Using Stochastic Collocation for Generalized Polynomial Chaos in RAVEN

RAVEN Training

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Idaho National Laboratory

Nomenclature

- $u(Y)$ is the response from a model (e.g., PCT)
- Uncertain inputs to model: $Y = (y_1, \ldots, y_n, \ldots, y_N)$
  - Material Properties
  - Boundary Conditions
  - Initial Conditions
- Goal: What range of values can $u(Y)$ have as a result of uncertainties in Y?

Stochastic Polynomials

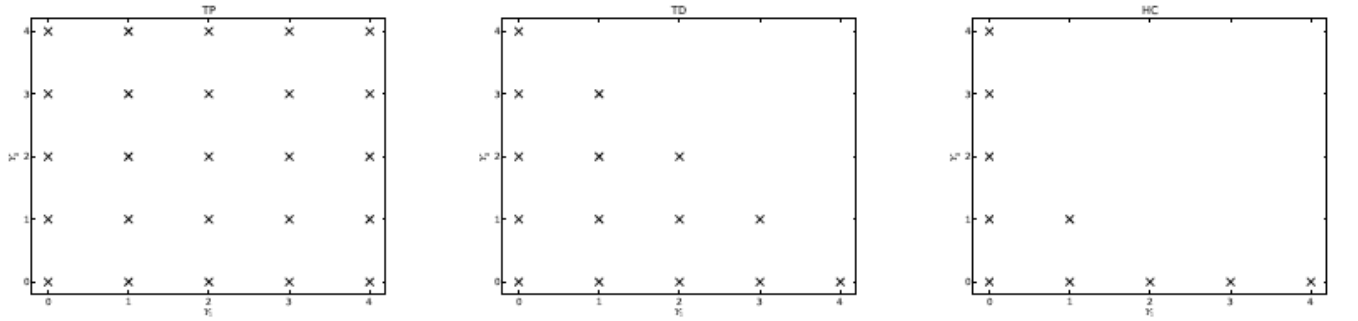
- Expand model as combination of polynomials

- Polynomial Representation
  - Given a quantity of interest: $u(Y)$
  - Represent as combination of polynomials: $\phi_k(Y)$
    - Simpler to evaluate
    - Easy to get statistical moments
    - Less effort and more accurate than Monte Carlo

$$u(Y) \approx \sum_{k \in \Lambda} c_k \Phi_k(Y)$$

Stochastic Collocation

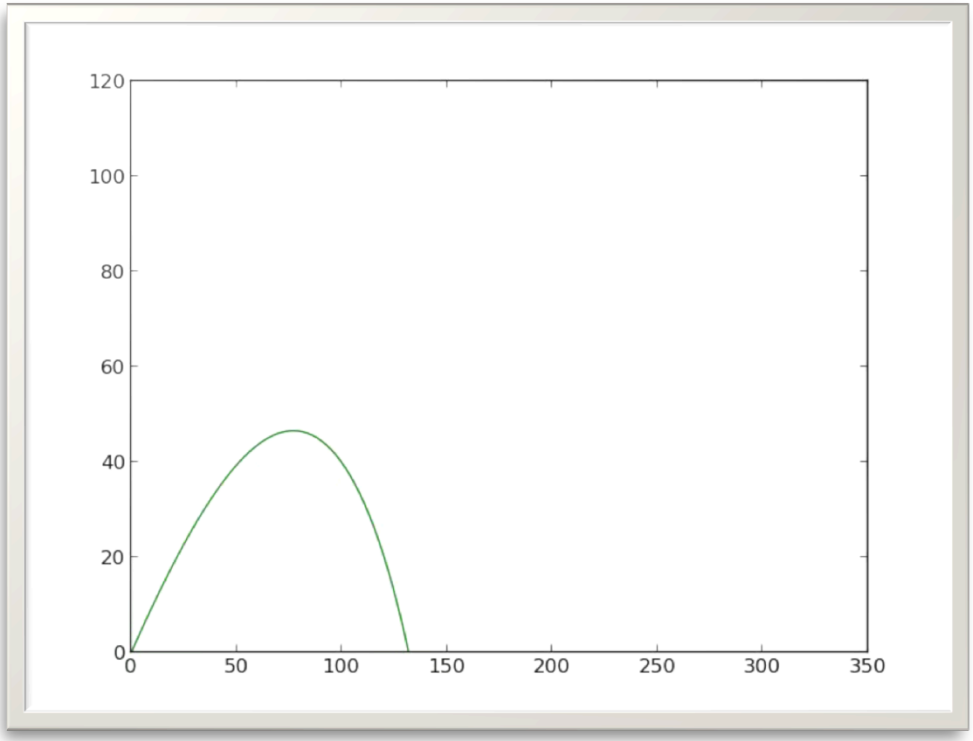
- Effectiveness depends on:
  - Regularity of quantity of interest $u(Y)$
  - Polynomial expansion order L
  - Polynomial combination indices $\Lambda(L)$
  - Sparse Grid quadrature types (Gauss, Clenshaw)
  - Number of uncertain inputs $N = |Y|$
- Tensor Product: Overkill
- Total Degree: For Analytic Responses
- Hyperbolic Cross: For Irregular Responses



