

# RATIO, PROPORTION AND VARIATION

## RATIO

Ratio is the comparison between two quantities in terms of their magnitudes. The ratio of two quantities is equivalent to a fraction that one quantity is of the other.

For example, let Swati has 5 note books and Priya has 7 note books. Then the ratio of the number of books that have with Swati to the number of books that have with Priya is 5 is to 7. This ratio is expressed as 5 : 7 or  $\frac{5}{7}$ , which is a quotient of 5 and 7.

Ratio of any two numbers  $a$  and  $b$  is expressed as  $a : b$  or  $\frac{a}{b}$ . The numbers that form the ratio is called the terms of the ratio. The numerator of the ratio is called the antecedent and the denominator is called the consequent of the ratio.

## DECIMAL AND PERCENTAGE VALUE OF A RATIO

A ratio can be expressed in decimal and percentage.

$$\text{Decimal value of } \frac{3}{5} = 0.6$$

To express the value of a ratio as a percentage, we multiply the ratio by 100.

$$\text{Hence } \frac{3}{5} = \frac{3}{5} \times 100\% = 60\%$$

To find the decimal value of any ratio, you may calculate the percentage value using the percentage rule (discussed in the chapter Percentage) and then shift the decimal point 2 places towards left. Hence the decimal value of a ratio whose percentage value is 54.82% will be 0.5482.

## PROPERTIES OF RATIOS

$$(i) \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \dots = \frac{a_1 + a_2 + a_3 + \dots}{b_1 + b_2 + b_3 + \dots}$$

This means that if two or more ratios are equal, then the ratio whose numerator is the sum of the numerators of all the ratios and denominator is the sum of the denominators of all the ratios is equal to the original ratio.

$$\text{Since } \frac{35}{50} = \frac{7}{10}$$

$$\therefore \frac{35}{50} = \frac{7}{10} = \frac{35 + 7}{50 + 10} = \frac{42}{60}$$

(ii) If  $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \dots, \frac{a_n}{b_n}$  are unequal ratios (or fractions), then

$$\frac{a_1 + a_2 + a_3 + \dots + a_n}{b_1 + b_2 + b_3 + \dots + b_n} \text{ lies between the lowest and the}$$

highest of these ratios.

(iii) If the ratio  $\frac{a}{b} > 1$  and  $k$  is a positive number, then

$$\frac{a+k}{b+k} < \frac{a}{b} \text{ and } \frac{a-k}{b-k} > \frac{a}{b}$$

Similarly, if  $\frac{a}{b} < 1$ , then

$$\frac{a+k}{b+k} > \frac{a}{b} \text{ and } \frac{a-k}{b-k} < \frac{a}{b}$$

(iv) If  $\frac{c}{d} > \frac{a}{b}$ , then  $\frac{a+c}{b+d} > \frac{a}{b}$

and if  $\frac{c}{d} < \frac{a}{b}$ , then  $\frac{a+c}{b+d} < \frac{a}{b}$

**Illustration 1:** Salaries of Rajesh and Sunil are in the ratio of 2 : 3. If the salary of each one is increased by ₹ 4000 the new ratio becomes 40 : 57. What is Sunil's present salary ?

- (a) ₹ 17000 (b) ₹ 20000  
(c) ₹ 25500 (d) None of these

**Solution: (d)** Let the salaries of Rajesh and Sunil be ₹  $2x$  and ₹  $3x$  respectively.

$$\text{Then, } \frac{2x+4000}{3x+4000} = \frac{40}{57}$$

$$\text{or } 114x + 228000 = 120x + 160000$$

$$\text{or } 6x = 68000$$

$$\text{or } 3x = ₹ 34000$$

**Illustration 2:** The ratio between the present ages of  $P$  and  $Q$  is 5 : 8. After four years, the ratio between their ages will be 2 : 3. What is  $Q$ 's age at present ?

