

### Question 3

3.1, 3.2, 3.3

Add each vertex's starting cost and finishing cost together in a separate array taking  $O(n)$  time. Find the minimum of this array taking  $O(n)$  time. This is likely the smallest score of any path in the graph.

We will use the Dijkstra's algorithm using an augmented heap outlined in the lecture, with a few minor changes:

- In the beginning,  $d_s = s_s + f_s$  and is added to a separate array in a tuple with 's' as the starting and finishing vertex
- Instead of deleting the distance from the min-heap, we put it into the separate array in a tuple along with start and end vertices.
  - o When we do this, we will also add the finishing cost of the end vertex.
- will run it on each vertex in the given graph

We search the  $n^2$  size list for the smallest distance, taking  $O(n^2)$  time.

Since the algorithm runs in  $O(m \log n)$  time on  $n$  vertices, the algorithm's time will be dominated by  $O(mn \log n)$  time. Since  $m \leq (n-1)$  as the graph is simple, this can be rewritten as  $O(m^2 \log n)$ .

Now,  $O((n+m)^2 \log(n+m))$  is equivalent to  $O(m^2 \log(n+m))$  for the same reason and the log can be simplified to  $O(m^2 \log n)$ .