CHAPTER 10

TIME, SPEED AND DISTANCE

MOTION OR MOVEMENT

The relation between speed (S), distance (D) and time (T) is given below :

$$\text{Distance} = \text{Speed} \times \text{Time}$$

or, $\text{Speed} \times \text{Time} = \text{Distance}$ *i.e.* $S \times T = D$

In the above relation, the unit used for measuring the distance (D) covered during the motion and the unit of time (T) i.e. duration to cover the distance (D) will be the same as in numerator and denominator respectively of the unit used for the speed.

CONVERSION OF KMPH (KILOMETER PER HOUR) TO M/S (METRE PER SECOND) AND VICE-VERSA

$$1 \text{ kmph or } 1 \text{ km/h} = \frac{1 \text{ km}}{1 \text{ hr}} = \frac{1000 \text{ m}}{60 \times 60 \text{ sec}} = \frac{5 \text{ m}}{18 \text{ sec}} = \frac{5}{18} \text{ m/s}$$

$$\Rightarrow \quad x \text{ kmph} = \frac{5x}{18} \text{ m/s} \quad \text{and vice-versa } x$$

$$\text{m/s} = \frac{18x}{5} \text{ kmph} \quad \text{or} \quad \frac{18x}{5} \text{ km/h}$$

i.e. to convert km/hr to m/sec, multiply by $\frac{5}{18}$ and to convert m/sec to km/hr multiply by $\frac{18}{5}$.

Illustration 1: Convert 90 km/h into m/s.

Solution: 90 km/h = $90 \times \frac{5}{18} = 25$ m/s.

Illustration 2: The driver of a Maruti car driving at the speed of 68 km/h locates a bus 40 metres ahead of him. After 10 seconds, the bus is 60 metres behind. The speed of the bus is

(a) 30 km/h **(b) 32 km/h**
(c) 25 km/h **(d) 38 km/h**

Solution: (b) Let speed of Bus = S_B km/h.

Now, in 10 sec., car covers the relative distance

$$= (60 + 40) \text{ m} = 100 \text{ m}$$

$$\therefore \text{ Relative speed of car} = \frac{100}{10} = 10 \text{ m/s}$$

$$= 10 \times \frac{18}{5} = 36 \text{ km/h}$$

$$\therefore \quad 68 - S_B = 36$$

$$\Rightarrow \quad S = 32 \text{ km/h}$$

AVERAGE SPEED

Average speed is defined as the ratio of total distance covered to the total time taken by an object *i.e.*

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

If an object travels $d_1, d_2, d_3, \ldots, d_n$ distances with different speeds $s_1, s_2, s_3, \ldots, s_n$ in time $t_1, t_2, t_3, \ldots, t_n$ respectively; then average speed (S_a) is given by

$$\boxed{S_a = \frac{d_1 + d_2 + d_3 + \ldots + d_n}{t_1 + t_2 + t_3 + \ldots + t_n}} \qquad \ldots (1)$$

Since, Distance = Speed × Time

$$\therefore \; d_1 = s_1 t_1, \quad d_2 = s_2 t_2, \quad d_3 = s_3 t_3, \ldots, d_n = s_n t_n$$

Hence from (1),

$$\boxed{S_a = \frac{s_1 t_1 + s_2 t_2 + s_3 t_3 + \ldots + s_n t_n}{t_1 + t_2 + t_3 + \ldots + t_n}}$$

$$\text{Since} \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\therefore \quad t_1 = \frac{d_1}{s_1}, \quad t_2 = \frac{d_2}{s_2}, \quad t_3 = \frac{d_3}{s_3}, \ldots, t_n = \frac{d_n}{s_n}$$

Hence from (1),

$$\boxed{S_a = \frac{d_1 + d_2 + d_3 + \ldots + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \ldots + \frac{d_n}{s_n}}}$$

Special Cases

In chapter of Averages, we studied that

(i) If with two different speeds s_1 and s_2 the same distance d is covered, then

$$\text{Average Speed} = \frac{2s_1 \cdot s_2}{s_1 + s_2}$$

(ii) If with three different speeds s_1, s_2 and s_3 the same distance d is covered, then

$$\text{Average Speed} = \frac{3s_1 \cdot s_2 \cdot s_3}{s_1 \cdot s_2 + s_2 \cdot s_3 + s_3 \cdot s_1}.$$

Illustration 3: A car moves 300 km at a speed of 45 km/h and then it increases its speed to 60 km/h to travel another 500 km. Find average speed of car.

