GEOMETRY

CHAPTER 18

POINTS, LINES, LINE SEGMENT, RAY AND PLANE

Point: A point is like a dot marked by a very sharp pencil on a plane paper. A point is named by a capital letter like P. In the figure P is a point. Length, breadth and height of a point are negligible and hence cannot be measured.

. P

Line: A line is defined as a group of points. Which are straight one after another. Each line is extended infinitely in two directions. Examples:

(i) (ii)

A line is named by either any two points on it or by a single small letter. In figure (i), AB is a line. In figure (ii), l is a line.

Arrows on both sides of a line indicate that the line is extended both sides infinitely. A line has only length. It does not have any width or height.

Line Segment: If a part of the line is cut out, then this cut out piece of the line is called a line segment. A line segment has no arrow at its any end.

This means that no line segment is extended infinitely in any direction.

Ray: A ray is a part of a line extended infinitely in any one direction only. Example:

A ray is named by two points, one of which is the end point on the ray called initial point and other point is any point on the ray.

In the figure, AB is a ray. The point A is called the initial point. Arrow of the ray indicates that the ray is extended infinitely towards arrow head.

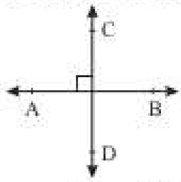
Plane: It is a flat surface extended infinitely. It has only length and breadth but no thickness. Surface of a black board, surface of a wall, surface of a table are some examples of parts of planes because they are flat surfaces but not extended infinitely.

LINES AND ANGLES

Intersecting Lines: If two or more lines intersect each other, then they are called intersecting lines. In the figure AB and CD are intersecting lines.

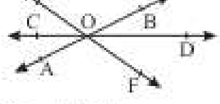
If two lines intersect at right angles, then two lines are called perpendicular lines

In the following figure AB and CD are perpendicular lines.



Symbolically it is represented as $AB \perp BC$ or $BC \perp AB$.

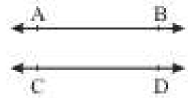
Concurrent Lines: If three or more lines pass through a point, then they are called concurrent lines and the point through which these all lines pass is called point of concurrent.



In the figure, AB, CD and EF are concurrent lines and point O is the point of concurrent.

Parallel Lines: Two straight lines are parallel if they lie in the same plane and do not intersect even if they produced.

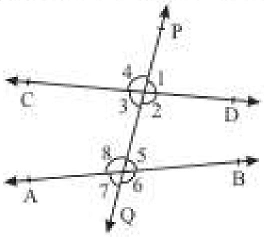
Perpendicular distances between two parallel lines are the same at all places.



In the figure AB and CD are parallel lines.

Symbol for parallel lines is ||. Hence parallel lines AB and CD represented symbolically as $AB \parallel CD$.

Transversal Line: A line which intersects two or more given lines at distinct points is called a transversal of the given lines.



In figure straight lines AB and CD are intersected by a transversal PQ.

(i) Corresponding angles: In the figure $\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 8$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$ are four pairs of corresponding angles.

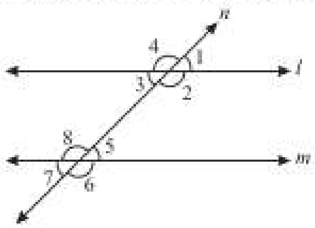
(ii) Alternate interior angles: $\angle 3$ and $\angle 5$, $\angle 2$ and $\angle 8$, are two pairs of alternate interior angles.

(iii) Alternate exterior angles: $\angle 1$ and $\angle 7$, $\angle 4$ and $\angle 6$ are two pairs of alternate exterior angles.

(iv) Consecutive interior angles: In the figure, $\angle 2$ and $\angle 5$, $\angle 5$ and $\angle 8$, $\angle 8$ and $\angle 3$, $\angle 3$ and $\angle 2$ are four pairs of consecutive interior angles.

Interior angles on the same side of a transversal are called cointerior angles. In the fig. $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$ are two pairs of cointerior angles.

When a transversal intersects two parallel lines:



In the figure two parallel lines l and m are intersected by a transversal line n, then

(a) Two angles of each pair of corresponding angles are equal i.e. $\angle 1 = \angle 5$; $\angle 2 = \angle 6$; $\angle 4 = \angle 8$; $\angle 3 = \angle 7$

(b) Two angles of each pair of alternate interior angles are equal i.e.
$\angle 2 = \angle 8$; $\angle 3 = \angle 5$

(c) Two angles of each pair of alternate exterior angles are equal i.e.
$\angle 1 = \angle 7$; $\angle 4 = \angle 6$

(d) Any two consecutive interior angles are supplementary. i.e. their sum is 180°. Hence
$\angle 2 + \angle 5 = 180°$; $\angle 5 + \angle 8 = 180°$; $\angle 8 + \angle 3 = 180°$; $\angle 3 + \angle = 180°$

Note that

(i) If two angles of any pair of corresponding angles are equal, then the two lines are parallel.

(ii) If two angles of any pair of alternate interior angles are equal, then the two lines are parallel.

(iii) If two angles of any pair of alternate exterior angles are equal, then the two lines are parallel.

(iv) If any two consecutive interior angles are supplementary (i.e. their sum is 180°), then the two lines are parallel.

Acute angle: An angle is said to be acute angle if it is less than 90°.

Here 0° ∠ θ ∠ 90°, hence θ is acute angle.

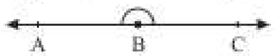
Right angle: An angle is said to be right angle if it is of 90°.

Here θ is right angle.

Obtuse angle: An angle is said to be obtuse angle if it is of more than 90°.

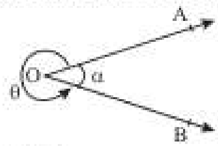
Here θ is obtuse angle.

Straight angle: An angle is said to be straight angle if it is of 180°.



Here θ is a straight angle.

Reflex angle: An angle is said to be reflex angle if it is of greater than 180°.

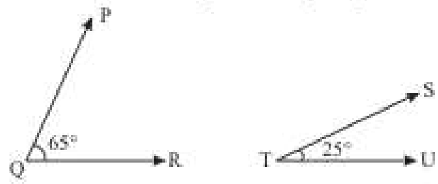


Here θ is the reflex angle.
Reflex angle θ is written as

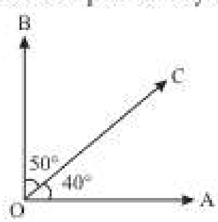
$$\theta = 360° - \angle AOB \text{ (or } 360° - \alpha)$$

Here $\angle AOB$ or α is less than 180°

Complementary angles: Two angles, the sum of whose measures is 90°, are called the complementary angles.

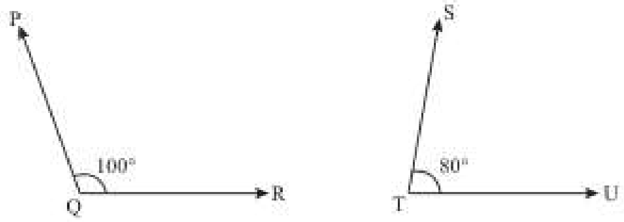


$\angle PQR$ and $\angle STU$ are complementary angles.

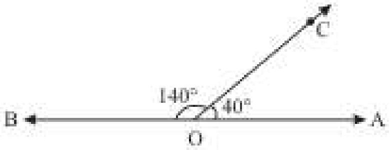


In figure $\angle AOC$ and $\angle BOC$ are also complementary angles.

Supplementary angles: Two angles, the sum of whose measures is 180°, are called the supplementary angles.



In figure, $\angle PQR$ and $\angle STU$ are supplementary angles.



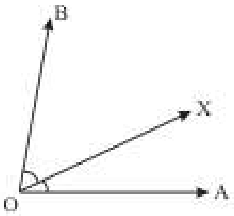
In figure, $\angle AOC$ and $\angle BOC$ are also supplementary angles.

Adjacent angles: Two angles are called adjacent angles, if

(i) they have the same vertex

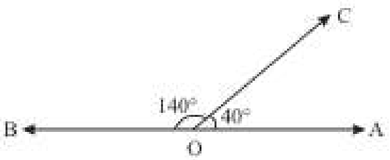
(ii) they have a common arm and

(iii) non-common arms are on either side of the common arm.



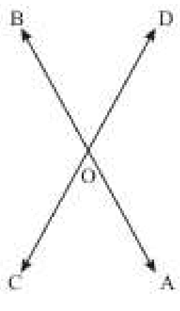
In figure, $\angle AOX$ and $\angle BOX$ are adjacent angles because O is the common vertex, OX is common arm, non-common arm OA and OB are on either side of OX.

Linear pair of angles: Two adjacent angles are said to form a linear pair of angles, if their non common arms are two opposite rays. In other words if the sum of two adjacent angles is 180°, then they are said to form a linear pair of angles.



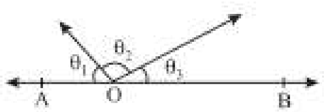
In figure, $\angle AOC$ and $\angle BOC$ are linear pair angles.

Vertically opposite angles: Two angles are called a pair of vertically opposite angles, if their arms form two intersecting lines.



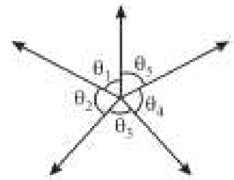
In figure, $\angle AOC$ and $\angle BOD$ form a pair of vertically opposite angles. Also $\angle AOD$ and $\angle BOC$ from a pair of vertically opposite angles.

Angles on one side of a line at a point on the line: Sum of all the angles on any one side of a line at a point on the line is always 180°.



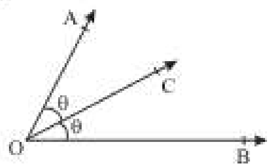
Here AOB is a straight line, hence in figure, $\theta_1 + \theta_2 + \theta_3 = 180°$.

Angle around a point: Sum of all the angles around a point is always 360°.



Here θ_1, θ_2, θ_3, θ_4 and θ_5 are the angles around a point. Hence $\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 = 360°$

Angle bisector: An angle bisector is a ray which bisects the angle whose initial point be the vertex of the angle.



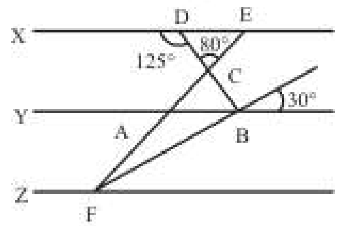
Since $\angle AOC = \angle BOC = \theta$

Hence ray OC is the bisector of $\angle AOB$.

Illustration 1: Three straight lines, *X*, *Y* and *Z* are parallel and the angles are as shown in the figure above. What is ΔAFB equal to?

