

Question 4

4.1

Ask the question:

“What is the maximum value amongst $A[1], A[2], \dots, A[n-1], A[n]$?”

This will obtain the value of the largest element in the entire array.

4.2

Ask the questions:

“What is the maximum value amongst $A[1], A[2], \dots, A[i-2], A[i-1]$?”

“What is the maximum value amongst $A[i+1], A[i+2], \dots, A[n-1], A[n]$?”

Then choose whichever one is greatest.

4.3

First ask the question from 4.1 to obtain the value of the largest element in the array.

Then ask:

“What is the maximum value amongst $A[1], A[2], \dots, A[n/2 - 1], A[n/2]$?”

If the maximum value is in this first half, recursively ask the same question on this half.

If the maximum value is different to the largest element in the array, recursively ask the same question on the other half.

Since we are halving n at each step, it will take $O(\log n)$ time to find the exact index of the largest element in the array.

Now ask the questions from 4.2 and you will have found the second largest element in the array in $O(\log n)$ time.

4.4

In the algorithm in 4.3, the difference between ‘ l ’ and ‘ r ’ for each recursion will always roughly halve in size, as we are decreasing the problem array size by $n/2$ at each step. Therefore, the algorithm outlined in 4.3 will satisfy the requirements given in 4.4 as the value of “ $r - l$ ” is always decreasing.