Solution:

$$\text{Average speed} = \frac{d_1 + d_2}{\frac{d_1}{s_1} + \frac{d_2}{s_2}} = \frac{300 + 500}{\frac{300}{45} + \frac{500}{60}} = \frac{800}{\frac{45}{3}} = \frac{160}{3}$$

$$= 53\frac{1}{3} \text{ km/h}$$

Illustration 4: A covers 1/3rd of the journey at the speed of 10 km/h and half of the rest at the speed of 20 km/h and rest at the speed of 30 km/h. What is the average speed of A ?

Solution:

Distance covered at 10 km/h = 1/3rd of the whole journey

Distance covered at 20 km/h = $\left(\left(1 - \frac{1}{3}\right) \times \frac{1}{2} = \frac{1}{3}\right)$ rd of the whole journey

Distance covered at 30 km/h = $\left(1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}\right)$ rd of the whole journey

Since the distances covered with each of the three given speeds are the same, therefore

$$\text{Average speed} = \frac{3s_1 \cdot s_2 \cdot s_3}{s_1 \cdot s_2 + s_2 \cdot s_3 + s_3 \cdot s_1}$$

$$= \frac{3 \times 10 \times 20 \times 30}{10 \times 20 + 20 \times 30 + 30 \times 10}$$

$$= 16\frac{4}{11} \text{ km/h.}$$

Illustration 5: A man makes his upward journey at 16 km/h and downward journey at 28 km/h. What is his average speed ?

(a) 32 km/h **(b) 56 km/h**

(c) 20.36 km/h **(d) 22 km/h**

Solution: (c) Let the distance travelled during both upward and down-ward journey be x km.

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{x + x}{\frac{x}{16} + \frac{x}{28}} = \frac{2}{\frac{28+16}{28 \times 16}}$$

$$= \frac{2 \times 28 \times 16}{44} = 20.36 \text{ km/h}$$

RELATIVE SPEED

Generally, when we talk about the speed of a body, we mean the speed of the body with respect to a stationary point (or object), which we have already discussed. In many cases, we need to determine the speed of a body with respect to an independent moving point (or body). In such cases, we have to take into account the speed of the independent body with respect to which we want to find the speed of another body.

The speed of a body 'A' with respect to an independent moving body 'B' is called relative speed of the body A with respect to the body 'B'.

Formulae of Relative Speed

(i) If two bodies are moving in opposite directions at speeds s_1 and s_2 respectively, then relative speed of any one body with respect to other body is $(s_1 + s_2)$.

(ii) If two bodies are moving in the same direction at speeds s_1 and s_2 respectively, then relative speed of any one body with respect to other body is given by

$s_1 - s_2$, when s_1 is greater than s_2

and $s_2 - s_1$ when s_2 is greater than s_1.

Illustration 6: A car X starts from Delhi and another car Y starts from Moradabad at the same time to meet each other. Speed of car X is 40 km/h while speed of car Y is 50 km/h. If the distance between Delhi and Moradabad is 210 kms, when will they meet ?

Solution: Effective speed = Relative speed = 40 + 50

= 90 km/h

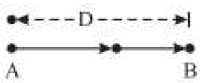
$$\text{Time taken} = \frac{210}{90} = 2\frac{1}{3} \text{ hrs.}$$

TO AND FRO MOTION IN A STRAIGHT LINE BETWEEN TWO POINTS A AND B

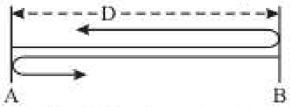
Two and fro motion in a straight line between two points A and B means motion of one or more bodies between two fixed points A and B such that when any body reached at any end point A or B, they start moving towards the opposite end point.