- (a) 36 years (b) 20 years  
(c) 24 years (d) None of these

Solution: (d)  $\frac{P}{Q} = \frac{5}{8}$  or  $P = \frac{5Q}{8}$  ... (1)

$$\frac{P+4}{Q+4} = \frac{2}{3}$$

or  $3P + 12 = 2Q + 8$

or  $2Q - 3P = 4$  ... (2)

Putting value of  $P$  from eq. (1),

$$2Q - 3 \times \frac{5}{8}Q = 4 \Rightarrow Q = 32.$$

## USES OF RATIOS

### (i) As a Bridge between three or more Quantities

If  $a : b = N_1 : D_1$

$b : c = N_2 : D_2$

$c : d = N_3 : D_3$

and  $d : e = N_4 : D_4$

Then  $a : b : c : d : e = N_1 N_2 N_3 N_4 : D_1 N_2 N_3 N_4 : D_1 D_2 N_3 N_4 : D_1 D_2 D_3 N_4 : D_1 D_2 D_3 D_4$

Here

$a$  is correspond to the product of all four numerators

$(N_1 N_2 N_3 N_4)$

$b$  is correspond to the first denominator and the last three numerators  $(D_1 N_2 N_3 N_4)$

$c$  is correspond to the first two denominators and the last two numerators  $(D_1 D_2 N_3 N_4)$

$d$  is correspond to the first three denominators and the last numerators  $(D_1 D_2 D_3 N_4)$

$e$  is correspond to the product of all four denominators  $(D_1 D_2 D_3 D_4)$

This method is applied for any three or more ratios.

This can be understood by following illustrations:

**Illustration 3:** Ratio of the age of  $A$  and  $B$  is  $3 : 5$  and ratio of the age of  $B$  and  $C$  is  $4 : 7$ . Find the ratio of the age of  $A$  and  $C$ .

**Solution:**  $A : B = 3 : 5 ; B : C = 4 : 7$

$$\Rightarrow A : B : C = 3 \times 4 : 5 \times 4 : 5 \times 7 = 12 : 20 : 35$$

Here

$A$  is correspond to the product of both numerators  $(3 \times 4)$

$B$  is correspond to the product of first denominator and second numerator  $(5 \times 4)$

and  $C$  is correspond to the product of both denominators  $(5 \times 7)$

Hence ratio of the age of  $A$  and  $C = 12 : 35$

### Conventional Method

LCM of 5 and 4 (the two values corresponding  $B$ 's amount) is 20.

Now convert  $B$ 's value in both ratio to 20.

Hence  $A : B = 3 \times 4 : 5 \times 4 = 12 : 20$

$B : C = 4 \times 5 : 7 \times 5 = 20 : 35$

$$\Rightarrow A : B : C = 12 : 20 : 35$$

$$\Rightarrow A : C = 12 : 35$$

This conventional method will be long for more than three ratios.

**Illustration 4:** If  $A : B = 4 : 5 ; B : C = 3 : 7 ; C : D = 6 : 7$   
 $D : E = 12 : 17$

then find the value of ratio  $A : E$ .

**Solution:**  $A : B : C : D : E = (4 \times 3 \times 6 \times 12) : (5 \times 3 \times 6 \times 12) : (5 \times 7 \times 6 \times 12) : (5 \times 7 \times 7 \times 12) : (5 \times 7 \times 7 \times 17)$

$$\therefore A : E = (4 \times 3 \times 6 \times 12) : (5 \times 7 \times 7 \times 17) = 864 : 4165$$

Note that here we have found the ratio of  $A : E$  directly without finding the consolidate ratio  $(A : B : C : D : E)$  of  $A, B, C, D$  and  $E$ .

## COMPARISON OF RATIOS

The value of a ratio is directly related to the value of numerator but inversely related to the value of denominator i.e. if (only numerator decrease)/(only denominator increases)/(numerator decreases and denominator increases) then the value of the ratio decreases and vice-versa.

