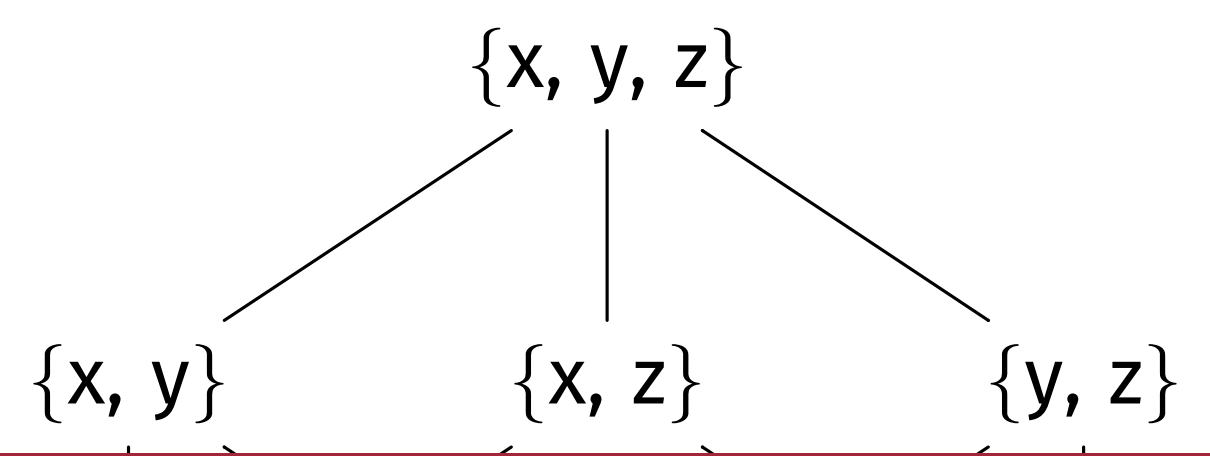

Abstract Interpretation

Software Engineering
Programming Languages

universität ulm

The Why

```
public static void main(String[] args) {
    int a = 1;
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$\{ a \in \{1\} \}$

$\{ r \in [0..10) \}$

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    double r = Math.random() * 10;   ⟅ a ∈ {1} ⟆
    if (r > 5) {                     ⟅ r ∈ [0..10) ⟆
       a = 2;
    }                                ⟅ a ∈ {2} ⟆
    System.out.println(a);
}
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- We want to proof, that a program satisfies certain properties

