CHAPTER 19

MENSURATION

BASIC CONVERSION OF UNITS

(i) Length:

1 m = 10 dm = 100 cm = 1000 mm
1 dm = 10 cm = 100 mm
1 cm = 10 mm
1 feet (ft) = 12 inches
1 inch = 2.54 cm
1 yard (y) = 3 feet (ft)
1 m = 1.094 yard (y) = 39.37 inches
1 yard (y) = 0.914 metre (m)
1 km = 1000 m = $\frac{5}{8}$ miles
1 mile = 1760 yards (y) = 5280 feet (ft)
1 nautical mile (knot) = 6080 feet (ft)

(ii) Surface Area:

Surface areas are measured in square units.
1 square metre = 1m × 1m = 100 cm × 100 cm = 10000 cm^2
1 square yard = 1y × 1y = 3 ft × 3 ft = 9 ft^2
1 acre = 4047 m^2 (approx.)
1 hectare = 10000 m^2

(iii) Mass:

1 kg = 1000 grams (g) = 2.2 pounds (approx.)
1 gram = 10 miligram (mg)
1 quintal = 100 kg
1 tonne = 10 quintal = 1000 kg

(iv) Volume:

Volumes are measured in cubic units.
1 litre = 1000 cm^3 or cc
1 m^3 = 10000 litres (= 10^4 l) = 10^7 cm^3

Note that

$\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$,

$\sqrt{6} = 2.45$, $\pi = \frac{22}{7}$ or 3.14

PLANE FIGURES

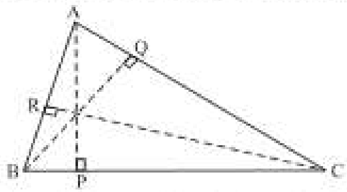
We have already dealt with plane figures (Triangles, Quadrilaterals and Circles) in geometry chapter. In this chapter, we will deal with perimeter and area of plane figures.

Perimeter: The perimeter of a plane geometrical figure is the total length of sides (or boundary) enclosing the figure. Units of measuring perimeter can be cm, m, km, etc.

Area: The area of any figure is the amount of surface enclosed within its bounding lines. Area is always expressed in square units.

AREA OF A TRIANGLE

1. If in a triangle, we draw a perpendicular AP from vertex A on opposite side BC then AP is called altitude (or height) of the triangle ABC corresponding to base BC.

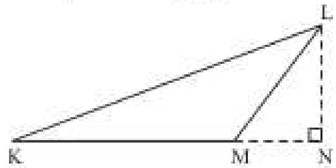


Similarly, BQ and CR are altitude of ΔABC corresponding to, bases AC and AB respectively.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{corresponding altitude}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AP = \frac{1}{2} \times AC \times BQ = \frac{1}{2} \times AB \times CR$$

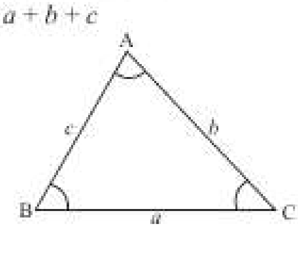
Note that in ΔKLM, LN is the perpendicular on KM produced.



Here, LN is the altitude corresponding to the base KM of ΔKLM.

$\therefore$ Area of $\Delta KLM = \frac{1}{2} \times KM \times LN$

2. Let in ΔABC, $BC = a$, $AC = b$ and $AB = c$; then perimeter of $\Delta ABC = a + b + c$



Semi-perimeter of $\Delta\, ABC$'s $= \dfrac{a+b+c}{2}$

Area of $\Delta\, ABC = \sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)

3. Area of $\Delta\, ABC = \dfrac{1}{2}$ × (Product of two sides) × (Sine of the included angle)

$$= \frac{1}{2}\, ac \sin B \text{ or } \frac{1}{2}\, ab \sin C \text{ or } \frac{1}{2}\, bc \sin A$$

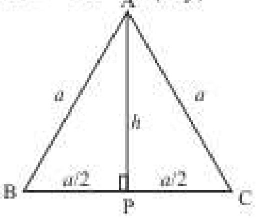
Note that $\sin 30^\circ = \dfrac{1}{2}$, $\sin 45^\circ = \dfrac{1}{\sqrt{2}}$,

$$\sin 60^\circ = \frac{\sqrt{3}}{2},\ \sin 90^\circ = 1$$

Area of an Equilateral Triangle

Since, $\Delta\, ABC$ is an equilateral triangle.

$\therefore \quad AB = BC = CA = a$ (say)



From $\Delta\, APC$,

$$AP^2 = AC^2 - PC^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$$AP = \frac{\sqrt{3}}{2}a \Rightarrow h = \frac{\sqrt{3}}{2}a$$

Area of an equilateral $\Delta = \dfrac{1}{2} \times a \times \dfrac{\sqrt{3}}{2}a = \dfrac{\sqrt{3}}{4}a^2$,

where a is the length of its one side

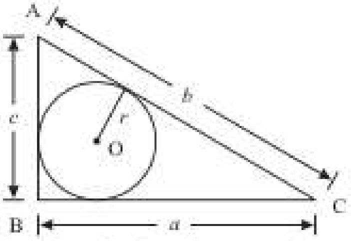
Note that

(i) among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.

(ii) For a given area of triangle, the perimeter of equilateral triangle is minimum.

Area of Incircle and Circumcircle of a Triangle

(i) If a circle touches all the three sides of a triangle, then it is called incircle of the triangle.



Area of incircle of a triangle = $r.\,s$, where r is the radius of the incircle and s is the half of the perimeter of the triangle.

If a, b, c are the length of the sides of $\Delta\, ABC$, then

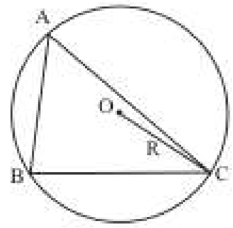
$$s = \frac{a+b+c}{2}$$

For an equilateral triangle,

$$r = \frac{\text{Length of a side of the triangle}}{2\sqrt{3}} = \frac{h}{3},$$

where h is the height of the triangle.

(ii) If a circle passess through the vertices of a triangle, then the circle is called circumcircle of the triangle.



Area of the circumcircle = $\dfrac{abc}{4R}$, where R is the radius of the circumcircle and a, b, c are the length of sides of the triangle.

For an equilateral triangle,

$$R = \frac{\text{Length of a side of the triangle}}{\sqrt{3}} = \frac{2h}{3},$$

where h is the height or altitude of the equilateral triangle.

