CHAPTER

7

INTEREST

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If an agency (i.e. an individual, a firm or a bank etc.) borrow some money from any other agency, then the first agency is called the *borrower* and the second agency is called the *lender*. The borrowed money is called the *principal*.

If the borrower has to pay some additional money together with the borrowed money for the benefit of using borrowed money for a certain time period is called *loan period*, then this additional money is called the *interest* and the principal together with the interest is called the *amount* (i.e. Amount = Principal + Interest). When we deposite money in a bank, we earn interest, interest is calculated according to an agreement which specifies the rate of interest. Generally the rate of interest is taken as "percent per annum" which means "per ₹ 100 per year". For example, a rate of 10% per annum means ₹ 10 on ₹ 100 for 1 year.

SIMPLE INTEREST (S.I.)

If the principal remains the same for whole loan period, then the interest is called the simple interest.

$$\text{S.I.} = \frac{\text{PRT}}{100}$$

where P = Principal, R = Rate of interest in percent per annum, T = Loan period (or whole time period in years)

In the formula of simple interest, by putting the value of any three unknowns out of the four unknowns *S.I., P, R, T*; you can find the remaining fourth unknown.

Simple rate of interest is generally written as rate of interest only i.e. if it is not mentioned whether the interest is simple or compound, then we should assume it as simple interest.

Illustration 1: At what rate percent by simple interest, will a sum of money double itself in 5 years 4 months ?

Solution: Let $P = ₹\, x$

Then $A = ₹\, 2x$

$\therefore$ $\text{S.I.} = A - P = ₹\, 2x - ₹\, x = ₹\, x$

$$T = 5 \text{ years } 4 \text{ months} = 5\frac{4}{12} \text{ years}$$

$$= 5\frac{1}{3} \text{ year} = \frac{16}{3} \text{ years}$$

Let R be the rate percent per annum.

$$\text{Using } R = \frac{\text{S.I.} \times 100}{\text{P} \times \text{T}}, \text{ We get } R = \frac{x \times 100}{x \times \frac{16}{3}} = \frac{300}{16} = 18.75.$$

Hence required rate = 18.75 % p.a.

Illustration 2: Find the SI on ₹ 1800 from 21st Feb. 2003 to 12th April 2003 at 7.3% rate per annum.

Solution: $P = ₹\, 1800$; $R = 7.3\%$; $I = ?$

No. of days = 7 + 31 + 12 = 50 days.

$$T = \frac{50}{365} \text{ years}.$$

$$I = \frac{PTR}{100} = \frac{1800 \times \frac{50}{365} \times 7.3}{100} = ₹\, 18.$$

COMPOUND INTEREST (C.I.)

If the borrower and the lender agree to fix up a certain interval of time (a year, a half year or a quarter of a year etc.) called conversion period, so that the amount (= principal + interest) at the end of an conversion period becomes the principal for the next conversion period, then the total interest over the whole loan period calculated in this way is called the compound interest.

Note: The main difference between the simple interest and the compound interest is that the principal in the case of simple interest remains constant throughout the loan period whereas in the case of compound interest, the principal changes periodically (i.e. after each conversion period) throughout the loan period.

Rate of interest is always given annually but it can be compounded annually, half yearly, quaterly or monthly.

Interest compounded annually means conversion period is one year and hence amount at the end of every one year becomes the principal for the next conversion period.

Interest compounded half yearly means conversion period is half year and hence amount at the end of every half year becomes the principal for the next conversion period.

Interest compounded quarterly means conversion period is a quarter of a year and hence amount at the end of every quarter of a year becomes the principal for the next conversion period.

Similarly, interest compounded monthly means conversion period is one month and hence amount at the end of every one month becomes the principal for the next conversion period.

1. Computation of Compound Interest When Interest is Compounded Annually

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$\therefore \quad \text{C.I.} = A - P = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

Here A is the amount,
P is the principal, r is the rate of interest in percent per conversion period and n is the number of conversion periods in the whole loan period.

In the formula of compound interest by putting the value of any three unknowns out of the four unknowns A, P, r and n; you can find the remaining fourth unknown.

Illustration 3: Roohi deposited ₹ 7,000 in a finance company for 3 years at an interest of 15% per annum compounded annually. What is the compound interest and the amount that Roohi gets after 3 years ?

