CHAPTER 16

LOGARITHMS

DEFINITION

If $x = a^m$, then $\log_a x = m$, where 'a' and 'x' both are positive real numbers but 'a' not equal to 1 i.e., $a, x > 0$, but $a \neq 1$.

Here log is the short form of logarithm.

$\log_a x$ is read as log of x to the base a.

For example,

(i) Since, $10 = 10^1$, $100 = 10^2$, $1000 = 10^3$, etc.

Hence, $\log_{10}10 = 1$, $\log_{10}100 = 2$, $\log_{10}1000 = 3$, etc.

(ii) Since, $8 = 2^3$, $16 = 2^4$, $32 = 2^5$, etc.

Hence, $\log_2 8 = 3$, $\log_2 16 = 4$, $\log_2 32 = 5$, etc.

(iii) Since, $\frac{1}{8} = (2)^{-3}$, $\frac{1}{16} = (2)^{-4}$, etc.

Hence, $\log_2\left(\frac{1}{8}\right) = -3$, $\log_2\left(\frac{1}{16}\right) = -4$, etc.

(iv) Since, $0.01 = (10)^{-2}$, $0.001 = (10)^{-3}$, etc.

Hence $\log_{10}(0.01) = -2$, $\log_{10}(0.001) = -3$, etc.

LAWS OF LOGARITHM

(i) $\log_a(m \times n) = \log_a m + \log_a n$

In general,

$\log_a(m \times n \times p \times ....) = \log_a m + \log_a n + \log_a p + ...$

For example:

$\log_2(4 \times 5 \times 6) = \log_2 4 + \log_2 5 + \log_2 6$

Note that

$\log_a m + \log_a n \neq \log_a(m + n)$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

For example:

$\log_4\left(\frac{8}{15}\right) = \log_4 8 - \log_4 15$

Note that

$\log_a m - \log_a n \neq \log_a(m - n)$

(iii) $\log_a(m)^n = n \log_a m$

For example:

$\log_3(5)^4 = 4\log_3 5$

(iv) $\log_b a = \frac{\log_c a}{\log_c b}$ [Change of base rule]

For example,

$\log_5 20 = \frac{\log_2 20}{\log_2 5} = \frac{\log_4 20}{\log_4 5} = \frac{\log_7 20}{\log_7 5} = ...$ etc.

(v) $\log_b a = \frac{1}{\log_a b}$

For example,

$\log_{10} 100 = \frac{1}{\log_{100} 10}$

(vi) $\log_b a \,.\, \log_c b = \log_c a$ [Chain Rule]

In general,

$\log_b a \,.\, \log_c b \,.\, \log_d c ....\log_n m = \log_n a$

For example,

$\log_{24} 256 \,.\, \log_{10} 24 \,.\, \log_2 10 = \log_2 256$

Illustration 1: $\log_a 4 + \log_a 16 + \log_a 64 + \log_a 256 = 10$. Then $a = ?$

(a) 4 **(b)** 2
(c) 8 **(d)** 5

Solution: (a) The given expression is:

$\log_a(4 \times 16 \times 64 \times 256) = 10$

i.e. $\log_a 4^{10} = 10$

Thus, $a = 4$.

Illustration 2: Find x if $\log x = \log 1.5 + \log 12$

(a) 12 **(b)** 8
(c) 18 **(d)** 15

Solution: (c) $\log x = \log 18 \Rightarrow x = 18$

Illustration 3: Find x, if $\log(2x - 2) - \log(11.66 - x) = 1 + \log 3$

(a) 452/32 **(b)** 350/32
(c) 11 **(d)** 11.33

Solution: (c) $\log(2x - 2)/(11.66 - x) = \log 30$

$\Rightarrow (2x - 2)/(11.66 - x) = 30$

$2x - 2 = 350 - 30x$

Hence, $32x = 352 \Rightarrow x = 11$.

Illustration 4: **Solve for x:**

$$\log \frac{75}{35} + 2\log \frac{7}{5} - \log \frac{105}{x} - \log \frac{13}{25} = 0$$

(a) 90 **(b)** 65
(c) 13 **(d)** 45

Solution: **(c)** $(75/35) \times (49/25) \times (x/105) \times (25/13) = 1$
$\Rightarrow x = 13$

SOME IMPORTANT PROPERTIES

(i) $x = a^m \Rightarrow \log_a x = m$
and $\log_a x = m \Rightarrow x = a^m$
Here equation $x = a^m$ is in exponential form and equation $\log_a x = m$ is in logarithmic form.

(ii) If base of log is not mentioned, then we assume the base as 10.
$\therefore \log m = \log_{10} m$
log to the base 10 is called common log.

(iii) Since, $10000 = (100)^2 = (10)^4$
$\therefore \log_{100} 10000 = 2, \log_{10} 10000 = 4$
Thus value of log of a number on different bases is different i.e., value of log of a number depends on its base.

(iv) (a) Since, $a = a^1$, hence $\log_a a = 1$
For example, $\log_5 5 = 1$, $\log_{10} 10 = 1$
Thus log of any number to the same base is always 1.
(b) Since, $1 = a^0$, hence, $\log_a 1 = 0$
For example, $\log_8 1 = 0$
Thus log of 1 to any base always equal to 0.

(v) $a^{(\log_a x)} = x$
For example,

$$20^{(\log_{20} 50)} = 50$$

(vi) (a) log of zero and negative numbers is not defined.
(b) Base of log is always positive but not equal to 1.

Illustration 5: **Find x, if $0.01^x = 2$**

(a) log 2/2 **(b)** 2/log 2
(c) – 2/log 2 **(d)** – log 2/2

Solution: **(d)** $x = \log_{0.01} 2 = -\log 2/2$.

Illustration 6: **If log 3 = .4771, find $\log (.81)^2 \times \log \left(\frac{27}{10}\right)^{\frac{2}{3}} \div \log 9$.**

(a) 2.689 **(b)** – 0.0552
(c) 2.2402 **(d)** 2.702

Solution: **(b)** $2 \log (81/100) \times 2/3 \log (27/10) \div \log 9$
$= 2 [\log 3^4 - \log 100] \times 2/3 [(\log 3^3 - \log 10)] \div 2 \log 3$
$= 2 [\log 3^4 - \log 100] \times 2/3 [(3\log 3 - 1)] \div 2 \log 3$
Substitute $\log 3 = 0.4771 \Rightarrow -0.0552$.