1. When two bodies start moving towards each other from two points A and B

(a) If distance between A and B is D, then the two bodies together have to cover D unit of distance for the first meeting.



(b) For the next number of meeting (*i.e.* second, third, fourth meeting and so on) both A and B together have to cover $2D$ distance more from the previous meeting.

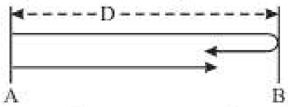


Hence to meet the fifth time they have to cover together $D + (4 \times 2D) = 9D$ unit of distances. Similarly for the ninth meeting they have to cover together $D + (8 \times 2D) = 17D$ units of distance. Thus, for the nth meeting they have to cover together $D + (n - 1) \times 2D$ *i.e.* $(2n - 1) D$ units of distance.

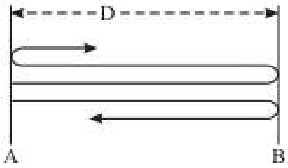
(c) At any point of time ratio of the distances covered by the two bodies will be equal to the ratio of their speeds.

2. When two bodies start moving towards the same direction from the point *A*

(a) Since the faster body reaches the next end (or opposite end) first than the slower body and the faster body starts returning before the slower body reaches the same opposite end and hence the two bodies meet somewhere between the two ends. For the first meeting after they start to move they have to cover $2D$ distance, where D is the distance between two particular end points (*i.e.* A and B)



(b) For every subsequent meeting they have to cover together $2D$ unit distance more from the previous meeting.



Thus, for the nth meeting they have to cover together $(n \times 2D)$ units of distance.

(c) At any point of time ratio of the distances covered by the two bodies will be equal to the ratio of their speeds.

Illustration 7: Two runners Shiva and Abhishek start running to and fro between opposite ends A and B of a straight road towards each other from A and B respectively. They meet first time at a point $0.75D$ from A, where D is the distance between A and B. Find the point of their 6th meeting.

Solution: At the time when Shiva and Abhishek meet first time,

Ratio of their speeds = Ratio of distance covered by them

$$= 0.75 : 0.25$$
$$= 3 : 1$$

Total distance covered by Shiva and Abhishek together till they meet at 6th time $= D + 5 \times 2D = 11D$

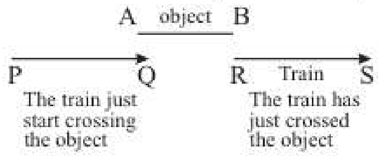
Total distance covered by Shiva till he meets Abhishek 6th time $= \dfrac{3}{3+1} \times 11D = 8.25D$

After covering a distance of $8.25D$, Shiva will be at a point at a distance of $0.25D$ from A or $0.75D$ from B.

CONCEPT RELATED TO MOTION OF TRAINS

The following things need to be kept in mind before solving questions on trains.

(i) For the train is crossing a moving object, the speed of the train has to be taken as the relative speed with respect to the object.



$$\begin{pmatrix}\text{Relative speed of the train}\\ \text{with respect to the object}\end{pmatrix} \times \begin{pmatrix}\text{Time taken}\\ \text{by the train}\\ \text{to cross the}\\ \text{object}\end{pmatrix} = \begin{pmatrix}\text{Distance}\\ \text{travelled}\\ \text{by the}\\ \text{train}\end{pmatrix}$$

(ii) For object moving in opposite direction of the train,

$$\begin{pmatrix}\text{Relative speed of the train}\\ \text{with respect to the object}\end{pmatrix} = \begin{pmatrix}\text{Speed of}\\ \text{the train}\end{pmatrix} + \begin{pmatrix}\text{Speed of}\\ \text{the object}\end{pmatrix}$$

(iii) For object moving in the same direction of the train,

(a) $$\begin{pmatrix}\text{Relative speed of the train}\\ \text{with respect to the object}\end{pmatrix} = \begin{pmatrix}\text{Speed of}\\ \text{the train}\end{pmatrix} - \begin{pmatrix}\text{Speed of}\\ \text{the object}\end{pmatrix}$$

(b) (Distance travelled by the train when crossing the object)

= Distance travelled by the engine from Q to S

$= QR + RS$

$= AB + RS$

= Length of the object + Length of the train

In the case of a train crossing a man, tree or a pole, the length of the man, tree or pole is actually its diameter (or width) which is generally considered as negligible *i.e.* a man, a tree, a pole or a point etc. has no length.

