

Assignment 1: Introduction to Probability

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Dobrow Chapter 1

1.6

- (a) $\{X + Y = 4\}$ solution: $\{13, 22, 31\}$
- (b) $\{X + Y = 9\}$ solution: $\{45, 36, 63, 54\}$
- (c) $\{Y = 3\}$ solution (assuming that X value does not matter): $\{13, 23, 33, 43, 53, 63\}$
- (d) $\{X = Y\}$ solution: $\{11, 22, 33, 44, 55, 66\}$
- (e) $\{X > 2Y\}$ solution: $\{31, 52\}$

1.8

If a couple plans on having children until they have 1 girl or 6 boys, the sample space or Ω would be the following (G = Girl, B = Boy):

$\{G\}$

$\{BG\}$

$\{BBG\}$

$\{BBBG\}$

$\{BBBBG\}$

$\{BBBBBG\}$

$\{BBBBBB\}$

A reasonable random variable for having a girl is 0.5, $P(G) = 0.5$. The same probability can be associated with a boy, $P(B) = 0.5$. This random variable was selected because the outcomes of gender is 1 of 2 possibilities.

1.10

In order for the random experiment with three possible outcomes a, b, and c, with $P(a) = p$, $P(b) = p^2$, and $P(c) = p$ then the three probabilities when added together must = 1.

A possible probability for the $p = 27/64$.

1.16

A license plate can be two, three, four, or five letters long and taken from the alphabets A to Z. All letters are possible, including repeats.

(A) The probability of the plate A-R-R is:

$$(1/26) * (1/26) * (1/26) * (1/4) = 0.00001422394$$

The $1/4$ is also multiplied because the plate probability has to be taken into account. There is a .25 percentage chance that the three letter plate is chosen.

(B) The probability that the four letter plate is chosen is $1/4$ or .25. The reason for this is that there are four types of lengths for license plates that can be chosen.

(C) Probability of a plate being a palindrome depends on the exact requirements of the palindrome. If looking for a three letter plate and a palindrome such as DAD, the probability could be as follows:

$$(1/26) * (1/26) * (1/26) * (1/4) = 0.00001422394$$

$(1/4)$ = selecting the correct plate $(1/26)$ = chances of selecting letters

If looking for a four letter plate and palindrome such as CIVIC, the probability would be as follows:

$$(1/4) * (1/26) * (1/26) * (1/26) * (1/26) = 0.0000005470$$

(D) The probability of the plate having one R is $1/26$ no matter the type of plate being selected.

1.22

$$P(A \cup B) = 0.6 \text{ and } P(A \cup B^c) = 0.8$$

$$P(A \cup B^c) = [1 - P(A \cup B)] + P(A)$$

$$0.8 = [1 - 0.6] + P(A)$$

$$0.8 = 0.4 + P(A)$$

$$P(A) = 0.4$$

1.37

Random Integer between 1 and 5000 divisible by 4,7,10

$$P(D4UD7UD10) = P(D4) + P(D7) + P(D10) - P(D4D7) - P(D4D10) - P(D7D10) + P(D4D7D10)$$

$$P(D4) = [5000/4]/5000 \quad P(D7) = [5000/7]/5000 \quad P(D10) = [5000/10]/5000 \quad P(D4D7) = [5000/28]/5000$$

$$P(D4D10) = [5000/40]/5000 \quad P(D7D10) = [5000/70]/5000 \quad P(D4D7D10) = [5000/280]/5000$$

$$P(D4UD7UD10) = 0.40$$

1.44

```
require(dice)
```

```
## Loading required package: dice
## Loading required package: gtools
```

```
getEventProb(nrolls = 5,
             ndicePerRoll = 1,
             nsidesPerDie = 4,
             eventList = list(2))
```

```
## [1] 0.7626953
```

1.45

```
X <- c(1, 4, 8, 16)
sample(X, 10, prob = c(0.1, 0.2, 0.3, 0.4), replace = TRUE)
```

