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Question 4

4.1

Go through every player and calculate S, taking O(n) total time.

Since all s_i are either 0 or 1, S will equal the number of players that have a 1 as a rating and 2n - S will equal the number of players that have a 0 as a rating. Once we sort players by order of skill taking O(nlogn), we will know the indexes of where the 0-rated players start and the 1-rated players start.

Begin matching 0-rated players with 1-rated players until m is reached, keeping track of how many pairs have been matched, taking at most O(n) time, since at most there will be n pairs formed.

If m is not reached and there are only 1-rated or only 0-rated players remaining, it is not possible, as further pairings will not increase total imbalance, making m unreachable.

Once m is reached:

If the number of remaining 0-rated players is odd and 1-rated players is odd, it is not possible. This is because additional imbalances must be made in further pairings as at least 1 0-rated player must be matched with a 1-rated player, making total imbalance equal at least m + 1.

If the number of remaining 0-rated players is even and 1-rated players is even, it is possible, as further pairings can keep total imbalances at m, if each 0-rated player is matched with another 0-rated player with the same for 1-rated players.

Time complexity is O(nlogn) as sorting dominates all other steps.

4.2

<u>Subproblem:</u>

$0 \leq i,j \leq 2n$

$0 \leq T \leq S$

$s_i + s_{i-1} \leq 2s_i$

<u>Recurrence:</u>

$m(i) = m(i - 2) + |s_{i-1} - s_i|$

1. Both people play each other
2. Each person plays another person and those displaced people play each other
3. Each person plays

Algorithm relies on 3 things, necessitating a 3d array to store values.

Sum of all $2*\max(s_i, s_j)$ must be less than T, where $1 \leq T \leq S$.

Two cases:

1. If m is reached and there are pairs still left, the pairings thereafter must have a difference of 0
2. If m is not reached and there are no pairs left, discard the current pairings and begin again

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Base case:

If T = 0, i = 0, j = 0

Order of computation:

Final answer:

Time Complexity:

Time is in $O(n^2S)$ 2n things iterated over 2n times S times.