EERF 6351 FDTD Project: FINITE-DIFFERENCE TIME-DOMAIN

Objectives:

- Understand and program the FDTD equations in 1D
- Observe CW and Pulsed time domain data
- Observe numerical dispersion
- Understand and program the Mur 1st order absorbing boundary conditions
- Understand the relationship between time domain and frequency domain data and use this to calculate reflection coefficient

1. Differential Equations (3D to 1D)

Starting with Maxwell's equations in the time domain (Ampere's and Faraday's laws), differential form, write the 6 coupled differential equations. (Take the cross products and equate vector components.)

Convert these equations to the 1-dimensional TE-to-z case by setting d/dy = d/dz = 0 and Ez = 0. This represents a plane wave propagating in the x-direction. You should end up with equations for Ey and Hz. (The TM-to-z case would have similar equations for Ez and Hy.)

2. FDTD Equations (1D TE-to-z case)

Convert the 1D TE differential equations above to their FDTD difference form. (Use the central difference formula to approximate the derivatives, and solve for Ey ^ (n+1) and Hz ^ (n+1/2).)

Use the 1D FDTD lattice shown below:

```
                      |←----cell I----→|
                      |←----dx-------→|
-------+---------o---------+---------o---------+---------o----------+---------
  Ey(I-1)  Hz(I-1)  Ey(I)   Hz(I)   Ey(I+1)  Hz(I+1)  Ey(I+2) -→I
```

Let the E fields be defined at times n, n+1, n-1, etc.
Let the H fields be defined at times n-1/2, n+1/2, etc.

3. Program the FDTD Equations (1D TE case)

Program the equations in (2) for the geometry shown below. Use a forced CW source on Ey: Ey(I=inc) = sin(ωt). Note: although all the test cases here are in air, write your code for arbitrary materials.

```
|------------------------------|------------------------------|
0                          Ey(inc)                           nx
```

4. Test the FDTD Equations and observe CW Time Domain Data:

Use F = 2GHz, dx = wavelength/20, dt = dx/(2c), nx=120, inc=60.

Plot the Ey and Hz fields at points A,B,C,D as a function of time for 100 time steps. (Note that this stops the simulation just before the wave touches the lack of boundary conditions at the ends.) Give one plot of the four E fields, and another of the four H fields. Store the E fields at point C for use in problem 7.

Plot the Ey field at point D against the analytical value: $Ey(x) = \sin(\omega t-\beta x)$, where x is the distance from the source.

```
                      A  B  C  D
                     Ey(inc)
|---------------------|-------------------------|
0                    inc                        nx
                     60                        120
```

A is located at I=60, at source
B is located at I=63, 3 cells from source
C is located at I=67, 7 cells from source
D is located at I=90, 30 cells from source

5. Observe Pulsed Time Domain Data

Change the source to a raised cosine pulse:

$$Ey(inc) = \begin{cases} 1-\cos(\omega t) & o<t<1/Fmax \\ 0 & t>1/Fmax \end{cases}$$

Use Fmax = 2GHz, dx = wavelength/20, dt=dx/(2c), nx=120, inc=60.

Plot the Ey fields at points A,B,C,D as a function of time for 100 time steps. (Notes: If you run more than 120 time steps you will see the waves reflect off the ends of the FDTD mesh.)

6. Observe Numerical Dispersion

Use the raised cosine pulsed source, Fmax = 2GHz, dt=dx/(2c), nx=220, inc=110. Run for 200 time steps.

Plot the Ey fields as a function of time 30 cells from the source for dx = wavelength/60, wavelength/20, wavelenth/10, and wavelength/5.

Plot the Ey fields 30 cells from the source for the CW source using dx=wavelength/5 and compared to the values observed at point D in part 4.

7. Mur 1st order boundary conditions

a) Make sure you understand the derivation from class: What approximation is made to make the boundary conditions "first order"?

b) Write the difference form of the 1st order boundary conditions for Ey.

c) Program the 1st order boundary conditions for Ey on both boundaries.

d) Test the boundary conditions:

Incident Fields: Use the Ey fields you stored 7 cells from the source in problem 4 as the "incident" fields. There is no reflection in these fields, because the waves have not yet hit the boundary.

Total Fields: Rerun your simulation using the same parameters as problem 4 except nx=30, inc=15, for 100 time steps. Store the Ey fields 7 cells from the source. These are the "total" fields, because they contain the incident field plus any fields which might be reflected from the boundaries.

Reflected Fields: = Total fields - Incident Fields

Plot the reflected field as a function of time 7 cells from the source. They should be tiny! They should look like noise, possibly increasing slightly with time.

e) Run the simulation with boundary conditions for 10,000 time steps to be sure your simulation does not blow up. Plot the last 3 cycles of the Ey fields 7 cells from the source. Does it still look like a clean sine wave?

8. Added Source

Program the "added" source in your simulation and test to be sure that it works the same way as the "forced" source in air.

9. Numerical Stability

Test your CW simulations with several values of dt = dx / K to verify the stability criterion. For 1D you expect your simulations to become unstable when dt > dx / c

10. Standing Wave

Replace the Mur boundary condition at nx with a metal boundary :

|------|------------------------||

Mur Ey(5) = added source Ey(nx=120) = 0.

Metal boundary

a) Observe reflected fields:

Using a pulsed source with Fmax = 2GHz, dt = dx/(2c), dx = wavelength/20, plot the Ey fields as a function of time at I=40 and I=100. You should see the incident and reflected pulses clearly.

b) Verify Propagation Time:

How many time steps will it take for your wave to propagate to the conductor and back to the source using dt=dx/(2c)?______________ using dt = dx/3c?______________

Verify this using a pulsed source with Fmax = 2GHz, dx = wavelength/20, dt = dx/(2c) and dt = dx/3c

c) Observe and understand a standing wave:

Using a CW source with F = 2GHz, dt = dx/(2c), dx = wavelength/20, plot the Ey fields as a function of time at I=40 and I=100. Indicate the magnitudes (peak value) of these fields on your graph.

Now have your program find the magnitudes (peak values) of the fields at every point. Verify the values for I=40 and I=100, compared to those you observed by hand above. Plot the magnitudes of Ey as a function of I (at every point). You should see the classic 'standing wave" pattern.

Where is the first zero of the standing wave in front of the wall? ______

Where is the first peak? ____________

Plot Ey as a function of time at these points. This is the total field

Using the method in part 7, find and plot the incident and reflected fields at these two points as a function of time. Verify that the sum of the incident and reflected fields gives the total fields observed above.

BONUS) **Use your program to compute the reflection coefficient** from a quarter-wave dielectric transformer as a function of frequency. A quarter-wave dielectric transformer is a layer of material that can be used to "match" two different materials (such as air and fluid). The transformer has a characteristic impedance of $\eta_t = \text{sqrt}(\eta_1\eta_2)$. The transformer is a quarter of a wavelength (in the transformer material) long. Design a transformer to match air ($\varepsilon_r = 1.0$) and water ($\varepsilon_r = 40.0$) at 1 MHz. The reflection coefficient should be zero at 1 MHz. Evaluate the reflection coefficient from 0.5 to 2 MHz.

TURN IN:

[ ] Hard copy of your code

[ ] Derivations of all formulas used in your code (FDTD, boundary conditions, etc.)

[ ] Plots

[ ] Answers

[ ] Summarize and comment on your results

Parts 1–6 due 3/16

Parts 7–10 due 3/30