(a) Tensor Product (b) Total Degree (c) Hyperbolic Cross

Model: Ballistic Motion

- Quantity of Interest
  - Range of Projectile
- Variables
  - Initial launch angle
    - Low: 25 degrees
    - High: 65 degrees
  - Initial Velocity
    - Low: 15 m/s
    - High: 45 m/s



Three Examples

- Total Degree index set
- First, Second, Third order polynomials

1. Sample points for ROMs
2. Train ROMs
3. Write ROMs to file

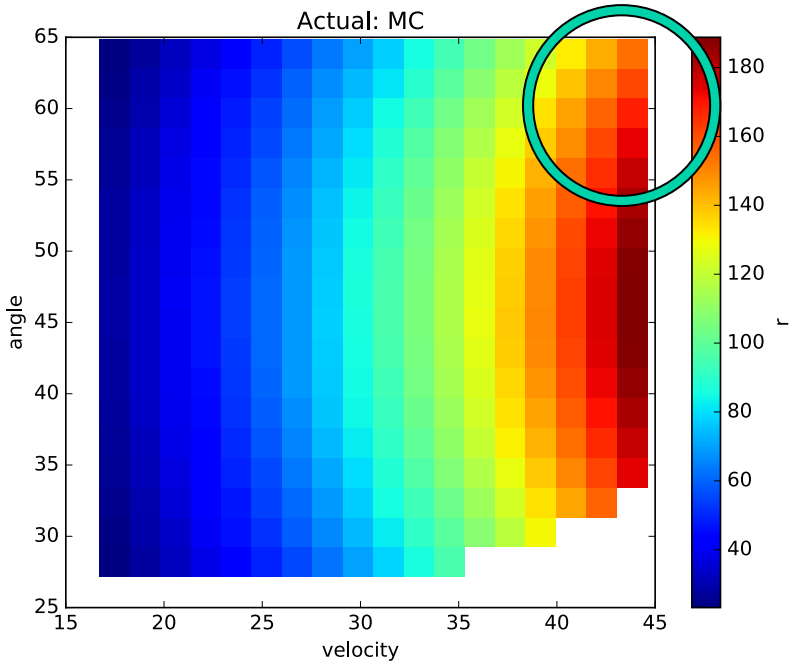
- See “scgpc_trainer_1.xml” (and 2, and 3)