S. No.	Situations	Basic Formulae	Expended Form of Basic Formulae	Expended Formulae in Symbolic Form
1.	When a train crossing a moving object with length in opposite direction	Relative Speed × Time = Distance	$\left[\begin{pmatrix}\text{Speed}\\ \text{of the}\\ \text{train}\end{pmatrix} + \begin{pmatrix}\text{Speed}\\ \text{of the}\\ \text{object}\end{pmatrix}\right] \times \begin{pmatrix}\text{Time taken by}\\ \text{the train to cross}\\ \text{the moving object}\end{pmatrix} = \begin{pmatrix}\text{Length}\\ \text{of the}\\ \text{train}\end{pmatrix} + \begin{pmatrix}\text{Length}\\ \text{of the}\\ \text{object}\end{pmatrix}$	$(S_T + S_0) \times t = (L_T + L_0)$
2.	When a train crossing a moving object with length in the same direction	Relative Speed × Time = Distance	$\left[\begin{pmatrix}\text{Speed}\\ \text{of the}\\ \text{train}\end{pmatrix} - \begin{pmatrix}\text{Speed}\\ \text{of the}\\ \text{object}\end{pmatrix}\right] \times \begin{pmatrix}\text{Time taken by}\\ \text{the train to cross}\\ \text{the moving object}\end{pmatrix} = \begin{pmatrix}\text{Length}\\ \text{of the}\\ \text{train}\end{pmatrix} + \begin{pmatrix}\text{Length}\\ \text{of the}\\ \text{object}\end{pmatrix}$	$(S_T - S_0) \times t = (L_T + L_0)$
3.	When a train crossing a moving object without length like a man, a tree, a pole, a point etc. in opposite direction	Relative Speed × Time = Distance	$\left[\begin{pmatrix}\text{Speed}\\ \text{of the}\\ \text{train}\end{pmatrix} + \begin{pmatrix}\text{Speed}\\ \text{of the}\\ \text{object}\end{pmatrix}\right] \times \begin{pmatrix}\text{Time taken by}\\ \text{the train to cross}\\ \text{the moving object}\end{pmatrix} = \begin{pmatrix}\text{Length}\\ \text{of the}\\ \text{train}\end{pmatrix}$	$(S_T + S_0) \times t = L_T$

S. No.	Situations	Basic Formulae	Expended Form of Basic Formulae	Expended Formulae in Symbolic Form
4.	When a train crossing a moving object without length in the same direction	Relative Speed × Time = Distance	$\left[\begin{pmatrix}\text{Speed}\\\text{of the}\\\text{train}\end{pmatrix} - \begin{pmatrix}\text{Speed}\\\text{of the}\\\text{object}\end{pmatrix}\right] \times \begin{pmatrix}\text{Time taken by}\\\text{the train to cross}\\\text{the moving object}\end{pmatrix} = \begin{pmatrix}\text{Length}\\\text{of the}\\\text{train}\end{pmatrix}$	$(S_t - S_0) \times t = L_T$
5.	When a train crossing a stationary object with length	Speed × Time = Distance	$\begin{pmatrix}\text{Speed}\\\text{of the}\\\text{train}\end{pmatrix} \times \begin{pmatrix}\text{Time taken to cross}\\\text{the stationary object}\end{pmatrix} = \left[\begin{pmatrix}\text{Length}\\\text{of the}\\\text{train}\end{pmatrix} + \begin{pmatrix}\text{Length}\\\text{of the}\\\text{object}\end{pmatrix}\right]$	$S_T \times t = L_T + L_0$
6.	When a train crossing a stationary object without length	Speed × Time = Distance	$\begin{pmatrix}\text{Speed}\\\text{of the}\\\text{train}\end{pmatrix} \times \begin{pmatrix}\text{Time taken to cross}\\\text{the stationary object}\end{pmatrix} = \begin{pmatrix}\text{Length}\\\text{of the}\\\text{train}\end{pmatrix}$	$S_T \times t = L_T$
S_T = Speed of the train, S_0 = Speed of the object, L_T = Length of the train, L_0 = Length of the object, t = time taken by the train to cross the object				

Illustration 8: A train passes an electric pole in 10 seconds and a platform 120 m long in 18 seconds. Find the length of the train.