There are eight cases in which we have to compare two ratios. In six out of these eight cases, we can easily compare the two ratios by keeping the above mentioned facts related to ratios in mind as shown in the following table.

S.No.	Cases	Comparison of Ratios	Comparison of Ratios (Example)
(i)	Numerator : Decreases Denominator : Fixed	(First Ratio) > (Second Ratio)	$\frac{5}{8} > \frac{3}{8}$
(ii)	Numerator : Increases Denominator : Fixed	(First Ratio) < (Second Ratio)	$\frac{4}{9} < \frac{7}{9}$
(iii)	Numerator : Fixed Denominator : Decreases	(First Ratio) < (Second Ratio)	$\frac{6}{7} < \frac{6}{5}$
(iv)	Numerator : Fixed Denominator : Increases	(First Ratio) > (Second Ratio)	$\frac{5}{8} > \frac{5}{9}$
(v)	Numerator : Decreases Denominator : Increases	(First Ratio) > (Second Ratio)	$\frac{6}{7} > \frac{5}{8}$
(vi)	Numerator : Increases Denominator : Decreases	(First Ratio) < (Second Ratio)	$\frac{3}{7} < \frac{5}{4}$



In the remaining two cases, we cannot compare the two ratios just by looking them.

The remaining two cases are

(vii) Numerator : Decreasing

Denominator : Decreasing

(viii) Numerator : Increasing

Denominator : Increasing

In both the remaining two cases (vii) and (viii), we can compare the two ratios by any one of the following two methods.

### Method-I: Cross Multiplication Method

$$\frac{a}{b} > \frac{c}{d}, \text{ if } ad > bc$$

and  $\frac{a}{b} < \frac{c}{d}, \text{ if } ad < bc$

For example  $\frac{6}{7} > \frac{3}{5}$  because  $6 \times 5 > 7 \times 3$

and  $\frac{4}{5} < \frac{7}{8}$  because  $4 \times 8 < 5 \times 7$

### Method-II: Denominator Equating Method

By making the denominator of each ratio equal to the LCM of the denominators of both ratios, we can compare the two ratios by checking their numerators.

**Illustration 5:** Which of the two ratios  $\frac{5}{6}$  and  $\frac{8}{9}$  is greater.

**Solution:** LCM of 6 and 9 = 18

$$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}$$

$$\frac{8}{9} = \frac{8 \times 2}{9 \times 2} = \frac{16}{18}$$

Since numerator of second ratio is greater than the numerator of first ratio,

$$\therefore \frac{16}{18} > \frac{15}{18} \Rightarrow \frac{8}{9} > \frac{5}{6}$$

## PROPORTION

When two ratios are equal, the four quantities composing them are said to be proportionals. Hence, if  $\frac{a}{b} = \frac{c}{d}$ , then  $a, b, c, d$  are in proportional and is written as

$$a : b :: c : d$$

The terms  $a$  and  $d$  are called extremes while the terms  $b$  and  $c$  are called the means.

$$a : b :: c : d \Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$$

Hence product of extremes = Product of means

**Illustration 6:** What must be added to each of the four numbers 10, 18, 22, 38 so that they become in proportion?

**Solution:** Let the number to be added to each of the four numbers be  $x$ .

By the given condition, we get

$$(10 + x) : (18 + x) :: (22 + x) : (38 + x)$$

$$\Rightarrow (10 + x)(38 + x) = (18 + x)(22 + x)$$

$$\Rightarrow 380 + 48x + x^2 = 396 + 40x + x^2$$

Cancelling  $x^2$  from both sides, we get

$$380 + 48x = 396 + 40x$$

$$\Rightarrow 48x - 40x = 396 - 380$$

$$\Rightarrow 8x = 16 \Rightarrow x = \frac{16}{8} = 2$$

Therefore, 2 should be added to each of the four given numbers.

