IS 606: Statistics and Probability for Data Analytics
Hands-On Laboratory Series
The Continuous Uniform Distribution

Overview

This exercise is designed to give you useful practice at working with a commonly used family of distributions: the continuous uniform distribution.

Prerequisites

You should have a good understanding of the two fundamentals labs for continuous distributions. We will make use of those fundamentals and apply them to this specific context.

Materials

This lab exercise is self-contained.

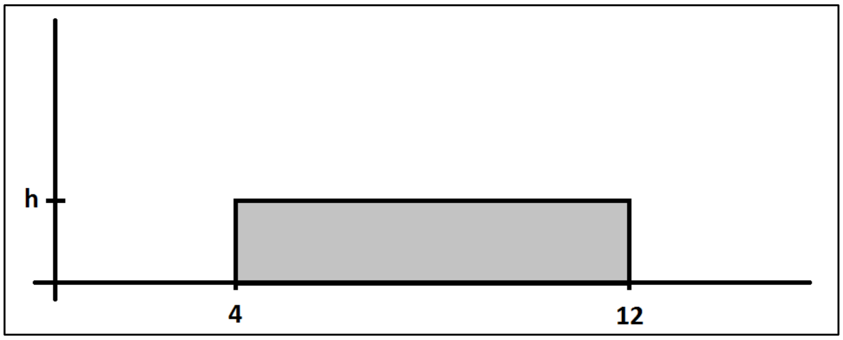
Instructions

This lab exercise is to be completed step by step according to the instructions given. If you are struggling with a particular step, then our recommendation is that you look to the solution ***for only that step*** for help. Once you have sorted out the details of the step in question, proceed to the next task.

Interspersed within the instructions are some short, basic tutorials. Further reading is available in chapter 6 of the Dobrow textbook.

Introducing the Continuous Uniform Distribution

The continuous uniform distribution is best understood graphically. Here is a plot of the PDF of a continuous uniform distribution spanning the interval from 4 to 12:



As you can see, the likelihood is "uniform" (identical) across the entire interval, hence the name. Before we get into the technical details, let's answer a few questions using some common sense and intuition. For the following three questions, use the above distribution.

1. What is the height of the rectangle that would make this a legitimate probability distribution?
2. What is the probability that an observation falls in the interval from 6 to 8?
3. What is the five-number summary of this distribution?
4. What do you think the expected value is for this distribution? Explain.

In the next few sections, we'll formalize the details that let us answer these questions and more.

The PDF and CDF

As you can see in the image above, the PDF of a continuous uniform distribution is a horizontal line, which means that $f(x) = c$ for some constant.

5. Assuming that a continuous uniform distribution has arbitrary endpoints a and b, derive the PDF of the distribution.
6. Using the result in the previous question, derive the CDF for a continuous uniform distribution on the interval from a to b.
7. In terms of a and b, derive the five-number summary of the distribution using the CDF. Does this match with your geometric intuition?

Expected Value and Variance

Next, let's derive formulas for the expected value and variance of a continuous uniform distribution:

8. Using the arbitrary endpoints a and b again, derive a formula for the expected value of the continuous uniform distribution.
9. Using the arbitrary endpoints a and b again, derive a formula for the variance of the continuous uniform distribution.
10. Recall the rule we used previously to describe "typical" outcomes. Using the $E(X) \pm 2\,SD(X)$ approach, and in terms of endpoints a and b again, what would constitute unusual outcomes? Comment on the practical meaning of the result you obtain. (You may wish to calculate the result for a specific case or two to gain some intuition.)

Applications

Here are two practice questions that use the continuous uniform distribution:

11. A machine produces sheets of steel of different thickness. The thickness of a sheet of steel is uniformly distributed between 200 mm and 250 mm.
    a. What is the probability that a sheet of steel is less than 210 mm in thickness?
    b. What is the expected value of a sheet of steel? With what standard deviation?
    c. What is the five-number summary of the thicknesses of sheets of steel?
    d. Suppose sheets of steel need to be between 205 mm and 245 mm in thickness. What percentage of sheets of steel will need to be discarded?
12. The time of arrival of a delivery service is uniformly distributed between 10 a.m. and 3 p.m.
    a. What is the probability of arrival before noon?
    b. What is the expected arrival time?
    c. Are there any unusual arrival times in the interval? Explain.

Challenge Problem

This problem goes above and beyond the basics of the uniform distribution. See what you make of it!

13. The lifetime of a lightbulb is uniformly distributed on the interval (2 , 12). The lightbulb will be replaced upon failure or upon reaching the age 10, whichever comes first. What are the expected value and standard deviation of the age of a light bulb at the time of replacement?

Summary

The uniform distribution isn't used quite as often as some other distributions, but it is sometimes useful. It also provides a good training ground for fundamental concepts of continuous distributions, as we've seen in this lab.