Origins

Based on the amazing “Tutorial on Static Inference of Numeric Invariants by Abstract Interpretation” by Miné [Min17], https://www.di.ens.fr/~cousot/AI/, and [Bal+18; GR22]

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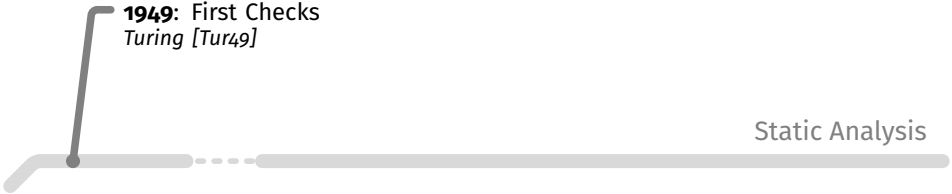

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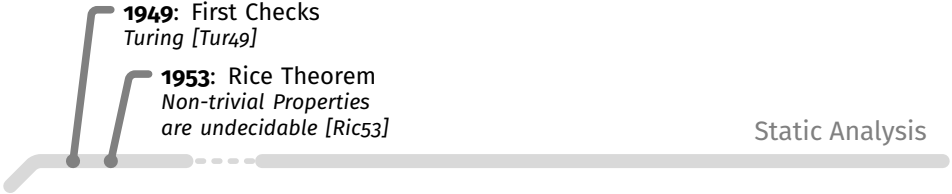

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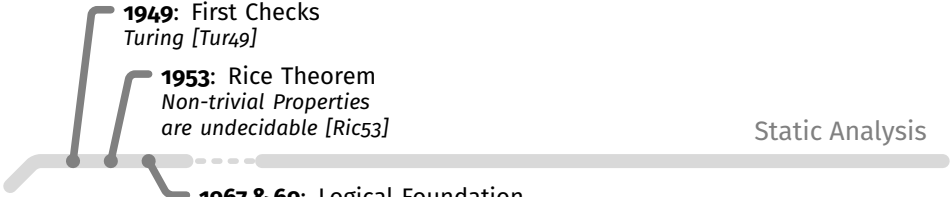

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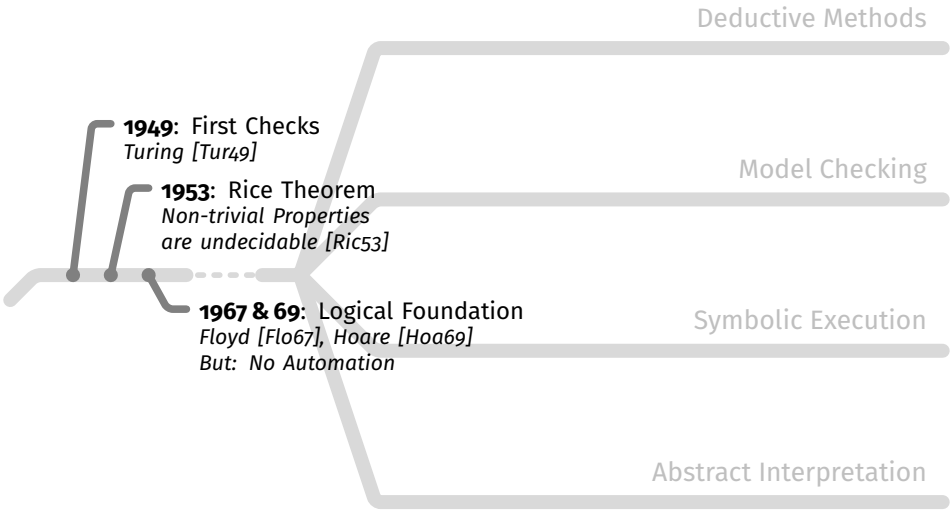

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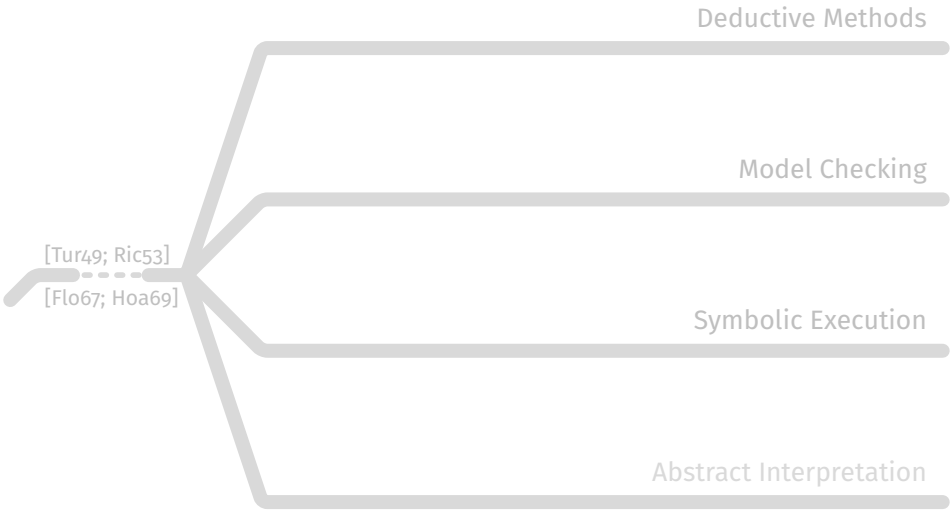