Solution: Let the length of the train be = x m

$$\text{Speed} = \frac{x}{10} = \frac{120 + x}{18} \Rightarrow x = 150$$

Hence length of the train = 150 m.

Illustration 9: A train of length 100 m takes 1/6 hour to pass over another train 150 m long coming from the opposite direction. If the speed of first train is 60 km/h, then find speed of the second train.

Solution: Let speed of the second train be x km/h.

$$\text{Relative Speed} = \text{Sum of speed of two trains}$$

$$= (60 + x) \text{ km/h} = (60 + x)\ \frac{5}{18}\ \text{m/s}$$

$$\text{Time} = \frac{\text{Sum of length of two trains}}{\text{Relative Speed}}$$

$$\Rightarrow \quad 10 = \frac{250 \times 18}{(60 + x) \times 5} \Rightarrow x = 30 \text{ km/h.}$$

Illustration 10: Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

(a) 10 sec **(b) 12 sec**
(c) 14 sec **(d) cannot be determined**

Solution: (b) Relative speed of the trains

$$= (42 + 48) \text{ kmph} = 90 \text{ kmph}$$

$$= \left(90 \times \frac{5}{18}\right) \text{m/sec} = 25 \text{ m/sec.}$$

Time taken by the trains to pass each other

= Time taken to cover (137 + 163) m at 25 m/sec

$$= \left(\frac{300}{25}\right) \text{sec} = 12 \text{ seconds.}$$

Illustration 11: A train 110 m in length travels at 60 km/h. How much time does the train take in passing a man walking at 6 km/h against the train?

(a) 6 s **(b) 12 s**
(c) 10 s **(d) 18 s**

Solution: (a) Relative speeds of the train and the man

$$= (60 + 6) = 66 \text{ km/h} = \frac{66 \times 5}{18} \text{ m/s}$$

Distance = 110 m

Therefore, time taken in passing the men

$$= \frac{110 \times 18}{66 \times 5} = 6\text{s}$$

BOATS AND STREAMS

In still water, a boat moves with its own speed which is called speed of the boat in still water (S_B).

When the boat is moving against the flow of the water or with the flow of the water, the speed of movement of the boat depends on the speed of flow of water [*i.e.* speed of stream (S_S)].

Speed of the boat moving against the flow of water (*i.e.* moving in upstream)

= Speed of boat in still water – Speed of stream

$= S_B - S_S$

Speed of the boat moving with flow of water (*i.e.* moving in downstream)

= Speed of boat in still water + Speed of stream

$= S_B + S_S$

The basic formula used for solving the problems of boats and streams is

Speed × Time = Distance

Illustration 12: A man can row a boat in downstream at 12 km/h and in upstream at 8 km/h. Find the speed of the boat that the man can row in still water.

Solution:

$$S_B + S_S = 12 \quad \text{... (1)}$$

$$S_B - S_S = 8 \quad \text{... (2)}$$

On adding (1) and (2), we get

$$2\,S_B = 20 \;\Rightarrow\; S_B = 10$$

Hence speed of boat in still water = 10 km/h.

Illustration 13: A boat covers 48 km in upstream and 72 km in downstream in 12 hours, while it covers 72 km in upstream and 48 km in downstream in 13 hours. Find the speed of the stream.

