs.

$$\text{RE}(m_{\text{ttc}}, m) = \frac{\text{Acc}_{\text{pure}}(m_{\text{ttc}}) - \text{Acc}_{\text{pure}}(m)}{\text{Acc}(m_{\text{ttc}}) - \text{Acc}(m)} = \frac{\Delta_{\text{pure}}}{\Delta}.$$

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RQ3: Fine-tuning on math/code I

FT Corpus	Base Model (FT Model)	Size	Acc	FT Acc	Δ	Acc_{pure}	FT Acc_{pure}	Δ_{pure}
Code	Gemma (CodeGemma)	7B	48.94	53.19	+ 4.25	7.45	12.23	+ 4.78
	DeepSeek-V2 (-Coder)	16B	29.79	55.32	+ 25.53	7.98	15.96	+ 7.98
		236B	38.30	80.85	+ 42.55	11.17	36.70	+ 25.53
	Mistral (Codestral)	22B	61.17	63.30	+ 2.13	19.68	11.17	- 8.51
	Qwen2.5 (-Coder)	1.5B	30.32	36.70	+ 6.38	6.91	9.04	+ 2.13
		7B	65.96	61.17	- 4.79	21.28	21.28	0.00
		32B	74.47	82.45	+ 7.98	36.17	31.91	- 4.26
Math	Mistral (Mathstral)	7B	45.21	47.34	+ 2.13	7.99	15.43	+ 7.44
	Qwen2 (-Math)	7B	40.43	43.09	+ 2.66	3.19	10.64	+ 7.45
		72B	63.83	71.28	+ 7.45	21.81	33.51	+ 11.7

Table 3: Result comparison of fine-tuning. FT Corpus: the corresponding fine-tuning corpus. Δ, Δ_{pure}: absolute increase in accuracy after fine-tuning.

RQ3: Fine-tuning on math/code II

- fine-tuning on code sometimes leads to a - decline in performance,
- fine-tuning on math consistently results in performance + gains,
- fine-tuning on code exhibit smaller or negative improvements on TF-Bench$_{\text{pure}}$, i.e. $\text{RE} < 0$,
- the *same models* fine-tuned on math demonstrate greater improvements on TF-Bench$_{\text{pure}}$, i.e. $\eta > 1$, although not as significant as TTC.

Observation: Fine-tuning on math might enhance the models' reasoning ability, which also translates effectively tasks related to code.

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Background

Design of TF-Bench

Experimental results

References

Backup slides

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Formal reasoning in deductive systems

Definition

A deductive system (or inference system) is specified by [2]

- a collection of judgments/assertions/validations,
- a collection of steps (inference rules) that move from validation to validation, and finally to the proposition.

Natural deduction is a deductive system that *reason from assumptions*.

[2]This is not a completely standard definition, but is an illustrative description.

Effects of different rewrite operators

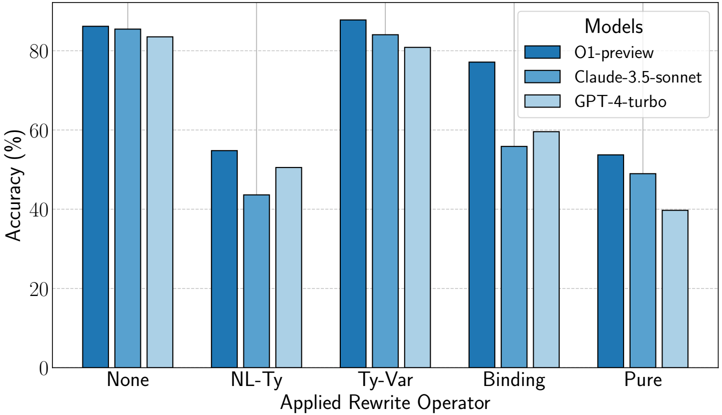

Figure 6: Accuracy on TF-Bench with different rewrite operators. None: the original TF-Bench. NL-Ty: rewriting NL types. Ty-Var: rewriting type variables. Binding: rewriting binding names. Pure: TF-Bench$_{\text{pure}}$.