(a) 20° **(b) 15°**

(c) 30° **(d) 10°**



Solution: (b) $\angle CDE = 180° - 125° = 55°$

In ΔDCE,

$\angle CED = 180° - 55° - 80° = 45°$

and $\angle ABF = 30°$ (vertically opposite)

Also, $\angle ABF = \angle BFM = 30°$ (alternate angle)

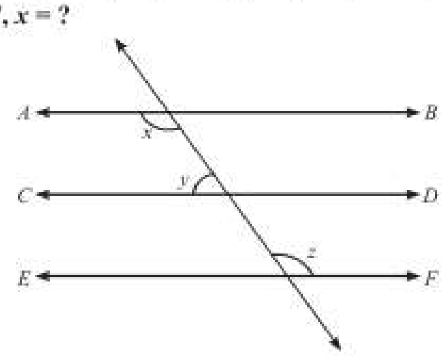
and, $\angle DEF = \angle EFM$ (alternate angle)

$\angle EFM = 45°$

$\Rightarrow \angle EFB + \angle BFM = 45° \Rightarrow \angle EFB = 45° - 30°$

$\Rightarrow \angle AFB = 15°$

Illustration 2: In figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, $x = ?$



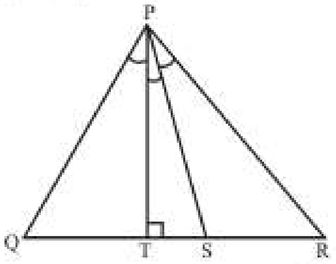
(a) 112° **(b) 116°**

(c) 96° **(d) 126°**

Solution: (d) As $y + z = 180°$, $\therefore y = 54°$

$$x + y = 180°$$
$$x = 180 - 54 = 126°$$

Illustration 3: In the ΔPQR, PS is the bisector of $\angle P$ and $PT \perp QR$, then $\angle TPS$ is equal to



(a) $\angle Q + \angle R$ (b) $90° + \frac{1}{2}\angle Q$

(c) $90° - \frac{1}{2}\angle R$ (d) $\frac{1}{2}(\angle Q - \angle R)$

Solution: (d) PS is the bisector of $\angle QPR$

$\therefore$ $\angle 1 + \angle 2 = \angle 3$ ...(1)

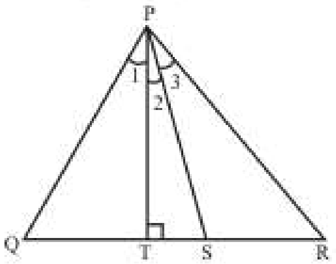
$\Rightarrow$ $\angle Q = 90° - \angle 1$

$\angle R = 90° - \angle 2 - \angle 3$

So, $\angle Q - \angle R = [90° - \angle 1] - [90° - \angle 2 - \angle 3]$

$\Rightarrow$ $\angle Q - \angle R = \angle 2 + \angle 3 - \angle 1$

$= \angle 2 + (\angle 1 + \angle 2) - \angle 1$ [From Eq. (1)]



$\Rightarrow$ $\angle Q - \angle R = 2\angle 2 \Rightarrow \frac{1}{2}(\angle Q - \angle R) = \angle TPS$

POLYGONS

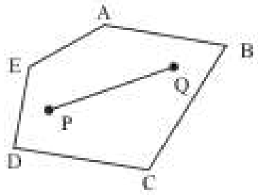
Polygons are closed plane figures formed by series of line segments, e.g. triangles, rectangles, etc.

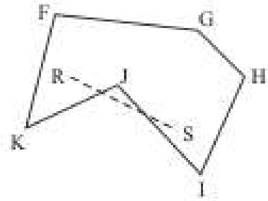
Polygons can also be classified into convex and concave polygons.

A convex polygon is a polygon in which any line segment joining any two points of the polygon always lies completely inside the polygon, otherwise the polygon is concave polygon.

ABCDE is a convex polygon because any line segment joining any two points of the polygon completely lies inside the polygon.

FGHIJK is a concave polygon because line segment joining two points *R* and *S* of the polygon does not lie completely inside the polygon.





Convex polygons can be classified into regular and irregular polygons.

(a) Regular polygon: A convex polygon whose all the sides are equal and also all the angles equal is called a regular polygon.
A regular polygon is simply called polygon.

(b) Irregular Polygon: A convex polygon in which all the sides are not equal or all the angles are not of the same measure is called an irregular polygon.

Polygons can also be divided on the basis of number of sides they have

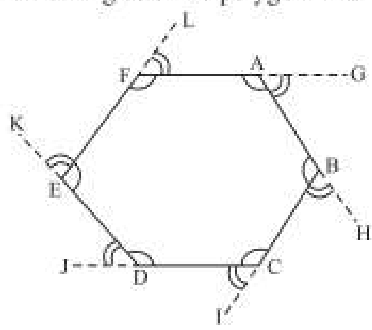
No. of sides of the polygon	Name of the polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
:	:
etc.	etc.

Interior and Exterior Angles of a Polygon

An angle inside a polygon between any two adjacent sides at a vertex of the polygon is called an interior angle of the polygon. An angle outside a polygon made by a side of the polygon with the its adjacent side produced is called an exterior angle of the polygon.

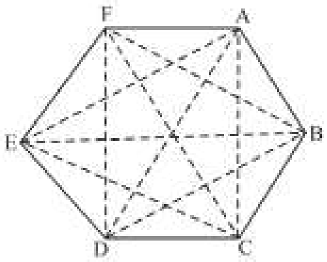
In the figure *ABCDEF* is a polygon.

$\angle FAB$, $\angle ABC$, $\angle BCD$, $\angle CDE$, $\angle DEF$ and $\angle EFA$ are interior angles of the polygon *ABCDEF*.



$\angle BAG$, $\angle CBH$, $\angle DCI$, $\angle EDJ$, $\angle FEK$ and $\angle AFL$ are exterior angles of the polygon *ABCDEF*.

Diagonals of a Polygon



A diagonal of a polygon is a line segment connecting two non-consecutive vertices of the Polygon.

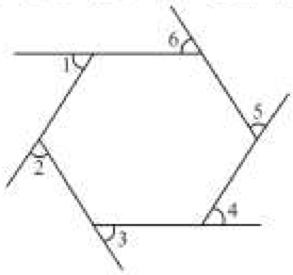
In the figure, diagonals are drawn by dotted line segments.

Properties of Polygons

(i) Sum of all the interior angles of a polygon with 'n' sides $= (n-2)\ 180°$

(ii) Sum of all the exterior angles of a polygon = 360°

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$



(iii) Perimeter of a regular polygon with a side length of $a = n \times a$

(iv) No. of sides of a regular polygon = an exterior

$$= \frac{360°}{\text{An exterior angle}}$$

(v) Number of diagonals of a polygon with n sides

$$= \frac{n(n-3)}{2}$$

Illustration 4: An interior angle of a regular polygon is 135°. Find the number of sides of the polygon.

Solution: Since interior angle of the regular polygon = 135°, hence exterior angle = 180° – 135° = 45°

$$\therefore \quad \text{No. of sides} = \frac{360°}{\text{An exterior angle}} = \frac{360°}{45°} = 8$$

$$\therefore \quad \text{No. of sides} = 8$$

Illustration 5: An interior angle of a regular polygon is 100° more than its an exterior angle. Find the number of sides the polygon.

Solution: Let measure of each exterior angle be $x°$.

Then measure of each interior angle $= (x + 100)$

Now $\quad x + (x + 100) = 180$

$\Rightarrow \quad 2x = 80 \;\Rightarrow\; x = 40$

$$\text{Now} \quad \text{number of sides} = \frac{360}{\text{An exterior angle}} = \frac{360}{40} = 9.$$

TRIANGLES

A triangle is a convex polygon having three sides.

A triangle is represented by the symbol Δ.

Triangles can be classified on the basis of their sides or angles.

On the basis of sides, triangles are of the following types

(a) **Equilateral triangle:** All the three sides are equal

(b) **Isosceles triangle:** Two sides are equal

(c) **Scalene triangle:** All the three sides are unequal.

On the basis of angles, triangles are of the following types

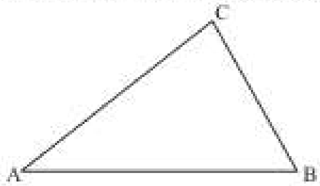
(a) **Acute angled triangle:** Each interior angle is less than 90°.

(b) **Right angled triangle:** One of the interior angle is equal 90°.

(c) **Obtuse angled triangle:** One of the interior angle is more than 90°.

BASIC PROPERTIES AND SOME IMPORTANT THEOREMS OF TRIANGLES

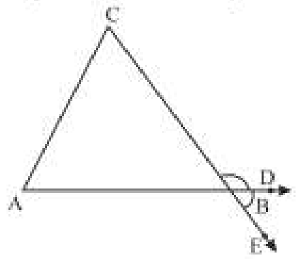
1. Sum of measures of the interior angles of a triangle is 180°.



In ΔABC, $\angle CAB + \angle ABC + \angle ACB = 180°$

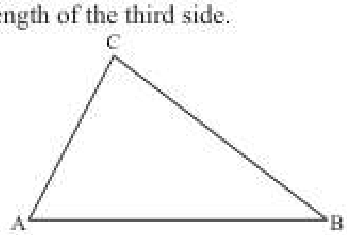
or $\quad \angle A + \angle B + \angle C = 180°$

2. The exterior angle of a triangle is equal to the sum of the opposite (not adjacent) interior angles



In ΔABC, $\angle CBD = \angle A + \angle C = \angle ABE$

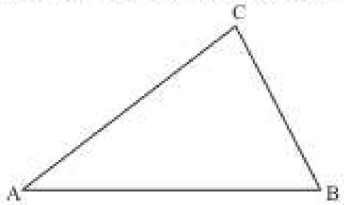
3. Sum of the lengths of any two sides of a triangle is greater than the length of the third side.



(i) $AB + AC > BC$ (ii) $AC + BC > AB$

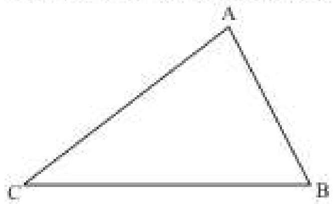
(iii) $AB + BC > AC$

4. Difference between the lengths of any two sides of a triangle is smaller than the length of the third side.



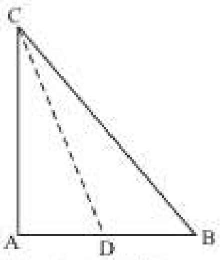
(i) $|AB - BC| < AC$ (ii) $|AC - AB| < BC$
(iii) $|AC - BC| < AB$

5. In any triangle, side opposite to greatest angle is largest and side opposite to smallest angle is smallest.



In ΔABC, if $\angle A > \angle B > \angle C$, then BC is the largest side and AB is the smallest side.

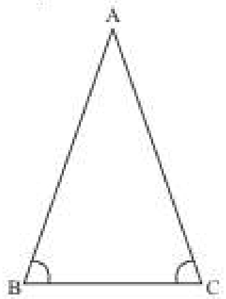
6. In any triangle line joining any vertex to the mid point of its opposite side is called a median of the opposite side of the triangle.



In $\Delta ABCD$ is the mid point of AB
Hence CD is a median of ΔABC.
A triangle can have 3 medians.

Any median of a triangle divides the triangle into two triangles of equal areas.

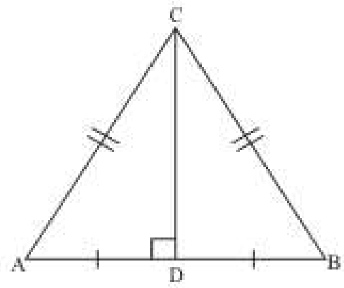
7. Sides opposite to equal angles in a triangle are equal.



In ΔABC, $\angle B = \angle C$
$\therefore$ $AB = AC$

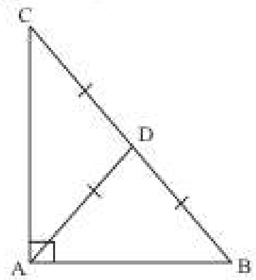
Converse of this property is also true.

8. In an isosceles triangle, if a perpendicular is drawn to unequal side from its opposite vertex, then
   (a) The perpendicular is the median
   (b) The perpendicular bisects the vertex angle.



ΔABC is an isosceles triangle in which $AC = BC$.
CD is perpendicular to AB, hence CD is a median and $\angle ACD = \angle BCD$

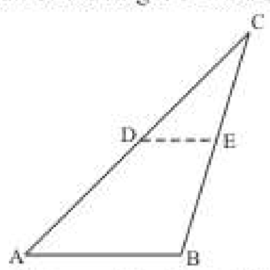
9. In a right angled triangle, the line joining the vertex of the right angle to the mid point of the hypotenuse is half the length of the hypotenuse.



