

# Assignment 8

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1. A professor is constructing a multiple-choice history quiz. Quiz has ten questions each with four possible answers. Minimum passing score on the quiz is 60% (6 out of 10).
  - a. Student has not studied and guesses randomly on all ten questions. What is the probability that the student is able to pass the quiz.

## Notes

Each question is independent Student must get six correct in order to pass correct answer on guessing =  $1/4$   
 $P(Y \geq 6)$

## Answer

$$10 \text{ choose } 6 * .25^6 * .75^4 = 0.016$$

$$10 \text{ choose } 7 * .25^7 * .75^3 = 0.003$$

$$10 \text{ choose } 8 * .25^8 * .75^2 = 0.0003$$

$$10 \text{ choose } 9 * .25^9 * .75^1 = 0.00002$$

$$10 \text{ choose } 10 * .25^{10} * .75^0 = 0.0000009$$

$$= 0.0193209$$

- b. How much lower would his chance of passing be if each question had five possible answers instead of four.

## Notes

Each question is independent need to get six correct correct answer on guessing =  $1/5$

## Answer

$$10 \text{ choose } 6 * .20^6 * .80^4 = 0.005$$

$$10 \text{ choose } 7 * .20^7 * .80^3 = 0.0007$$

$$10 \text{ choose } 8 * .20^8 * .80^2 = 0.00007$$

$$10 \text{ choose } 9 * .20^9 * .80^1 = 0.000004$$

$$10 \text{ choose } 10 * .20^{10} * .80^0 = 0.0000001$$

$$= 0.0057741$$

- c. Using (a) as the probability a randomly selected unprepared student passes using part(a) method of guess. 100 student taking the quiz, how many do you expect to pass this quiz out of 100.

$$(0.0193209) * 100$$

## [1] 1.93209

- d. I would not recommend that the professor take the extra work to make the number of questions five instead of four. The probability that a student will pass the quiz by guessing on each question is 0.019. This is a low probability. Though it is true that if 100 students are taking the quiz, there will be 1.9 students, which is the equivalent of 1 student that could pass by guessing all the questions. The chances of 100 students guessing on 10 questions with four questions is low, it is more likely that the number of students guessing on every question is less than 10.

2. Worksite injuries at a large manufacturing plant occur at an average rate of 1.5 per month. Accidents typically occur independent of one another.

(Poisson Distribution) a.  $P(X=3) = p(3) = (e^{-\lambda} \lambda^x) / x!$

$$= (e^{-1.5} (1.5^3)) / 3!$$

$$(0.2231)(1.5^3) / (3 * 2 * 1)$$

$$(0.2231 * 3.75) / 6$$

```
## [1] 0.1394375
```

- b. Suppose you are looking out for months with unusually high numbers of accidents. How many accidents would have to occur in a month for you to take special notice?

```
ppois(1.5, lambda=5)
```

```
## [1] 0.04042768
```

5 accidents occurring a month has less than a 5 percent chance of happening. 4 accidents a month happens around 10 percent of the time.

```
ppois(1.5, lambda=4)
```

```
## [1] 0.09157819
```

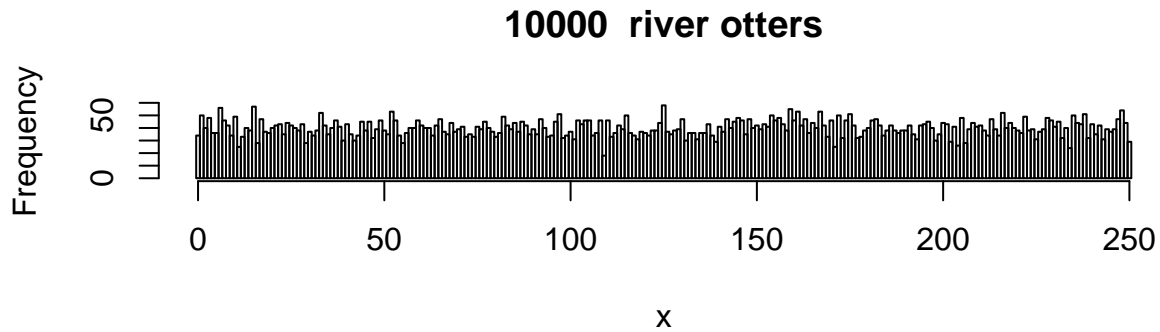
- c. What is the probability that there are at most two accidents in a particular month?

```
ppois(1.5, lambda=2)
```