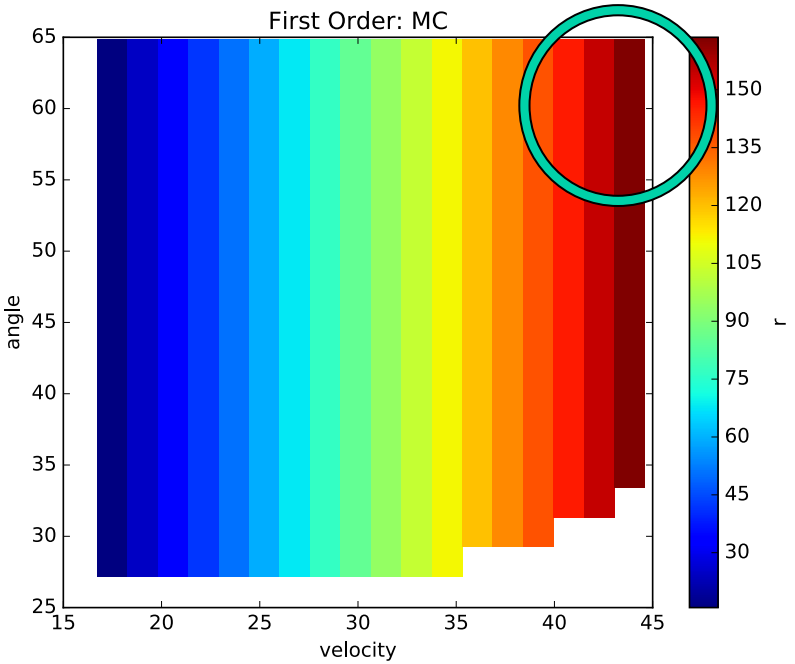
Validate and Compare

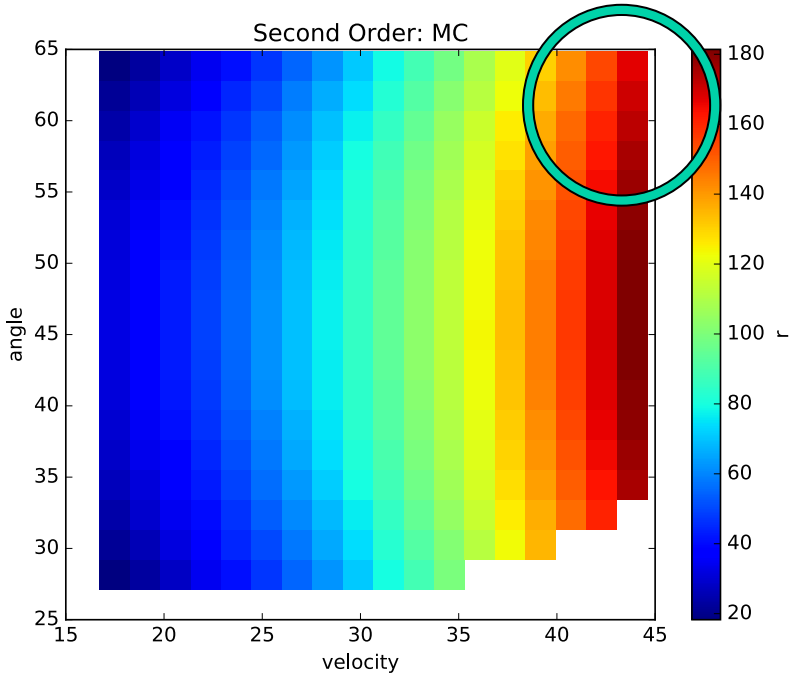
- Check validity of ROMs against original model