Solution: $\dfrac{48}{S_B - S_S} + \dfrac{72}{S_B + S_S} = 12$

$$\frac{72}{S_B - S_S} + \frac{48}{S_B + S_S} = 13$$

Let $\dfrac{1}{S_B - S_S} = x$ and $\dfrac{1}{S_B + S_S} = y$

Then $48x + 72y = 12$ ... (1)

and $72x + 48y = 13$ ... (2)

On adding (1) and (2),

$$120x + 120y = 25 \;\Rightarrow\; x + y = \frac{5}{24} \quad \text{... (3)}$$

On subtracting (2) from (1),

$$24y - 24x = -1 \;\Rightarrow\; x - y = \frac{1}{24} \quad \text{... (4)}$$

On adding (3) and (4),

$$2x = \frac{1}{4} \;\Rightarrow\; x = \frac{1}{8} \;\Rightarrow\; S_B - S_S = 8 \quad \text{... (5)}$$

On subtracting (4) from (3),

$$2y = \frac{1}{6} \;\Rightarrow\; y = \frac{1}{12} \;\Rightarrow\; S_B + S_S = 12 \quad \text{... (6)}$$

Subtracting (5) from (6),

$$2\,S_S = 4 \;\Rightarrow\; S_S = 2$$

Hence speed of stream = 2 km /h.

Illustration 14: A motor boat takes 12 hours to go downstream and it takes 24 hours to return the same distance. Find the ratio of the speed of boat in still water to the speed of stream.

Solution: Distance = Speed × Time

Distance travelled in downstream

= Distance travelled in upstream

$$(S_B + S_S) \times 12 = (S_B - S_S) \times 24$$

$$\Rightarrow \quad S_B + S_S = 2\,S_B - 2\,S_S$$

$$\Rightarrow \quad 3\,S_S = S_B \;\Rightarrow\; \frac{S_B}{S_S} = \frac{3}{1} \;\Rightarrow\; S_B : S_S = 3 : 1$$

Hence required ratio = 3 : 1.

CIRCULAR MOTION

When two bodies start moving from a place on a circular track simultaneously in the same direction, the faster body keeps increasing the distance by which the slower body is behind the faster body. When the distance by which the faster body is in front of the slower body becomes equal to the circumference of the track, the faster body meets the slower body first time *i.e.* faster body comes in line with the slower body.

(i) When two bodies are moving in the opposite directions, their relative speed is equal to the sum of their individual speeds.

(ii) When two bodies are moving in the same direction, their relative speed is equal to the difference of the speeds of the two bodies.

First Meeting

(i) Let A and B are two runners.

Time taken by A and B to meet for the first time

$$= \frac{\text{Circumference of the circular Track}}{\text{Relative speed}}$$

(ii) When there are more than two runners, suppose A is the fastest runner and A meets B first time in time t_{AB}, A meets C first time in time t_{AC}, A meets D first time in time t_{AD} and so on. Then time taken by all of them to meet for the first time is the LCM of t_{AB}, t_{AC}, t_{AD}, etc.

First Meeting at the Starting Point

Let A take, t_A time, B takes t_B time, C takes t_C times and so on, to complete one round, then the time taken to meet all the runners for the first time at the starting point

= LCM of t_A, t_B, t_C etc.

Illustration 15: The jogging track in a sports complex is 726 metres in circumference. Pradeep and his wife start from the same point and walk in opposite directions at 4.5 km/h and 3.75 km/h, respectively. They will meet for the first time in

(a) 5.5 min **(b) 6.0 min**

(c) 5.28 min **(d) 4.9 min**

Solution: (c) Let the husband and the wife meet after x minutes 4500 metres are covered by Pradeep in 60 minutes.

In x minutes, he will cover $\dfrac{4500}{60}x$ metres.

Similarly,

In x minutes, his wife will cover $\dfrac{3750}{60}x$ m.

Now, $\dfrac{4500}{60}x + \dfrac{3750}{60}x = 726$

$$\Rightarrow \quad x = \frac{726 \times 60}{8250} = 5.28\,\text{min}$$

Illustration 16: A and B run on a circular track of circumference 800 m in the opposite direction. Speeds of A and B are 50 m/s and 30 m/s respectively. Initially A and B are diametrically opposite to each other.

(i) When will they meet for the first time ?

(ii) What is the ratio of distances covered by each one to meet for the first time ?