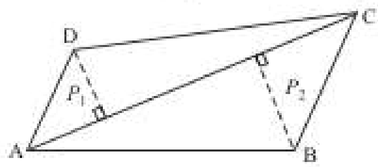
Hence for an equilateral triangle, $R = 2r$.

Note that an equilateral triangle inscribed in a circle will have the maximum area compared to other triangles inscribed in the same circle.

AREA OF A QUADRILATERAL

1. Area of quadrilateral $ABCD$

$= \dfrac{1}{2}$ × (Length of the longest diagonal) × (Sum of length of perpendicular to the longest diagonal from its opposite vertices)



$= \dfrac{1}{2} \times d \times (p_1 + p_2)$, where $d = AC$ (i.e. longest diagonal)

2. If length of four sides and one of its diagonals of quadrilateral $ABCD$ are given, then

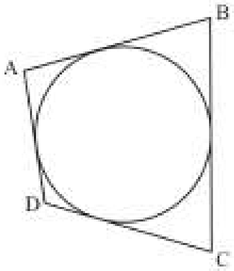
Area of the quadrilateral $ABCD$

= Area of $\Delta\, ABC$ + Area of $\Delta\, ADC$

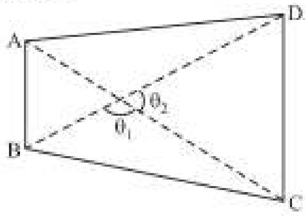
3. Area of circumscribed quadrilateral

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where $\quad s = \dfrac{a+b+c+d}{2}$ and a, b, c, d are length of sides of quadrilateral $ABCD$.



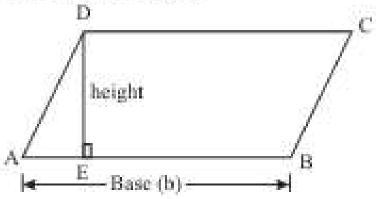
4. If θ_1 and θ_2 are the angles between the diagonals of a quadrilateral, then



Area of the quadrilateral = $\frac{1}{2}\, d_1 d_2 \sin\theta_1$ or $\frac{1}{2}\, d_1 d_2 \sin\theta_2$

Here d_1 and d_2 are the length of the diagonals of the quadrilateral.

Area of a Parallelogram



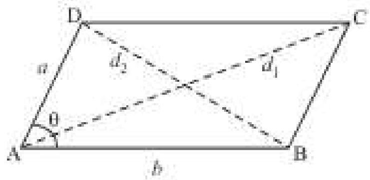
Area of parallelogram = Base × Corresponding height

$$A = b \times h$$

Perimeter of a parallelogram = $2(a + b)$, where a and b are length of adjacent sides.

If θ be the angle between any two adjacent sides of a parallelogram whose length are a and b, then

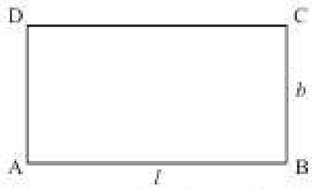
Area of parallelogram = $ab \sin\theta$



Note that in a parallelogram sum of squares of two diagonals = 2 (sum of squares of two adjacent sides)

i.e., $d_1^2 + d_2^2 = 2\,(a^2 + b^2)$

Area of a Rectangle

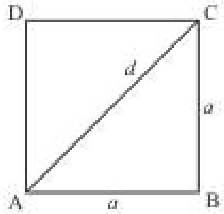


Area of a rectangle = Length × Breadth = $l \times b$

[If any one side and diagonal is given]

Perimeter of a rectangle = $2(l + b)$

Area of a Square



Area of square = side × side = $a \times a = a^2$

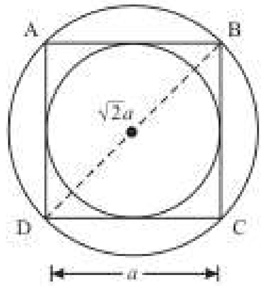
Length of diagonal $(d) = a\sqrt{2}$ (by Pythagoras theoram)

Hence area of the square = $\left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$

Perimeter of square = 4 × side = $4 \times a$

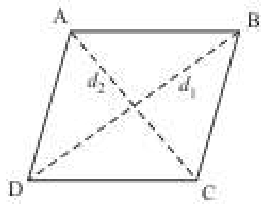
For a given perimeter of a rectangle, a square has maximum area.

Note that the side of a square is the diameter of the inscribed circle and diagonal of the square is the diameter of the circumscribing circle.



Hence inradius = $\frac{a}{2}$ and circumradius = $\frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

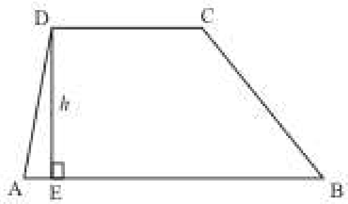
Area of a Rhombus



Area of a rhombus = $\frac{1}{2}$ × product of diagonals

$= \frac{1}{2} \times d_1 \times d_2$

Area of a Trapezium



Distance between parallel sides of a trapezium is called height of trapezium.

In fig. $ABCD$ is a trapezium, whose sides AB and CD are parallel,

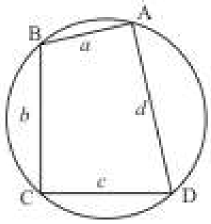
$DE = h$ = Height of the trapezium
= Distance between ∥ sides.

Area of trapezium = $\frac{1}{2}$ (sum of ∥ sides) × height

$= \frac{1}{2} \times (AB + CD) \times DE$

Area of a Cyclic Quadrilateral

For a given quadrilateral *ABCD* inscribed in a circle with sides measuring *a*, *b*, *c*, and *d*;



$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where $$s = \frac{a+b+c+d}{2}$$

Illustration 1: A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet?

(a) 46 **(b) 81**
(c) 126 **(d) 252**

Solution: (c) Clearly, we have : $l = 9$ and $l + 2b = 37$ or $b = 14$.
∴ Area = $(l \times b) = (9 \times 14)$ sq. ft. = 126 sq. ft.

Illustration 2: A square carpet with an area 169 m² must have 2 metres cut-off one of its edges in order to be a perfect fit for a rectangular room. What is the area of rectangular room?

