CHAPTER

11

PROGRESSIONS

ARITHMETIC PROGRESSIONS (A.P.)

A sequence of numbers which are either continuously increased or continuously decreased by a common difference found by subtracting any term of the sequence from the next term.

The following sequences of numbers are arithmeticprogressions:

(i) 5, 8, 11, 14, ...

(ii) – 6, – 1, 4, 9, 14, ...

(iii) 10, 7, 4, 1, – 2, – 5, ...

(iv) $p, p + q, p + 2q, p + 3q, ...$

In the arithmetic progression (i); 5, 8, 11 and 14 are first term, second term, third term and fourth term respectively. Common difference of this A.P. is found out either by subtracting 5 from 8, 8 from 11 or 11 from 14. Thus common difference = 3. Similarly, common difference of arithmetic progression (ii), (iii) and (iv) are 5, – 3 and q respectively. First term and common difference of an A.P. are denoted by a and d respectively. Hence

d of (i) A.P.= 3, d of (ii) A.P.= 5,

d of (iii) A.P. = – 3 and d of (iv) A.P. = q

n^{th} TERM OF AN A.P.

To find an A.P. if first term and common difference are given, we add the common difference to first term to get the second term and add the common difference to second term to get the third term and so on.

The standard form of an A.P. is

$$a, a + d, a + 2d, a + 3d, ...$$

Here 'a' is the first term and 'd' is the common difference. Also we see that coefficient of d is always less by one than the position of that term in the A.P. Thus n^{th} term of the A.P. is given by

$$T_n = a + (n - 1)\,d \qquad ...(1)$$

This equation (1) is used as a formula to find any term of the A.P.

If l be the last term of a sequence containing n terms, then

$$l = T_n = a + (n - 1)\,d$$

To find any particular term of any A.P., generally we put the value of a, n and d in the formula (i) and then calculate the required term.

For example to find the 25th term of the A.P. 6, 10, 14, 18, ... ; using the formula (i), we put the value of $a = 6$, $n = 25$ and $d = 4$ in formula and calculate as

Illustration 1: In an A.P. if $a = -7.2$, $d = 3.6$, $a_n = 7.2$, then find the value of n.

Solution: $a_n = a + (n - 1)\,d$

$\Rightarrow \quad 7.2 = -7.2 + (n - 1)(3.6)$

$\Rightarrow \quad 14.4 = (n - 1)(3.6)$

$\Rightarrow \quad n - 1 = 4 \Rightarrow n = 5.$

Illustration 2: Which term of the A.P. 21, 42, 63, ... is 420 ?

Solution: $420 = a_n = a + (n - 1)\,d$

[Here $a = 21$, $d = 42 - 21 = 21$]

$= 21 + (n - 1)\,21$

$= 21n$

$\therefore \quad n = \dfrac{420}{21} = 20$

$\therefore$ required term is 20th term.

Illustration 3: Is – 150 a term of the A.P. 11, 8, 5, 2, ... ?

Solution: Here $a = 11$, $d = -3$

$-150 = a_n = a + (n - 1)\,d$

$= 11 + (n - 1)(-3)$

$= 11 - 3n + 3$

$= 14 - 3n$

$3n = 14 + 150$

$n = \dfrac{164}{3} = 54\dfrac{2}{3}$,

which is not possible because n is +ve integer.

$\therefore$ – 150 is not a term of the given A.P.

SUM OF FIRST n TERMS OF AN A.P.

Sum of first n terms means sum of from first term to n^{th} term.

Consider an A.P. whose first term and common difference are 'a' and 'd' respectively. Sum of first n terms S_n of this A.P. is given by

$$S_n = \frac{n}{2}\,[2a + (n - 1)\,d] \qquad ...(1)$$

If last term of an A.P. containing n terms be l, then n^{th} term $= l = a + (n-1)\,d$.

$$\therefore \quad S_n = \frac{n}{2}\,[2a + (n-1)\,d] = \frac{n}{2}\,[a + \{a + (n-1)\,d\}]$$

$$\Rightarrow \quad S_n = \frac{n}{2}\,(a + l) \qquad ...(2)$$

CONSIDERING THE TERMS IN AN A.P.

If sum of three consecutive terms of an A.P. is given, then if required consider the three consecutive terms as $(a - d)$, a and $(a + d)$. This reduces one unknown d thereby making the solution easier.

Similarly, we consider the four consecutive terms as $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$ and five consecutive terms as $(a - 2d)$, $(a - d)$, a, $(a + d)$ and $(a + 2d)$; if their sums are given otherwise consider three terms as a, $a + d$, $a + 2d$; four terms as a, $a + d$, $a + 2d$, $a + 3d$ and five terms as a, $a + d$, $a + 2d$, $a + 3d$, $a + 4d$.

USEFUL RESULTS

(i) **(a)** Sum of first n natural numbers

$$= 1 + 2 + 3 + ... + n = \frac{n\,(n+1)}{2}$$

(b) Sum of first n odd natural numbers

$$= 1 + 3 + 5 + ... + (2n - 1) = n^2$$

(c) Sum of first n even natural numbers

$$= 2 + 4 + 6 + ... + 2n = n\,(n + 1)$$

(d) Sum of odd numbers $\le n$

$$= \begin{cases} \left(\frac{n+1}{2}\right)^2, & \text{if } n \text{ is odd} \\ \left(\frac{n}{2}\right)^2, & \text{if } n \text{ is even} \end{cases}$$

(e) Sum of even numbers $\le n$

$$= \begin{cases} \frac{n}{2}\left(\frac{n}{2} + 1\right), & \text{if } n \text{ is odd} \\ \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right), & \text{if } n \text{ is even} \end{cases}$$

(ii) **(a)** Sum of squares of first n natural numbers

$$= 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n\,(n+1)\,(2n+1)}{6}$$

(b) Sum of cubes of first n natural numbers

$$= 1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n\,(n+1)}{2}\right]^2$$

= Square of the sum of first n natural numbers.

(iii) **(a)** $T_n = S_n - S_{n-1}$

(b) For A.P., $d = S_2 - 2S_1$

(iv) **(a)** In an A.P., the sum of terms equidistant from the beginning and end is constant and equal to the sum of first term and last term.

(b) If in an A.P. sum of p terms is equal to sum of q terms, then sum of $(p + q)$ terms is zero.

(c) If in an A.P., p^{th} term is q and q^{th} term is p then n^{th} term is $(p + q - n)$.

(d) If in an A.P., sum of p terms is q and sum of q terms is p, then sum of $(p + q)$ terms is $-(p + q)$.