In ΔABC, $\angle BAC = 90°$ and D is the mid point of BC, then

$$AD = \frac{1}{2}BC = BD = CD$$

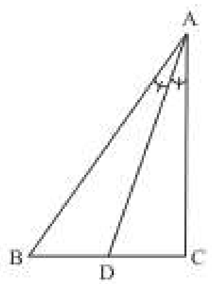
10. **Mid-point theorem:** In any triangle, line segment joining the mid points of any two sides is parallel to the third side and equal to half of the length of third side.



In ΔABC, D and E are mid points of sides AC and BC, then

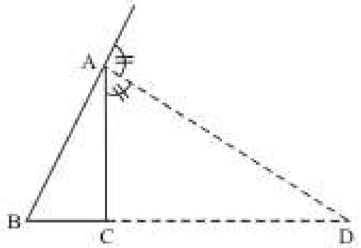
DE is parallel to AB i.e. $DE \parallel AB$ and $DE = \frac{1}{2} AB$

11. **Angle Bisector Theorem:** Bisector of an angle (internal or external) of a triangle divides the opposite side (internally or externally) in the ratio of the sides containing the angle.
For example:



In figure AD is the bisector of exterior $\angle BAC$

$$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$$

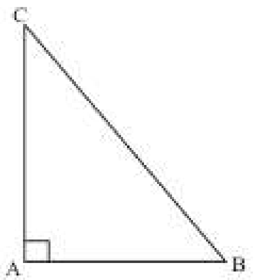


In figure AD is the bisector of exterior $\angle BAC$.

$$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$$

Converse of the angle bisector theorem is also true.

12. Pythagoras Theorem: In a right angled triangle.
Square of longest or hypotenuse = Sum of square of other two sides.

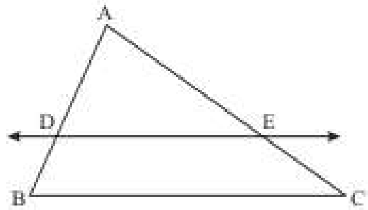


In figure ΔABC is a triangle right angled at A.

$$\therefore \quad (BC)^2 = (AB)^2 + (AC)^2$$

Converse of this theorem is also true.

13. Basic Proportionality Theorem (BPT): If a line is drawn parallel to one side of a triangle which intersects the other two sides in distinct points, the other two sides are divided in the same ratio.



In ΔABC, $DE \parallel BC$,

Then, $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

This theorem is also known as Thalse theorem.
Converse of this theorem is also true.

Illustration 6: In a triangle ABC, $\angle A = x$, $\angle B = y$, and $\angle C = y + 20$.

If $4x - y = 10$, then the triangle is :

(a) Right-angled **(b) Obtuse-angled**
(c) Equilateral **(d) None of these**

Solution: (a) We have, $x + y + (y + 20) = 180$

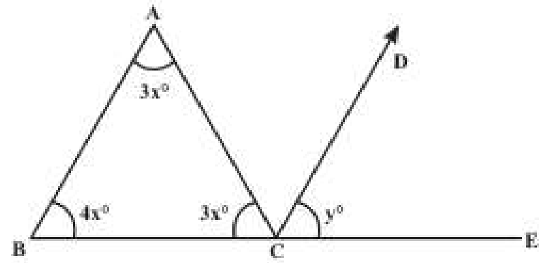
or $\quad x + 2y = 160 \quad$ ...(1)

and $\quad 4x - y = 10 \quad$ ...(2)

From (1) and (2), $y = 70$, $x = 20$

Angles of the triangles are 20°, 70°, 90°. Hence the triangle is a right angled.

Illustration 7: In the given figure, $CD \parallel AB$. Find y.



(a) 79° **(b) 72°**
(c) 74° **(d) 77°**

Solution: (b) In ΔABC,

$\angle ABC + \angle BCA + \angle CAB = 180°$

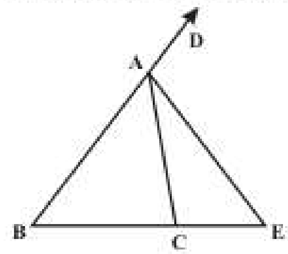
$\Rightarrow \quad 4x + 3x + 3x = 180° \Rightarrow 10°x = 180° \Rightarrow x = 18°$

Now, $\quad \angle ABC = \angle DCE$

(corresponding angles are equal)

$\Rightarrow \quad \angle DCE = 4x° \Rightarrow y = 4 \times 18° = 72°$

Illustration 8: In the adjoining figure, AE is the bisector of exterior $\angle CAD$ meeting BC produced in E. If $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm, then CE is equal to



(a) 6 cm **(b) 12 cm**
(c) 18 cm **(d) 20 cm**

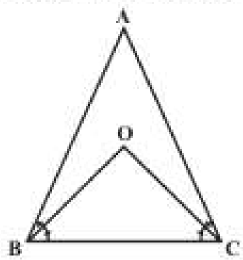
Solution: (c) $\dfrac{BE}{CE} = \dfrac{AB}{AC}$ as AE is an exterior angle bisector.

Let $\quad CE = x$, $BE = BC + EC = 12 + x$

$\Rightarrow \quad \dfrac{12 + x}{x} = \dfrac{10}{6} \quad \Rightarrow (12 + x)\,6 = 10x$

$\Rightarrow \quad 72 + 6x = 10x \Rightarrow 4x = 72 \Rightarrow x = 18$ cm

Illustration 9: OB and OC are respectively the bisectors of $\angle ABC$ and $\angle ACB$. Then, $\angle BOC$ is equal to



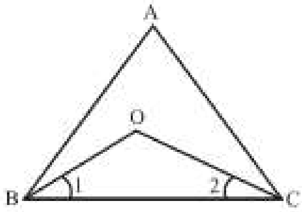
(a) $90° - \dfrac{1}{2}\angle A$ **(b) $90° + \angle A$**

(c) $90° + \dfrac{1}{2}\angle A$ **(d) $180° - \dfrac{1}{2}\angle A$**

Solution: (c) In ΔBOC,

$\angle 1 + \angle 2 + \angle BOC = 180°$

$\angle A + \angle B + \angle C = 180°. \quad$ .... (1)



$$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90^\circ$$

$$\Rightarrow \quad \frac{1}{2}(\angle A) + \angle 1 + \angle 2 = 90^\circ \Rightarrow \angle 1 + \angle 2 = 90^\circ - \frac{1}{2}\angle A$$

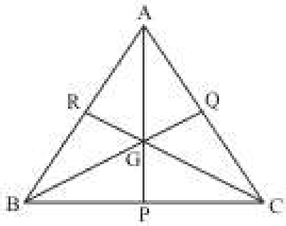
Put $\angle 1 + \angle 2$ in Eq. (1), we get

$$\angle BOC = 180^\circ - 90^\circ - \left(90^\circ - \frac{1}{2}\angle A\right)$$

$$= 90^\circ + \frac{1}{2}\angle A$$

IMPORTANT TERMS RELATED TO A TRIANGLE

1. **Medians and Centroid:** We know that a line segment joining the mid point of a side of a triangle to its opposite vertex is called a median.



AP, BQ and CR are medians of ΔABC where P, Q and R are mid points of sides BC, CA and AB respectively.

(i) Three medians of a triangle on concurrent. The point of concurrent of three medians is called Centroid of the triangle denoted by G.

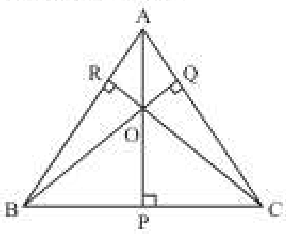
(ii) Centroid of the triangle divides each median in the ratio 2 : 1

i.e. $AG : GP = BG : GQ = CG : GR = 2 : 1$, where G is the centroid of ΔABC.

2. **Altitudes and Orthocentre:** A perpendicular drawn from any vertex of a triangle to its opposite side is called altitude of the triangle. There are three altitudes of a triangle.

In the figure, AP, BQ and CR are altitudes of ΔABC.

The altitudes of a triangle are concurrent (meet at a point) and the point of concurrency of altitudes is called Orthocentre of the triangle, denoted by O.



In figure, AP, BQ and CR meet at O, hence O is the orthocentre of the triangle ABC.

Note: The angle made by any side at the orthocentre and at the vertex opposite to the side are supplementary angle.

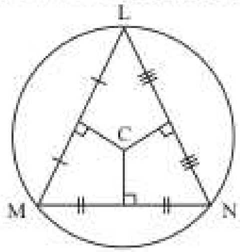
Hence, $\angle BAC + \angle BOC = \angle ABC + \angle AOC = \angle ACB + \angle AOB = 180^\circ$.

3. **Perpendicular Bisectors and Circumcentre:** A line which is perpendicular to a side of a triangle and also bisects the side is called a perpendicular bisector of the side.

(i) Perpendicular bisectors of sides of a triangle are concurrent and the point of concurrency is called circumcentre of the triangle, denoted by 'C'.

(ii) The circumcentre of a triangle is centre of the circle that circumscribes the triangle.

(iii) Angle formed by any side of the triangle at the circumcentre is twice the vertical angle opposite to the side.



In figure, perpendicular bisectors of sides LM, MN and NL of ΔLMN meets at C. Hence C is the circumcentre of the triangle LMN.

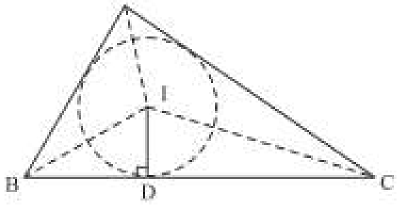
$\angle MCN = 2\ \angle MLN$.

4. **Angle Bisectors and Incentre:** Lines bisecting the interior angles of a triangle are called angle bisectors of triangle.

(i) Angle bisectors of a triangle are concurrent and the point of concurrency is called Incentre of the triangle, denoted by I.

(ii) With I as centre and radius equal to length of the perpendicular drawn from I to any side, a circle can be drawn touching the three sides of the triangle. So this is called incircle of the triangle. Incentre is equidistant from all the sides of the triangle.

(iii) Angle formed by any side at the incentre is always 90° more than half the vertex angle opposite to the side.

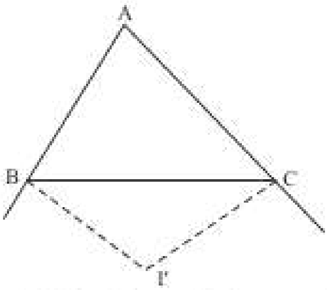


In figure AI, BI, CI are angle bisectors of ΔABC.

Hence I is the incentre of the ΔABC and

$$\angle BIC = 90^\circ + \frac{1}{2}\angle A,\ \angle AIC = 90^\circ + \frac{1}{2}\angle B$$

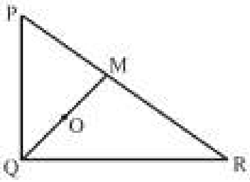
and $$\angle AIB = 90^\circ + \frac{1}{2}\angle C$$



If BI' and CI' be the angle bisectors of exterior angles at B and C, then

$$\angle BI'C = 90° - \frac{1}{2}\angle A.$$

Illustration 10: If in the given figure $\angle PQR = 90°$, O is the centroid of ΔPQR, $PQ = 5$ cm and $QR = 12$ cm, then OQ is equal to



(a) $3\frac{1}{2}$ **cm** **(b)** $4\frac{1}{3}$ **cm**

(c) $4\frac{1}{2}$ **cm** **(d)** $5\frac{1}{3}$ **cm**

Solution: (b) By Pythagoras theorem,

$PR = \sqrt{PQ^2 + QR^2} = \sqrt{5^2 + 12^2} = 13$ cm

$\therefore$ O is centroid $\Rightarrow$ QM is median and M is mid-point of PR.