(a) 180 m² **(b) 164 m²**
(c) 152 m² **(d) 143 m²**

Solution: (d) Side of square carpet $\sqrt{\text{Area}} = \sqrt{169} = 13$ m
After cutting of one side,
Measure of one side = 13 – 2 = 11 m
and other side = 13 m (remain same)
∴ Area of rectangular room = 13 × 11 = 143 m²

Illustration 3: The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km/hr completes one round in 8 minutes, then the area of the park (in sq. m) is:

(a) 15360 **(b) 153600**
(c) 30720 **(d) 307200**

Solution: (b) Perimeter = Distance covered in 8 min.

$$= \left(\frac{12000}{6} \times 8\right) \text{m} = 1600 \text{ m.}$$

Let length = $3x$ metres and breadth = $2x$ metres.
Then, $2(3x + 2x) = 1600$ or $x = 160$.
∴ Length = 480 m and Breadth = 320 m.
∴ Area = (480 × 320) m² = 153600 m².

Illustration 4: The length and breadth of a playground are 36m and 21 m respectively. Poles are required to be fixed all along the boundary at a distance 3m apart. The number of poles required will be

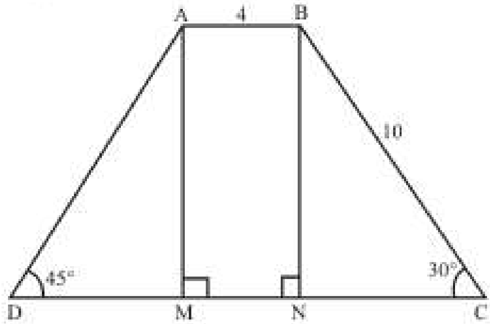
(a) 39 **(b) 38**
(c) 37 **(d) 40**

Solution: (b) Given, playground is rectangular.
Length = 36 m, Breadth = 21 m
Now, perimeter of playground = 2(21 + 36) = 114
Now, poles are fixed along the boundary at a distance 3 m.
∴ Required no. of poles = $\frac{114}{3} = 38$.

Illustration 5: Find the area of the trapezium *ABCD*.

(a) $5/2\,(13 + 2\sqrt{3})$ **(b)** $\frac{5\sqrt{3}\,(13 + 5\sqrt{3})}{2}$
(c) $13\,(13 + 2\sqrt{3})$ **(d) None of these**

Solution: (d)



AB and *DC* are the parallel sides
Height = *AM* = *BN*
AB = *MN* = 4
Δ*BNC* and Δ*AMD* are right angled triangles

In Δ*BNC* ⇒ $\sin 30 = \frac{BN}{10} \Rightarrow BN = 5$

Using Pythagoras theorem $NC = \sqrt{10^2 - 5^2} = 5\sqrt{3}$

In Δ*ADM*; *AM* = 5; $\tan 45 = \frac{AM}{DM} = 1 \frac{5}{DM}$

⇒ $DM = 5$

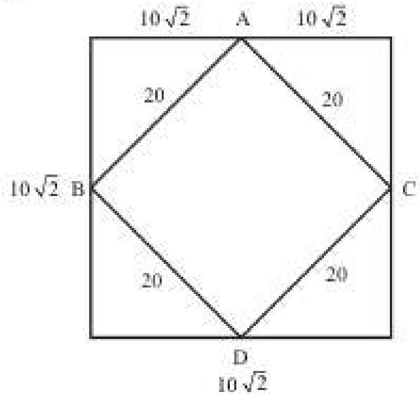
Area of trapezium ⇒ $\frac{1}{2}$ (Sum of parallel sides) × height

$$\Rightarrow \frac{1}{2}(4 + 4 + 5\sqrt{3} + 5) \times 5 = \frac{5(13 + 5\sqrt{3})}{2}$$

Illustration 6: Two goats tethered to diagonally opposite vertices of a field formed by joining the mid-points of the adjacent sides of another square field of side $20\sqrt{2}$. What is the total grazing area of the two goats?

(a) $10\,\pi\text{m}^2$ **(b)** $50(\sqrt{2} - 1)\,\pi\text{m}^2$
(c) $100\,\pi\,(3 - 2\sqrt{2})\text{m}^2$ **(d)** $200\,\pi\,(2 - \sqrt{2})\text{m}^2$

Solution: (a)



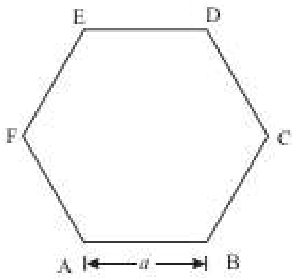
The length of rope of goat = $10\sqrt{2}$ m

Then the two goats will graze an area = Area of a semicircle with radius $10\sqrt{2}$ m.

So total area grazed = $\frac{\pi r^2}{2} \Rightarrow 100\ \pi\text{m}^2$

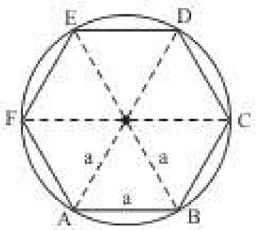
AREA OF A REGULAR HEXAGON

Area = $\frac{3\sqrt{3}}{2}a^2$, where 'a' is the length of each side of the regular hexagon.



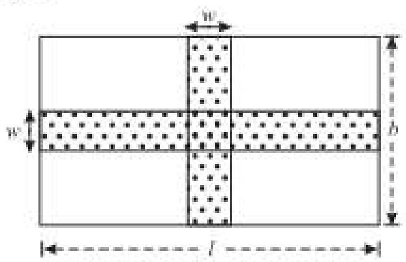
Diagonals of a hexagon divide it into six equelateral triangle. Hence, radius of the circumcircle of the hexagon

= Length of a side of the hexagon = a



PATHS

1. Pathways Running Across the Middle of a Rectangle

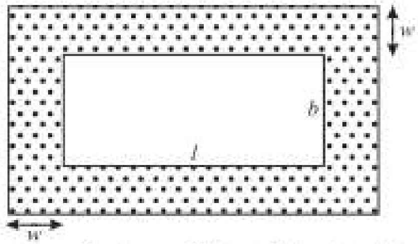


Area of the path = $l.w + b.w - w.w$
$= (l + b - w).w$

Perimeter of the path
$= 2l + 2b - 4w$
$= 2(l + b - 2w)$

Here w is the width of the path.

2. Pathways Outside a Rectangle

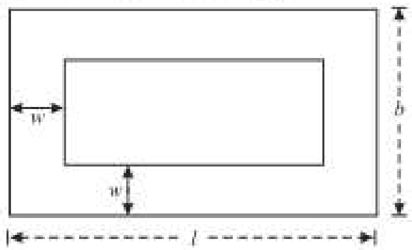


Area of path = $2(lw) + 2(b.w) + 4(w.w)$
$= (l + b + 2w)2w$

Perimeter of path
= (Internal perimeter) + (External perimeter)
$= 2(l + b) + 2(l + b + 4w)$
$= 4(l + b + 2w)$

Here w is the width of the path.

