CHAPTER 1

FUNDAMENTALS

'BODMAS' RULE

A given series of calculations or operations is done in a specific order as each letter of BODMAS in order represent.

B → Brackets and order of operation of brackets is ( ), { }, [ ]
O → Of (Calculation is done the same as multiplication)
D → Division
M → Multiplication
A → Addition
S → Subtraction

So, first of all we solve the inner most brackets moving outwards. Then we perform 'of' which means multiplication, then division, addition and subtraction.

- Addition and subtraction can be done together or separately as required.
- Between any two brackets if there is not any sign of addition, subtraction and division it means we have to do multiplication $(20 \div 5)\ (7 + 3 \times 2) + 8 = 4\ (7 + 6) + 8$
$= 4 \times 13 + 8 = 52 + 8 = 60$

BRACKETS

They are used for the grouping of things or entities. The various kind of brackets are:

(i) '–' is known as line (or bar) bracket or vinculum.
(ii) ( ) is known as parenthesis, common bracket or small bracket.
(iii) { } is known as curly bracket, brace or middle bracket.
(iv) [ ] is known as rectangular bracket or big bracket.

The order of eliminating brackets is:

(i) line bracket
(ii) small bracket (i.e., common bracket)
(iii) middle bracket (i.e., curly bracket)
(iv) big bracket (i.e., rectangular bracket)

Illustration 1: Find the value of

$$\left[5-\left\{6-(5-\overline{4-3})\right\}\right] \text{ of } \frac{1+\frac{1}{2}}{1-\frac{1}{2}} \div \frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-\frac{1}{3}}$$

Solution: $\left[5-\left\{6-(5-\overline{4-3})\right\}\right] \text{ of } \dfrac{1+\frac{1}{2}}{1-\frac{1}{2}} \div \dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-\frac{1}{3}}$

$$= [5-\{6-(5-1)\}] \text{ of } \frac{\frac{3}{2}}{\frac{1}{2}} \surd \frac{\frac{5}{6}}{\frac{1}{6}}$$

$$= \{5-(6-4)\} \text{ of } \left(\frac{3}{2}\times\frac{2}{1}\right) \div \left(\frac{5}{6}\times\frac{6}{1}\right)$$

$$= (5-2) \text{ of } 3 \div 5$$

$$= 3 \text{ of } 3 \div 5 = 3\times\frac{3}{5} = \frac{9}{5}$$

FACTORIAL

The product of n consecutive natural numbers (or positive integers) from 1 to n is called as the factorial 'n'. Factorial n is denoted by $n!$.

i.e., $n! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \ldots (n-2)\ (n-1)\ n$
$4! = 1 \times 2 \times 3 \times 4 = 4 \times 3 \times 2 \times 1$
$5! = 1 \times 2 \times 3 \times 4 \times 5 = 5 \times 4 \times 3 \times 2 \times 1$
$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Note: $0! = 1$ and $1! = 1$

Properties

(i) $n!$ is always an even number if $n \geq 2$.
(ii) $n!$ always ends with zero if $n \geq 5$.

ROMAN NUMBERS

In this system there are basically seven symbols used to represent the whole Roman number system. The symbols and their respective values are given below.

I = 1, V = 5, X = 10, L = 50,
C = 100, D = 500 and M = 1000

In general, the symbols in the numeral system are read from left to right, starting with the symbol representing the largest value; the same symbol cannot occur continuously more than three times; the value of the numeral is the sum of the values of the symbols.

For example LX VII = 50 + 10 + 5 + 1 + 1 = 67.

An exception to the left to the right reading occurs when a symbol of smaller value is followed immediately by a symbol of greater value, then the smaller value is subtracted from the larger. For example.

CDXL VIII = (500 – 100) + (50 – 10) + 5 + 1 + 1 + 1 = 448.

Illustration 2: The value of the numeral MCDLXIV is:

(a) 1666 (b) 664 (c) 1464 (d) 656

Solution: MCDLXIV = 1000 + (500 – 100) + 50 + 10 + (5 – 1) = 1464

Hence (c) is the correct option.

Illustration 3: Which of the following represents the numeral for 2949

(a) MMMIXL (b) MMXMIX
(c) MMCMIL (d) MMCMXLIX

Solution: 2949 = 2000 + 900 + 40 + 9
= (1000 + 1000) + (1000 – 100) + (50 – 10) + (10 – 1)
= MMCMXLIX

Hence (d) is the correct option.

IMPORTANT CONVERSION

1 trillion = 10^{12} = 1000000000000
1 billion = 10^9 = 1000000000
1 million = 10^6 = 1000000
1 crore = 10^7 = 100 lakh
10 lakh = 10^6 = 1 million
1 lakh = 10^5 = 100000 = 100 thousand
1 thousand = 10^3 = 1000

ABSOLUTE VALUE OR MODULUS OF A NUMBER

Absolute value of a number is its numerical value irrespective of its sign.

If x be a real number N then | N | indicates the absolute value of N.

