CHAPTER

4

ALLIGATIONS

ALLIGATION

Alligation is the simplified, faster technique to solve the problems based on weighted average. This method plays a vital role in saving the time in solving the problems related to weighted average situation.

We know that

$$\text{Weighted Average} = \frac{\text{Sum total of all numbers of all groups}}{\text{Total number of numbers in all groups together}}$$

Therefore weighted average A_w of two groups having n_1 and n_2 numbers with averages A_1 and A_2 respectively is

$$A_w = \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2}$$

$$\Rightarrow \quad (n_1 + n_2) A_w = n_1 A_1 + n_2 A_2$$

$$\Rightarrow \quad n_1 (A_w - A_1) = n_2 (A_2 - A_w) \quad \Rightarrow \quad \frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$$

Equation $\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$ is called Alligation Formula.

For convenient, we take $A_1 < A_2$. Hence $A_1 < A_w < A_2$.

SOLVING THE PROBLEMS OF ALLIGATIONS USING ALLIGATION FORMULA

Illustration 1: 10 kg of wheat costing ₹ 12 per kg and 15 kg of wheat costing ₹ 20 per kg are mixed. Find the average cost of the mixture per kg.

Solution: $\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1} \Rightarrow \frac{10}{15} = \frac{20 - A_w}{A_w - 12}$

$$\Rightarrow \quad \frac{2}{3} = \frac{20 - A_w}{A_w - 12} \Rightarrow 5A_w = 84$$

$$\Rightarrow \quad A_w = \frac{84}{5} = 16.8$$

Hence average cost of the mixture = ₹ 16.8 per kg.

Illustration 2: A mixture worth ₹ 3.25 per kg is formed by mixing two types of salts, one costing ₹ 3.10 per kg while the other ₹ 3.60 per kg. In what ratio must they have been mixed?

Solution: $\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1} \Rightarrow \frac{n_1}{n_2} = \frac{3.60 - 3.25}{3.25 - 3.10} = \frac{35}{15}$

$$\Rightarrow \quad n_1 : n_2 = 7 : 3$$

Hence required ratio = 7 : 3.

GRAPHICAL REPRESENTATION OF ALLIGATION– CROSS METHOD

The alligation formula $\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$ is graphically represented by the following cross diagram:

$$\begin{array}{ccc} A_1 & & A_2 \\ & \searrow \quad \swarrow & \\ & A_w & \\ & \swarrow \quad \searrow & \\ \end{array}$$

$$\begin{bmatrix} A_2 - A_w & : & A_w - A_1 \\ n_1 & : & n_2 \end{bmatrix}$$

The ratios in the bracket [ ] are equal i.e.

$$n_1 : n_2 = A_2 - A_w : A_w - A_1.$$

In the above graphical representation five variables A_1, A_2, A_w, n_1 and n_2 are involved.

Based on the problem situation, one of the following three cases may occur with respect to the known and the unknown out of the five variables A_1, A_2, A_w, n_1 and n_2 involved in the problem.

Case	Known	Unknown
I	(a) A_1, A_2, A_w	(a) $n_1 : n_2$
	(b) A_1, A_2, A_w, n_1	(b) n_2 and $n_1 : n_2$
II	A_1, A_2, n_1, n_2	A_w
III	A_1, A_w, n_1, n_2	A_2

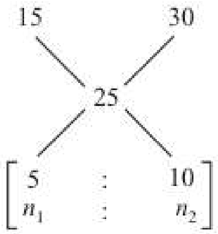
Solving the problem using graphical representation of alligation is called cross method.

Let us solve some problems in each of the three cases using cross method.

Case I: When A_1, A_2, A_w are known and one of n_1 and n_2 may be also known then to find $n_1 : n_2$ and n_2 if n_1 is known OR n_1 if n_2 is known.

Illustration 3: If the average weight of the students of a class is 15kg, the average weight of the students of another class is 30kg and average weight of the students of both the classes is 25kg, then find the ratio of the number of students of the first class to the another class.