3. Pathway Inside a Rectangle



Area of path = $2(l.w) + 2(b.w) - 4(w.w)$
$= (l + b - 2w).2w$

Perimeter of path
= Length of outer path + Length of inner path
$= 2(l + b) + 2(l + b - 4w)$
$= 4(l + b - 2w)$

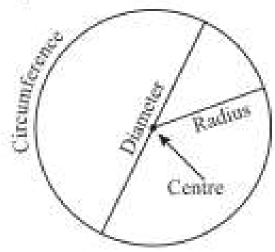
AREA RELATED TO A CIRCLE

Circle

Set of all points in a plane which are at a fixed distance from a fixed point in the same plane is called a circle.

The fixed point is called centre of the circle and the fixed distance is called radius of the circle.

Circumference or perimeter of a circle of radius r is

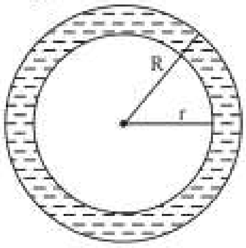


$c = 2\pi r = \pi d$ ($2r = d$ = diameter)

Area of the circle = $\pi r^2 = \frac{\pi d^2}{4} = \frac{c^2}{4\pi} = \frac{1}{2} \times c \times r$

Circular Ring

Region enclosed between two concentric circles of different radii in a plane is called a ring.



Area of the ring = $\pi R^2 - \pi r^2 = \pi (R^2 - r^2)$

Circumference of the ring

= (External circumference) + (Internal circumference)

= $2\pi R + 2\pi r = 2\pi(R + r)$

Semi-circle

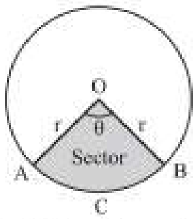
A semi-circle is a figure enclosed by a diameter and one half of the circumference of the circle.

Area of the semi-circle = $\frac{\pi r^2}{2}$

Circumference of the semi-circle = $\pi r + 2r = r(\pi + 2)$

Sector of a Circle

Sector of a circle is the portion of a circle enclosed by two radii and an arc of the circle. *OACB* is a sector of the circle.



Length of arc *ACB* (which make angle θ at the centre)

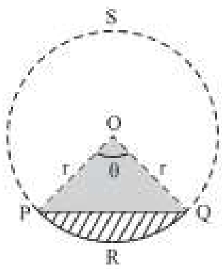
$$= (2\pi r) \times \frac{\theta}{360} = \frac{\pi r\theta}{180}$$

Perimeter of the sector *OACB* = $2r + \frac{\pi r\theta}{180}$

Area of sector *OACB* = $(\pi r^2) \times \frac{\theta}{360}$

Segment of a Circle

A segment of a circle is a region enclosed by a chord and an arc of the circle.



Any chord of a circle which is not a diameter divides the circle into two segments, one of which is the major segment and other is minor segment.

Perimeter of the segment *PRQP*

= Length of the arc *PRQ* + Length of *PQ*

$$= \frac{\pi r\theta}{180} + 2r \sin\frac{\theta}{2}$$

Area of (minor) segment *PQR*

= Area of sector *OPRQO* – Area of ΔOPQ

Area of (major) segment *PSQ*

= Area of circle – Area of segment *PQR*

Illustration 7: A circular grass lawn of 35 metres in radius has a path 7 metres wide running around it on the outside. Find the area of path.

(a) 1694 m² **(b) 1700 m²**
(c) 1598 m² **(d) None of these**

Solution: (a) Radius of a circular grass lawn (without path) = 35 m

$\therefore$ Area = $\pi r^2 = \pi (35)^2$

Radius of a circular grass lawn (with path)

= 35 + 7 = 42 m

$\therefore$ Area = $\pi r^2 = \pi(42)^2$

$\therefore$ Area of path = $\pi(42)^2 - \pi(35)^2$

$= \pi(42^2 - 35^2)$

$= \pi(42 + 35)(42 - 35)$

$= \pi \times 77 \times 7 = \frac{22}{7} \times 77 \times 7 = 1694 \text{ m}^2$

Illustration 8: A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be:

(a) 3520 cm² **(b) 6400 cm²**
(c) 7744 cm² **(d) 8800 cm²**

Solution: (c) Length of wire = $2\pi \times R = \left(2 \times \frac{22}{7} \times 56\right)$ cm = 352 cm.

Side of the square = $\frac{352}{4}$ cm = 88 cm.

Area of the square = (88×88) cm² = 7744 cm².

Illustration 9: There are two concentric circular tracks of radii 100 m and 102 m, respectively. A runs on the inner track and goes once round on the inner track in 1 min 30 sec, while B runs on the outer track in 1 min 32 sec. Who runs faster?

(a) Both A and B are equal
(b) A
(c) B
(d) None of these

Solution: (b) Radius of the inner track = 100 m

and time = 1 min 30 sec = 90 sec.

Also, Radius of the outer track = 102 m

and time = 1 min 32 sec = 92 sec.

Now, speed of *A* who runs on the inner track

$$= \frac{2\pi(100)}{90} = \frac{20\pi}{9} = 6.98$$

And speed of B who runs on the outer track

$$= \frac{2\pi(102)}{90} = \frac{51\pi}{23} = 6.96$$

Since, speed of A > speed of B

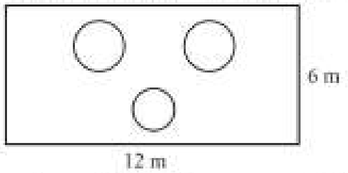
$\therefore$ A runs faster than B.

Illustration 10: A rectangular plate is of 6 m breadth and 12 m length. Two apertures of 2 m diameter each and one apertures of 1 m diameter have been made with the help of a gas cutter. What is the area of the remaining portion of the plate?

(a) 68.5 sq. m. **(b) 62.5 sq m**
(c) 64.5 sq. m **(d) None of these**

Solution: (c) Given, Length = 12 m and Breadth = 6 m

$\therefore$ Area of rectangular plate = $12 \times 6 = 72$ m^2



Since, two apertures of 3 m diameter each have been made from this plate.