Thus | 6 | = 6, | – 6 | = 6, | 0 | = 0, | 1 | = 1, | 3.4 | = 3.4, | – 6.8 | = 6.8, etc.

| – 6 | = 6 can also be written as | – 6 | = – (– 6) = 6. Thus, if x is a negative number, then $|x| = -x$ and if x is non-negative number, then $|x| = x$

Hence $|x| = \begin{cases} x, \text{ if } x \ge 0 \\ -x, \text{ if } x < 0 \end{cases}$

PROPERTIES OF A MODULUS

(i) $|a| = |-a|$ (ii) $|ab| = |a||b|$

(iii) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ (iv) $|a+b| \le |a| + |b|$

(The sign of equality holds only when the sign of a and b are same)

(v) If $|a| \le k \Rightarrow -k \le a \le k$

(vi) If $|a-b| \le k \Rightarrow -k \le a-b \le k \Rightarrow b-k \le a \le b+k$

Illustration 4: Solution of the equation | x – 2 | = 5 is

(a) 3, – 7 (b) – 3, 7
(c) 3, 6 (d) None of these

Solution: $|x-2| = 5 \Rightarrow x-2 = 5$ or $x-2 = -5$
$\Rightarrow$ $x = 7$ or $x = -3$

Hence (b) is the correct option.

Illustration 5: The minimum value of the expression | 17x – 8 | – 9 is

(a) 0 (b) – 9
(c) $\frac{8}{17}$ (d) none of these

Solution: The value of expression $|17x-8|-9$ is minimum only when $|17x-8|$ is minimum. But the minimum value of | k | is zero.

Hence minimum value of $|17x-8|-9 = 0-9 = -9$

Hence (b) is the correct answer.

POWERS OR EXPONENTS

When a number is multiplied by itself, it gives the square of the number. i.e., $a \times a = a^2$ (Example $5 \times 5 = 5^2$)

If the same number is multiplied by itself twice we get the cube of the number i.e., $a \times a \times a = a^3$ (Example $4 \times 4 \times 4 = 4^3$)

In the same way $a \times a \times a \times a \times a = a^5$

and $a \times a \times a \times \ldots$ upto n times $= a^n$

There are five basic rules of powers which you should know:

If a and b are any two real numbers and m and n are positive integers, then

(i) $a^m \times a^n = a^{m+n}$ (Example: $5^3 \times 5^4 = 5^{3+4} = 5^7$)

(ii) $\frac{a^m}{a^n} = a^{m-n}$, if $m > n$ $\left(\text{Example: } \frac{6^5}{6^2} = 6^{5-2} = 6^3\right)$

$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, if $m < n$ $\left(\text{Example: } \frac{4^3}{4^8} = \frac{1}{4^{8-3}} = \frac{1}{4^5}\right)$

and $\frac{a^m}{a^n} = a^0 = 1$, if $m = n$ $\left(\text{Example: } \frac{3^4}{3^4} = 3^{4-4} = 3^0 = 1\right)$

(iii) $(a^m)^n = a^{mn} = (a^n)^m$ (Example: $(6^2)^4 = 6^{2\times 4} = 6^8 = (6^4)^2$

(iv) (a) $(ab)^n = a^n \cdot b^n$ (Example: $(6 \times 4)^3 = 6^3 \times 4^3$)

(b) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \ne 0$ $\left(\text{Example: } \left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4}\right)$

(v) $a^{-n} = \frac{1}{a^n}$ $\left(\text{Example: } 5^{-3} = \frac{1}{5^3}\right)$

(vi) For any real number a, $a^0 = 1$

Illustration 6: $\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2} = ?$

Solution: $\frac{5^n \times 5^3 - 6 \times 5^n \times 5}{5^n (9 - 2^2)}$

$= \frac{5^n (5^3 - 6 \times 5)}{5^n \times 5}$

$= \frac{125 - 30}{5}$

$= \frac{95}{5} = 19$

Illustration 7: $\left\{\left(\sqrt[3]{(81)^2}\right)^{3/2}\right\}^{1/4} = ?$

Solution: $\left\{\left(\sqrt[3]{(81)^2}\right)^{3/2}\right\}^{1/4}$

$= \left\{\left(81^2\right)^{1/3 \times \frac{3}{2}}\right\}^{1/4}$

$= (81)^{2\times\frac{1}{3}\times\frac{3}{2}\times\frac{1}{4}} = \left(3^4\right)^{\frac{1}{4}} = 3$

ALGEBRAIC IDENTITIES

Consider the equality $(x + 2)(x + 3) = x^2 + 5x + 6$

Let us evaluate both sides of this equality for some value of variable x say $x = 4$

LHS $= (x + 2)(x + 3) = (4 + 2)(4 + 3) = 6 \times 7 = 42$

RHS $= (4)^2 + 5 \times 4 + 6 = 16 + 20 + 6 = 42$

So for $x = 4$, LHS = RHS

Let us calculate LHS and RHS for $x = -3$

LHS $= (-3 + 2)(-3 + 3) = 0$

RHS $= (-3)2 + -(-3) + 6 = 9 - 15 + 6 = 0$

$\therefore$ for $x = -3$, LHS = RHS

If we take any value of variable x, we can find that LHS = RHS

Such an equality which is true for every value of the variable present in it is called an identity. Thus $(x + 2)(x + 3) = x^2 + 5x + 6$, is an identity.