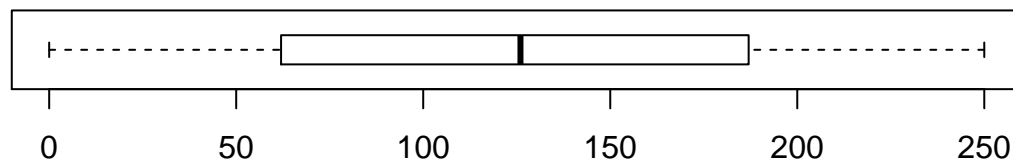
```
## [1] 0.4060058
```

3. For a particular population of river otters, the age (in months) is uniformly distributed between 0 months and 250 months. Imagine an otter is chosen at random from this population.

```
numcases <- 10000
min <- 0
max <- 250
x <- as.integer(runif(numcases,min,max+1) )           #generate random uniform numcases numbers from min to max
                                                    #as.integer truncates, round converts to integers, add .5 for e
par(mfrow=c(2,1))                                   #stack two figures above each other
hist(x,main=paste( numcases," river otters"),breaks=seq(min-.5,max+.5,1))
boxplot(x, horizontal=TRUE,range=1)                  # and the boxplot
title("boxplot of a uniform random distribution for river otters")
```



**boxplot of a uniform random distribution for river otters**



- a. Is there any age you would find surprising? Explain. Since this is a uniformly distributed, that means the probability are all equal for finding the same age. I would be most surprised to find a river otter that is of a young age, though because it is a uniform distribution, it would not surprise me that much.
- b. What is the probability that the otter is less than 24 months old?

$$P(X < 24)$$

$$(1/250) * 24$$

```
## [1] 0.096
```

- c. What is the expected age of the otter?

$$E[X(a,b)] = b + a/2$$

$$= 250 + 0/2$$

$$250/2$$

```
## [1] 125
```

- d. Which is more likely for the above population of river otters – that you select an otter less than one year old or that you select an otter more than 20 years old? Explain, giving the probability for each possibility.

$$P(X < 12 \text{ months})$$

```
(1/250)*12
```

```
## [1] 0.048
```

$P(X > 240 \text{ months})$

```
1/250*10
```

```
## [1] 0.04
```

The more likely is finding a river otter that is less than one year old (probability = 0.048) whereas finding one that is 240 months old is 0.04.

4. Consider a normal distribution with mean 80 and standard deviation 10.

a. What is the probability that an observation falls between the cutoffs of 65 and 75

```
pnorm(75,mean=80,sd=10)-pnorm(65,mean=80,sd=10)
```

```
## [1] 0.2417303
```

b. What is the probability that an observation falls above the value of 92?

```
pnorm(92,mean=80, sd=10, lower.tail=FALSE)
```

```
## [1] 0.1150697
```

c. What is the probability that an observation falls below the value of 68?

```
pnorm(68,mean=80, sd=10)
```

```
## [1] 0.1150697
```

The probabilities of a value lower than 68 and higher than 92 are the same. It is because they are the exact same distance away from the mean of 80.

d. What is the cutoff that separates the bottom 30 percent from the top 70 percent?

```
qnorm(0.30,mean=80,sd=10)
```

```
## [1] 74.75599
```

e. What is the 80th percentile?

```
qnorm(0.80,mean=80,sd=10)
```

```
## [1] 88.41621
```

f. What are the cutoffs that contain the middle 60%?

```
qnorm(0.80,mean=80,sd=10)
```

```
## [1] 88.41621
```

```
qnorm(0.20,mean=80,sd=10)
```

```
## [1] 71.58379
```

Between 71.58 and 88.41.

