CHAPTER

20

COORDINATE GEOMETRY

RECTANGULAR COORDINATE AXES

Let XOX' be a horizontal straight line and YOY' be a vertical straight line drawn through a point O in the plane of the paper. Then

the line XOX' is called x-axis

the line YOY' is called y-axis

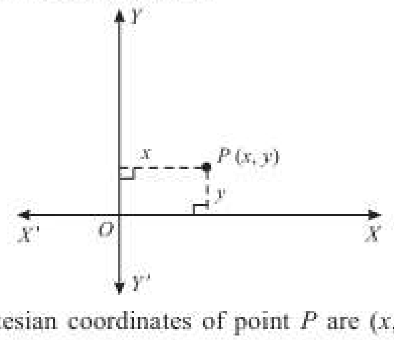
plane of paper is called xy-plane or cartesian plane.

x-axis and y-axis together are called co-ordinate axes or axis of reference.

The point O is called the origin.

Cartesian Coordinates

Position of any point in a cartesian plane can be described by their cartesian coordinates. The ordered pair of perpendicular distances first from y-axis and second from x-axis of a point P is called cartesian coordinates of P.



If the cartesian coordinates of point P are (x, y), then x is called abscissa or x-coordinate of P and y is called the ordinate or y-coordinate of point P.

SIGN CONVENTIONS IN THE *xy*-PLANE

(i) All the distances are measured from origin (o).

(ii) All the distances measured along or parallel to x-axis but right side of origin are taken as $+ve$.

(iii) All the distances measured along or parallel to x-axis but left side of origin are taken as $-ve$.

(iv) All the distances measured along or parallel to y-axis but above the origin are taken as $+ve$.

(v) All the distances measure along or parallel to y-axis but below the origin are taken as $-ve$.

According to the Above Sign Conventions

(i) Coordinate of origin is (0, 0)

(ii) Coordinate of any point on the x-axis but right side of origin is of the form $(x, 0)$, where $x > 0$.

(iii) Coordinate of any point on the x-axis but left side of origin is of the form $(-x, 0)$, where $x > 0$.

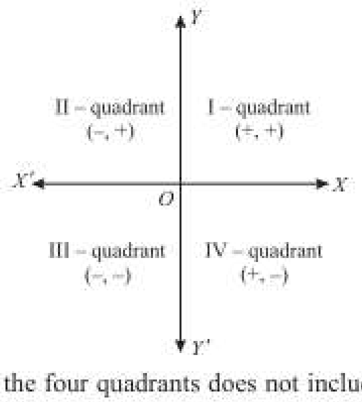
(iv) Coordinate of any point on the y-axis but above the origin is of the form $(0, y)$, where $y > 0$.

(v) Coordinate of any point on the y-axis but below the origin is of the form $(0, -y)$, where $y > 0$.

QUADRANTS OF xy-PLANE AND SIGN OF x AND y-COORDINATE OF A POINT IN DIFFERENT QUADRANTS

x and y-axis divide the xy-plane in four parts. Each part is called a quadrant.

The four quadrants are written as I-quadrant (XOY), II-quadrant (YOX'), III-quadrant ($X'OY'$) and IV-quadrant ($Y'OX$). Each of these quadrants shows the specific quadrant of the xy-plane as shown below:



(i) Any of the four quadrants does not includes any part of x or y-axis.

(ii) In the first quadrant both x and y-coordinates of any point are $+ve$.

(iii) In second quadrant x-coordinate of any point is $-ve$ but y-coordinate of any point is $+ve$.

(iv) In third quadrant, both x and y-coordinates of any point are $-ve$.

(v) In fourth quadrant, x-coordinate of any point is $+ve$ but y-coordinate of any point is $-ve$ as shown in the above diagram.

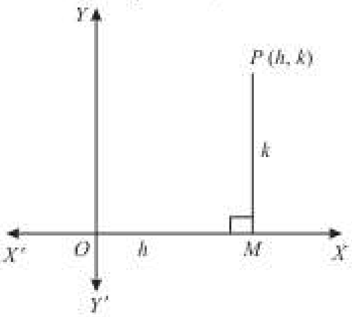
PLOTTING A POINT WHOSE COORDINATES ARE KNOWN

The point can be plotted by measuring its proper distances from both the axes. Thus, any point P whose coordinates are (h, k) can be plotted as follows:

(i) Measure OM equal to h (i.e. x-coordinate of point P) along the x-axis.

(ii) Now perpendicular to OM equal to k.

Mark point P above M such that PM is parallel to y-axis and $PM = k$ (i.e. y-coordinate of point P)



In this chapter, now we shall study to find the distance between two given points, section formula, mid-point formula, slope of a line, angles between two straight lines and equation of a line in different forms etc.

DISTANCE FORMULA

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \left|\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right| \text{ or } \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

Distance of point $P(x, y)$ from the origin $= \left|\sqrt{x^2 + y^2}\right|$

Illustration 1: If distance between the point $(x, 2)$ and $(3, 4)$ is 2, then find the value of x.

Solution:

$$2 = \left|\sqrt{(x-3)^2 + (2-4)^2}\right| \Rightarrow 2 = \left|\sqrt{(x-3)^2 + 4}\right|$$

Squaring both sides

$$4 = (x-3)^2 + 4 \Rightarrow x - 3 = 0 \Rightarrow x = 3$$

Illustration 2: Find the distance between each of the following points :

$A(-6, -1)$ and $B(-6, 11)$

Solution: Here the points are $A(-6, -1)$ and $B(-6, 11)$

By using distance formula, we have

$$AB = \sqrt{\{-6-(-6)\}^2 + \{11-(-1)\}^2} = \sqrt{0^2 + 12^2} = 12$$

Hence, $AB = 12$ units.