$\therefore$ Area of these two apertures $= \pi(1)^2 + \pi(1)^2$
$= \pi + \pi = 2\pi$

Area of 1 aperture of 1m diameter $= \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$

$\therefore$ Total area of aperture $= 2\pi + \frac{\pi}{4} = \frac{9\pi}{4} = \frac{9}{4} \times \frac{22}{7}$

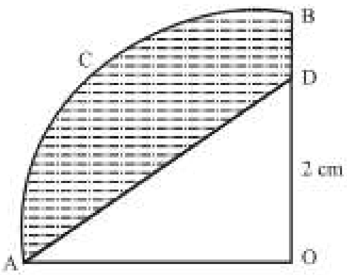
$$= \frac{99}{14}$$

$\therefore$ Area of the remaining portion of the plate

$$= 72 - \frac{99}{14} \text{ sq. m} = \frac{909}{14} \text{ sq. m} \approx 64.5 \text{ sq.m}$$

Illustration 11: In the adjoining figure, *AOBCA* represents a quadrant of a circle of radius 3.5 cm with centre *O*. Calculate the area of the shaded portion.

(a) 35 cm^2 **(b) 7.875 cm^2**
(c) 9.625 cm^2 **(d) 6.125 cm^2**



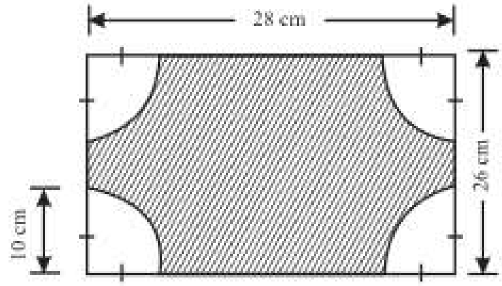
Solution: (d)

Area of shaded portion = Area of quadrant – Area of triangle

$$\Rightarrow \frac{\pi r^2}{4} - \frac{1}{2} \times 3.5 \times 2 = \frac{3.14 \times (3.5)^2}{4} - 3.5$$

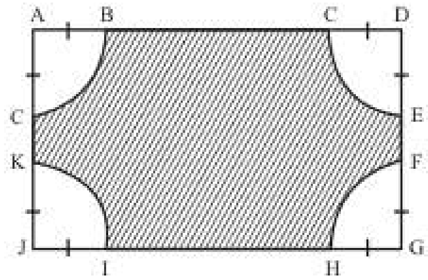
$\Rightarrow 6.125$ cm^2

Illustration 12: Find the perimeter and area of the shaded portion of the adjoining diagram:



(a) 90.8 cm, 414 cm^2 **(b) 181.6 cm, 423.7 cm^2**
(c) 90.8 cm, 827.4 cm^2 **(d) 181.6 cm, 827.4 cm^2**

Solution: (a)



KJ = radius of semicircles = 10 cm

4 quadrants of equal radius = 1 circle of that radius

Area of shaded portion ⇒ Area of rectangle – Area of circle

$(28 \times 26) - (3.14 \times 102) \Rightarrow 414$ cm^2

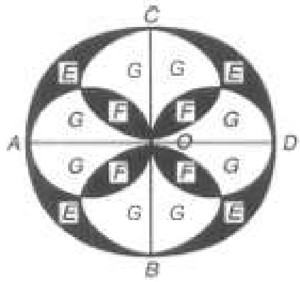
$BC = 28 - (10 + 10) = 8$ and $EF = 26 - (10 + 10) = 6$

Perimeter of shaded portion = 28 cm + $2\pi r$

Answer ⇒ 414 cm^2 = Area and

Perimeter = 90.8

Illustration 13: *ABDC* is a circle and circles are drawn with *AO*, *CO*, *DO* and *OB* as diameters. Areas *E* and *F* are shaded *E*/*F* is equal to



(a) 1/1 **(b) 1/2**
(c) 1/2 **(d) π/4**

Solution: (a)

$AO = CO = DO = OB$ = radius of bigger circle = r (let)

Then area of $(G + F) = \frac{\pi r^2}{2}$

Area of $2(G + F) = \pi r^2$. Also area of $2G + F + E = \pi r^2$

i.e. $2G + F + F = 2G + F + E \Rightarrow F = E$

So the ratio of areas E and F = 1 : 1

SURFACE AREA AND VOLUME OF SOLIDS

Solid

A solid body has three dimensions namely length, breadth (or width) and height (or thickness). The surfaces that bind it are called faces and the lines where faces meet are called edges.

The area of the surface that binds the solid is called its surface area.

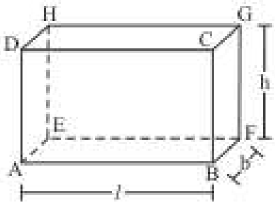
We measure the size of a solid body in terms of its volume.

The amount of space that any solid body occupies is called its volume.

Surface areas are measured in square units and volumes are measured in cubic units.

Cuboid

A cuboid is like a three dimensional box. It is defined by its length (l), breadth (b) and height (h). A cuboid can also be visualised as a room. It has six rectangular faces. It is also called rectangular parallelopiped.



A cuboid is shown in the figure with length 'l', breadth 'b' and height 'h'. 'd' denotes the length of a diagonal (AG, CE, BH or DF) of the cuboid.

Total surface area of a cuboid = 2 ($lb + bh + hl$)

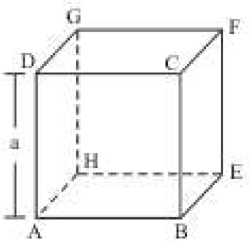
Lateral surface area (i.e., total area excluding area of the base and top) = $2h\ (l + b)$

Length of a diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Volume of a cuboid = Space occupied by cuboid
= Area of base × height
= $(l \times b) \times h = lbh$

Cube

A cube is a cuboid whose all edges are equal i.e.,
length = breadth = height = a (say)



Area of each face of the cube is a^2 square units.

Total surface area of the cuboid = Area of 6 square faces of the cube
$= 6 \times a^2 = 6a^2$

Lateral surface area of cube i.e., total surface area excluding top and bottom faces = $4a^2$

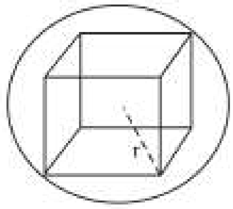
Length of diagonal (d) of the cube

$$= \sqrt{a^2 + a^2 + a^2}$$

$$= \sqrt{3a^2} = a\sqrt{3}$$

Volume of the cube (V) = Base area × Height
$= a^2 \times a = a^3$

Note that if a cube of the maximum volume is inscribed in a sphere of radius 'r', then the edge of the cube = $\dfrac{2r}{\sqrt{3}}$



Cylinder

A cylinder is a solid object with circular ends of equal radius and the line joining their centres perpendicular to them. This line is called axis of the cylinder. The length of axis between centres of two circular ends is called height of the cylinder.

