1 Forward propagation

第 l 层的正向函数:

cost function:

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} L(\hat{y}^{(1)}, y^{(1)}) \tag{1}$$

$$\begin{aligned} z_1^{[i]} = w_1^{[i]}x + b_1^{[i]}, a_1^{[i]} = g_1^{[i]}(z_1^{[i]}) \\ z_2^{[i]} = w_2^{[i]}x + b_2^{[i]}, a_2^{[i]} = g_1^{[i]}(z_2^{[i]}) \\ z_3^{[i]} = w_3^{[i]}x + b_3^{[i]}, a_3^{[i]} = g_3^{[i]}(z_3^{[i]}) \end{aligned} \tag{2}$$

After vectorization:

$$[z^{[i]}] = \begin{bmatrix} w_1^{[i]} \\ w_2^{[i]} \\ w_3^{[i]} \end{bmatrix} X + \begin{bmatrix} b_1^{[i]} \\ b_2^{[i]} \\ b_3^{[i]} \end{bmatrix} = W^{[i]}X + b^{[i]} \tag{3}$$

$$[A^{[i]}] = \begin{bmatrix} g_1^{[i]}(z_1^{[i]}) \\ g_2^{[i]}(z_1^{[i]}) \\ g_3^{[i]}(z_1^{[i]}) \end{bmatrix} \tag{4}$$

2 Backward propagation

反向是对$J(w,b)$，中的w,b进行反向

$$\begin{aligned} dz_1^{[i]} = da_1^{[i]'} \times g_1^{[i]'}(z_1^{[i]}) \\ dz_2^{[i]} = da_2^{[i]'} \times g_2^{[i]'}(z_2^{[i]}) \\ dz_3^{[i]} = da_3^{[i]'} \times g_3^{[i]'}(z_3^{[i]}) \end{aligned} \tag{5}$$

$$dZ^{[i]} = \sum_{k=1}^{m} dz_k^{[i]} = dA^{[i]} \times g^{[i]'}(Z^{[i]}) \tag{6}$$

$$\begin{aligned} dw_1^{[i]} = dz_1^{[i]'} \times a_1^{[i-1]} \\ dw_2^{[i]} = dz_2^{[i]'} \times a_2^{[i-1]} \\ dw_3^{[i]} = dz_3^{[i]'} \times a_3^{[i-1]} \end{aligned} \tag{7}$$

$$
\begin{aligned}
dW^{[i]} &= \frac{1}{m}\sum \frac{\partial J}{\partial z}\frac{\partial z}{\partial w} \\
&= \frac{1}{m}\sum dz^{[i]}a^{[i-1]} \\
&= \frac{1}{m}dZ^{[i]}dA^{[i-1]}
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
db_1^{[i]} &= dz_1^{[i]} \\
db_2^{[i]} &= dz_2^{[i]} \\
db_3^{[i]} &= dz_3^{[i]}
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
db^{[i]} &= \frac{1}{m}\sum \frac{\partial J}{\partial z}\frac{\partial z}{\partial b} \\
&= \frac{1}{m}\sum dz^{[i]} \\
&= \frac{1}{m}np.sum(dZ^{[i]}, axis = 1, keepdims = True)
\end{aligned} \tag{10}
$$

$$
[dA^{[i]}] = \begin{bmatrix} g_1^{[i]'}(z_1^{[i]}) \\ g_2^{[i]'}(z_1^{[i]}) \\ g_3^{[i]'}(z_1^{[i]}) \end{bmatrix} \tag{11}
$$

是对矩阵中的各个元素求导**注意**