Mathematics
UNIVERSITY OF TORONTO

High School Advanced Math Club 2023: Mathematical Modelling in Science
Dynamical Systems in Action

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1 Course Description

In this course, we'll study connections between concrete scientific problems and dynamical systems, a subfield of mathematics dedicated to understanding oscillations, chaos, and other complex behaviours associated to quantities that change over time according to simple prescribed rules. As a group, we will use ideas from dynamical systems theory to carefully understand the scientific merit of mathematical models. We'll also examine basic strategies from control theory for stabilizing dynamical systems that model rudimentary mechanical devices. Time permitting, we'll investigate the use of basic probabilistic Markov chain models for predicting certain natural outcomes and explore computer simulations of the models. A background in coding is not required (though it is strongly encouraged!).

Note: This course is open to Grade 9 - 12 students, but the class will be taught at a Grade 10 - 12 level. Students in Grade 9 with a strong interest and background in mathematics may join this course.

2 Course Goals

By the end of the course, students will...

- have a basic familiarity with mathematical modelling, the process by which applied mathematicians translate real-life information into mathematics problems;
- understand both the power and the limitations of mathematical models (what you can *predict*, you can *control*);
- see how computer simulations can be used to gain insight into mathematical models;
- understand the concept of a discrete-time dynamical system, and be able to explain why dynamical systems provide useful mathematical models of various real-life problems;

- be able to discuss deeper concepts in dynamical systems such as equilibria, stability, periodicity, and bifurcations;
- develop a heuristic notion of chaotic behaviour in dynamical systems, and provide examples of chaos in biological and physical problems;
- learn how to control a dynamical system's behaviour to meet some desired goal via feedback.

3 Meeting Information

Classes will be held in-person.

- **Meeting Time:** Saturdays 10:00 AM - 1:00 PM EST from November 4 to December 2 (inclusive). I'll stay around until 1:15 PM if you have additional questions.
- **Meeting Locations:** Room locations change week to week, and will be provided in the first meeting.

4 Reference Material + Course Website

There are no required textbooks for this course. However, I'll provide citations to all the references used to prepare my lectures, so you can dive further into a topic if you wish.

Code demonstrations will be posted on my Github page in the "AdvMathClub2023" repository linked here: https://github.com/ageorgemorgan/AdvMathClub2023 (click in the PDF version). These code demos will be integrated into Jupyter notebooks, so they also include lecture notes! Occasionally, additional lecture notes may also be posted on Github.

5 Tentative Schedule of Lectures

- November 4: Introduction & overview: why bother with mathematical modelling? Modelling the growth of a single species [5, 6, 9, 12, 16]. Introduction to dynamical systems theory [5, 6, 13].
- November 11: More dynamical systems theory [2, 5, 6, 13]. Bifurcations and chaos in the discrete logistic equation [2, 6, 14].
- November 18: Models of interacting populations: host-parasitoid dynamics and intraspecies predation [3, 5, 11]. Observing chaos in the laboratory [1–4].
- November 25: Dynamics on the circle and chronobiology [10, 15].

- December 2: An introduction to control theory. Feedback control of biological and mechancial systems. [7, 8, 17].

6 Classroom Expectations

Please familiarize yourself with the student code of conduct, available by clicking the link here: https://www.mathematics.utoronto.ca/Outreach/Student-Code-of-Conduct (click in the PDF version). Some particularly important guidelines for our course:

- During classroom discussions or problem-solving sessions, please engage with your classmates in good faith: never be cruel, and make sure to voice your disagreements respectfully.
- While it's 100% OK to interrupt the instructor, please don't interrupt other students when they are asking a question or explaining something. If you have a question while another student is speaking, please raise your hand.
- If you have concerns of any nature about the course, you may email your instructor (adam.morgan@mail.utoronto.ca) or the program administrator
  Kyle Tran (outreach@math.toronto.edu) and we will do our best to take care of your issues quickly.

References

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