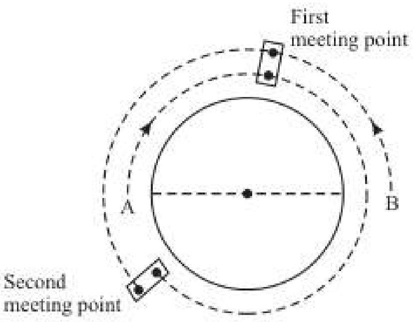
Solution:

(i) Relative speed of A with respect to $B = 50 + 30 = 80$ m/s

Initially A and B are diametrically opposite to each other means B is 400 m ahead of A in the race.

Time taken by A to meet B first time = $\dfrac{400}{80}$ = 5 s

(ii) To meet second time A and B have to cover 800 m



Hence time taken to meet second time $= \frac{800}{80} = 10$ seconds

CLOCKS

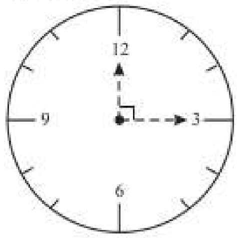
Problems on clocks are based on the movement of the minute hand and hour hand. We consider the dial of a clock as a circular track having a circumference of 60 km. minute hand and hour hand are two runners running with the speed of 60 km/h and 5 km/hr respectively in the same direction. Hence relative speed of minute hand with respect to hour hand is 55 km/h. This means that for every hour elapsed, the minute hand goes 55 km more than the hour hand.

Degree Concept of a Clock

Total angle subtended at the centre of a clock = 360°

Angle made by hour hand at the centre = 30° per hour

= 0.5° per minute



Angle made by minute hand at the centre = 360° per hour

= 6° per minute

Number of Right Angles and Straight Angles Formed by Minute Hand and Hour Hand

A right angle is formed by hour hand and minute hand when distance between tip of hour hand and tip of minute hand is 15 km. A straight line is formed by hour hand and minute hand when distance between their tips is 30 km.

A clock makes two right angles in every hour. Thus there are 2 right angles between marked 1 to 2, 2 to 3, 3 to 4 and so on the dial.

Two straight lines are formed by hour hand and minute hand in every hour.

Thus two straight lines are formed by hour hand and minute hand between marked 1 to 2, 2 to 3, 3 to 4 and so on.

(iii) Hour hand and minute hand of a clock are together after every $65\frac{5}{11}$ minutes. So, if hour hand and minute hand of a clock are meeting in less than $65\frac{5}{11}$ minutes, then the clock is running fast and if hour hand and minute hand are meeting in more than $65\frac{5}{11}$ minutes,then clock is running slow.

Illustration 17: Between 5 O' clock and 6 O' clock, when hour hand and minute hand of a clock overlap each other ?

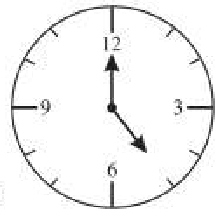
Solution: At 5 O' clock, distance between tips of two hands

= 25 km

Relative speed = 55 km/h

Required time to overlap the two hands

$$= \frac{25\text{ km}}{55\text{ km/h}} = \frac{5}{11}\text{h}$$



$$= \frac{5 \times 60}{11}\text{min}$$

$$= 27\text{ min} + \frac{3 \times 60}{11}\text{ sec}$$

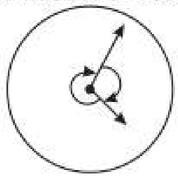
= 27 min + 16 sec.

= 27 minutes 16 seconds.

Illustration 18: Mrs. Veena Gupta goes for marketing between 5 P.M. and 6 P.M. When she comes back, she finds that the hour hand and the minute hand have interchanged their positions. For how much time was she out of her house ?

Solution: Since two hands are interchange their positions, so sum of the angles subtended at the centre by hour hand and minute hand = 360°

Let us suppose that she was out of house for 't' minutes.



So, the sum of the angles subtended at the centre by the hour hand and minute hand = $(0.5 \times t)° + (6t)°$

$\because \quad 0.5t + 6t = 360$

$\Rightarrow \quad 6.5t = 360 \Rightarrow t = 55.4$ (app.)

Hence required time = 55.4 minutes.