Identities differ from equations in the following manners.

An equation is a statement of equality of two algebraic expression involving one or more variables and it is true for certain values of the variable.

For example:

$4x + 3 = x - 3$ ... (1)

$\Rightarrow$ $3x = -6 \Rightarrow x = -2$

Thus equality (1) is true only for $x = -2$, no other value of x satisfy equation (1).

Standard Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(v) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Some More Identities

We have dealt with identities involving squares. Now we will see how to handle identities involving cubes.

(i) $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\Rightarrow (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(ii) $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$\Rightarrow (a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(iii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(iv) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(v) $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

2. Multiplication of Two Numbers Using Formulae $(a - b)(a + b) = a^2 - b^2$

If the difference between two numbers x and y is a small even number, then the smaller is express as $(a - b)$ whereas larger is expressed as $(a + b)$, then the product of x and y is found out by the formulae $x \cdot y$ i.e., $(a - b)(a + b) = a^2 - b^2$

Here a should be such that a^2 is very easily calculated.

For example:

(i) $38 \times 42 = (40 - 2) \times (40 + 2) = (40)^2 - (2)^2 = 1600 - 4 = 1596$

(ii) $66 \times 74 = (70 - 4) \times (70 + 4) = (70)^2 - (4)^2 = 4900 - 16 = 4884$

(iii) $2094 \times 2106 = (2100 - 6) \times (2100 + 6) = (2100)^2 - (6)^2 = 4410000 - 36 = 4409964$

If the difference between the two numbers is not even, still this method is used by modify as

$47 \times 54 = 47 \times 53 + 47$

$= (50 - 3) \times (50 + 3) + 47$

$= (50)^2 - (3)^2 + 47$

$= 2500 - 9 + 47 = 2538$

SQUARES

When a number is multiplied by itself, then we get the square of the number.

For example, square of $5 = 5 \times 5$ (or 5^2) $= 25$

Square of 2 and 3 digits numbers and cube of 2 digits numbers are very useful in CAT and CAT like competitions.

For this it is advised to learn the square of 1 to 30 as given in the table:

Number	Square	Number	Square
1	1	16	256
2	4	17	289
3	9	18	324
4	16	19	361
5	25	20	400
6	36	21	441
7	49	22	484
8	64	23	529
9	81	24	576
10	100	25	625
11	121	26	676

Number	Square	Number	Square
12	144	27	729
13	169	28	784
14	196	29	841
15	225	30	900

SQUARE ROOTS

If $b = a \times a$ or a^2, then a is called square root of b and it is represented as $\sqrt{b} = a$ or $(b)^{1/2} = a$.

Since, $16 = 4 \times 4$ or 4^2, therefore $\sqrt{16} = 4$

And $25 = 5 \times 5$ or 5^2, therefore $\sqrt{25} = 5$

There are two methods for finding the square root of a number.

(i) Prime Factorisation Method

To find the square root by this method, we first factorise the given number into prime numbers as given below for the number 3136.

$$\begin{array}{r|r} 2 & 3136 \\ \hline 2 & 1568 \\ \hline 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$3136 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$

Now pair the same prime factor like

$3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

Now product of prime numbers staken one number from each pair of prime factors is the square root of the given number

$\therefore \sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

If we write, $3136 = (2)^6 \times (7)^2$

Then square root of 3136 is the product of prime factors 2 and 7 with the powers half of the powers raised on 2 and 7 respectively.

i.e., $\sqrt{3136} = (2)^3 \times 7 = 56$

(ii) Division Method

In this method first of all pair the digits of the given number from right side. But there may be left a single digit at the left end of the number. Further process is shown below for the number 2304.

$\sqrt{2304} = 48$

$$\begin{array}{r|l} & 48 \\ \hline 4 & \underline{23}\,\underline{04} \\ 4 & 16 \\ \hline 88 & 704 \\ 8 & 704 \\ \hline & \times\times\times \end{array}$$

Illustration 8: Find the square root of 15625.

Solution: $\sqrt{15625} = 125$

$$\begin{array}{r|l} & 125 \\ \hline 1 & \underline{1}\,\underline{56}\,\underline{25} \\ 1 & 1 \\ \hline 22 & 56 \\ 2 & 44 \\ \hline 245 & 1225 \\ 5 & 1225 \\ \hline & \times\times\times\times \end{array}$$

CUBES

When a number multiplies itself three times, we get the cube of the number.

Cube of $4 = 4 \times 4 \times 4 = 64$

Cubes of large numbers are rarely used. It is advised to you to learn the cube of the integers from 1 to 10.

Number	1	2	3	4	5	6	7	8	9	10
Cube	1	8	27	64	125	216	343	512	729	1000