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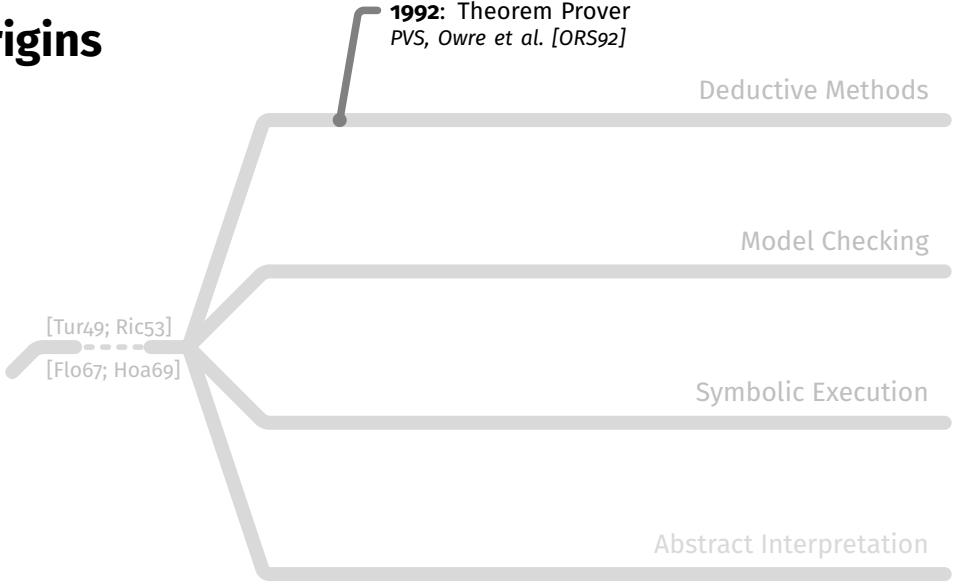

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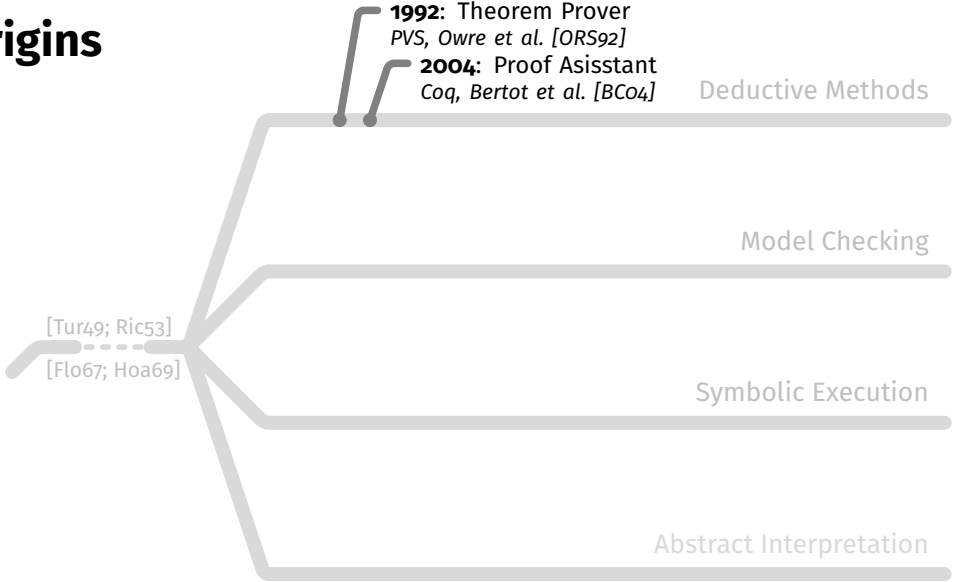

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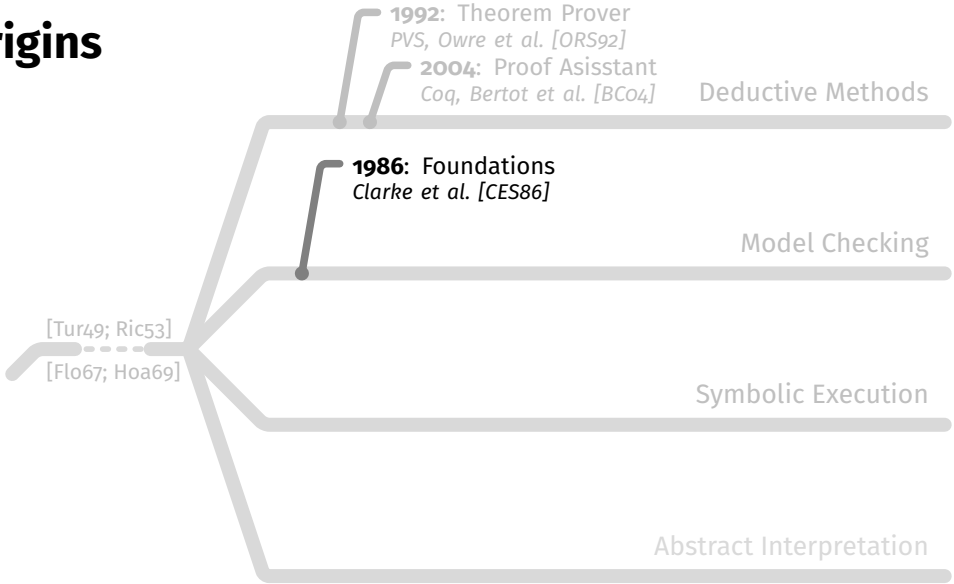

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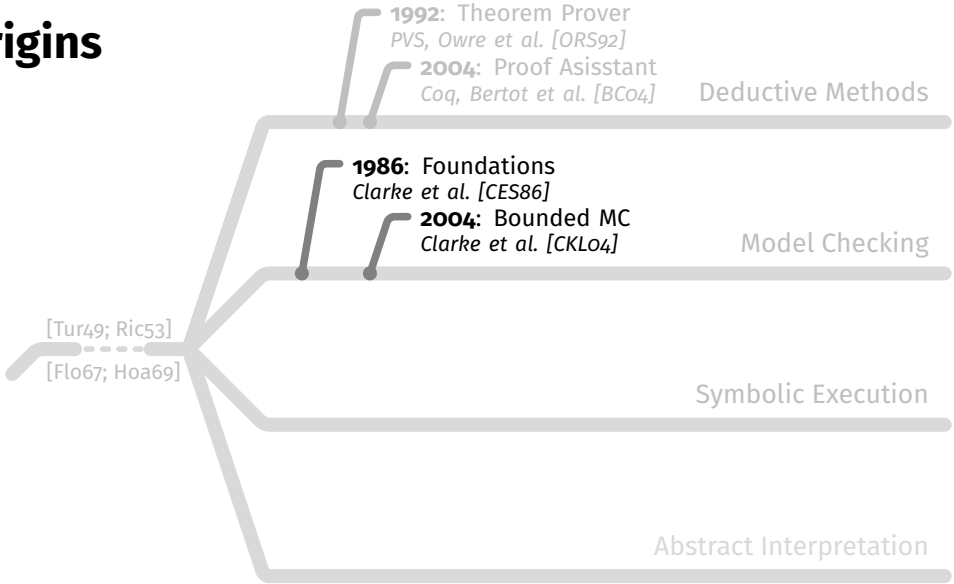

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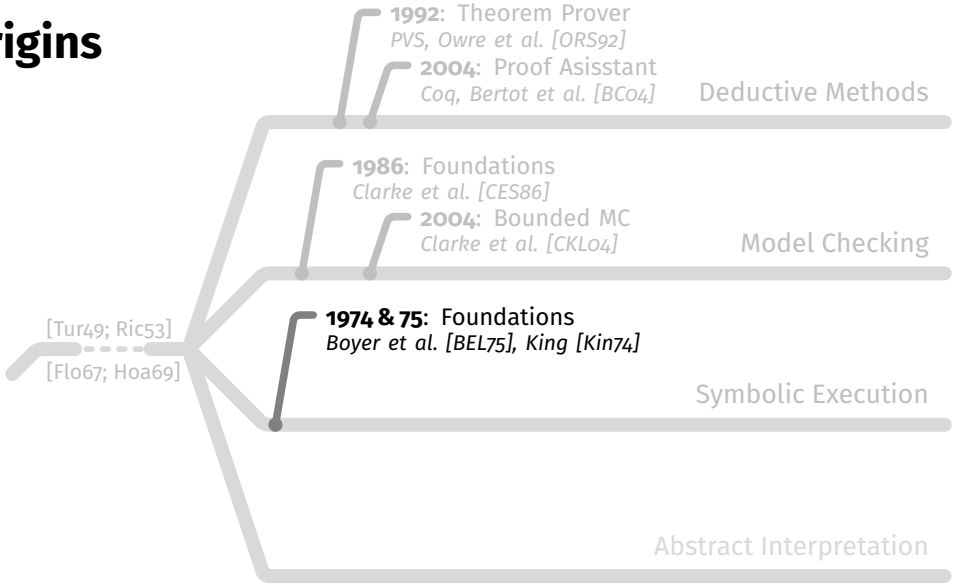

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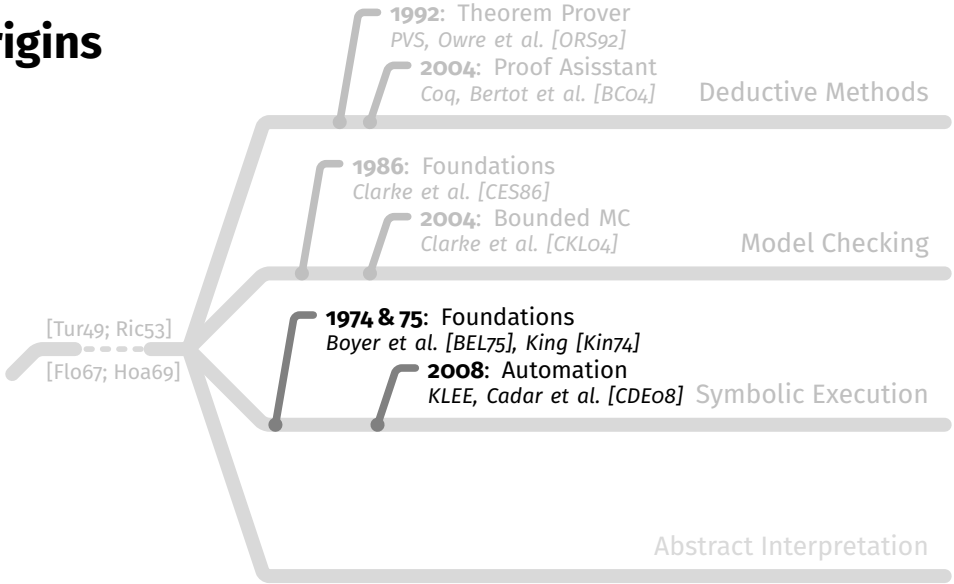

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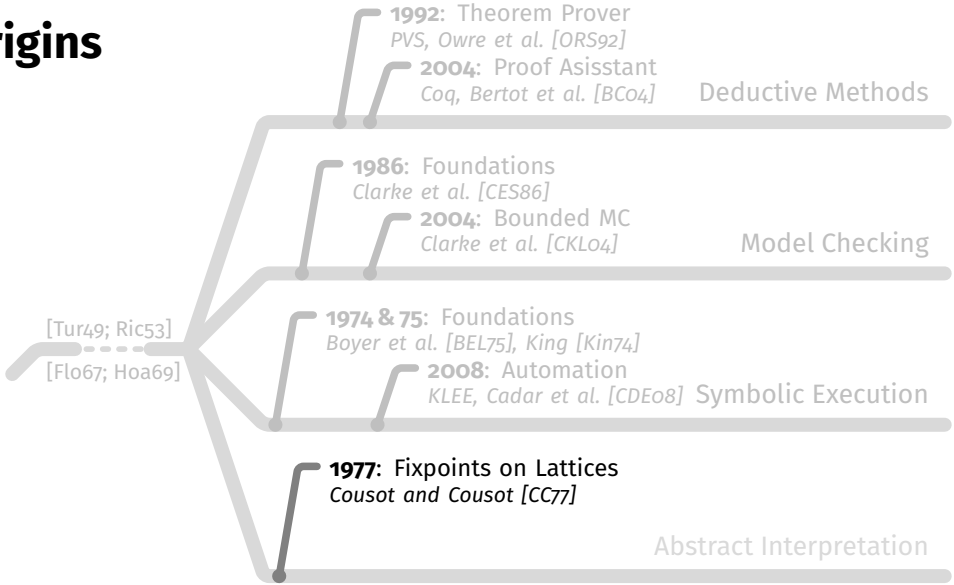