CALENDAR

INTRODUCTION

The solar year consists of 365 days, 5 hrs 48 minutes, 48 seconds. In 47 BC, Julius Ceasar arranged a calendar known as the Julian calendar in which a year was taken as $365\frac{1}{4}$ days and in order to get rid of the odd quarter of a day, an extra day was added once in every fourth year and this was called as leap year or Bissextile. Nowadays, the calendar, which is mostly used, is arranged by Pope Gregory XII and known as Gregorian calendar.

In India, number of calendars were being used till recently. In 1952, the Government adopted the National Calendar based on Saka era with Chaitra as its first month. In an ordinary year, Chaitra 1 falling on March 22 of Gregorian Calendar and in a leap year it falls on March 21.

Remember

- ★ In an ordinary year,
  1 year = 365 days = 52 weeks + 1 day
- ★ In a leap year,
  1 year = 366 days = 52 weeks + 2 days

NOTE : *First January 1 A.D. was Monday. So we must count days from Sunday.*

- ★ 100 years or one century contains 76 ordinary years and 24 leap years.

$\Rightarrow$ [76 × 52 weeks + 76 odd days] + [24 × 52 weeks + 24 × 2 odd days]

= (76 + 24) × 52 weeks + (76 + 48) odd days

= 100 × 52 weeks + 124 odd days

= 100 × 52 weeks + (17 × 7 + 5) odd days

= (100 × 52 + 17) weeks + 5 odd days

$\therefore$ 100 years contain 5 odd days.

Similarly, 200 years contain 3 odd days,

300 years contain 1 odd days,

400 years contain 0 odd days.

Year whose non-zero numbers are multiple of 4 contains no odd days; like 800, 1200, 1600 etc.

The number of odd days in months

The month with 31 days contains (4 × 7 + 3) ie. 3 odd days and the month with 30 days contains (4 × 7 + 2) ie. 2 odd days.

NOTE : *February in an ordinary year gives no odd days, but in a leap year gives one odd day.*

Illustration 19: What day of the week was 15th August 1949?

Sol. 15th August 1949 means

1948 complete years + first 7 months of the year 1949 + 15 days of August.

1600 years give no odd days.

300 years give 1 odd day.

48 years give {48 + 12} = 60 = 4 odd days.

[$\because$ For ordinary years $\rightarrow$ 48 odd days and for leap year 1 more day (48 ÷ 4) = 12 odd days; 60 = 7 × 8 + 4]

From 1st January to 15th August 1949

Odd days :

January – 3

Feburary – 0

March – 3

April – 2

May – 3

June – 2

July – 3

August – 1

17 $\Rightarrow$ 3 odd days.

$\therefore$ 15th August 1949 $\rightarrow$ 1 + 4 + 3 = 8 = 1 odd day.

This means that 15th Aug. fell on 1st day. Therefore, the required day was Monday.

Illustration 20: How many times does the 29th day of the month occur in 400 consecutive years?

Sol. In 400 consecutive years, there are 97 leap years. Hence, in 400 consecutive years, February has the 29th day 97 times and the remaining eleven months have the 29th day 400 × 1100 = 4400 times

$\therefore$ The 29th day of the month occurs (4400 + 97) or 4497 times.

Illustration 21: Today is 5th February. The day of the week is Tuesday. This is a leap year. What will be the day of the week on this date after 5 years?

Sol. This is a leap year. So, next 3 years will give one odd day each. Then leap year gives 2 odd days and then again next year give 1 odd day.

Therefore (3 + 2 + 1) = 6 odd days will be there.

Hence the day of the week will be 6 odd days beyond Tuesday, i.e., it will be Monday.

Illustration 22: What day of the week was 20th June 1837 ?

Sol. 20th June 1837 means 1836 complete years + first 5 months of the year 1837 + 20 days of June.

1600 years give no odd days.

200 years give 3 odd days.

36 years give (36 + 9) or 3 odd days.

1836 years give 6 odd days.

From 1st January to 20th June there are 3 odd days.

Odd days :

January : 3

February : 0

March : 3

April : 2

May : 3

June : 6

17

Therefore, the total number of odd days = (6 + 3) or 2 odd days.

This means that the 20th of June fell on the 2nd day commencing from Monday. Therefore, the required day was Tuesday.