CHAPTER

9

TIME AND WORK

CONCEPT OF EFFICIENCY

Efficiency means rate of doing work. This means that more the efficiency, less will be the number of days required to complete a certain work and less the efficiency, more will be the number of days required to complete a certain work.

Aliza is twice as efficient as Binny.

$\Rightarrow$ Aliza does twice as much work as Binny in the same time interval

$\Rightarrow$ Aliza will require half the time as required by Binny to do the same work.

CONCEPT OF NEGATIVE WORK

Suppose two persons A and B are working to build a wall while C is working to demolish the wall. If we consider the work as the building of the wall, then breaking the wall (by C) is negative work.

The concept of negative work generally appears in the problems based on pipes and cisterns, where there are inlet pipes and outlet pipes/leaks, which are working against each other.

If we consider the work of filling a tank, the inlet pipe does positive work while the outlet pipe/leaks does negative work.

Illustration 1: *A* can build a wall in 15 days and *B* can build it in 10 days, while *C* can completely demolish the wall in 12 days. If they start working at the same time, in how many days will the work be completed.

Solution: Work per day by $A = \frac{1}{15}$

Work per day by $B = \frac{1}{10}$

Work per day by $C = -\frac{1}{12}$

(negative sign is taken for negative work)

The net combined work per day by A, B and C

$$= \frac{1}{15} + \frac{1}{10} - \frac{1}{12} = \frac{1}{12}$$

Since, Total work done = (Work done per day) × (No. of days required to complete the work)

$\therefore$ No. of days required to complete the work

$$= \frac{\text{Total work done}}{\text{Work done per day}} = \frac{1}{\frac{1}{12}} = 12$$

CONCEPT OF MAN-DAYS

If 'M' men working together can complete a work in 'D' days, then the product of number of men (M) and number of days (D) i.e. $M \times D$ is known as the number of MAN-DAYS.

Number of man days to complete a specific task always remains constant.

Suppose 30 persons working together for 20 days to complete a job, then the total work done is equal to ($30 \times 20 = 600$) man-days. If we change the number of days in which the work is to be completed, then the other factor i.e. the number of persons will change accordingly, so that the product of the factors becomes equal to 600 man-days.

WORK DONE

Consider a whole work as the unit work.

1. Work Done by Two Persons

Let A can do a whole work in x days and B can do the same one unit work in y days.

Hence work done by A in one day $= \frac{1}{x}$

and work done by B in one day $= \frac{1}{y}$

Then work done in one day when A and B work together

$$= \frac{1}{x} + \frac{1}{y}$$

$$= \frac{y+x}{xy} \text{ or } \frac{x+y}{xy}$$

Whole work = (Work done in one day) × (Number of days required to complete the whole work)

Hence, number of days required to complete the whole work

$$= \frac{\text{Whole work}}{\text{Work done in one day}}$$

$\Rightarrow$ Number of days required to complete the whole work when A and B are working together

$= \dfrac{1}{\dfrac{x+y}{xy}} = \dfrac{xy}{x+y}$, because a whole work is considered as one unit of work.

Illustration 2: If A can do a work in 10 days and B can do the same work in 15 days, then how many days will they take to complete the work both while working together?

Solution: Work done by A in one day $= \dfrac{1}{10}$

Work done by B in one day $= \dfrac{1}{15}$

Work done in one day when A and B work together

$$= \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{5}{30} = \frac{1}{6}$$

Hence required number of days $= \dfrac{1}{\dfrac{1}{6}} = 6$

We can find the required number of days directly by using the formula,

Number of days $= \dfrac{xy}{x+y}$, derived above

$$= \frac{10 \times 15}{10+15} = \frac{150}{25} = 6.$$

Illustration 3: 'A' completes a work in 12 days. 'B' completes the same work in 15 days. 'A' started working alone and after 3 days B joined him. How many days will they now take together to complete the remaining work?