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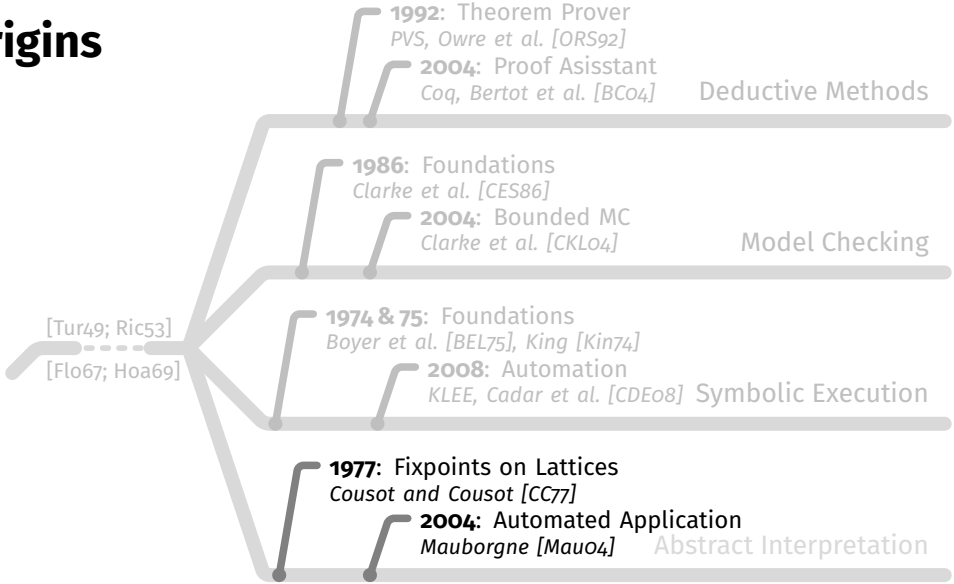

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Recommended Resources

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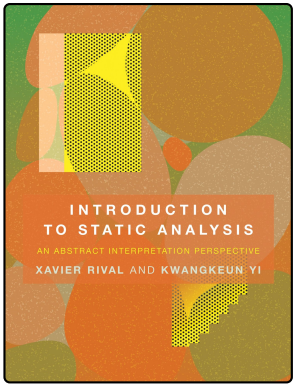
Using Analyses [RY20]



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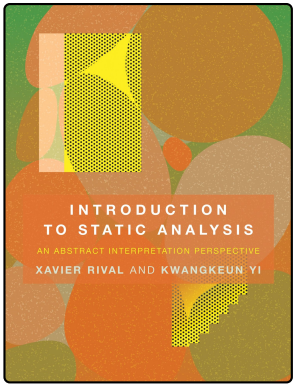
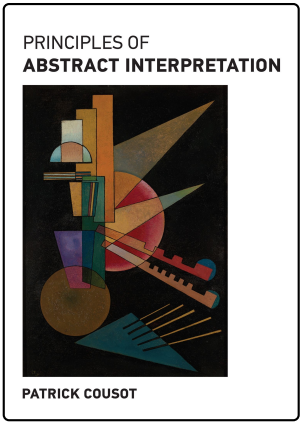


Formal Foundations [Cou21]



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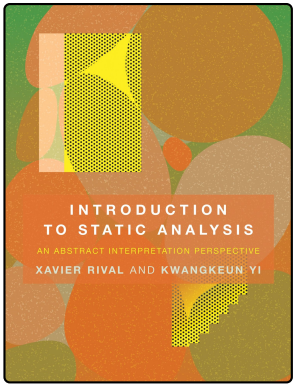

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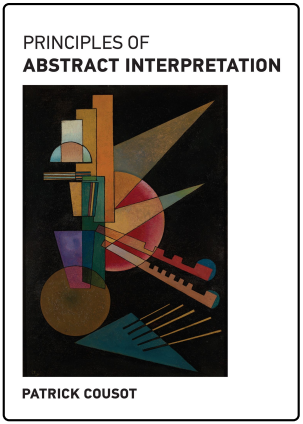