Solution:



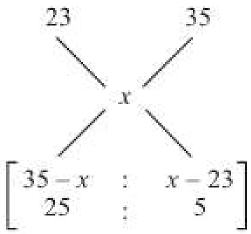
$\therefore n_1 : n_2 = 5 : 10 = 1 : 2$

Hence required ratio = 1 : 2

Case II: When A_1, A_2, n_1, n_2 are known and A_w is unknown then to find A_w.

Illustration 4: 5 kg of superior quality of sugar is mixed with 25 kg of inferior quality sugar. The price of superior quality and inferior quality sugar is ₹ 35 and ₹ 23 respectively. Find the average price per kg of the mixture.

Solution:



$$\therefore \quad \frac{35-x}{x-23} = \frac{25}{5}$$

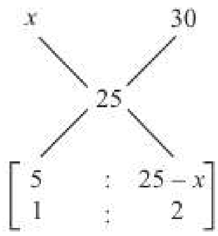
$$\Rightarrow \quad 30x = 175 + 575 \Rightarrow x = \frac{750}{30} = 25$$

Hence average price per kg of the mixture = ₹ 25

Case-III: When A_1, A_w, n_1, n_2 are known and A_2 is unknown, then to find the value of A_2.

Illustration 5: The ratio of number of girls to number of boys is 1 : 2. If the average weight of the boys is 30 kg and the average weight of both the boys and girls is 25 kg, then find the average weight of the girls.

Solution:

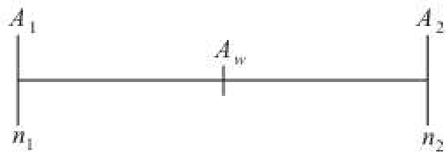


$$\frac{5}{25-x} = \frac{1}{2} \Rightarrow x = 15$$

Hence average weight of the girls = 15 kg.

THE STRAIGHT LINE APPROACH TO SOLVE THE PROBLEMS RELATED TO ALLIGATIONS

The straight line approach is actually the cross method.



The above diagram is the straight line diagram in which the symbols A_1, A_2, A_w, n_1 and n_2 denote the same quantity as shown in cross method. Here $A_1 < A_w < A_2$.

In the above diagram,

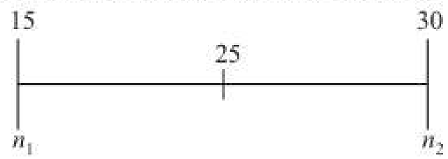
(a) n_1 corresponds to $(A_2 - A_w)$

(b) n_2 corresponds to $(A_w - A_1)$

(c) $(n_1 + n_2)$ corresponds to $(A_2 - A_1)$

Now, we again solve the examples 3, 4 and 5 given in case-I, II and III respectively of cross method using straight line approach.

Sol. of illustration 3 by straight line approach.



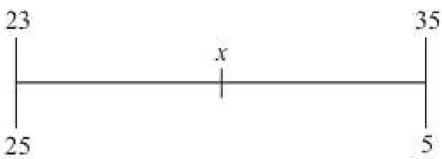
n_1 corresponds to 5 (= 30 – 25)

and n_2 corresponds to 10 (= 25 – 15)

$\therefore n_1 : n_2 = 5 : 10 = 1 : 2$

Hence required ratio = 1 : 2

Sol. of illustration 4 by straight line approach.



25 corresponds to (35 – x)

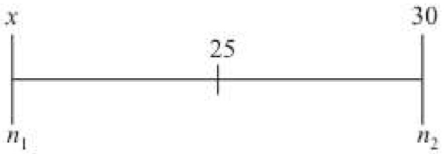
Also 5 corresponds to (x – 23)

$$\frac{25}{5} = \frac{35-x}{x-23}$$

$$\Rightarrow \quad x = 25$$

Hence average price per kg of mixture = ₹ 25

Sol. of illustration 5 by straight line approach.



Here $n_1 : n_2 = 1 : 2$

Now, n_1 corresponds to 5 (= 30 – 25)

and n_2 corresponds to $(25 - x)$

$$\therefore \frac{n_1}{n_2} = \frac{5}{25 - x} \Rightarrow \frac{1}{2} = \frac{5}{25 - x} \Rightarrow x = 15$$

Hence average weight of the girls = 15 kg

RECOGNITION OF DIFFERENT SITUATIONS WHERE ALLIGATION CAN BE USED

There are many types of situations where alligation can be used, which must be recognised by the students. Here you are given some situations (or problems) which help you to recognise different alligation situations and identify A_1, A_2, n_1, n_2 and A_w in each alligation situation.