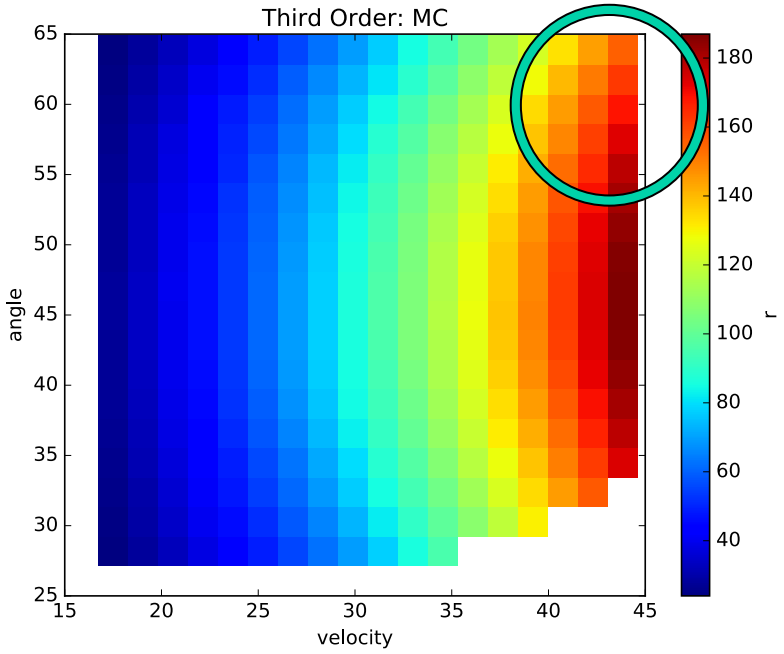
1. Sample original model (MC, Grid, LHS)
2. Load ROMs from file
3. Sample ROMs with same samplers
4. Plot results

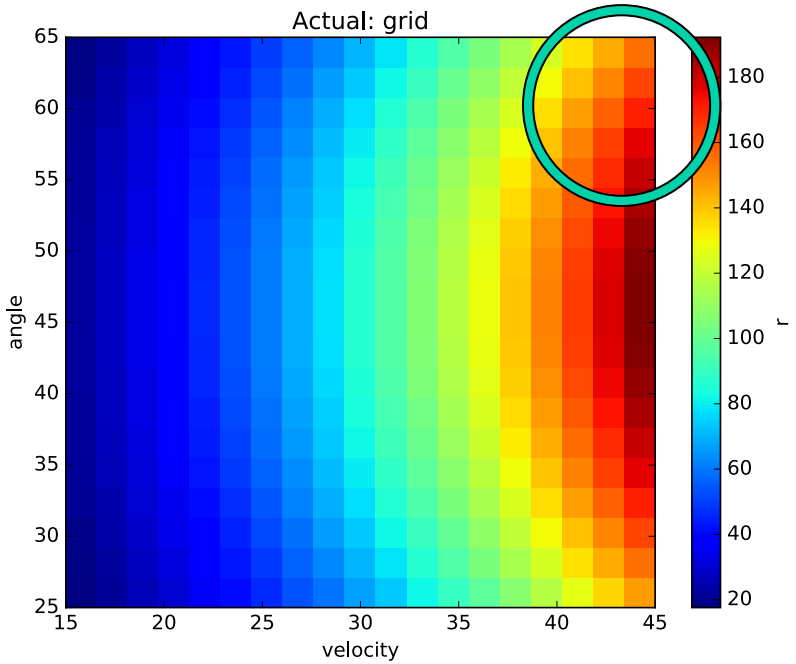
- See “mc.xml”, “grid.xml”, “lhs.xml”