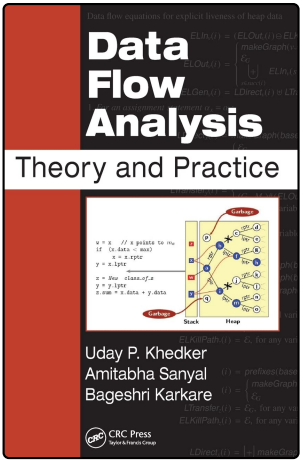


Dataflow Perspective [RY20]



Abstract Interpretation

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See “A casual introduction to Abstract Interpretation” [Cou12]

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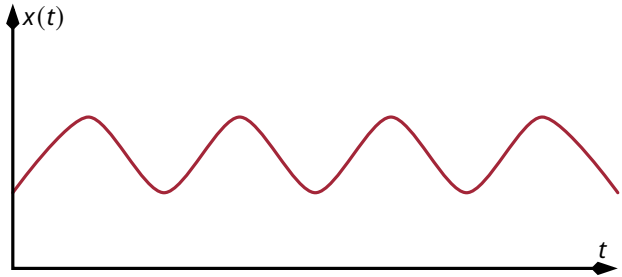

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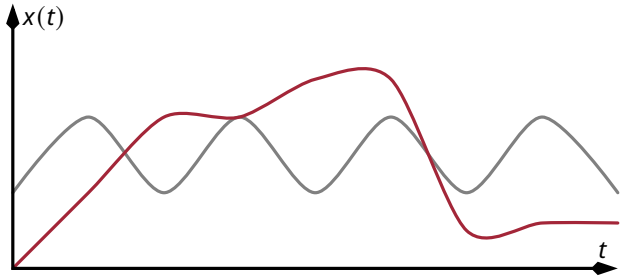

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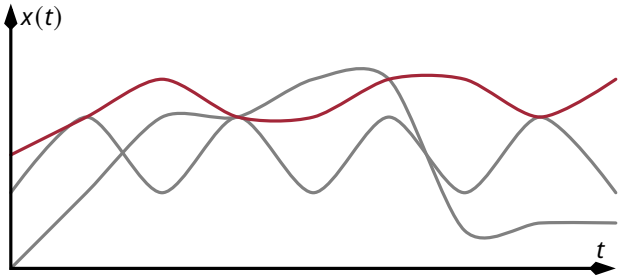

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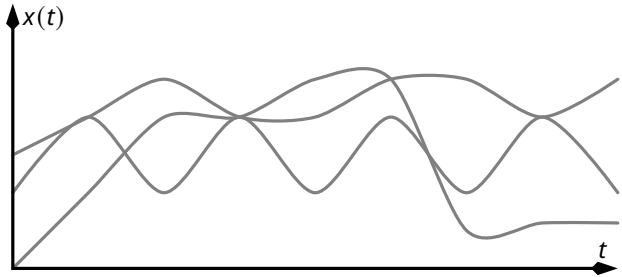

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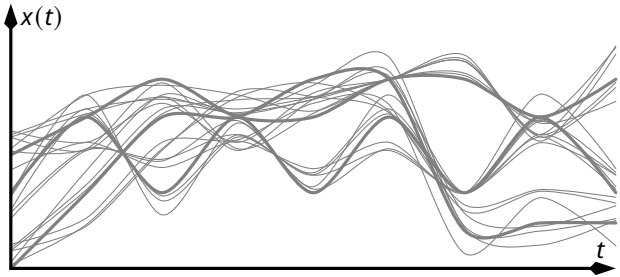