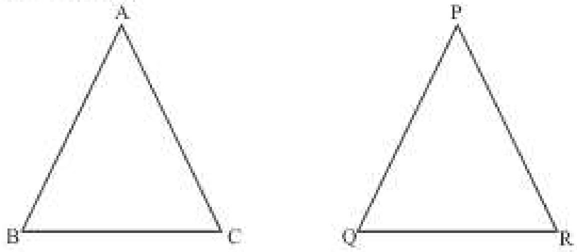
$QM = PM = \dfrac{13}{2}$

$\therefore$ Centroid divides median in ratio 2 : 1.

$\therefore OQ = \dfrac{2}{3}\ QM = \dfrac{2}{3} \times \dfrac{13}{2} = \dfrac{13}{3}$ $\therefore OQ = 4\dfrac{1}{3}$ cm

CONGRUENCY OF TWO TRIANGLES

Two triangles are congruent if they are of the same shape and size i.e .if any one of them can be made to superpose on the other it will cover exactly.



If two triangles ABC and PQR are congruent then 6 elements (i.e. three sides and three angles) of one triangle are equal to corresponding 6 elements of other triangle.

(i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

(ii) $AB = PQ$, $BC = QR$, $AC = PR$

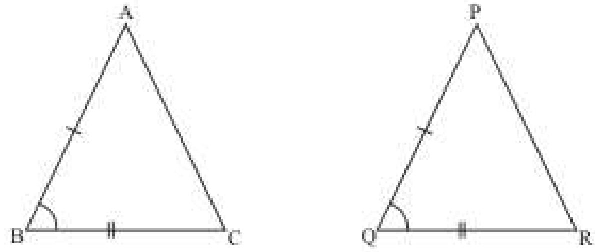
This is symbolically written as $\Delta ABC \cong \Delta PQR$

Note: In two congruent triangles, sides opposite to equal angles are corresponding sides and angles opposite to equal sides are corresponding angles.

Conditions of Congruency

There are 4 conditions of congruency of two triangles.

1. **SAS (Side-Angle-Side) Congruency:** If two sides and the included angle between these two sides of one triangle is equal to corresponding two sides and included angle between these two sides of another triangle, then the two triangles are congruent.



In ΔABC and ΔPQR

$AB = PQ$,
$BC = QR$

and $\angle ABC = \angle PQR$

$\therefore$ $\Delta ABC \cong \Delta PQR$ [by SAS congruency]

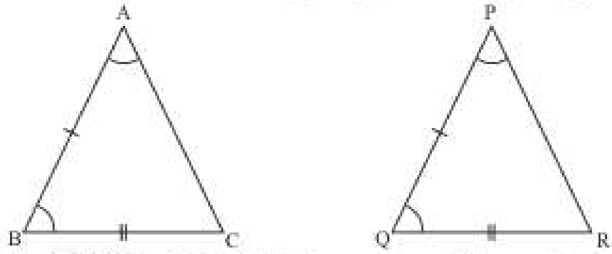
Here $\cong$ is the sign of congruency.

2. **ASA (Angle-Side-Angle) Congruency:** If two angles and included side between these two angles of one triangle are equal to corresponding angles and included side between these two angles of another triangle, then two triangles are congruent.

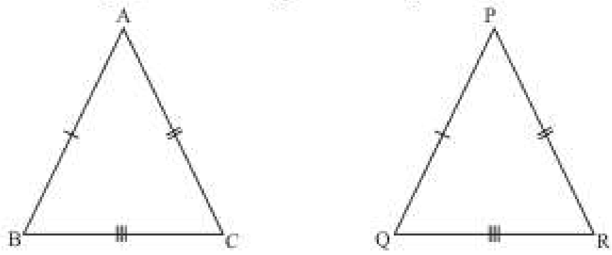
In ΔABC and ΔPQR

$\angle A = \angle P$
$\angle B = \angle Q$
$AB = PQ$

$\therefore$ $\Delta ABC \cong \Delta PQR$ [by ASA congruency]



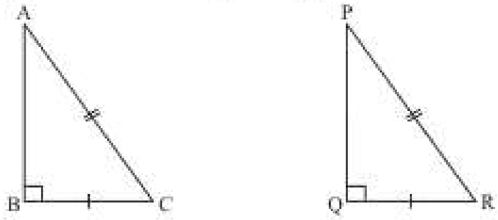
3. **SSS (Side-Side-Side) Congruency:** If three sides of one triangle are equal to corresponding three sides of another triangle, the two triangles are congruent.



In ΔABC and ΔPQR

$$AB = PQ$$
$$BC = QR$$
$$CA = RP$$
$\therefore \quad \Delta ABC \cong \Delta PQR$ [by SSS congruency]

4. **RHS (Rightangle-Hypotenuse-Side) Congruency:** Two right angled triangles are congruent to each other if hypotenuse and one side of one triangle are equal to hypotenuse and corresponding side of another triangle.



In ΔABC and ΔPQR

$$\angle ABC = \angle PQR = 90°$$
$$AC = PR$$
$$BC = QR$$
$\therefore \quad \Delta ABC \cong \Delta PQR$ [by RHS congruency]

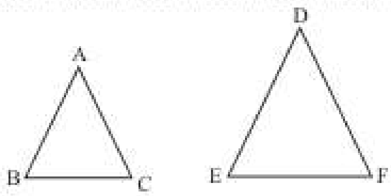
SIMILARITY OF TWO TRIANGLES

Two triangles are said to be similar, if their shapes are the same but their size may or may not be equal.

When two triangles are similar, then

(i) all the corresponding angles are equal and
(ii) all the corresponding sides are in the same ratio (or proportion)

Note: In two similar triangles, sides opposite to equal angles are called corresponding sides. And angles opposite to side proportional to each other are called corresponding angles.



If ΔABC and ΔDEF are similar, then

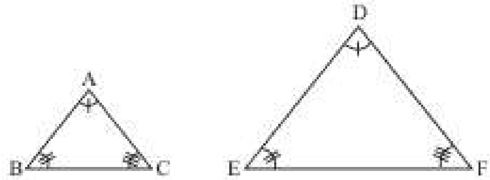
$$\angle A = \angle D$$
$$\angle B = \angle E$$
$$\angle C = \angle F$$
and $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$

$\Delta ABC \sim \Delta DEF$, read as triangle ABC is similar to triangle DEF. Here ~ is the sign of similarity.

Conditions of Similarity

There are 4 conditions of similarity.

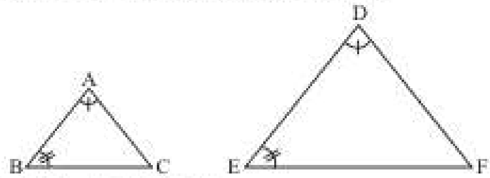
1. **AAA (Angle–Angle–Angle) Similarity:** Two triangles are said to be similar, if their all corresponding angles are equal. For example:



In ΔABC and ΔDEF, if

$$\angle A = \angle D$$
$$\angle B = \angle E$$
$$\angle C = \angle F$$
Then $\Delta ABC \sim \Delta DEF$ [By AAA Similarity]

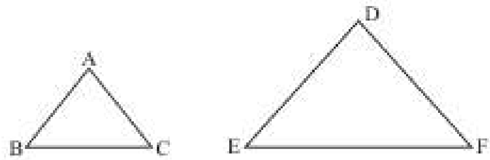
Corollary AA (Angle-Angle) Similarity: If two angles of one triangle are respectively equal to two angles of another triangles, then two triangles are similar.



In ΔABC and ΔDEF, if

$$\angle A = \angle D$$
$$\angle B = \angle E$$
then $\Delta ABC \sim \Delta DEF$ [By AA Similarity]

2. **SSS (Side–Side–Side) Similarity:** Two triangles are said to be similar, if sides of one triangle are proportional (or in the same ratio of) to the sides of the other triangle:
For example:

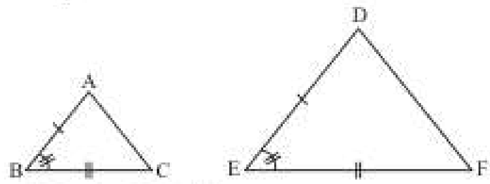


In ΔABC and ΔDEF, if

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

Then $\Delta ABC \sim \Delta DEF$ [By SSS Similarity]

3. **SAS (Side–Angle–Side) Similarity:** Two triangles are said to be similar if two sides of a triangle are proportional to the two sides of the other triangle and the angles included between these sides of two triangles are equal.
For example:

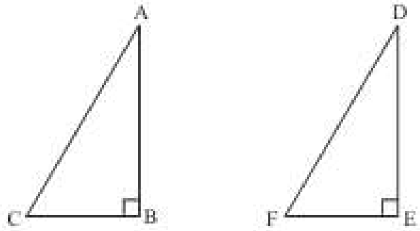


In ΔABC and ΔDEF, if

$$\frac{AB}{DE} = \frac{BC}{EF}$$
and $\angle B = \angle E$

Then, $\Delta ABC \sim \Delta DEF$ [By SAS Similarity]

4. RHS (Rightangle-Hypotenuse-Side) Similarity: Two triangles are said to be similar if one angle of both triangle is right angle and hypotenuse of both triangles are proportional to any one other side of both triangles respectively.
   For example:



In ΔABC and ΔDEF, if

$$\angle B = \angle E\ [= 90°]$$

$$\frac{AC}{DF} = \frac{AB}{DE}$$

Then $\Delta ABC \sim \Delta DEF$ [By RHS similarity]

Note: In similar triangles,
Ratio of medians = Ratio of corresponding heights
= Ratio of circumeradii
= Ratio of inradii

Theorem

If two triangles are similar, then ratio of areas of two similar triangle is equal to the ratio of square of corresponding sides.

Illustration 11: *D* and *E* are the points on the sides *AB* and *AC* respectively of a ΔABC and *AD* = 8 cm, *DB* = 12 cm, *AE* = 6 cm and *EC* = 9 cm, then *BC* is equal to

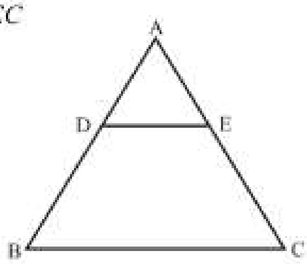
(a) $\frac{2}{5}DE$ (b) $\frac{5}{2}DE$

(c) $\frac{3}{2}DE$ (d) $\frac{2}{3}DE$

Solution: (b) As in ΔADE and ΔABC

$$\frac{AD}{AB} = \frac{8}{20} = \frac{2}{5}, \frac{AE}{AC} = \frac{6}{15} = \frac{2}{5}$$

So, $\frac{AD}{AB} = \frac{AE}{EC}$



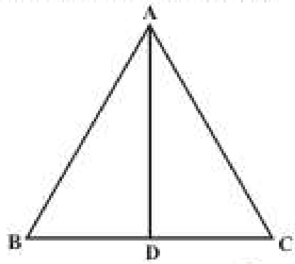
and $\angle A = \angle A$ (common)

$\Delta ADE \sim \Delta ABC$

$$\therefore \quad \frac{DE}{BC} = \frac{AD}{AB} \Rightarrow \frac{DE}{BC} = \frac{2}{5}$$

$$\Rightarrow \quad BC = \frac{5}{2}DE$$

Illustration 12: In a right angled ΔABC in which $\angle A = 90°$. If $AD \perp BC$, then the correct statement is