(a) 5 **(b)** 8

(c) 6 **(d)** 4

Solution: (a) Work done by 'A' in 3 days $= \dfrac{1}{12} \times 3 = \dfrac{1}{4}$

$\therefore$ Remaining work $= 1 - \dfrac{1}{4} = \dfrac{3}{4}$

Work done by A and B together $= \dfrac{12 \times 15}{27} = \dfrac{20}{3}$

$\therefore$ Remaining work done by A and B together in

$$= \frac{3}{4} \times \frac{20}{3} = 5 \text{ days}$$

2. Work Done by Three Persons

As we derived the formula for two persons, you can also derived the formula for three persons in the same way.

If A, B, C can do a work in x, y and z days respectively, then all of them working together can finish the work in $\dfrac{xyz}{xy + yz + zx}$ days.

Illustration 4: If A, B, C can do a work in 12, 15 and 20 days respectively, then how many days will they take to complete the work when all the three work together.

Solution:

Required number of days $= \dfrac{xyz}{xy + yz + zx}$

$$= \frac{12 \times 15 \times 20}{12 \times 15 + 15 \times 20 + 20 \times 12}$$

$$= \frac{3600}{180 + 300 + 240} = \frac{3600}{720} = 5$$

3. If A and B Together Can do a Work in x Days and A Alone can do it in y Days, then B alone can do the Work in $\frac{xy}{y-x}$ Days

Illustration 5: A and B can do a work in 8 days and A alone can do it in 12 days. In how many days can B alone do it?

Solution:

Work done by A and B working together in one day $= \dfrac{1}{8}$

Work done by A in one day $= \dfrac{1}{12}$

$\therefore$ Work done by B in one day $= \dfrac{1}{8} - \dfrac{1}{12} = \dfrac{3-2}{24} = \dfrac{1}{24}$

Hence number of days in which B alone can do the whole work

$$= \frac{1}{\frac{1}{24}} = 1 \times \frac{24}{1} = 24$$

You can find required number of days directly by using the above formula as

Required number of days $= \dfrac{xy}{y-x} = \dfrac{8 \times 12}{12 - 8} = \dfrac{8 \times 12}{4} = 24.$

EXTENSION OF THE CONCEPT OF TIME AND WORK

1. Pipes and Cisterns

Problems related to Pipes and Cisterns are almost the same as those of Time and Work. Statement 'pipes A and B can fill a tank in 2 hours and 3 hours working individually' is similar to the statement 'A and B can do a work in 2 hours and 3 hours respectively working individually'.

If a pipe fills a tank in 3 hours, then the pipe fills $\dfrac{1}{3}$rd of the same tank in 1 hour.

The only difference with the pipes and cisterns problems is that there are inlets as well as outlets. Inlet is a pipe connected with a tank (or a cistern or a reservoir) that fills it. Outlet is a pipe connected with a tank (or a cistern or a reservoir) that empties it.

Hence, if we consider filling the tank by inlet as positive work, then empting the tank by outlet will be considered as negative work.

(a) Let a pipe fill a tank in x hours and another pipe can empty the full tank in y hours. Then the net part of the tank filled in 1 hour, when both the pipes are opened, if x is less than y.

$$= \frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$$

$\therefore$ time taken to fill the tank, when both the pipes are opened

$$= \frac{1}{\frac{y-x}{xy}} = \frac{xy}{y-x}.$$

(b) Let a pipe fill a tank in x hours while another fills the same tank in y hours but a third one empties the full tank in z hours. If all the three pipes are opened together, then the net part of the tank filled in 1 hour $= \frac{1}{x} + \frac{1}{y} - \frac{1}{z}$

$$= \frac{yz + zx - xy}{xyz}$$

$\therefore$ time taken to fill the tank $= \dfrac{xyz}{yz + zx - xy}$

(c) Let a pipe fill a tank in x hours but due to the leak in the bottom, the tank is filled in y hours and when the tank is filled, the time taken by the leak to empty the tank is z hours.