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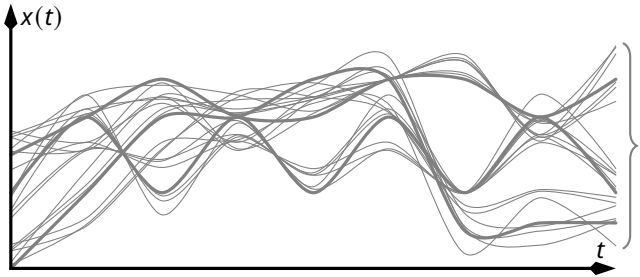

Collecting Semantics[Cou21, p. 91]

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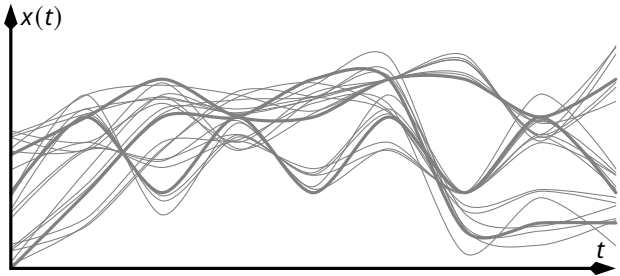

Collecting Semantics[Cou21, p. 91]

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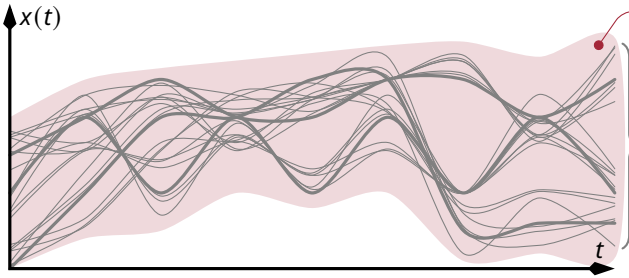

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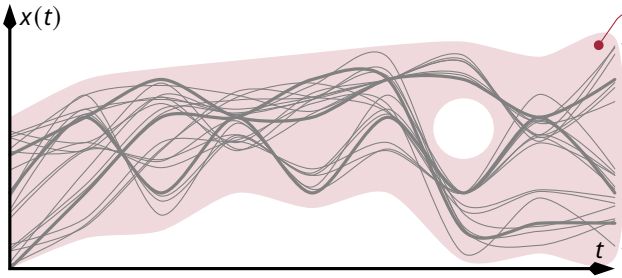

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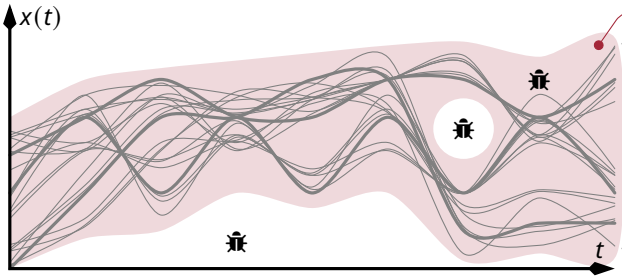

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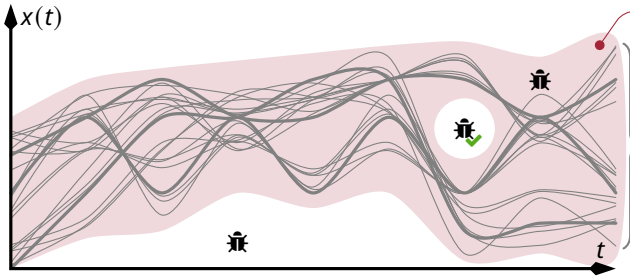