(a) $AB^2 = BD \times DC$ (b) $AB^2 = BD \times AD$

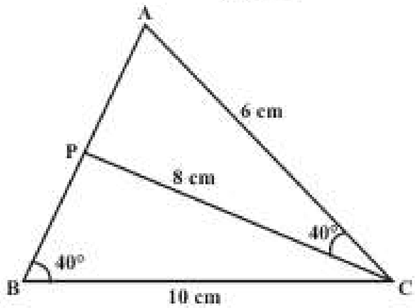
(c) $AB^2 = BC \times DC$ (d) $AB^2 = BC \times BD$

Solution: (d) Clearly, $\Delta ABD \sim \Delta CBA$

$$\Rightarrow \quad \frac{AB}{BD} = \frac{CB}{BA}$$

$$\Rightarrow \quad AB^2 = BC \times BD$$

Illustration 13: From the adjoining diagram, calculate
(i) *AB* **(ii) *AP***



Solution: In ΔAPC and ΔABC,

$$\angle ACP = \angle ABC$$

$$\angle A = \angle A$$

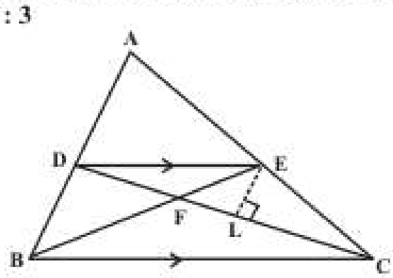
$$\Rightarrow \quad \Delta ACP \sim \Delta ABC \Rightarrow \frac{AP}{AC} = \frac{PC}{BC} = \frac{AC}{AB}$$

$$\therefore \quad \frac{AP}{6} = \frac{8}{10} = \frac{6}{AB}$$

$$\Rightarrow \quad AP = 6 \times \frac{8}{10} = 4.8 \text{ and } AB = \frac{60}{8} = 7.5$$

$$\Rightarrow \quad AP = 4.8 \text{ cm and } AB = 7.5 \text{ cm}$$

Illustration 14: In the adjoining figure, *DE* || *BC* and *AD* : *DB* = 4 : 3



Find $\frac{AD}{AB}$ and then $\frac{DE}{BC}$

Solution: Since the sides of similar triangles are proportional, we have

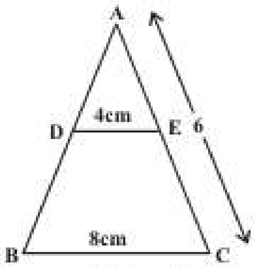
$$\frac{AD}{AB} = \frac{DE}{BC}$$

But, $\frac{AD}{DB}=\frac{4}{3} \Rightarrow \frac{AD}{AD+DB}=\frac{4}{4+3} \Rightarrow \frac{AD}{AB}=\frac{4}{7}$

$\therefore \quad \frac{DE}{BC}=\frac{AD}{AB}=\frac{4}{7}$

Illustration 15: In the given figure, DE parallel to BC. If AD = 2 cm, DB = 3 cm and AC = 6 cm, then AE is

(a) 2.4 cm **(b) 1.2 cm**
(c) 3.4 cm **(d) 4.8 cm**



Solution: (a) The triangles ADE and ABC are similar.

$$\Rightarrow \quad \frac{AD}{AB}=\frac{AE}{AC}$$

or $$\frac{2}{5}=\frac{AE}{6}$$

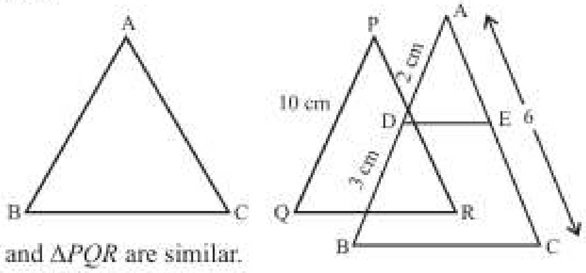
$$\therefore \quad AE=\frac{12}{5}$$

$$= 2.4\text{cm}$$

Illustration 16: The perimeters of two similar triangles ABC and PQR are 36 cm, and 24 cm, respectively. If PQ = 10 cm, then the length of AB is :

(a) 16 cm **(b) 12 cm**
(c) 14 cm **(d) 15 cm**

Solution: (d)



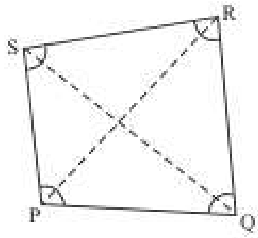
ΔABC and ΔPQR are similar.

$$\frac{AB}{PQ}=\frac{\text{Perimeter of }\Delta ABC}{\text{Perimeter of }\Delta PQR} \Rightarrow \frac{AB}{PQ}=\frac{36}{24}$$

or $$AB=\frac{36}{24}\times 10=15$$

QUADRILATERALS

Quadrilateral is a plane figure bounded by four straight lines. The line segment which joins the opposite vertices of a quadrilateral is called diagonal of the quadrilateral. In figure, $PQRS$ is a quadrilateral and PR, QS are its two diagonals.

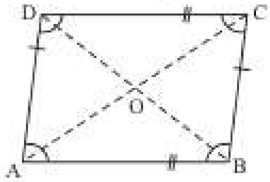


Sum of angles of a quadrilateral = 360°

i.e. $\angle P+\angle Q+\angle R+\angle S=360°$

Types of Quadrilaterals

1. **Parallelogram:** A parallelogram is a quadrilateral with opposite sides parallel and equal.



In figure, $ABCD$ is a parallelogram in which AC and BD as diagonals which intersect each other at O.

Properties:

(i) Opposite sides are equal i.e.
$$AB = DC, \quad AD = BC$$

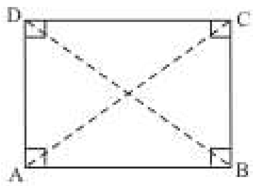
(ii) Opposite sides are parallel i.e.
$$AB \parallel DC \quad \text{and} \quad AD \parallel BC$$

(iii) Opposite angles are equal i.e.
$$\angle BAD = \angle BCD \quad \text{and} \quad \angle ABC = \angle ADC$$

(iv) Diagonals bisect each other, i.e.
$$OA = OC, \quad OB = OD$$

(v) Sum of pair of consecutive angles is 180° i.e.,
$$\angle A+\angle B=180°, \quad \angle B+\angle C=180°,$$
$$\angle C+\angle D=180°, \angle D+\angle A=180°.$$

2. **Rectangle:** A rectangle is a parallelogram with all angles equal to 90°.



In figure, $\angle A=\angle B=\angle C=\angle D=90°$

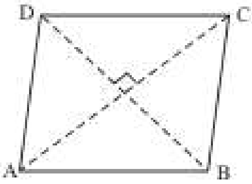
Properties:

(i) In a rectangle
Length of diagonal, are equal i.e.
$$AC=\sqrt{AB^2+BC^2}=BD$$

(ii) In a rectangle diagonals bisect each other.

(iii) All rectangles are parallelogram but all parallelograms are not rectangles.

3. **Rhombus:** A parallelogram is a rhombus if its all sides are equal.



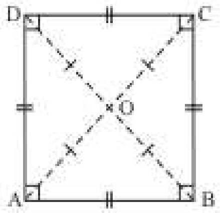
In rhombus $ABCD$, $AB = BC = CD = DA$

Properties:

(i) In a rhombus diagonals bisect each other at right angles i.e. angle between AC and DB is 90°.

(ii) All rhombus are parallelogram but all parallelograms are not rhombus.

4. **Square:** A parallelogram is a square if all the four sides are equal and also all the four angles are equal (i.e. 90°).



In figure, $ABCD$ is a square in which $AB = BC + CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$

Properties:

(i) In a square diagonals are equal i.e. $AC = BD$

(ii) In a square diagonals bisect each other at right angle, i.e.

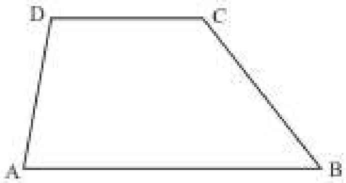
$$OA = OC, \quad OB = OD$$

and $\quad \angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$.

(iii) All square are rhombus but rhombus may or may not be a square.

5. **Trpaezium:** A quadrilateral is a trapezium if one pair of opposite sides are parallel.

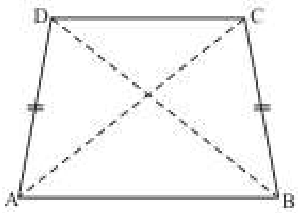
In trapezium $ABCD$, $AB \parallel DC$.



If lateral sides (i.e. non-parallel sides) of a trapezium are equal, then it is called isosceles trapezium.

Properties of isosceles trapezium

In the figure ABCD is an isosceles trapezium, then



(i) $AB \parallel DC$

(ii) $AD = BC$

(iii) Diagonals are equal i.e. $AC = BD$

Diagonal Properties of all Parallelograms

Sr. No.	Diagonal Properties	Type of Parallelogram			
		Parallelogram	Rectangle	Rhombus	Square
1	Diagonals bisect each other	✓	✓	✓	✓
2	Diagonals are equal	×	✓	×	✓
3	Diagonals are at 90° to each other	×	×	✓	✓

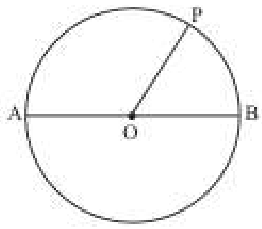
CIRCLES

A circle is a locus i.e. path of a point in a plane which moves in such a way that its distance from a fixed point always remains constant.

In figure, 'O' is the fixed point and P is a moving point in the same plane. The path traced by P is called a circle. Fixed point O is the centre of the circle and the constant distance OP is called radius of the circle.

A diameter is a line segment passing through the centre and joins the two points on the circle in the figure.

AB is the diameter as it passes through the centre and joins the two points on the circle. Diameter = 2 × radius.

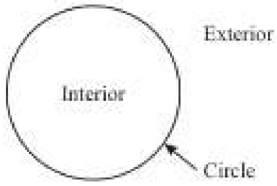


A circle divides the plane in which it lies into three parts.

(i) Inside the circle, called interior of the circle

(ii) The circle

(iii) Outside the circle, called the exterior of the circle.



The circle and its interior make up the circular region.

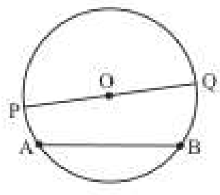
Circumference

Length of a complete circle is called its circumference.

In figure, AB is tangent to circle of radius 'r', which touches the circle at point P.

P is called the point of contact of tangent to the circle. Radius through the point of contact is always perpendicular to the tangent at the point of contact i.e. $OP \perp AB$.

Chord: A line segment joining any two points on the circle is called chord of the circle. A chord which passes through the centre is the diameter of the circle.

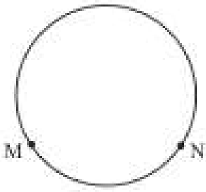


In the figure, O is the centre of the circle.

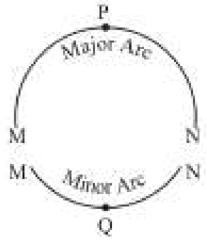
AB and PQ both are chords.

But PQ is the diameter (longest chord) also.

Arcs: A piece of a circle between two points is called an arc. Consider two points M and N on the circle. We find that there are two pieces of circle between M and N. One is longer and other is smaller.



The longer piece is called major arc and smaller piece is called minor arc

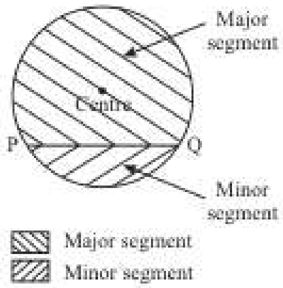


Major arc is denoted by $\widehat{MPN}$ and minor arc is denoted by $\widehat{MQN}$.