## Continue Proportion

(i) If  $\frac{a}{b} = \frac{b}{c}$ , then  $a, b, c$ , are said to be in continue proportion and vice-versa.

$$\text{Now } \frac{a}{b} = \frac{b}{c} \Rightarrow ac = b^2$$

Here  $b$  is called mean proportional and  $c$  is called third proportional of  $a$  and  $b$ .

(ii) If  $a, b, c$  and  $d$  are in continue proportion, then

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

$$\text{Also if } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k \text{ (let), a constant}$$

$$\begin{aligned} \text{Then } c &= dk \\ b &= ck = dk \cdot k = dk^2 \\ a &= bk = dk^2 \cdot k = dk^3 \end{aligned}$$

## PROPERTIES OF PROPORTION

(i) **Invertendo:** If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{b}{a} = \frac{d}{c}$

(ii) **Alternando:** If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a}{c} = \frac{b}{d}$

(iii) **Componendo:** If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a+b}{b} = \frac{c+d}{d}$

(iv) **Dividendo:** If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a-b}{b} = \frac{c-d}{d}$

(v) **Componendo and Dividendo:** If  $\frac{a}{b} = \frac{c}{d}$ , then

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

**Illustration 7:** Find the value of  $\frac{x+a}{x-a} + \frac{x+b}{x-b}$ , if  $x = \frac{2ab}{a+b}$

$$\text{Solution: } x = \frac{2ab}{a+b} \Rightarrow \frac{x}{a} = \frac{2b}{a+b}$$

By componendo – dividendo,

$$\frac{x+a}{x-a} = \frac{3b+a}{b-a}$$

Similarly,  $\frac{x}{b} = \frac{2a}{a+b}$

$$\Rightarrow \frac{x+b}{x-b} = \frac{3a+b}{a-b}$$

$$\begin{aligned} \therefore \frac{x+a}{x-a} + \frac{x+b}{x-b} &= \frac{3b+a}{b-a} + \frac{3a+b}{a-b} \\ &= \frac{-(3b+a)}{a-b} + \frac{3a+b}{a-b} = \frac{2a-2b}{a-b} = 2. \end{aligned}$$

## VARIATIONS

We come across many situations in our day to day life where we see change in one quantity bringing change in the other quantity. For example:

- If the number of items purchased increases, its cost also increases.
- If the number of workers working to complete a job increases then days required to complete the job will decrease.

Here we observe that change in one quantity leads to change in other quantity. This is called variation.

## TYPES OF VARIATIONS

There are three types of variations: Direct variation, Indirect variation and Compound variation.

### (i) Direct Variations

There is a direct variation in two quantities if they are related in such a way that an increase in one causes an increase in the other in the same ratio or a decrease in one causes a decrease in the other in the same ratio. This means that if one quantity becomes double then the other quantity also becomes double and if one quantity becomes half then the other quantity also becomes half etc. In other words if  $x$  and  $y$  are two variables then  $y$  varies

directly with  $x$  if the ratio  $\frac{y}{x}$  is a constant.

' $y$  varies directly with  $x$ ' is represented as  $y \propto x$

$y$  varies directly as  $x$  is simply say that  $y$  varies as  $x$ .

Here symbol ' $\propto$ ' means 'varies as'.

The representation  $y \propto x$  can be converted to an equation  $y = kx$ , where  $k$  is a positive constant and called constant of proportionality.

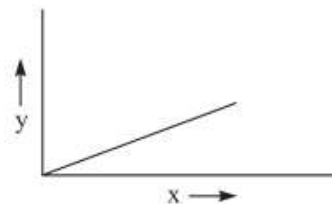
$$\text{Hence } \frac{y}{x} = \text{constant}$$

The equation  $\frac{y}{x} = k$ , means all ratios of a value of  $y$  with their corresponding value of  $x$  are equal.

If  $y_1, y_2$  are two values of  $y$  corresponding to two values  $x_1$  and  $x_2$  of  $x$ , then  $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ .

### Graph

If  $y$  varies directly as  $x$ , then graph between  $x$  and  $y$  will be as shown below:



### Some Examples of Direct Variations

- Number of persons  $\propto$  Amount of work done  
More number of persons, more work.
- Number of days  $\propto$  Amount of work  
More days, More work
- Working rate  $\propto$  Amount of work  
More working rate, more work
- Efficiency of worker  $\propto$  Amount of work  
More efficient worker, More work.

**Illustration 8:** A machine takes 5 hours to cut 120 tools. How many tools will it cut in 20 hours?

**Solution:** Here more time, more number of tools i.e. time and number of tools cut vary directly.

Let number of tools cut in 20 hours be ' $x$ ', then

$$\begin{aligned} \frac{5}{120} &= \frac{20}{x} & \left( \because \frac{y_1}{x_1} = \frac{y_2}{x_2} \right) \\ \Rightarrow x &= \frac{20 \times 120}{5} \\ x &= 480 \end{aligned}$$

Hence required number of tools = 480.

### (ii) Inverse Variations

There is an inverse variation in two quantities if they are so related that an increase in one causes a decrease in the other in the same ratio or vice-versa. This means that if one quantity becomes double then other quantity becomes half and if one quantity becomes one third then other quantity becomes thrice etc.

In other words if  $x$  and  $y$  are variables then  $y$  varies inversely with  $x$ , if  $xy$  is a constant.

' $y$  varies inversely with  $x$ ' is represented as  $y \propto \frac{1}{x}$ .

Here symbol ' $\propto$ ' means 'varies as'. The representation  $y \propto \frac{1}{x}$

can be converted to an equation  $y = \frac{k}{x}$  or  $xy = k$ , where  $k$  is a positive constant, called constant of proportionality.

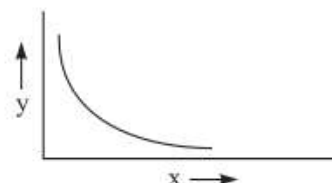
$$\Rightarrow xy = \text{constant}$$

The equation  $xy = \text{constant}$ , means all products of a value of  $y$  and their corresponding value of  $x$  are equal. That is if  $y_1, y_2$  are two values of  $y$  corresponding to the values  $x_1, x_2$  of  $x$  respectively, then

$$x_1 y_1 = x_2 y_2$$

### Graph

If  $y$  varies inversely as  $x$ , then graph between  $x$  and  $y$  will be as shown below:





**Illustration 9:** If 900 persons can finish the construction of a building in 40 days, how many persons are needed to complete the construction of building in 25 days.

**Solution:** Let the required number of persons be 'x'. As the number of days required to complete the job is less, so more number of persons will be required. It is a case of inverse variation.

$$\text{So } 900 \times 40 = x \times 25$$

$$\Rightarrow x = \frac{900 \times 40}{25} = 1440$$

Hence required number of persons = 1440.

## COMPOUND VARIATIONS

In real life, there are many situations which involve more than one variation, i.e. change in one quantity depends on changes in two or more quantities either directly or inversely or by both.

Let  $x$ ,  $y$  and  $z$  are variables, i.e.  $y \propto x$

(a)  $y$  varies directly as  $x$  when  $z$  is constant, i.e.,  $y \propto x$  and  $y$  varies directly as  $z$  when  $x$  is constant, i.e.  $y \propto z$ , then we say that  $y$  varies directly as the product of  $x$  and  $z$ .

Thus  $y \propto xz$

or  $y = k(xz)$ ,  $k$  is a positive constant

(b)  $y$  varies directly as  $x$  when  $z$  is constant, i.e.  $y \propto x$  and  $y$  varies inversely as  $z$  when  $x$  is constant i.e.  $y \propto \frac{1}{z}$ , then

$y \propto \frac{x}{z}$  or  $y = k\left(\frac{x}{z}\right)$ , where  $k$  is a positive constant.

(c)  $y$  varies inversely as  $x$  when  $z$  is constant i.e.  $y \propto \frac{1}{x}$  and

$y$  varies inversely as  $z$  when  $x$  is constant then  $y \propto \frac{1}{xz}$  or

$y = \frac{k}{xz}$ , where  $k$  is a positive constant.