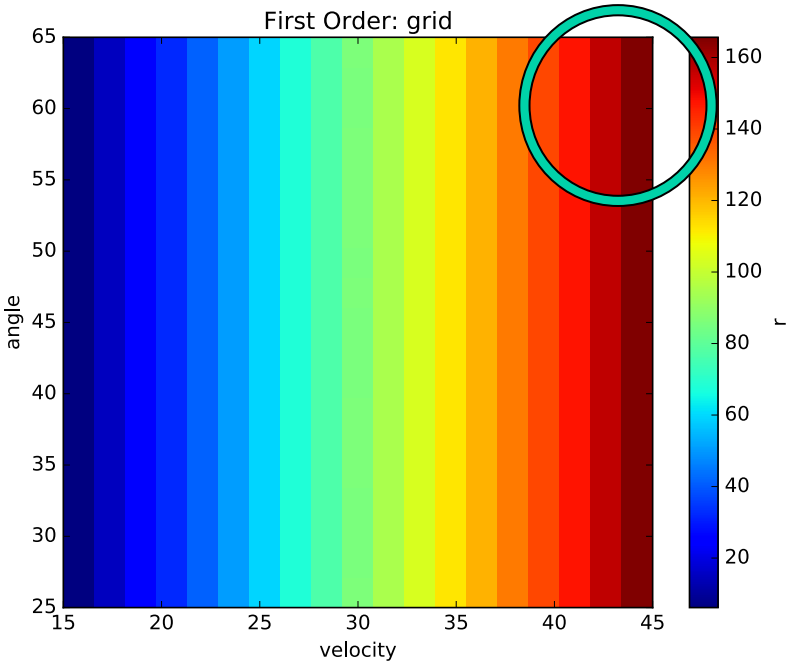

Actual: MC
angle
velocity
r

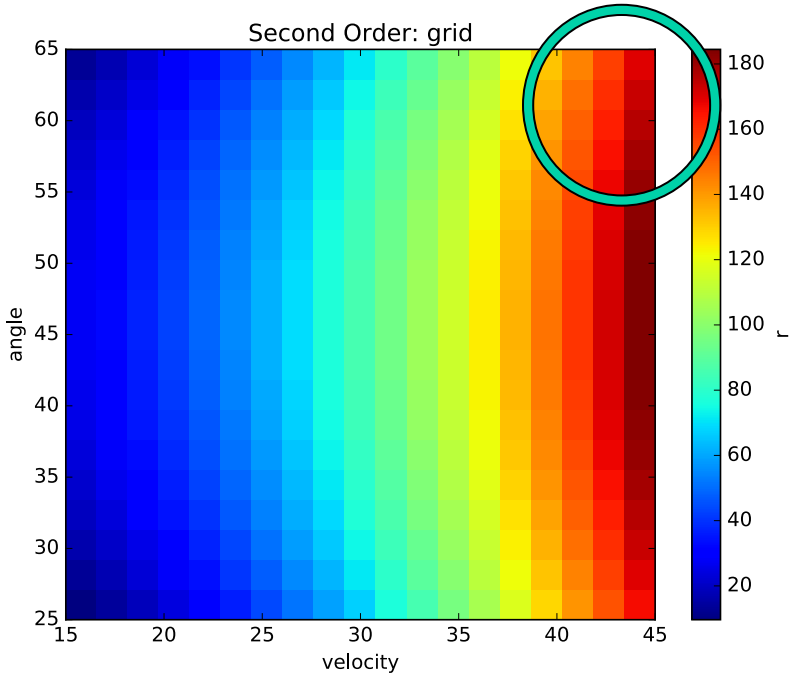

First Order: MC
angle
velocity
r

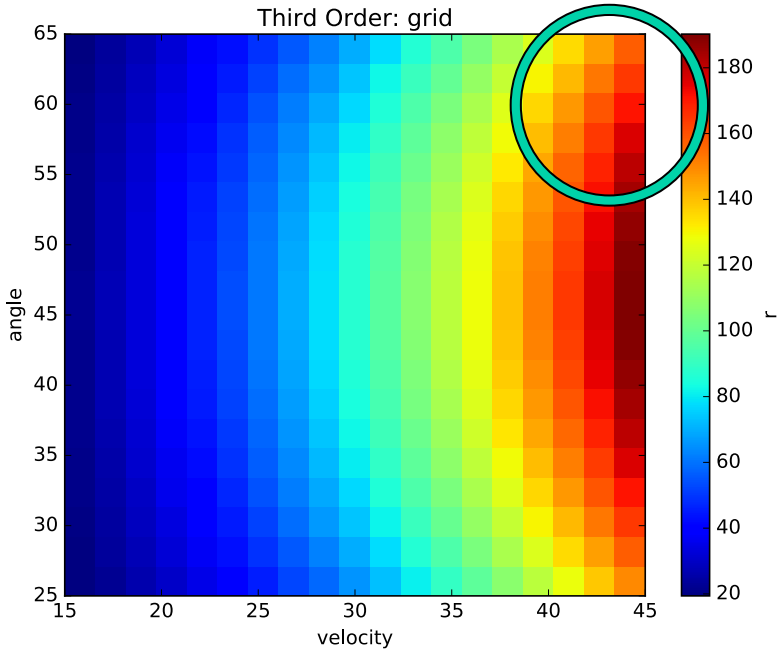

Second Order: MC
angle
velocity
r

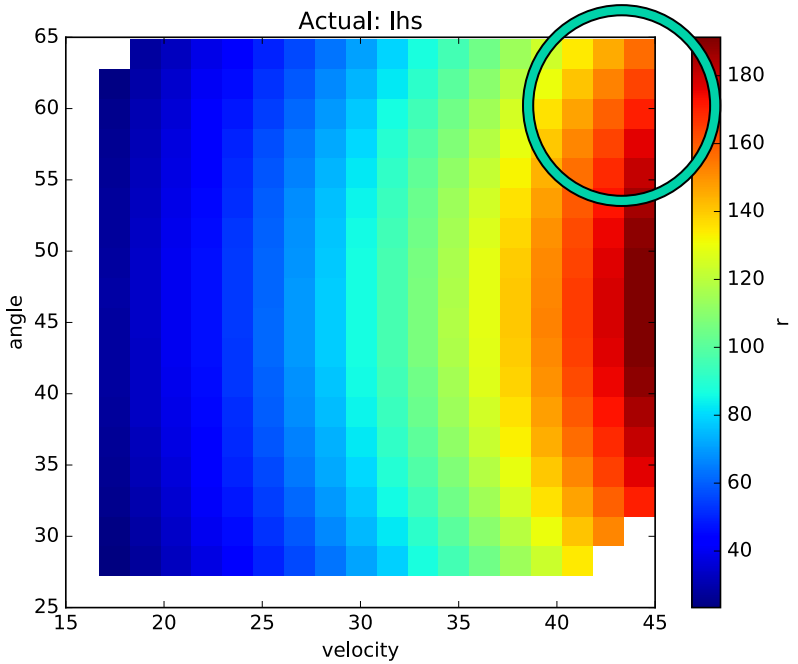

Third Order: MC
angle