When M and N are ends of a diameter then both the arcs are equal and both are called semicircle.

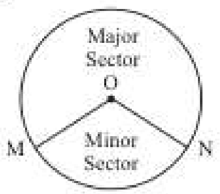
Segment: The region between a chord and an arc of a circle is called a segment.

There are two segments corresponding to two arcs, major segment and minor segment. Major segment is the segment enclosed by major arc. Centre of the circle lies in the major segment. Minor segment is the segment enclosed by minor arc. Centre of the circle does not lie in the minor segment.



If two arcs are equal, then both segments are semi-circles.

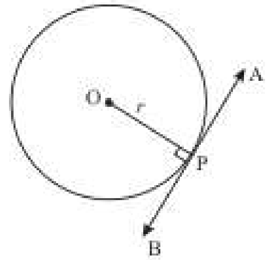
Sector: The region between an arc and the two radii joining the centre to the end point of the arc is called a sector. There are two sectors Minor and Major Sectors.



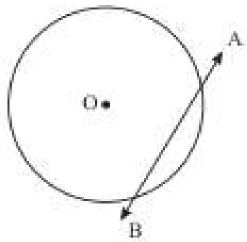
The sector which is larger than semicircular region is called major sector and the region less than the semicircular region is called minor sector.

If both sectors are equal, then each sector is a semi-circle.

Tangent: A tangent is a straight line which touches the circumference of a circle at only one point. A tangent does not intersect the circumference, if produced infinitely on either sides.

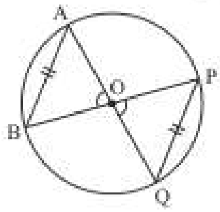


Secant: A secant is a straight line of infinite length which intersects the circumference of a circle at two different points. In figure, AB is a secant.



Basic Properties of a Circle

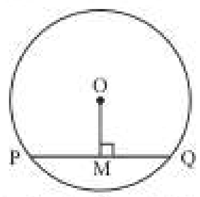
1. Equal chord of a circle subtend equal angles at the centre.



   If $AB = PQ$, then $\angle AOB = \angle POQ$

   The converse is also true.

2. The perpendicular from the centre of a circle to a chord of the circle bisects the chord.



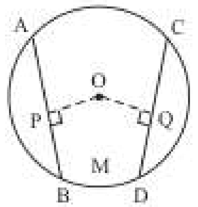
   In figure, PQ is chord of a circle with centre 'O', OM is perpendicular to PQ therefore $PM = MQ$. The converse is also true.

3. One and only one circle can pass through given three non-collinear points.

   If three or more points lie on a line, then they are called collinear points otherwise called non-collinear points.

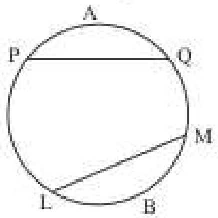
4. Equal chords of a circle are equidistant from the centre of the circle.

In the figure, if $AB = CD$, then $OP = OQ$



The converse is also true.

5. Two equal chords have equal corresponding arcs.

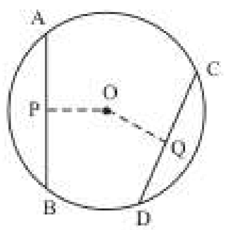


If $PQ = LM$ then

(a) $\widehat{PAQ} = \widehat{LBM}$ (Minor Arc)

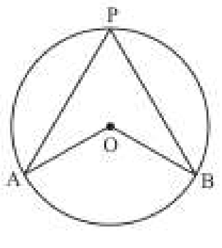
(b) $\widehat{PBQ} = \widehat{LAM}$ (Major Arc)

6. The greater of the two chords is nearer to the centre.



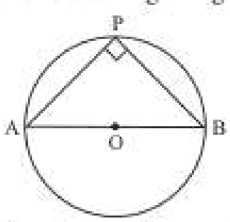
If $AB > CD$, then $OP < OQ$

7. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.



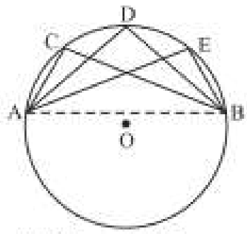
minor arc $\widehat{AB}$ subtend $\angle AOB$ at the centre O and also subtend $\angle APB$ at point P (situated on remaining part of circle). So $\angle AOB = 2\ \angle APB$

8. Angle in a semicircle is a right angle.



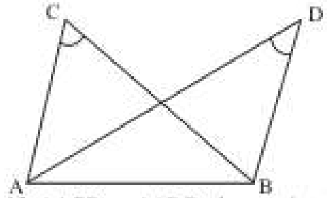
In figure, AOB is a diameter, hence $AOBPA$ is a semicircle, therefore $\angle APB = 90°$.

9. Angles in the same segment of a circle are equal. $\angle ACB$, $\angle ADB$, $\angle AEB$ are in the same segment $ACDEBA$ of the circle.



$\therefore \quad \angle ACB = \angle ADB = \angle AEB$.

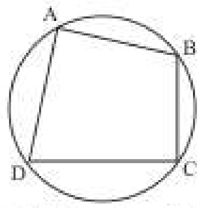
10. If in a plane a line segment joining two points subtends equal angles at two other points lying on the same side of a line containing the line segment, the four points lie on a circle i.e. they are concyclic.



In figure, if $\angle ACB = \angle ADB$, then points A, B, D, C lie on a circle.

11. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

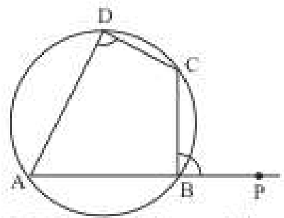
A cyclic quadrilateral is the quadrilateral whose four vertices are concyclic i.e. the four vertices lie on a circle. In figure, $ABCD$ is a cyclic quadrilateral,



$\therefore \quad \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

The converse is also true.

12. If a side of a cyclic quadrilateral is produced the exterior angle so formed is equal to the interior opposite angle.

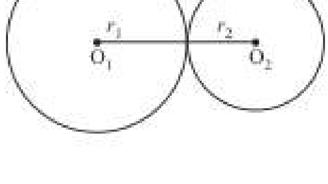


In figure, $ABCD$ is a cyclic quadrilateral,

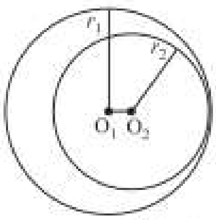
$\therefore \quad \angle CBP = \angle CDA$

13. Two circles C_1 with centre O_1, radius r_1 and C_2 with centre O_2, radius r_2 will touch

(a) Externally, if and only if $O_1O_2 = r_1 + r_2$



(b) Internally, if and only if $O_1O_2 = |r_1 - r_2|$

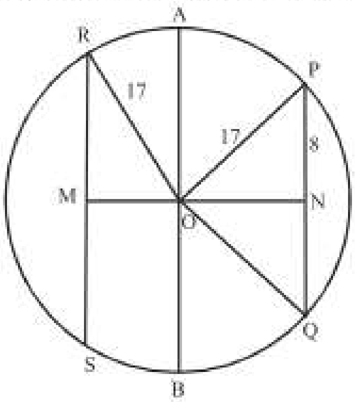


14. Two circles are congruent if their radii are equal.

Illustration 17: In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If length of one chord is 16 cm, then the length of the other one is :

(a) 15 cm **(b) 23 cm**
(c) 30 cm **(d) 34 cm**

Solution: (c) Let PQ and RS be two parallel chords of the circle on the opposite sides of the diameter AB = 16 cm



Now, PN = 8 (Since ON is the perpendicular bisector)

In ΔPON

$$ON^2 = OP^2 - PN^2$$
$$= (17)^2 - (8)^2 = 289 - 64 = 225$$

or $ON = 15 \Rightarrow \therefore OM = 23 - 15 = 8$

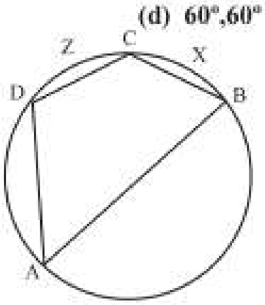
In ΔORM,

$$RM^2 = OR^2 - OM^2$$
$$17^2 - 8^2 = 289 - 64 = 225$$

or $RM = 15 \Rightarrow RS = 15 \times 2 = 30$ cm

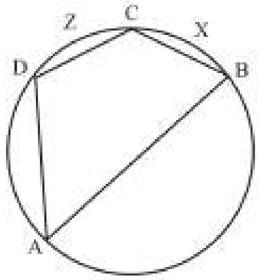
Illustration 18: In the cyclic quadrilateral $ABCD$, $\angle BCD$ = 120°, m (arc DZC) = 7°, find DAB and m (arc CXB).

(a) 60°, 70° **(b) 60°, 40°**
(c) 60°, 50° **(d) 60°,60°**



Solution: (c) m $\angle DAB$ + 180° – 120° = 60° ......(Opposite angles of a cyclic quadrilateral)

m (arc BCD) = 2m $\angle DAB$ = 120°.



$\therefore$ m (arc CXB) = m (BCD) – m (arc DZC)
= 120° – 70° = 50° .

BASIC PYTHAGOREAN TRIPLETS

A Pythagorean triplet is a set of three natural numbers a, b and c, which are length of the sides of a right angled triangle.

Hence, if $a^2 + b^2 = c^2$, $b^2 + c^2 = a^2$ or $c^2 + a^2 = b^2$, then the set of natural numbers a, b and c is a Pythagorean triplet.

Since $3^2 + 4^2 = 5^2$, hence 3, 4, 5 form a Pythagorean triplet.

General Rule To Find Pythagorean Triplet: If r and s are two natural numbers such that $r > s$, $r - s$ is odd and GCD of r and s is 1, then the Pythagorean triplet a, b, c are defined by

$$a = r^2 - s^2, \quad b = 2rs \quad \text{and} \quad c = r^2 + s^2.$$

Note: If each term of any Pythagorean triplet is multiplied or divided by such a positive number that the products or quotients obtained respectively are natural numbers then the new products or quotients are also form Pythagorean triplets.
Since 3, 4, 5 form a Pythagorean triplet, therefore 9, 12 and 15 also form a Pythagorean triplet.

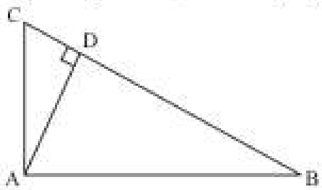
DETERMINATION OF NATURE OF TRIANGLE

Let length of three sides of a triangle are a, b and c.

- If c be the length of longest side and $c^2 = a^2 + b^2$, then the triangle is right-angled triangle.
- If c be the length of longest side and $c^2 > a^2 + b^2$, then the triangle is an obtuse-angled triangle.
- If c be the length of longest side and $c^2 < a^2 + b^2$, then the triangle is an acute-angled triangle.