In the figure, a cylinder with circular ends each of radius r and height h is shown.

Curved surface area of a cylinder
= Circumference of base × height
$= 2\pi r \times h = 2\pi rh$

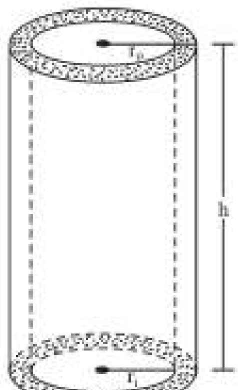
If cylinder is closed at both the ends then total surface area of the cylinder
= Curved surface area + Area of circular ends
$= 2\pi rh + 2 \times \pi r^2 = 2\pi r(h + r)$

Volume of the cylinder (V) = Base area × Height
$= \pi r^2 \times h = \pi r^2 h$

- Note that a cylinder can be generated by rotating a rectangle by fixing one of its sides.
- The curved surface of a cylinder is also called lateral surface.

Hollow Cylinder

A hollow cylinder is like a pipe.



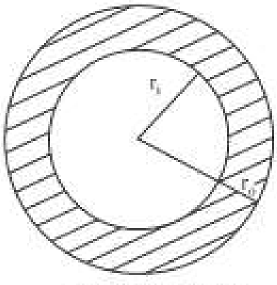
Inner radius = r_i and outer radius = r_o.

Hence $r_o - r_i$ = thickness of material of the cylinder.

Let length or height of the cylinder = h,

Curved surface area (C.S.A) of the hollow cylinder
= Outer curved surface area of the cylinder
+ Inner curved surface area of the cylinder
$= 2\pi r_o h + 2\pi r_i h = 2\pi h(r_o + r_i)$

Total surface area of hollow cylinder
= C.S.A. of hollow cylinder
+ Area of 2 circular end rings.



(one end of the pipe)

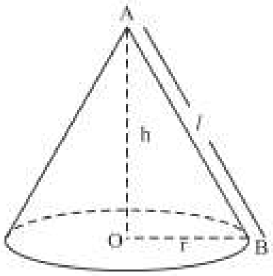
$= 2\pi h\,(r_o + r_i) + 2\pi\,(r_o^2 - r_i^2)$
$= 2\pi\,(r_o + r_i)\,(h + r_o + r_i)$

Volume of hollow cylinder = Volume of the material used in making the cylinder
$= \pi\,(r_o^2 - r_i^2)h$

Cone

A cone is a solid obtained by rotating a strip in the shape of a right angled triangle about its height. It has a circular base and a slanting lateral curved surface that converges at a point. Its dimensions are defined by the radius of the base (r), the height (h) and slant height (l).

A structure similar to cone is the ice-cream cone.



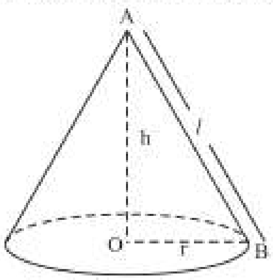
Height (AO) of cone is always perpendicular to base radius (OB) of the cone.

Slant height (l) = $\sqrt{h^2 + r^2}$

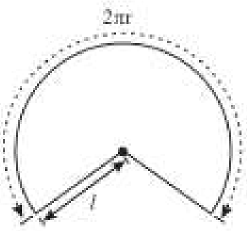
Volume of cone = $\frac{1}{3}$ × base area × height = $\frac{1}{3} \times \pi r^2 \times h$

Curved surface area (C.S.A.) = πrl
Total surface area (T.S.A.) = C.S.A. + Base area
$= \pi rl + \pi r^2 = \pi r(l + r)$

When a conical cup of paper (hollow cylinder) is unrolled, it forms a sector of a circle



Conical cup of paper

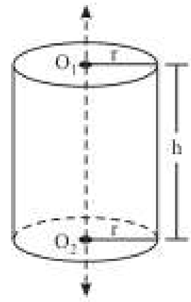


Unrolled conical cup, which is a sector of a circle.
Radius of this sector is equal to slant height of the cone.

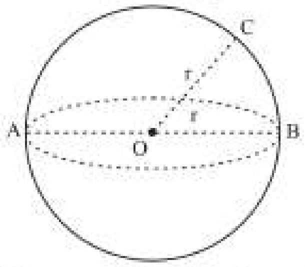
Length of curved edge of this sector is equal to the circumference of the base of the cone.

Sphere

A sphere is formed by revolving a semi-circle about its diameter. It has one curved surface which is such that all points on it are equidistant from a fixed point within it, called the centre.



Length of a line segment joining the centre to any point of the curved surface is called the radius (r) of the sphere.



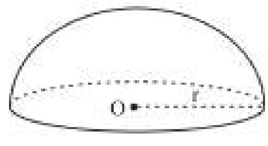
Any line segment passing through the centre and joining two points on the curved surface is called the diameter (d) of the sphere.

Centre = O
Radius = $OC = OA = OB = r$,
Diameter = AB
$= d = 2r$
Surface area of a sphere = $4\pi r^2$

Volume of a sphere (V) = $\frac{4}{3}\pi r^3$

Hemisphere

A plane through the centre of the sphere cuts the sphere into two equal parts. Each part is called a hemisphere.



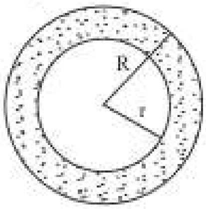
Volume of a hemisphere = $\frac{2}{3}\pi r^3$

Curved surface area (C.S.A.) of a hemisphere = $2\pi r^2$
Total surface area (T.S.A.) of a hemisphere
= C.S.A. + Base area
$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

Note that if a sphere is inscribed in a cylinder then the volume of the sphere is $\frac{2}{3}$rd of the volume of the cylinder.

Hollow Sphere or Spherical Shell

A rubber ball is an example of hollow sphere. In the rubber ball air is filled inside it. Thickness of the rubber in the ball is uniform. If outer and inner radii are R and r, then thickness of rubber or material used in hollow sphere = $R - r$.



Volume of the rubber or material used in hollow sphere

$$= \text{External volume} - \text{Internal volume}$$

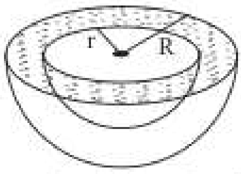
$$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(R^3 - r^3)$$

External surface area = $4\pi R^2$.

Hemispherical Bowl

When a spherical shell is cut off in two equal parts, then each part is called a hemispherical bowl as shown in the figure.