SECTION FORMULA

Co-ordinates of a point which divides the line segment joining two points $P(x_1, y_1)$ and $Q(x^2, y^2)$ in the ratio $m_1 : m_2$ are :

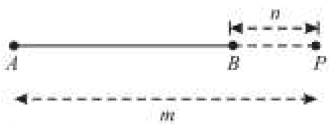
(i) $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$, for internal division.

P divides AB internally in the ratio $m : n$

If $m_1 = m_2$, then the point P will be the mid point of PQ whose co-ordinates $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

P is the mid-point of AB

(ii) $\left(\dfrac{m_1x_2 - m_2x_1}{m_1 - m_2}, \dfrac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$, for external division



P divides AB externally in the ratio $m : n$

(iii) When we need to find the ratio in which a point on a line segment divides it, we suppose the required ratio as $k : 1$ or $m/n : 1$.

Note:

(i) Co-ordinates of any point on the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are

$$\left(\dfrac{x_1 + \lambda x_2}{1 + \lambda}, \dfrac{y_1 + \lambda y_2}{1 + \lambda}\right), (\lambda \neq -1)$$

(ii) **Division by axes:** Line segment joining the points (x_1, y_1) and (x^2, y^2) is divided by

(a) x-axis in the ratio $-y_1 / y_2$

(b) y-axis in the ratio $-x / x_2$

If ratio is positive division internally and if ratio is negative division is externally.

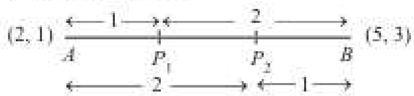
(iii) **Division by a line:** Line $ax + by + c = 0$ divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $\left(-\dfrac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right)$.

Illustration 3: Find the ratio in which the line $3x + 4y = 7$ divides the line segment joining the points $(1, 2)$ and $(-2, 1)$.

Solution: Ratio $= -\dfrac{3(1) + 4(2) - 7}{3(-2) + 4(1) - 7} = -\dfrac{4}{-9} = \dfrac{4}{9} = 4 : 9$.

Illustration 4: Find the points of trisection of line joining the points A (2, 1) and B (5, 3).

Solution:



$$P_1(x, y) = \left(\frac{1\times5+2\times2}{1+2}, \frac{1\times3+2\times1}{1+2}\right) = \left(3, \frac{5}{3}\right)$$

$$P_2(x, y) = \left(\frac{2\times5+1\times2}{2+1}, \frac{2\times3+1\times1}{2+1}\right) = \left(4, \frac{7}{3}\right).$$

Illustration 5: Prove that points A (1, 1), B (– 2, 7) and C (3, – 3) are collinear.

Solution: $AB = \left|\sqrt{(1+2)^2 + (1-7)^2}\right| = \left|\sqrt{9+36}\right| = 3\sqrt{5}$

$BC = \left|\sqrt{(-2-3)^2 + (7+3)^2}\right| = \left|\sqrt{25+100}\right| = 5\sqrt{5}$

$CA = \left|\sqrt{(3-1)^2 + (-3-1)^2}\right| = \left|\sqrt{4+16}\right| = 2\sqrt{5}$

Clearly, $BC = AB + AC$. Hence A, B, C are collinear.

Illustration 6: Find the ratio in which the join of (– 4, 3) and (5, –2) is divided by (i) x-axis (ii) y-axis.

Solution:

(i) x-axis divides the join of (x_1, y_1) and (x_2, y_2) in the ratio of $-y_1 : y_2 = -3 : -2 = 3 : 2$.

(ii) y-axis divides, in the ratio of $-x_1 : x_2 \Rightarrow 4 : 5$.

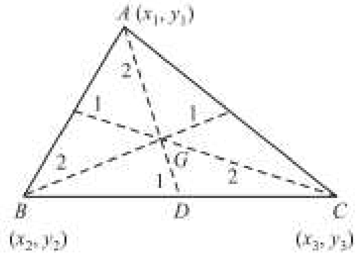
COORDINATES OF SOME PARTICULAR POINTS

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of any triangle ABC, then

Centroid

Centroid is the point of intersection of the medians of a triangle. Centroid divides each median in the ratio of 2 : 1.

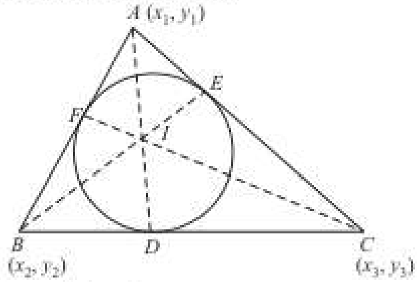
A median is a line segment joining the mid point of a side to its opposite vertex of a triangle.



Co-ordinates of centroid, $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Incentre

Incentre is the point of intersection of internal bisectors of the angles of a triangle. Also incentre is the centre of the circle touching all the sides of a triangle.



Co-ordinates of incentre,

$$I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right),$$

where a, b, c are length of the sides opposite to vertices A, B, C respectively of triangle ABC.

(i) Angle bisector divides the opposite sides in the ratio of the sides included in the angle. For example $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$.

(ii) Incentre divides the angle bisectors AD, BE and CF in the ratio $(b+c) : a$, $(c+a) : b$ and $(a+b) : c$ respectively.