IMPORTANT POINTS

1. In ΔABC right angled at A, if AD is perpendicular to BC.



$$\Delta ABC \sim \Delta DBA \sim \Delta DAC$$

Now $\Delta ABC \sim \Delta DBA$

$$\Rightarrow \frac{AB}{BC} = \frac{DB}{BA} \Rightarrow AB^2 = DB \times BC$$

And $\Delta ABC \sim \Delta DAC$

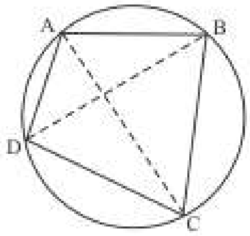
$$\Rightarrow \frac{AC}{BC} = \frac{DC}{AC} \Rightarrow AC^2 = DC \times BC$$

And $\Delta DBA \sim \Delta DAC$

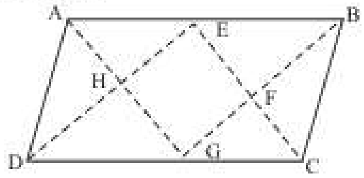
$$\Rightarrow \frac{DA}{DB} = \frac{DC}{DA} \Rightarrow DA^2 = DB \times DC$$

2. In a cyclic quadrilateral, product of the diagonals is equal to the sum of the products to the opposite sides,

$AC \times BD = (AD \times BC) + (AB \times CD)$

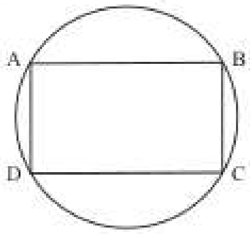


3. Bisectors of the angles of a parallelogram or a rectangle form a rectangle.

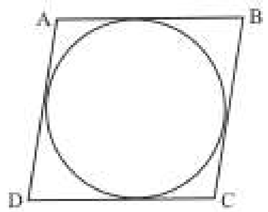


In parallelogram $ABCD$, AG, BG, CE and DE are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively. Hence in the figures $EFGH$ is a rectangle.

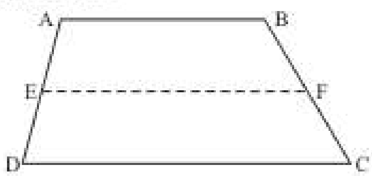
4. A parallelogram inscribed in a circle is a rectangle.
In figure, $ABCD$ is a rectangle.



5. A parallelogram circumscribed a circle is a rhombus. In figure, $ABCD$ is a rhombus.



6. Median of a trapezium is the line segment joining mid-points of non-parallel sides of the trapezium.
In the figure E and F are the mid points of non-parallel sides AB and CD respectively. Hence EF is the median of trapezium $ABCD$.

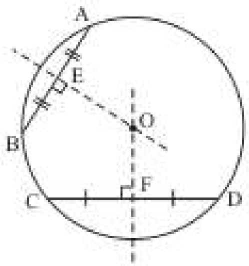


$$EF = \frac{1}{2}(AB + CD)$$

Also $$EF = \frac{a \times (AB) + b \times (DC)}{AD},$$

where $AE = a$ and $ED = b$

7. Perpendicular bisectors of two chords of a cricle intersect at its centre of the circle.

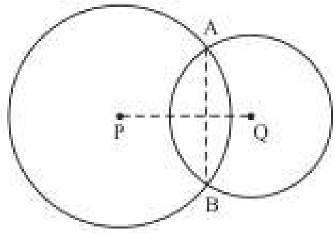


In figure, OE and OF are perpendicular bisectors of chords AB and CD, OE and OF meet at point O. Hence O is the centre of the circle.

8. If two circles intersect each other at two points then the line through the centres is the perpendicular bisectors of the common chord.

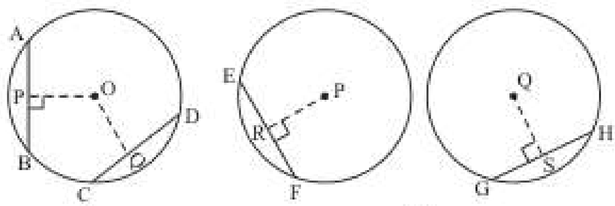
In figure, two circles with centre P and Q intersect each other at two points A and B.

Hence AB is the common chord of the two circles.



Therefore, PQ is the perpendicular bisector of common chord AB.

9. Equal chords of a circle or congruent circles are equidistant from the centre.

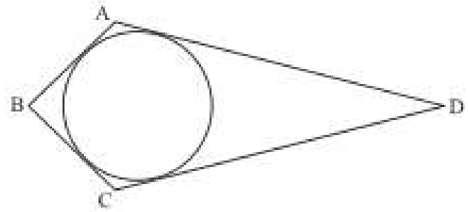


(i) (ii)

In figure (i), AB and CD are two equal chords of a circle, therefore their perpendicular distances OP and OQ respectively from the centre O are equal.

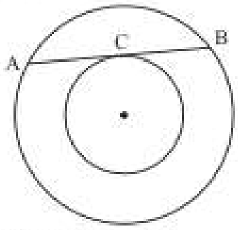
In figure (ii), two circles are congruent i.e. their radii are equal. EF and GH are two equal chords. Hence their perpendicular distances from centre P and Q respectively are equal.

10. If a circle touches all the four sides of a quadrilateral then the sum of the two opposite sides is equal to the sum of other two.



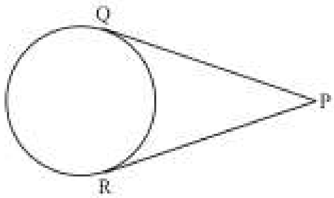
$AB + DC = AD + BC$

11. In two concentric circles, if a chord of the larger circle is also tangent to the smaller circle, then the chord is bisected at the point of contact.



Hence in the figure, $AC = CB$

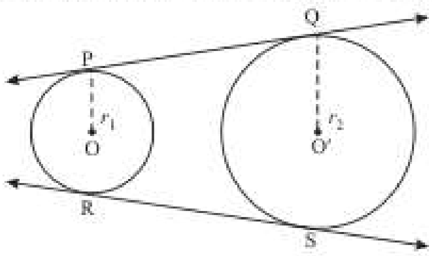
12. Length of two tangents from an exterior point to a circle are equal.



In figure PQ and PR are two tangents drawn from an exterior point to a circle.

$\therefore \quad PQ = PR$

13. **Direct common tangent:** A tangent to two circles are such that the two circles lies on the same side of the tangent, then the tangent is called direct tangent to the two circles.



In the figure, PQ and RS are two direct common tangent to the same two circles. Length of these two common tangents to the same two circles are equal.

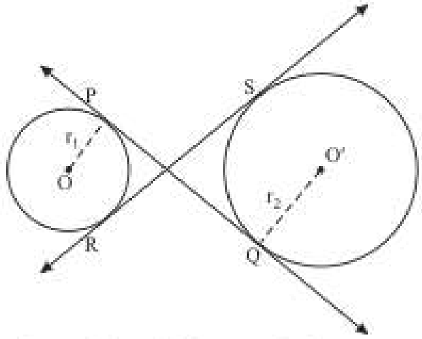
i.e. $\quad PQ = RS$

Also $\quad PQ = RS = \sqrt{(OO')^2 + (r_2 - r_1)^2}$

Here O, O' are the centres and r_1, r_2 are the radii of the two circles respectively. Also $r_2 > r_1$.

14. **Indirect or Transverse Common Tangent:** If a tangent to two circles is such that the two circles lie on opposite sides of the tangent, then the tangent is called indirect tangent.

Length of two indirect tangents to two circles is equal.



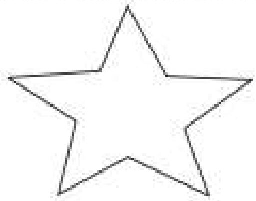
In the figure, PQ and RS are two indirect common tangents to the same two circles.

$\therefore \quad PQ = RS$

Also $\quad PQ = RS = \sqrt{(OO')^2 - (r_1 + r_2)^2}$

Here O, O' are centres r_1, r_2 are radii of the two circles respectively.

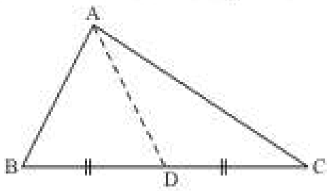
15. **Star:** A star has a shape like given in the figure.



If a star has n sides, then

Sum of its all angles $= (n - 4) \times 180°$.

16. In a triangle, the sum of the square of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side.

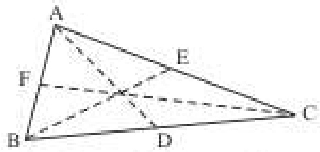


In the figure, AD is the median.

$$\therefore \quad AB^2 + AC^2 = 2\left[AD^2 + \left(\frac{BC}{2}\right)^2\right]$$

17. In a triangle,

$$3 \times \begin{pmatrix}\text{Sum of square of} \\ \text{three sides of} \\ \text{a triangle}\end{pmatrix} = 4 \times \begin{pmatrix}\text{Sum of the square of} \\ \text{three medians of} \\ \text{the triangle}\end{pmatrix}$$

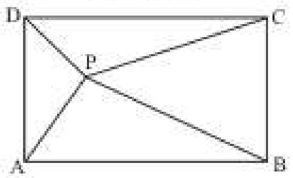


In figure AD, BE and CF are medians of ΔABC.

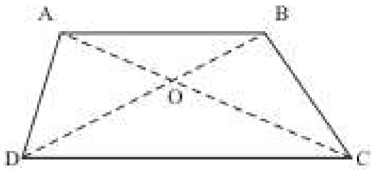
$\therefore \quad 3 \times (AB^2 + BC^2 + CA^2) = 4 \times (AD^2 + BE^2 + CF^2)$

18. In the figure given below, if P is any point inside the rectangle $ABCD$, then

$$PA^2 + PC^2 = PB^2 + PD^2$$

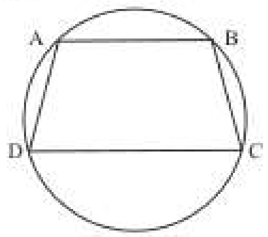


19. Diagonals of a trapezium divide each other in the ratio of the parallel sides of the trapezium. In trapezium $ABCD$, $AB \parallel DC$



$$\therefore \quad \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{CD}.$$

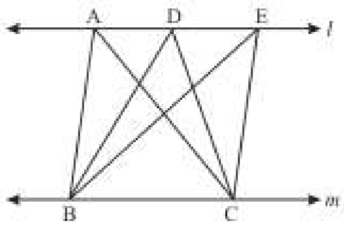
20. If a trapezium is inscribed inside a circle, then the trapezium is an isosceles trapezium i.e. its non-parallel sides are equal.



In the figure, $ABCD$ is a trapezium in which $AB \parallel CD$

$\therefore \quad AD = BC$

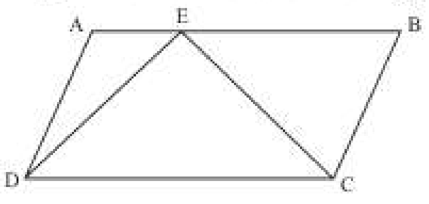
21. Area of triangles on the same base and lie between the same pair of parallel lines are equal.



In the figure, ΔABC, ΔDBC and ΔEBC are on the same base BC and lie between the same pair of parallel lines l and m.

$\therefore$ area of ΔABC = area of ΔDBC = area of ΔEBC.

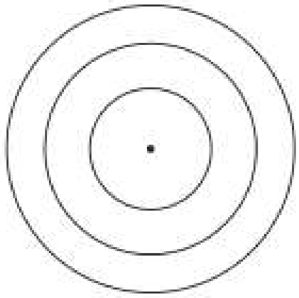
22. If a parallelogram and a triangle are on the same base and lie between the same pair of parallel lines, then area of the parallelogram is twice the area of the triangle.



In the figure, $ABCD$ a parallelogram and EDC a triangle are on the same base and lie between the same pair of parallel lines AB and CD.

$\therefore$ area of parallelogram $ABCD = 2 \times$ (area of ΔEDC).

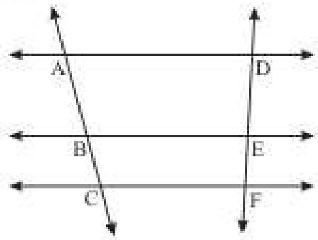
23. Concentric circles: Two or more circles in a plane arc said to be concentric, if they have the same centre.



Concentric circles

24. Intercepts made by three or more parallel lines on two or more lines are in the same ratios.

In the figure three parallel lines AD, BE and CF made intercepts AB, BC and DE, EF on two lines AC and DF respectively.