Net part of the tank filled in 1 hour by the pipe when there is the leak in the bottom $= \frac{1}{x} - \frac{1}{z} = \frac{z-x}{xz}$

Since the tank will be filled completely in y hours by the pipe when there is the leak in the bottom, therefore

$$\left(\frac{z-x}{xz}\right) \times y = 1 \Rightarrow y = \frac{xz}{z-x} \Rightarrow yz - xy = xz$$

$$\Rightarrow z(y-x) = xy \Rightarrow z = \frac{xy}{y-x}$$

Hence, if a pipe can fill a tank in x hours but due to the leak in the bottom, the tank is filled in y hours, then the fully filled tank will be emptied in $\dfrac{xy}{y-x}$ hours.

(d) Let a pipe A fill a tank in x hrs while pipe B can fill the tank in y hrs alone. When both the pipes are opened together, then time required to fill the tank $= \dfrac{xy}{x+y}$ hrs.

(e) Let pipes A, B and C fill a tank alone in x, y and z hrs respectively. When all the three pipes open together, then time required to fill the tank $= \dfrac{xyz}{xy + yz + zx}$ hrs.

Illustration 6: If a pipe fills a tank in 4 hrs and another pipe can empty the full tank in 6 hrs. When both the pipes are opened together, then find the time required to completely fill the tank.

Solution: Required time $= \dfrac{xy}{y-x}$

Here $x = 4,\ y = 6$

$\therefore$ Required time $= \dfrac{4 \times 6}{6-4} = 12$ hrs.

Illustration 7: Pipe A can fill a tank in 6 hrs while pipe B alone can fill it in 5 hrs and pipe C can empty the full tank in 8 hrs. If all the pipes are opened together, how much time will be needed to completely fill the tank?

Solution: Required time

$$= \frac{xyz}{yz + zx - xy} = \frac{6 \times 5 \times 8}{5 \times 8 + 8 \times 6 - 6 \times 5}$$

$$= \frac{6 \times 5 \times 8}{58} = \frac{120}{29} = 4\frac{4}{29} \text{ days.}$$

Illustration 8: A pipe can fill a tank in 10 hrs. Due to a leak in the bottom, it is filled in 15 hrs. If the tank is full, how much time will the leak take to empty it.

Solution: Required time $= \dfrac{xy}{y-x} = \dfrac{10 \times 15}{15-10} = 30$ hrs.

Illustration 9: If three pipes A, B and C can fill the tank alone in 5, 6 and 8 hrs, then when all the three pipes are opened together, find the time to fill the tank completely.

Solution: Required time

$$= \frac{xyz}{xy + yz + zx} = \frac{5 \times 6 \times 8}{5 \times 6 + 6 \times 8 + 8 \times 5}$$

$$= \frac{240}{30 + 48 + 40} = \frac{240}{118} = 2\frac{2}{59} \text{ hrs}$$

3. Alternate Work

In some problems two or more people of different efficiencies work alternatively or in some particular pattern. You can understand the method to solve these types of problems through the following illustration.

Illustration 10: Sanjeev can build a wall in 20 days and Parveen can demolish the same wall in 30 days. If they work on alternate days with Sanjeev starting the job on the 1st day, then in how many days will the wall be built for the first time ?

Solution: Let us assume the total units of work

= 60 units (i.e. LCM of 20 and 30)

So, the wall built by Sanjeev in one day = 3 units

And wall demolished by Parveen in one day = 2 units

So, effectively in two days, total wall built = 1 unit

Now, they work on alternate days, so days taken to built 57 units = 57 days

On 58th day Sanjeev will add another 3 units and so completing the construction of wall in 58 days.

(This problem can be understood well with another very traditional problem—A frog climbs up a pole 3 inches in 1 minute and slips 2 inches in next minute. If height of the pole is 120 inches, then how much time is taken by the frog to reach the top of the pole ?)