**Illustration 10:** 25 horses eat 5 bags of corn in 12 days, how many bags of corn will 10 horses eat in 18 days?

**Solution:** Here three quantities : number of horses ( $h$ ), number of bags ( $b$ ) and number of days ( $d$ ) are involved.

Number of bags increases as number of horses increases. Also, number of bags increases as number of days increases.

$$\text{Hence } b \propto hd \Rightarrow \frac{b}{hd} = \text{constant}$$

$$\Rightarrow \frac{b_1}{h_1 d_1} = \frac{b_2}{h_2 d_2} \Rightarrow b_2 = \frac{b_1 h_2 d_2}{h_1 d_1}$$

$$\therefore b_2 = \frac{5 \times 10 \times 18}{25 \times 12} = 3$$

Hence number of bags required by 10 horses in 18 days = 3 bags.

## PARTNERSHIP

A partnership is an association of two or more persons who invest their money in order to carry on a certain business.

A partner who manages the business is called the **working partner** and the one who simply invests the money is called the **sleeping partner**.

Partnership is of two kinds :

(i) Simple (ii) Compound.

**Simple partnership :** If the capitals of the partners are invested for the same period, the partnership is called simple.

**Compound partnership :** If the capitals of the partners are invested for different lengths of time, the partnership is called compound.

☒ If the period of investment is same for each partner, then the profit or loss is divided in the ratio of their investments.

If A and B are partners in a business investing for same period, then

$$\frac{\text{Investment of A}}{\text{Investment of B}} = \frac{\text{Profit of A}}{\text{Profit of B}} \text{ or } = \frac{\text{Loss of A}}{\text{Loss of B}}$$

If A, B and C are partners in a business, then

Investment of A : Investment of B : Investment of C

= Profit of A : Profit of B : Profit of C, or

= Loss of A : Loss of B : Loss of C

**Illustration 11:** Three partner Rahul, Puneet and Chandan invest ₹ 1600, ₹ 1800 and ₹ 2300 respectively in a business. How should they divide a profit of ₹ 399?

**Solution:** Profit is to be divided in the ratio 16 : 18 : 23

$$\begin{aligned} \text{Rahul's share of profit} &= \frac{16}{16+18+23} \times 399 \\ &= \frac{16}{57} \times 399 = ₹ 112 \end{aligned}$$

$$\text{Puneet's share of profit} = \frac{18}{57} \times 399 = ₹ 126$$

$$\text{Chandan's share of profit} = \frac{23}{57} \times 399 = ₹ 161$$

**Illustration 12:** A, B and C enter into a partnership by investing 1500, 2500 and 3000 rupees, respectively. A as manager gets one-tenth of the total profit and the remaining profit is divided among the three in the ratio of their investment. If A's total share is ₹ 369, find the shares of B and C.

**Solution:** If total profit is  $x$ , then

$$\text{A's share} = \frac{1}{10}x + \frac{15}{15+25+30} \text{ of the balance } \frac{9}{10}x$$

$$\Rightarrow \frac{1}{10}x + \frac{27x}{140} = 369$$

$$\Rightarrow 14x + 27x = 369 \times 140$$

$$\Rightarrow x = \frac{369 \times 140}{41} = 9 \times 140 = 1260$$

$$\text{B's share} = \frac{5}{14} \times \frac{9}{10} \times 1260 = ₹ 405$$

$$\text{C's share} = \frac{6}{14} \times \frac{9}{10} \times 1260 = ₹ 486$$

**Illustration 13:** A and B invested in the ratio 3 : 2 in a business. If 5% of the total profit goes to charity and A's share is ₹ 855, find the total profit.

**Solution:** Let the total profit be ₹ 100.

Then, ₹ 5 goes to charity.