Solution: Principal, P = ₹ 7000, n = 3 years, r = 15% per annum

Amount of C.I.

$$A = P\left(1+\frac{r}{100}\right)^n = 7000\left(1+\frac{15}{100}\right)^3 = 7000\left(\frac{115}{100}\right)^3$$

$$= 7000 \times \frac{115}{100}\times\frac{115}{100}\times\frac{115}{100} = 10646.125 = ₹\ 10646.13$$
$$= ₹\ 10646 \text{ (approx.)}$$

Compound interest = A – P = 10646 – 7000 = ₹ 3646

2. Computation of Compound Interest When Interest is Compounded *k* Times Every Year

If r be the rate of interest in percent per year, then the rate of interest in percent per conversion period is $\frac{r}{k}$.

If n be the number of years in the whole loan period (or whole time period), then the number of conversion period is nk.

$$\therefore \quad A = P\left(1+\frac{r}{100k}\right)^{nk}$$

$$\text{and} \quad \text{C.I.} = P\left[\left(1+\frac{r}{100k}\right)^{nk} - 1\right]$$

(a) In case of interest compounded half-yearly, $k = 2$

$$\therefore \quad A = P\left(1+\frac{r}{100\times 2}\right)^{2n}$$

$$\text{and} \quad \text{C.I.} = P\left[\left(1+\frac{r}{100\times 2}\right)^{2n} - 1\right]$$

(b) In case of interest compounded quarterly, $k = 4$

$$\therefore \quad A = P\left(1+\frac{r}{100\times 4}\right)^{4n}$$

$$\text{and} \quad \text{C.I.} = P\left[\left(1+\frac{r}{100\times 4}\right)^{4n} - 1\right]$$

(c) In case of interest compounded monthly, $k = 12$

$$\therefore \quad A = P\left(1+\frac{r}{100\times 12}\right)^{12n}$$

$$\text{and} \quad \text{C.I.} = P\left[\left(1+\frac{r}{100\times 12}\right)^{12n} - 1\right]$$

Illustration 4: A sum of money is lent out at compound interest rate of 20 % per annum for 2 years. It would fetch ₹ 482 more if interest is compounded half-yearly. Find the sum.

Solution: Suppose the sum is ₹ P.

C.I. when interest is compounded yearly

$$= P\left[1+\frac{20}{100}\right]^2 - P$$

C.I. when interest is compounded half-yearly

$$= P\left[1+\frac{10}{100}\right]^4$$

Now, we have,

$$P\left[1+\frac{10}{100}\right]^4 - P\left[1+\frac{20}{100}\right]^2 = 482$$

$$\Rightarrow \quad P\left[\{1.1\}^4 - \{1.2\}^2\right] = 482$$

$$\Rightarrow \quad P\left[\left\{(1.1)^2 - (1.2)\right\}\left\{(1.1)^2 + (1.2)\right\}\right] = 482$$

$$\Rightarrow \quad P\,[\{1.21 - 1.2\}\{1.21 + 1.2\}] = 482$$

$$\Rightarrow \quad P\,[(0.01)\,(2.41)] = 482$$

$$\therefore \quad P = \frac{482}{2.41\times 0.01} = ₹\ 20{,}000$$

Illustration 5: Lussy deposited ₹ 7500 in a bank which pays him 12% interest per annum compounded quarterly. What is the amount which she will receive after 9 months?

Solution: Here, P = ₹ 7500, r = 12% per annum and n = 9 months = $\frac{9}{12}$ years = $\frac{3}{4}$ years.

$$\therefore \quad \text{Amount after 9 months} = P\left(1+\frac{r}{400}\right)^{4n}$$

$$= ₹\ 7500 \times \left(1+\frac{12}{400}\right)^{4\times\frac{3}{4}} = ₹\ 7500 \times \left(1+\frac{3}{100}\right)^3$$

$$= ₹\ 7500 \times \frac{103}{100}\times\frac{103}{100}\times\frac{103}{100} = ₹\ 8195.45$$

Illustration 6: A sum of money placed at compound interest doubles itself in 4 years. In how many years will it amount to eight times itself ?