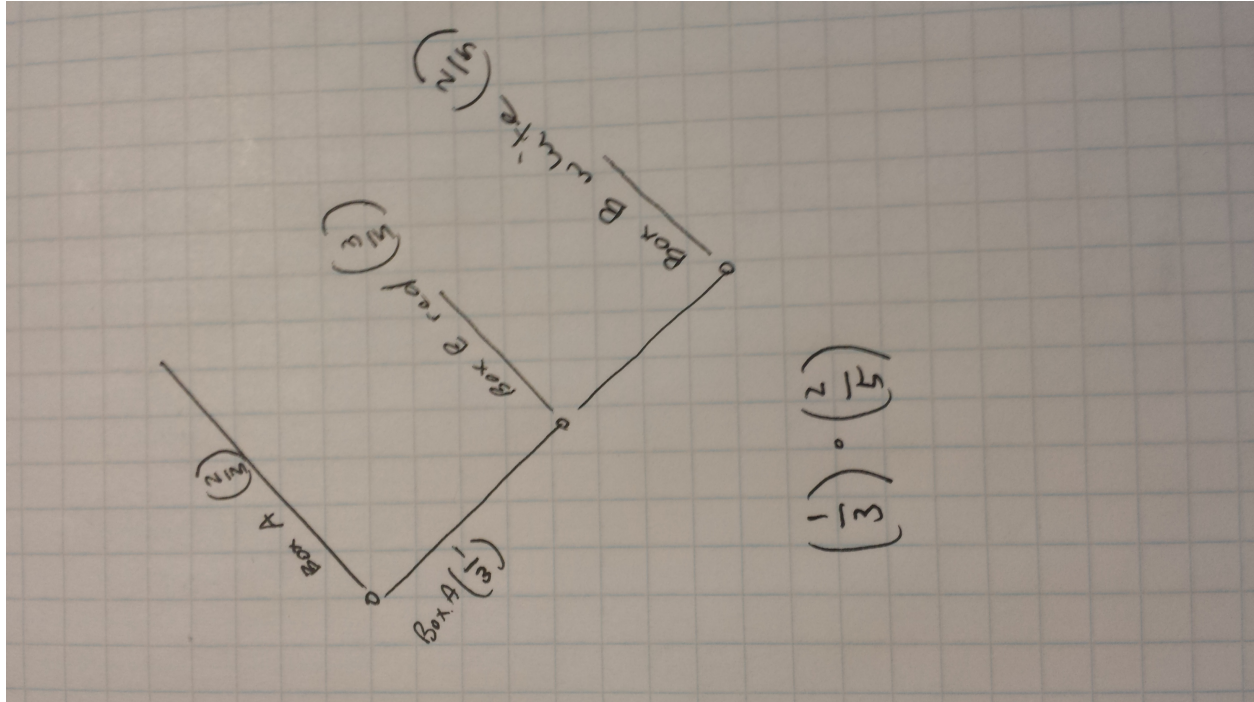
```
## [1] 8 8 8 1 8 8 8 16 1 4
```

Dobrow Chapter 2

2.10

Box A 1WB 2RB BOX B 1WB 3RB

Answer: 2/15 see tree diagram below:



2.12

$$P(A) = 1/2 \quad P(B^C|AC) = 1/3 \quad P(C|A) = 1/4$$

$$P(ABC) = P(A) * P(C|A) * P(B|AC)$$

$$1 - P(B^C|AC) = P(B|AC)$$

$$1/2 * 1/4 * 2/3$$

$$= 1/12$$

2.14

Using Taylor Series approximation, your resulting formula for Mars used is below:

$$P(\text{birthday}) = 1 - e^{-k^2/2 * 687}$$

Solving for k results in 31 maritians.

2.24

What is the probability that she has the disease?

D = has cancer S = test comes back positive

$$P(S|D) = .85$$

$$P(S^c|D) = .15$$

$$P(S|D^c) = .15$$

$$P(S^c|D^c) = .05$$

$$P(D) = .0238$$

Using Bayes Formula:

$$P(D|S) = (.85)(.0238)/((.85)(.0238) + (.15)(.9762))$$

$$P(D|S) = 0.1213849$$

2.26

Was the cab blue?

R = reliable

B = Blue

Y = Yellow

C = RB

$$P(C) = P(RB) + P(R^cY)$$

$$(.80)(.05) + (0.95)(.2)$$

$$P(C) = 0.23$$

2.30

Monty Hall problem with 4 envelopes and 100 dollar bill

```
envelopes <- c("A", "B", "C", "D")
xdata=c()

for(i in 1:1000){
  prize <- sample(envelopes)[1]
  pick <- sample(envelopes)[1]
  open <- sample(envelopes[which(envelopes != pick & envelopes !=prize)])[1]
  switchyes <- envelopes[which(envelopes != pick & envelopes != open)]
  if(pick==prize){xdata=c(xdata,"noswitchwin")}
  if(switchyes==prize){xdata=c(xdata,"switchwin")}
}

length(which(xdata == "switchwin"))
```

```
## [1] 374
```

```
length(which(xdata == "noswitchwin"))
```

```
## [1] 247
```

Based on the above code, you should switch.