Now, ₹ 95 is divided in the ratio 3 : 2.

$$\therefore \text{A's share} = \frac{95}{3+2} \times 3 = ₹ 57$$

But A's actual share is ₹ 855.

$$\therefore \text{Actual total profit} = 855 \left( \frac{100}{57} \right) = ₹ 1500$$

In a group of  $n$  persons invested different amount for different period then their profit or loss ratio is

$$At_1 : Bt_2 : Ct_3 : Dt_4 : \dots : Xt_n$$

[Here first person invested amount A for  $t_1$  period, second person invested amount B for  $t_2$  period and so on.]

**Illustration 14:** A and B start a business. A invests ₹ 600 more than B for 4 months and B for 5 months. A's share is ₹ 48 more than that of B, out of a total profit of ₹ 528. Find the capital contributed by each.

$$\text{Solution: B's profit} = \frac{528 - 48}{2} = ₹ 240$$

$$\text{A's profit} = 528 - 240 = ₹ 288$$

$$\frac{\text{A's capital} \times 4}{\text{B's capital} \times 5} = \frac{288}{240} = \frac{6}{5}$$

$$\therefore \frac{\text{A's capital}}{\text{B's capital}} = \frac{6}{5} \times \frac{5}{4} = \frac{3}{2}$$

$$\Rightarrow \frac{\text{B's capital} + 600}{\text{B's capital}} = \frac{3}{2}$$

$$\Rightarrow \text{B's capital} = ₹ 1200 \text{ and A's capital} = ₹ 1800$$

**Illustration 15:** Three persons A, B, C rent the grazing of a park for ₹ 570. A puts in 126 oxen in the park for 3 months, B puts in 162 oxen for 5 months and C puts in 216 oxen for 4 months. What part of the rent should each person pay?

**Solution:** Monthly equivalent rent of A =  $126 \times 3 = 378$

Monthly equivalent rent of B =  $162 \times 5 = 810$

Monthly equivalent rent of C =  $216 \times 4 = 864$

$\therefore$  Rent is to be divided in the ratio 378 : 810 : 864, i.e. 7 : 15 : 16

$$\therefore \text{A would have to pay } \frac{7}{7+15+16} \text{ of the rent} \\ = \frac{7}{38} \text{ of the rent} = \frac{7}{38} \times 570 = ₹ 105$$

$$\therefore \text{B would have to pay } \frac{15}{38} \text{ of the rent} = \frac{15}{38} \times 570 \\ = ₹ 225$$

$$\text{and C would have to pay } \frac{16}{38}, \text{ i.e. } \frac{8}{19} \text{ of the rent} \\ = \frac{8}{19} \times 570 = ₹ 240$$

**Illustration 16:** Shekhar started a business investing ₹ 25,000 in 1999. In 2000, he invested an additional amount of ₹ 10,000 and Rajeev joined him with an amount of ₹ 35,000. In 2002, Shekhar invested another additional amount of ₹ 10,000 and Jatin joined them with an amount of ₹ 35,000. What will be Rajeev's share in the profit of ₹ 1,50,000 earned at the end of 3 years from the start of the business in 1999?

(a) ₹ 45,000

(b) ₹ 50,000

(c) ₹ 70,000

(d) ₹ 75,000

**Solution:** (b) Ratio of Shekhar, Rajeev and Jatin's investments  
 $= 25000 \times 36 + 10000 \times 24 + 10000 \times 12 : 35000$   
 $\times 24 : 35000 \times 12,$

$$= 25 \times 36 + 10 \times 24 + 10 \times 12 : 35 \times 24 : 35 \times 12$$

$$= 25 \times 3 + 10 \times 2 + 10 \times 1 : 35 \times 2 : 35 \times 1$$

$$= 75 + 20 + 10 : 70 : 35$$

$$= 105 : 70 : 35, \text{ i.e. } 3 : 2 : 1$$

$$\therefore \text{Rajeev's share in the profit} = \frac{2}{6} \times 150000 = ₹ 50000$$