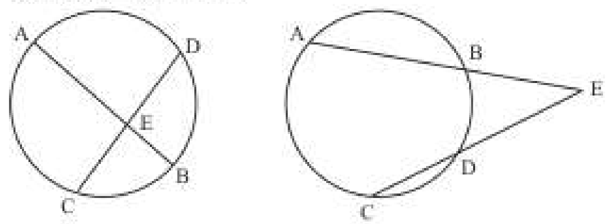
$$\therefore \quad \frac{AB}{BC} = \frac{DE}{EF}$$

25. (a) In an equilateral triangle centroid, incentre, circumcentre, orthocentre coincide at the same point.

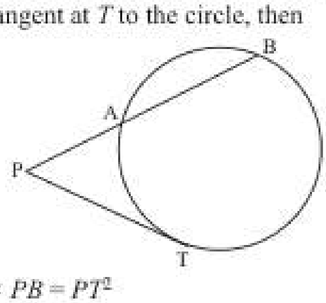
(b) Circumradius = 2 × in radius

26. A parallelogram is a rectangle if its diagonals are equal.

27. If two chords AB and CD of a circle intersect inside a circle (or outside a circle when produced) at point E, then $AE \times EB = CE \times ED$.

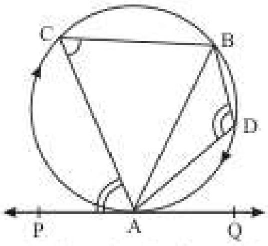


28. If PB is a secant which intersects the circle at A and B and PT is a tangent at T to the circle, then



$$PA \times PB = PT^2$$

29. Angles in the alternate segment:



In the figure, AB is a chord of a circle. PQ is a tangent at an end point A of the chord to the circle. C is any point on arc AB and D is any point on arc BA.

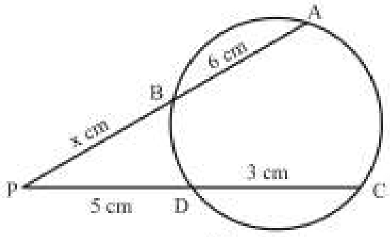
$\angle BAQ$ and $\angle ACB$ are angles in the alternate segments

$\angle BAP$ and $\angle ADB$ are angles in the alternate segments.

Angles in the alternate segments of a circle are equal i.e.

$\angle BAQ = \angle ACB$ and $\angle BAP = \angle ADB$

Illustration 19: In the given figure, chords AB and CD of a circle intersect externally at P. If AB = 6 cm, CD = 3 cm and PD = 5 cm, then PB = ?



(a) 5 cm **(b) 6.25 cm**
(c) 6 cm **(d) 4 cm**

Solution: (d) $PA \times PB = PC \times PD$ (According to property of circle)

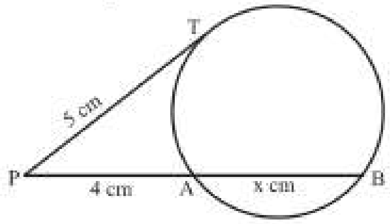
$\Rightarrow \quad (x + 6) \times x = 8 \times 5$

$\Rightarrow \quad x^2 + 6x - 40 = 0$

$\Rightarrow \quad (x + 10)(x - 4) = 0 \Rightarrow x = 4$

$\therefore \quad PB = 4$ cm

Illustration 20: In the given figure, PAB is a secant and PT is a tangent to the circle from P. If PT = 5 cm, PA = 4 cm and AB = x cm, then x is equal to



(a) 2.5 cm **(b) 2.6 cm**
(c) 2.25 cm **(d) 2.75 cm**

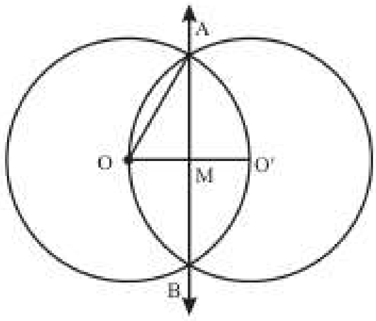
Solution: (c) $PA \times PB = PT^2 \Rightarrow 4 \times (4 + x) = 25$

$$\Rightarrow \quad 4 + x = \frac{25}{4} = 6.25 \Rightarrow x = 2.25 \text{ cm}$$

Illustration 21: Two equal circles pass through each other's centre. If the radius of each circle is 5 cm, what is the length of the common chord?

(a) $5\sqrt{3}$ **(b) $10\sqrt{3}$**
(c) $\frac{5\sqrt{3}}{2}$ **(d) 5**

Solution: (a)



Given, distance between the centres of two circle = 5 cm

$OO' = 5$ cm

$\therefore \quad OM = \frac{5}{2}$ cm

In ΔOAM,

$OA^2 = OM^2 + AM^2$

$(5)^2 = \left(\frac{5}{2}\right)^2 + AM^2$

$AM = \sqrt{25 - \frac{25}{4}} = \frac{5\sqrt{3}}{2}$ cm

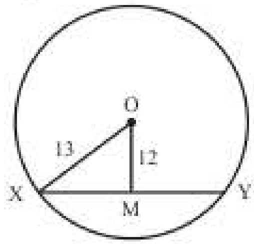
$\therefore$ The length of common chord, $AB = 2 \times AM$

$= 2 \times \frac{5\sqrt{3}}{2} = 5\sqrt{3}$ cm

Illustration 22: The radius of a circle is 13 cm and xy is a chord which is at a distance of 12 cm from the centre. The length of the chord is

(a) 12 cm **(b) 10 cm**
(c) 20 cm **(d) 15 cm**

Solution: (b) From figure,



$XM = \sqrt{13^2 - 12^2}$

$= \sqrt{169 - 144} = 5$

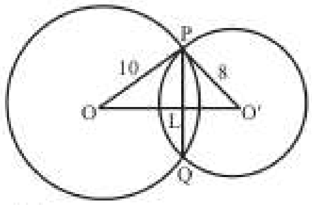
$\therefore$ Length of the chord $= 2 \times XM$

$= 2 \times 5 = 10$ cm

Illustration 23: Two circles of radii 10 cm and 8 cm. intersect and length of the common chord is 12 cm. Find the distance between their centres.

(a) 13.8 cm **(b) 13.29 cm**
(c) 13.2 cm **(d) 12.19 cm**

Solution: (b) Here, $OP = 10$ cm; $O'P = 8$ cm



$PQ = 12$ cm

$\therefore \quad PL = 1/2\ PQ \Rightarrow PL = \frac{1}{2} \times 12 \Rightarrow PL = 6$ cm

In rt. ΔOLP, $OP^2 = OL^2 + LP^2$

(using Pythagoras theorem)

$\Rightarrow \quad (10)^2 = OL^2 + (6)^2 \Rightarrow OL^2 = 64;\ OL = 8$

In $\Delta O'LP$, $(O'L)^2 = O'P^2 - LP^2 = 64 - 36 = 28$

$O'L^2 = 28 \Rightarrow O'L = \sqrt{28}$

$O'L = 5.29$ cm

$\therefore \quad OO' = OL + O'L = 8 + 5.29$

$OO' = 13.29$ cm

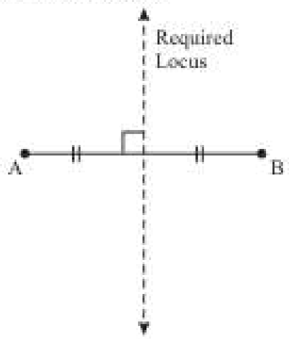
LOCUS

The locus of a point is the path traced out by a moving point under given geometrical conditions. Alternatively, the locus is the set of all those points which satisfy the given geometrical conditions.

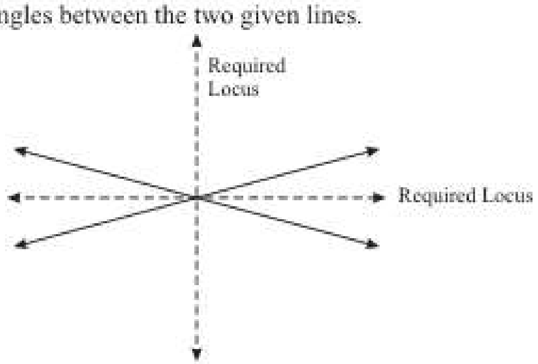
The plural of locus is loci and is read as 'Losai'.

The Locus of a Point in Different Conditions

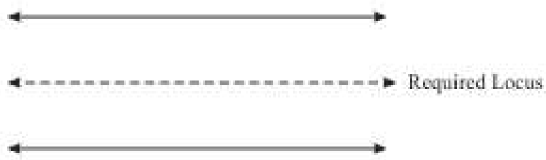
(i) The locus of a point which is equidistant from two fixed points is the perpendicular bisector of the line segment joining the two fixed points.



(ii) The locus of a point which is equidistant from two intersecting straight lines is a pair of straight lines which bisect the angles between the two given lines.

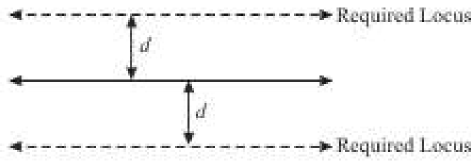


(iii) The locus of a point equidistant from two given parallel straight lines is a straight line parallel to the given straight lines and midway between them.



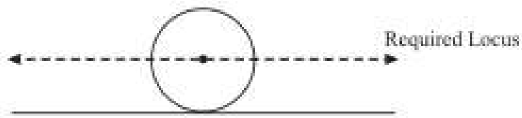
(iv) The locus of a point which is equidistant from a fixed point in a plane is a circle.

(v) The locus of a point, which is at a given distance from a given straight line, is a pair of parallel straight lines either side to the given line at a given distance from it.

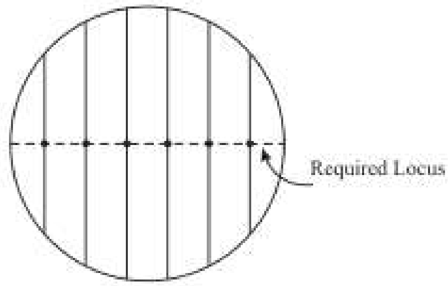


Here d is the given distance.

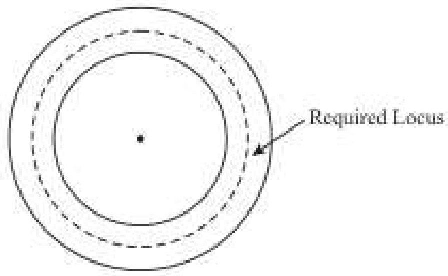
(vi) The locus of the centre of a wheel moving on a straight horizontal road, is a straight line parallel to the road and at a height equal to the radius of the wheel.



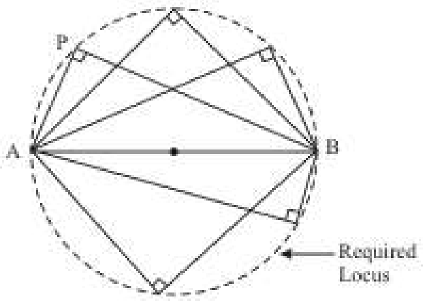
(vii) The locus of mid-points of all parallel chords of a circle, is the diameter of the circle which is perpendicular to the given parallel chords.



(viii) The locus of a point which is equidistant from two concentric circles is the circumference of the circle concentric with the given circles and midway between them.



(ix) If A and B are two fixed points, then the locus of a point P such that $\angle APB = 90°$, is the circle with AB as diameter.



(x) The locus of midpoints of all equal chords of a circle is the circumference of the circle concentric with the given circle and radius equal to the distance of equal chords from the centre of the given circle.