If R and r are external and internal radii of the hemisphere respectively, then

Volume of the material used in the hemispherical bowl

$$= \text{External volume} - \text{Internal volume}$$

$$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(R^3 - r^3)$$

External curved surface area = $2\pi R^2$

Internal surface area = $2\pi r^2$

Area of the cross-sectional ring = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

Total surface area

= (External curved surface area) + (Internal curved surface area) + (Area of cross-sectional ring)

$$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$$

$$= \pi(3R^2 + r^2)$$

Illustration 14: If the radius of a sphere is increased by 2 cm, then its surface area increases by 352 cm2. The radius of the sphere before the increase was:

(a) 3 cm **(b) 4 cm**
(c) 5 cm **(d) 6 cm**

Solution: (d) $4\pi(r + 2)^2 - 4\pi r^2 = 352$

$$\Rightarrow (r + 2)^2 - r^2 = \left(352 \times \frac{7}{22} \times \frac{1}{4}\right) = 28.$$

$$\Rightarrow (r + 2 + r)(r + 2 - r) = 28$$

$$\Rightarrow 2r + 2 = \frac{28}{2} \Rightarrow 2r + 2 = 14 \Rightarrow r = 6 \text{ cm}$$

Illustration 15: A cylindrical bucket of height 36 cm and radius 21 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed, the height of the heap being 12 cm. The radius of the heap at the base is :

(a) 63 cm **(b) 53 cm**
(c) 56 cm **(d) 66 cm**

Solution: (a) Volume of the bucket = volume of the sand emptied

Volume of sand = $\pi (21)^2 \times 36$

Let r be the radius of the conical heap.

Then, $\frac{1}{3}\pi r^2 \times 12 = \pi (21)^2 \times 36$

or $r^2 = (21)^2 \times 9$ or $r = 21 \times 3 = 63$ cm

Illustration 16: The length of the longest rod that can be placed in a room which is 12 m long, 9 m broad and 8 m high is

(a) 27 m **(b) 19 m**
(c) 17 m **(d) 13 m**

Solution: (c) Required length = length of the diagonal

$$= \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{ m}$$

Illustration 17: The internal measurements of a box with lid are 115 × 75 ×735 cm3 and the wood of which it is made is 2.5 cm thick. Find the volume of wood.

(a) 82,125 cm^3 **(b) 70,054 cm^3**
(c) 78,514 cm^3 **(d) None of these**

Solution: (a) Internal volume = $115 \times 75 \times 35 = 3{,}01{,}875$ cm^3

External volume = $(115 + 2 \times 2.5) \times (75 + 2 \times 2.5) \times (35 + 2 \times 2.5)$

$= 120 \times 80 \times 40 = 3{,}84{,}000$ cm^3

$\therefore$ Volume of wood = External volume – Internal volume

$= 3{,}84{,}000 - 3{,}01{,}875 = 82{,}125$ cm^3

Illustration 18: A rectangular tank is 225 m by 162 m at the base. With what speed must water flow into it through an aperture 60 cm by 45 cm that the level may be raised 20 cm in 5 hours ?

(a) 5000 m/hr **(b) 5400 m/hr**
(c) 5200 m/hr **(d) 5600 m/hr**

Solution: (b) Required speed of flow of water

$$= \frac{225 \times 162 \times 20}{5 \times 100} = \frac{60}{100} \times \frac{45}{100} \times h$$

$$\therefore \quad h = 5400$$

Illustration 19: A metallic sheets is of rectangular shape with dimensions 48 cm × 36 cm. From each one of its corners, a square of 8 cm is cut off. An open box is made of the remaining sheet. Find the volume of the box

(a) 5110 cm^3 **(b) 5130 cm^3**
(c) 5120 cm^3 **(d) 5140 cm^3**

Solution: (c) Volume of the box made of the remaining sheet

$= 32 \times 20 \times 8 = 5120$ cm^3

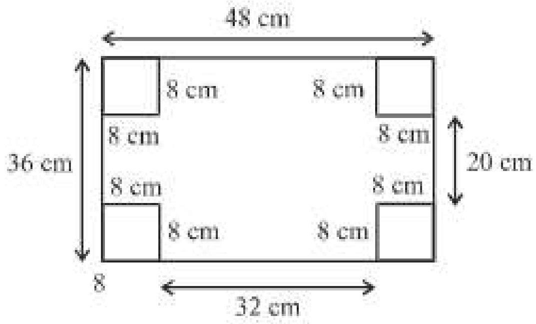


Illustration 20: The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres, what is the diameter of the base?

(a) 1.4 m **(b) 2.8 m**

(c) 14 m **(d) None of these**

Solution: (d) Volume of the tank = 246.4 litres = 246400 cm^3.

Let the radius of the base be r cm. Then,

$$\left(\frac{22}{7} \times r^2 \times 400\right) = 246400$$

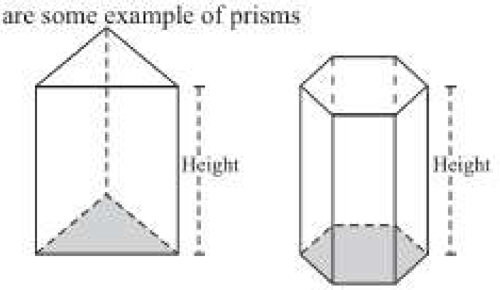
$$\Rightarrow r^2 = \left(\frac{246400 \times 7}{22 \times 400}\right) = 196 \Rightarrow r = 14.$$

$\therefore$ Diameter of the base = $2r$ = 28 cm = .28 m

Prism

A 'prism' is a solid having identical and parallel top and bottom (or base) faces. These identical faces are regular polygon of any number of sides. The side faces of a prism are rectangular and are known as lateral faces. Number of lateral faces is equal to the number of sides in the base.

Here are some example of prisms



Triangular base prism Hexagonal base prism

Lateral surface area of the prism

= (Perimeter of the base) × (Height)

Total surface area of the prism

= (Surface area of the top and bottom) + (Lateral surface area)

= 2 × Area of the base + Perimeter of base × Height

Volume of the prism = (Area of base) × (Height)

The actual formula used to find the surface area and volume will depend upon the number of sides in the base of the prism.

Pyramid

It is a three-dimensional body made up of a regular polygon shaped base and triangular lateral faces that meet at a point called vertex, which is also called the apex of the pyramid.