Solution: We have

$$P\left(1+\frac{r}{100}\right)^4 = 2P$$

$$\therefore \quad \left(1+\frac{r}{100}\right)^4 = 2$$

Cubing both sides, we get

$$\left(1+\frac{r}{100}\right)^{12} = 2^3 = 8$$

$$\therefore \quad P\left(1+\frac{r}{100}\right)^{12} = 8P$$

Hence required time = 12 years.

Shortcut Approach:

x becomes $2x$ in 4 yrs.

$2x$ becomes $4x$ in 4 yrs.

$4x$ becomes $8x$ in 4 yrs.

Thus x becomes $8x$ in 4 + 4 + 4 = 12 yrs.

Remember the follow result

If a sum becomes x times in y years at compound interest then it will be $(x)^n$ times in ny years

Thus if a sum becomes 3 times in y years at compound interest, it will be $(3)^2$ times in 2 × 3 = 6 years.

Illustration 7: If a sum deposited at compound interest becomes double in 4 years, when will it be 4 times at the same rate of interest ?

Solution: Using the above conclusion, we say that the sum will be $(2)^2$ times in 2 × 4 = 8 years.

3. Computation of Compound Interest When Interest is Compounded Annually but Rate of Interest in Percent being Different for Different Years

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)...\left(1+\frac{R_n}{100}\right),$$

where $R_1, R_2, ..., R_n$ are rate of interest in percent per year for different years.

Illustration 8: Ram Singh bought a refrigerator for ₹ 4000 on credit. The rate of interest for the first year is 5% and of the second years is 15%. How much will it cost him if he pays the amount after two years.

Solution: Here, P = ₹ 4000, R_1 = 5% per annum and R_2 = 15% per annum.

$$\therefore \quad \text{Amount after 2 years} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)$$

$$= ₹\ 4000 \times \left(1+\frac{5}{100}\right)\left(1+\frac{15}{100}\right)$$

$$= ₹\ 4000 \times \left(1+\frac{1}{20}\right)\left(1+\frac{3}{20}\right)$$

$$= ₹\ 4000 \times \frac{21}{20} \times \frac{23}{20} = ₹\ 4830$$

Thus, the refrigerator will cost ₹ 4830 to Ram Singh.

4. If *P* be the value of an article (or population of a town or a country etc.) at a certain time and *R*% per annum is the rate of depreciation, then the value *A* at the end of *n* years is given by

$$A = P\left(1-\frac{R}{100}\right)^n$$

Illustration 9: The population of a town 2 years ago was 62500. Due to migration to cities, it decreases every year at the rate of 4% per annum. Find its present population.

Solution: We have,

Population two years ago = 62500

Rate of decrease of population = 4% per annum.

$$\therefore \quad \text{Present population} = 62500 \times \left(1-\frac{4}{100}\right)^2$$

$$= 62500 \times \left(1-\frac{1}{25}\right)^2$$

$$= 62500 \times \left(\frac{24}{25}\right)^2$$

$$= 62500 \times \frac{24}{25} \times \frac{24}{25} = 57600$$

Hence, present population = 57600

5. If P be the population of a country (or value of an article etc.) at a certain time, which increases at the Rate R_1 % per year for first n_1 years and decreases at the rate of R_2 % per year for next n_2 years, then the population at the end of $(n_1 + n_2)$ years is given by

$$A = P\left(1+\frac{R_1}{100}\right)^{n_1} \cdot \left(1-\frac{R_2}{100}\right)^{n_2}$$

This formula can be extended for more than 2 different periods and rates.

Illustration 10: 10000 workers were employed to construct a river bridge in four years. At the end of first year, 10% workers were retrenched. At the end of the second year, 5% of the workers at the begining of the second year were retrenched. However to complete the project in time, the number of workers was increased by 10% at the end of the third year. How many workers were working during the fourth year ?

Solution: We have,

Initial number of workers = 10000

Reduction of workers at the end of first year = 10%

Reduction of workers at the end of second year = 5%

Increase of workers at the end of third year = 10%

$\therefore$ Number of workers working during the fourth year

$$= 10000\left(1-\frac{10}{100}\right)\left(1-\frac{5}{100}\right)\left(1+\frac{10}{100}\right)$$

$$= 10000 \times \frac{9}{10} \times \frac{19}{20} \times \frac{11}{10} = 9405$$

Hence, the number of workers working during the fourth year was 9405.