velocity
r

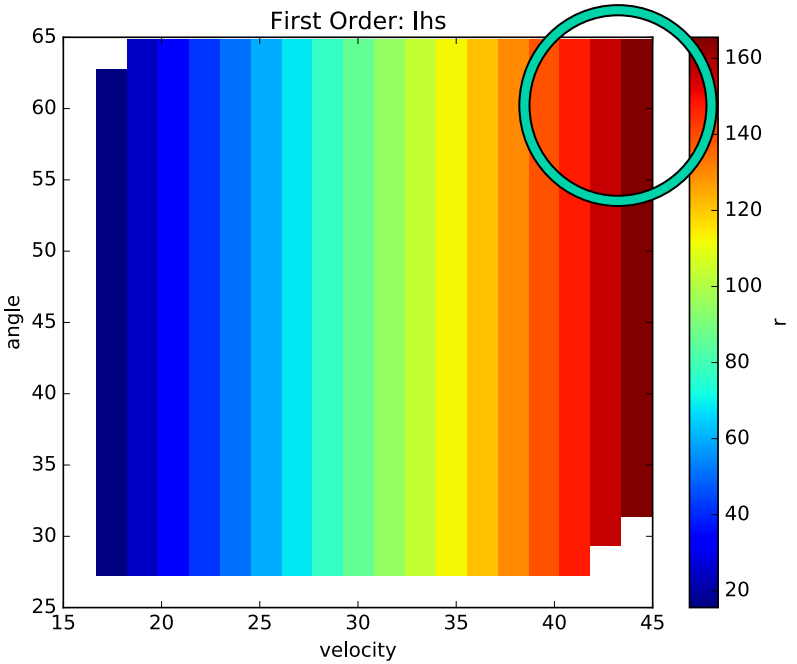

Actual: grid
angle
velocity
r

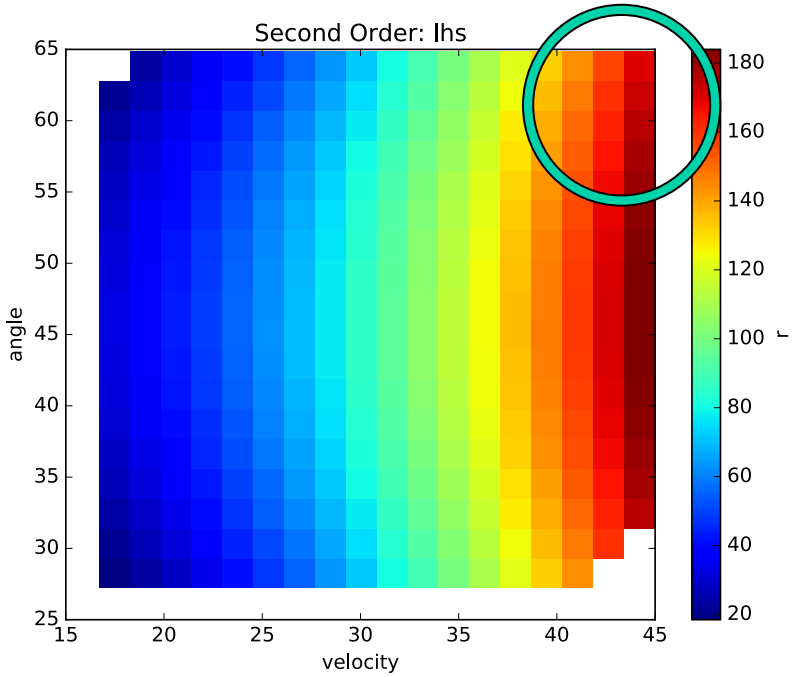

First Order: grid
angle
velocity
r

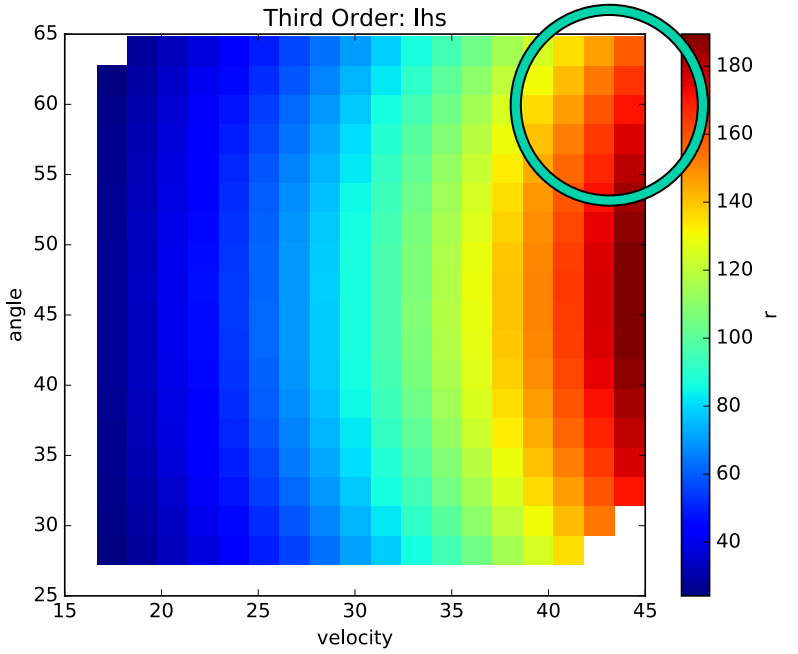

Second Order: grid
angle
velocity
r


Third Order: grid
angle
velocity
r


Actual: lhs
angle
velocity
r


First Order: lhs
angle
velocity
r


Second Order: lhs
angle
velocity
r


Third Order: lhs
angle
velocity
r

Conclusions

- Polynomial ROMs
  - *Cheap to create*
  - *Fast to evaluate*
  - *Accurate*

- Exceptions
  - *High dimension input space*
  - *Irregular response*