See "A casual introduction to Abstract Interpretation" [Cou12]

Abstract Interpretation



(Trace) Abstraction[Cou21, p. 92]
just one of many

Collecting Semantics[Cou21, p. 91]

- Maybe impossible to compute statically
- … or very expensive (▸ *dynamic*)
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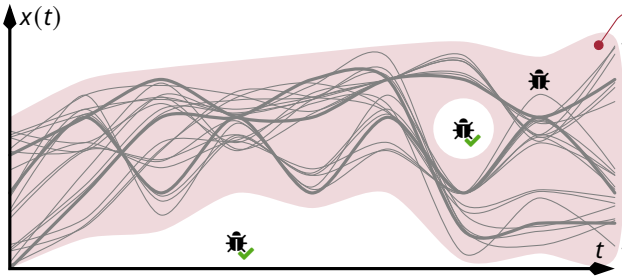
See “A casual introduction to Abstract Interpretation” [Cou12]

Abstract Interpretation



(Trace) Abstraction[Cou21, p. 92]
just one of many

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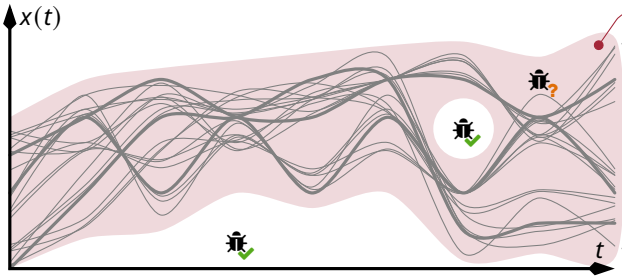
Abstract Interpretation



(Trace) Abstraction[Cou21, p. 92]
just one of many

Collecting Semantics[Cou21, p. 91]

- Maybe impossible to compute statically
- … or very expensive (▸ *dynamic*)
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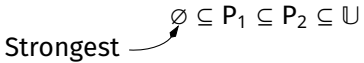
Terminology

- **Property** — Set of states/traces that satisfy that property
  Even integers: $\mathsf{P} = \{ z \in \mathbb{Z} \mid \exists k \in \mathbb{Z} : z = 2k \} = \{ 0, 2, 4, 6, \dots \} \subseteq \mathcal{P}(\mathbb{Z})$



What is a Property? Set basis Poset etc. I have to abstract! Galois, Semantics Principles of Abstract Interpretation book

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