2. The Tsai-Wu tensor polynomial theory for orthotropic materials is specialized to the expression given in Table 4-3 for orthotropic laminae under a condition of plane stress. The coefficient F_{12} is experimentally determined from test specimens under biaxial loading. This inconvenience along with the constraint that F_{12} satisfy a stability criterion of the form

   $$\left(\frac{1}{x_t x_c}\right)\left(\frac{1}{y_t y_c}\right) - F_{12}^2 > 0$$

   creates some complication in the use of this theory. Narayanaswami and Adelman (see Reference 1) have suggested that F_{12} be set to zero and the use of Hoffman's Theory or the Tsai-Wu theory with $F_{12} = 0$ are preferred alternatives. The analyst may provide a value of f_{12} to be used in the Tsai-Wu failure index if he wishes.

Hill's Theory (Failure Theory ID = "HILL")

$$\text{Failure Index} = \frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2}$$

where:

X	=is allowable stress in 1-direction
Y	=is allowable stress in 2-direction
S	=is allowable stress in shear

and $X = X_t$ if σ_1 is positive or $X = X_c$ if σ_1 is negative and similarly for Y and σ_2. For the interaction term $\sigma_1\sigma_2/X^2$, $X = X_t$ if $\sigma_1\sigma_2$ is positive or $X = X_c$ if $\sigma_1\sigma_2$ is negative.

Note that Hill's theory is used for materials that have the same strength in tension and compression.

A plot of the above equation obtained by setting the failure index to 1 on the σ_1-σ_2 plane yields an ellipse and is the anisotropic yield criterion of Hill (modified later by Tsai, and hence also sometimes known as the Tsai-Hill theory). Therefore, if the failure index so calculated is less than 1, the ply stresses are inside the yield ellipse and the ply is said to be "safe"; conversely, if the failure index is greater than 1, the ply stresses are outside the yield ellipse and the ply has failed.

Hoffman's Theory (Failure Theory ID = "HOFF")

The Hoffman's theory for an orthotropic lamina in a general state of plane stress is given by

$$\left(\frac{1}{X_t} - \frac{1}{X_c}\right)\sigma_1 + \left(\frac{1}{Y_t} - \frac{1}{Y_c}\right)\sigma_2 + \frac{\sigma_1^2}{X_t X_c} + \frac{\sigma_2^2}{Y_t Y_c} + \frac{\sigma_{12}^2}{S^2} - \frac{\sigma_1\sigma_2}{X_t X_c} = 1$$

The failure index is obtained by evaluating the left-hand side of the above equation.

Note that this theory takes into account the difference in tensile and compressive allowable stresses by using linear terms in the equation.

Tensor Polynomial Theory of Tsai-Wu (Failure Theory ID = "TSAI")

The theory of strength for anisotropic materials proposed by Tsai and Wu specialized to the case of an orthotropic lamina in a general state of plane stress is