In each of the following problems

$$A_1 = 20, A_2 = 35, n_1 = 20, n_2 = 40$$

and answer as $A_w = 30$.

1. An average weight of students of a class of 40 students is 35 kg and an average weight of students of a class of 20 students is 20 kg. Find the average weight of the students of both the combined classes. (30 kg)
2. 20 litres of one variety of soda water is mixed with 40 litres of other variety of soda water. The price of first variety of soda water is ₹ 20 per litre and price of other variety of soda water is ₹ 35 per litre. Find the cost of the mixture per litre. (₹ 30)
3. A car travels at 20 km/h for 20 minutes and at 35 km/h for 40 minutes. Find the average speed of the car for the whole journey. (30 km/hr)
4. A car agency sold 20 cars at 20% profit and 40 cars at 35% profit. Find the gain percent on the sale of all these cars. (30%)
5. A trader earns a profit of 20% on 20% of his goods sold while he earns a profit of 35% on 40% of his goods sold. Find the percentage profit on whole. (30%)
6. A 40 litres mixture of water and milk contains 35% of milk and in another 20 litres of mixture of water and milk contains 20% of milk. If a new mixture is formed by mixing the both mixtures, then find the percentage of milk in new mixture. (30%)
7. A shopkeeper sold the 40% hardware at the profit of 35% and 20% software at a profit of 20%. Find the average profit% on the whole goods sold, if he sells only these two kind of things. (30%)

Some Keys to Identify A_1, A_2 & A_w and Differentiate These from n_1 and n_2

1. Normally, there are 3 averages mentioned in the problem, while there are only 2 quantities. This is not foolproof. Sometimes the question might confuse the students by giving 3 values for quantities representing n_1, n_2 and $n_1 + n_2$ respectively.
2. A_1, A_2 and A_w are always rate units, while n_1 and n_2 are quantity units.
   Rate units are like ₹ x/kg, y km/hour, etc. and corresponding quantity units are kg, hour etc.
3. The denominator of the average unit corresponds to the quantity unit (i.e., unit for n_1 and n_2).
   For example, denominator kg and hour of rate units ₹ x/kg and y km/hour are the units of quantity corresponding to rates.

A TYPICAL PROBLEM

Let's discuss the solution of a typical problem given below:

Illustration 6: A person used to draw out 20% of the honey from a jar containing 10 kg honey and replaced it with sugar solution. He has repeated the same process three times.

Find the final amount of honey left in the jar and the final ratio of honey to sugar solution finally left in the jar.

Solution: In first step: Honey drawn out 20% of 10 kg from the jar and then 2 kg sugar solution is put in the jar.

Hence after first step,

Honey remains in the jar = 10 – 20% of 10 = 10 – 2 = 8 kg and sugar solution remains in the jar = 2 kg

In second step: 20% of (8 kg honey and 2 kg sugar solution) is drawn out from the jar and then 2 kg of sugar solution is put in the jar.

⇒ 20% of 8 kg honey and 20% of 2 kg sugar solution is drawn out from the jar and then 2 kg of sugar solution is put in the container.

Thus in each step of drawing, 20% of remaining honey is drawn out.

Hence honey left in the container after second draw

= 8 – 20% of 8 = 8 – 1.6 = 6.4 kg

Honey left in the container after third draw

= 6.4 – 20% of 6.4

= 6.40 – 1.28 = 5.12 kg

Hence the final amount of the honey left in the jar = 5.12 kg

The above whole process can be shown in a single line as

$10 - 20\% \text{ of } 10 \to 8 - 20\% \text{ of } 8 \to 6.4 - 20\% \text{ of } 6.4 \to 5.12$ kg

Now the final amount of sugar solution left in the jar

= 10 – 5.12 kg = 4.88 kg

Hence final ratio of honey to the sugar solution left in the jar

$$= \frac{5.12}{4.88} = 64 : 61.$$