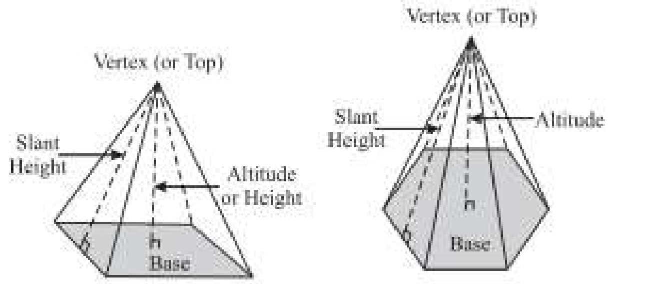
The number of triangular faces is equal to the number of sides in the base.

For example: A pyramid with a square base has four triangular faces, whereas a pyramid with a hexagonal face is made up of six triangular faces, and so on.

Lower face is called the base and the perpendicular distance of the vertex (or top) from the base is called the height or altitude of the pyramid.

The altitude of a lateral face of a pyramid is the slant height, which is the perpendicular distance of the vertex (or top) from the mid-point of any side of the base.

The lateral surface area of a regular pyramid is the sum of the areas of its lateral faces.



Square Based Pyramid Hexagonal Pyramid

Lateral surface area of a pyramid

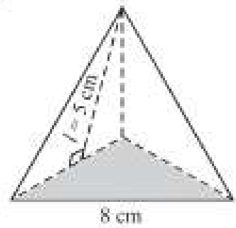
$$= \frac{1}{2} \times (\text{Area of the base}) \times (\text{Slant height})$$

Total surface area of a pyramid

$$= \frac{1}{2} \times (\text{Perimeter of the base}) \times (\text{Slant height}) + (\text{Area of the base})$$

Volume of a pyramid = $\frac{1}{3}$ × Area of base × Height

Illustration 21: Find the lateral surface area of a regular pyramid with triangular base, if each edge of the base measures 8 cm and slant height is 5 cm.

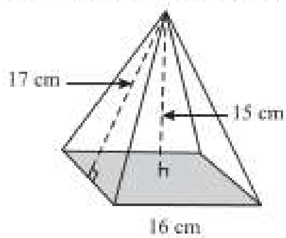


Solution: The perimeter of the base is the sum of the sides,

$$p = 3.(8) = 24 \text{ cm}$$

$$\text{L.S.A.} = \frac{1}{2} \times (24) \times (5) = 60 \text{ cm}^2$$

Illustration 22: Find the total surface area of a pyramid with a square base if each side of the base measures 16 cm, the slant height of a side is 17 cm and the altitude is 15 cm.



Solution: The perimeter of the base,

$$p = 4 \times 16 = 64 \text{ cm}$$

The area of the base

$$= 16^2 = 256 \text{ cm}^2$$

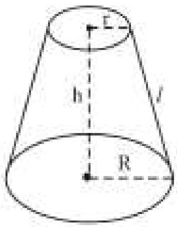
$$\text{T.S.A.} = \frac{1}{2}(64)(17) + 256$$

$$= 544 + 256 = 800 \text{ cm}^2$$

Frustum of a Cone

When top portion of a cone cut off by a plane parallel to the base of it, the left-over part is called the frustum of the cone.

In the figure, r and R are the radius of two ends, h is the height and l is the slant height of the frustum of cone.



Slant height, $l = \sqrt{(R-r)^2 + h^2}$

Curved surface area $= \pi(R + r)l$

Total surface area

$= $ (Curved surface area) + (Area of two circular ends)

$$= \pi(R + r)l + \pi R^2 + \pi r^2$$

$$= \pi(Rl + rl + R^2 + r^2)$$

Height of the original cone $= \dfrac{Rh}{R - r}$

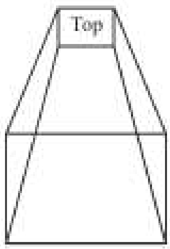
Volume of the frustum of cone

$$= \frac{\pi h}{3}\left(R^2 + r^2 + Rr\right)$$

Frustum of a Pyramid

When top portion of a pyramid is cut off by a plane parallel to the base of it, the left-over part is called the frustum of the pyramid.

If A_1, A_2 are of top and bottom face, P_1 and P_2 are the perimeters of top and bottom face, h is the height and l is the slant height of the frustum of the pyramid, then



Lateral surface area $= \dfrac{1}{2}(P_1 + P_2)\,l$

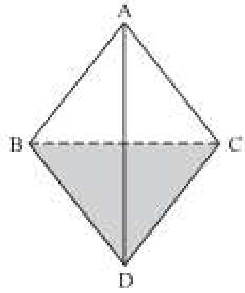
Total surface area $=$ Lateral surface area $+ A_1 + A_2$

$$= \frac{1}{2}(P_1 + P_2)\,l + A_1 + A_2$$

$$\text{Volume} = \frac{1}{3}\,h\,(A_1 + A_2 + \sqrt{A_1 . A_2}\,)$$

Tetrahedron (Only Shape)

A tetrahedron is a solid object which has 4 faces. All the faces of a tetrahedron are equilateral triangles. A tetrahedron has 4 vertices and 6 edges.



EULER'S RULE

For any regular shape solid (like cuboid, cube, cylinder, etc)

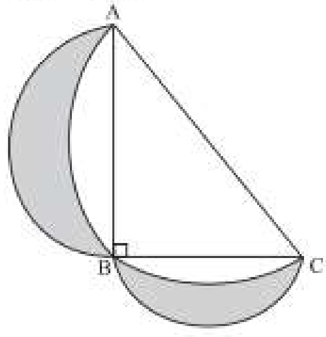
Number of faces (F) + Number of vertices (V)

= Number of edges (E) + 2

i.e., $F + V = E + 2$

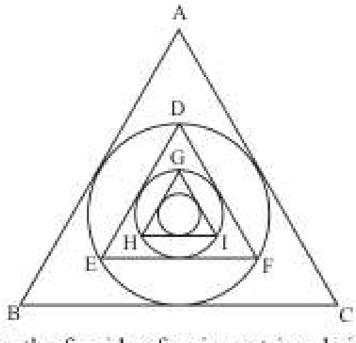
SOME OTHER IMPORTANT CONCEPTS

1. In the figure ABC is a triangle right angled at B. Three semi-circles are drawn taking the three sides AB, BC and CA as diameter. The region enclosed by the three semi-circles is shaded.



Area of the shaded region = Area of the right angled triangle.

2. In the figure given below all triangles are equilateral triangles and circles are inscribed in these triangles. If the side of triangle $ABC = a$, then the side of triangle $DEF = \dfrac{a}{2}$ and the side of triangle $GHI = \dfrac{a}{4}$



Thus length of a side of an inner triangle is half the length of immediate outer triangle. Similarly the radius of an inner circle is half the radius of immediate outer circle.