5. The arrival time (measured in minutes late) for a particular flight each day is normally distributed with a mean of 10 minutes late and a standard deviation of 20 minutes late.

a. What is the probability that a flight is between ten and twenty minutes late?

```
pnorm(20,mean=10,sd=20)-pnorm(10,mean=10,sd=20)
```

```
## [1] 0.1914625
```

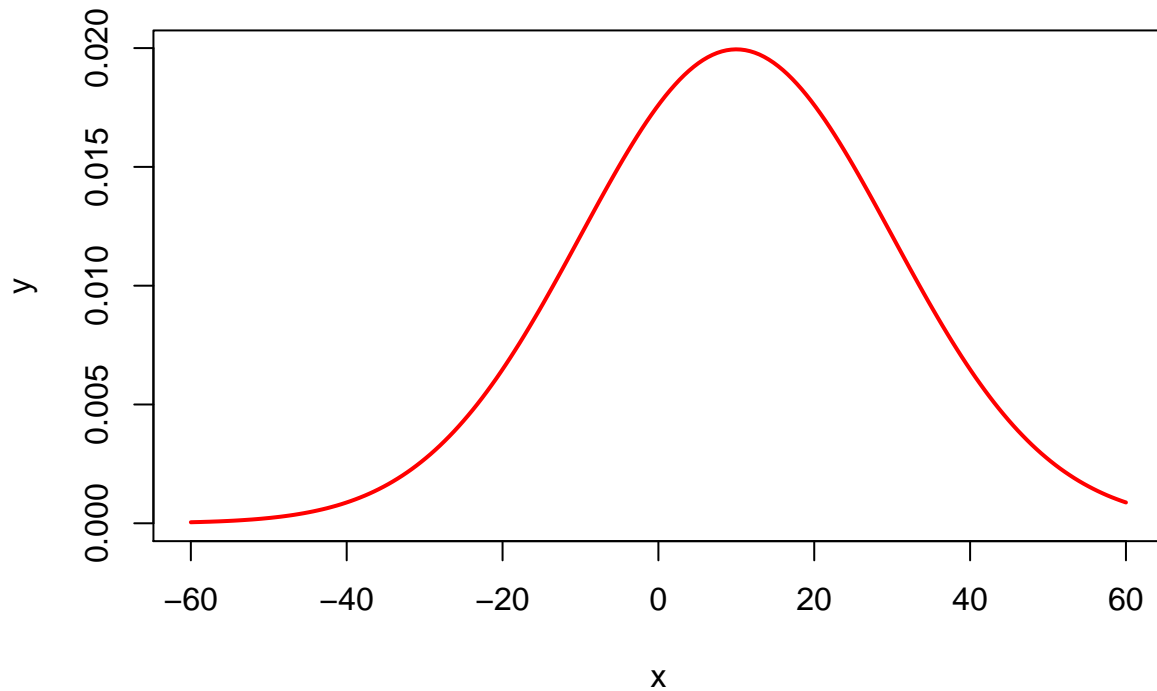
b. What is the probability that the flight is more than an hour late?

```
pnorm(60,mean=10, sd=20, lower.tail=FALSE)
```

```
## [1] 0.006209665
```

c. What percentage of the time would you expect the arrival time to be negative? What does this mean?

```
x=seq(-60,60,length=200)
y=dnorm(x,mean=10,sd=20)
plot(x,y,type="l",lwd=2,col="red")
```



```
pnorm(0, mean=10, sd=20)
```

```
## [1] 0.3085375
```

Would expect the arrival time to be negative around 30% of the time. This means that the plane arrives early.

d. What is the 25th percentile for arrival times? What is the 75th percentile?

```
qnorm(0.25, mean=10, sd=20)
```

```
## [1] -3.489795
```

```
qnorm(0.75, mean=10, sd=20)
```

```
## [1] 23.4898
```