

Averages

AVERAGE

An average is a simple concept of mathematics but its uses are very common in day-to-day life. In CAT and CAT like aptitude test exams at least one question is always asked, the nature of the question asked in CAT is applied and blended with logical reasoning.

An average of a group of numbers is a number that is the best representative of the group of numbers because it tells a lot about the entire numbers of the group.

In other words an average is a measure of central tendency called arithmetic mean of a group of numbers,

The formula for finding the average is

$$\text{Average} = \frac{\text{Sum of all numbers}}{\text{Number of numbers}}$$

Thus if A_v be the average of n numbers $x_1, x_2, x_3, \dots, x_n$ then

$$A_v = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$\Rightarrow x_1 + x_2 + x_3 + \dots + x_n = n \cdot A_v$$

$$\Rightarrow \text{Sum of } n \text{ numbers} = (\text{Number of numbers}) \times (\text{Average})$$

Illustration 1: The average of the first nine prime numbers is:

- (a) 9 (b) 11 (c) $11\frac{1}{9}$ (d) $11\frac{2}{9}$

Solution: (c) Average

$$= \frac{2+3+5+7+11+13+17+19+23}{9}$$

$$= \frac{100}{9} = 11\frac{1}{9}$$

Illustration 2: The average of five consecutive odd numbers is 61. What is the difference between the highest and lowest numbers?

- (a) 2 (b) 5
(c) 8 (d) Cannot be determined

Solution: (c) Let the numbers be $x, x+2, x+4, x+6$ and $x+8$.

$$\text{Then, } \frac{x+(x+2)+(x+4)+(x+6)+(x+8)}{5} = 61$$

$$\text{or } 5x + 20 = 305 \text{ or } x = 57.$$

$$\text{So, required difference} = (57+8) - 57 = 8.$$

WEIGHTED AVERAGE

If we have two or more groups of numbers whose individual averages are known, then combined average of all the numbers of all the groups is known as Weighted Average. Thus if there are k groups having number of numbers $n_1, n_2, n_3, \dots, n_k$ with averages $A_1, A_2, A_3, \dots, A_k$ respectively; then weighted average,

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + \dots + n_k A_k}{n_1 + n_2 + n_3 + \dots + n_k}$$

Illustration 3: The average score of a cricketer in two matches is 27 and in three other matches is 32. Then find the average score in all the five matches.

Solution:

$$\text{Average in 5 matches} = \frac{2 \times 27 + 3 \times 32}{2 + 3} = \frac{54 + 96}{5} = 30.$$

Illustration 4: The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is

- (a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 3 : 5

Solution: (b) Let the number of boys in a class be x .

Let the number of girls in a class be y .

$$\therefore \text{Sum of the ages of the boys} = 16.4x$$

$$\text{Sum of the ages of the girls} = 15.4y$$

$$\therefore 15.8(x+y) = 16.4x + 15.4y$$

$$\Rightarrow 0.6x = 0.4y \Rightarrow \frac{x}{y} = \frac{2}{3}$$

$$\therefore \text{Required ratio} = 2 : 3$$

Illustration 5: The average age of 30 boys of a class is equal to 14 yrs. When the age of the class teacher is included the average becomes 15 yrs. Find the age of the class teacher.

Solution: Total ages of 30 boys = $14 \times 30 = 420$ yrs.

Total ages when class teacher is included = $15 \times 31 = 465$ yrs.

\therefore Age of class teacher = $465 - 420 = 45$ yrs.

By direct formula,

Age of new entrant

= (No. of old members) \times (Increase in average) + (New average)

= $30(15 - 14) + 15 = 45$ yrs.

Illustration 6: The average weight of 45 students in a class is 52 kg. 5 of them whose average weight is 48 kg leave the class and other 5 students whose average weight is 54 kg join the class. What is the new average weight (in kg) of the class?

- (a) 52.6 (b) $52\frac{2}{3}$
 (c) $52\frac{1}{3}$ (d) None of these

Solution: (b) Total weight of 45 students = $45 \times 52 = 2340$ kg

Total weight of 5 students who leave

= $5 \times 48 = 240$ kg

Total weight of 5 students who join

= $5 \times 54 = 270$ kg

Therefore, new total weight of 45 students

= $2340 - 240 + 270 = 2370$

\Rightarrow New average weight = $\frac{2370}{45} = 52\frac{2}{3}$ kg.

Remember

(i) Ages and Averages

If the average age of a group of persons is x years today then after n years their average age will be $(x + n)$ years because for a group of people, 1 year is added to each person's age every year.

Similarly, n years ago their average age would have been $(x - n)$ years, because 1 year is subtracted from each person's age before every year.

(ii) Average Speed if Equal Distances are Travelled by Two Different Speeds

If a car travels at a speed S_1 from A to B and at a speed S_2 from B to A . Then

$$\text{Average speed} = \frac{2S_1 \cdot S_2}{S_1 + S_2}$$

The above formula can be found out as follows:

If distance between A and B is d , then

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance}}{\text{Total time}} = \frac{2d}{\frac{d}{S_1} + \frac{d}{S_2}} \\ &= \frac{2}{\frac{1}{S_1} + \frac{1}{S_2}} = \frac{2S_1 \cdot S_2}{S_2 + S_1} \end{aligned}$$

Similarly for three equal distances travelled by three different speeds S_1 , S_2 and S_3 :

$$\text{Average speed} = \frac{3S_1 \cdot S_2 \cdot S_3}{S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1}$$

Let's find the average speed of a car which goes from Delhi to Panipat at a speed of 60 kmph and returns at a speed of 90 kmph.

$$\begin{aligned} \text{Average speed} &= \frac{2S_1 \cdot S_2}{S_1 + S_2} = \frac{2 \times 60 \times 90}{60 + 90} = \frac{2 \times 60 \times 90}{150} \\ &= 72 \text{ kmph.} \end{aligned}$$

Illustration 7: The average age of a family of 6 members is 22 yrs. If the age of the youngest member be 7 yrs, then what was the average age of the family at the birth of the youngest member?

Solution: Total ages of all members = $6 \times 22 = 132$ yrs.

7 yrs. ago, total sum of ages = $132 - (6 \times 7) = 90$ yrs.

But at that time there were 5 members in the family.

\therefore Average at that time = $90 \div 5 = 18$ yrs.

Illustration 8: The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks of passed candidates is 39 and that of the failed candidates is 15, what is the number of candidates who passed the examination?

Sol. Let the number of passed candidates be x .

Then total marks = $120 \times 35 = 39x + (120 - x) \times 15$

or, $4200 = 39x + 1800 - 15x$ or $24x = 2400$

$\therefore x = 100$

\therefore number of passed candidates = 100.

Illustration 9: A cricketer has completed 10 innings and his average is 21.5 runs. How many runs must he make in his next innings so as to raise his average to 24?

Solution: Total of 10 innings = $21.5 \times 10 = 215$

Suppose he needs a score of x in 11th innings; then

$$\text{average in 11 innings} = \frac{215 + x}{11} = 24$$

or, $x = 264 - 215 = 49$.