Assignment 3 - More on Random Variables

Ben Arancibia

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Chapter 4

4.24

Find the variance of a fair die roll.

To find variance first you need to find the expected value:

Expected Value: (1 * 1/6) + (2 * 1/6) + (3 * 1/6) + (4 * 1/6) + (5 * 1/6) + (6 * 1/6) = 21/6 = 3.5

then take the variance of the expected value:

(1 - 3.5)^2 * 1/6 + (2 - 3.5)^2 * 1/6 + (3 - 3.5)^2 * 1/6 + (4 - 3.5)^2 * 1/6 + (5 - 3.5)^2 * 1/6 + (6 - 3.5)^2 * 1/6 = 70/24 = 2.91

4.36

Cor(X,Y) = -0.5 Find V[X+Y]

V[X+Y] = V[X] + V[Y] + 2 COR(X,Y)

V[X] + V[Y] + 2 * (-0.5)

V[X+Y] = variance since it is equal distribution, mean, STD, variance are all the same. =.25

4.40

E[X] = 1 E[X^2] = 2 E[X^3] = 5 E[X^15] = 15

E[Y] = 2 E[Y^2] = 6 E[Y^3] = 22 E[Y^4] = 94

V[3X^2 - Y] = 138 V[X^2] = (9*(2)^4 - 2^2) V[Y] = 6 - 4

E[X^{4Y}4] 15*94 =1410

COV(X, X2) V[X^2] = 9 - 2^2 V[X] = 1 - 1 6 - 3 =3

V[X^{2Y}2] V[X^2] = 6 V[Y^2]1296-36 6+1260 =1266

4.56

```r
sim <- sample(c(-10,-10,-10,0,0,14), 100000, replace = T)
mean(sim)
```

```
## [1] -2.62692
```

```r
var(sim)
```

```
## [1] 75.84077
```

4.58

```r
x <- rpois(100000, 1)
y <- rpois(100000, 2)
var(3*x^2 - y)
```

```
## [1] 101.3953
```

```r
mean(x^4 * y^4)
```

```
## [1] 1370.677
```

```r
cov(x, x^2)
```

```
## [1] 3.006644
```

```r
var(x^2*y^2)
```

```
## [1] 1230.063
```

Chapter 5

5.4 .01% chance of being defective

a) Find the probability that exactly 110 Components produced before defective one: P(X=k) = (1-p)$^{\text{k-1 ((1-.01)}}$109)*.01 =0.0033

b) Find the probability that takes at least 110 components before defective: P(X>110) (1-p)$^{\text{k ((1-.01)}}$109) =0.33

c) Find is the expected number of components that will be produced before a defective one: E[X] = 1/p 1/.01 =100

5.12

X ~ NgBin(R,p) p=0.5

2 (k -1 | 3) (1/2)^k

E[X] = 4(0.125) + 5(0.25) + 6(0.3125) + 7(0.3125) =5.8125

5.20 500 deer, sample of 50 deer caught and tagged.

20 are then caught.

a) 50/500 0.1*20 E[X] = 2

variance of E[X] = 1.8496

SD[X] = (1.8495)^1/2

=1.316

b)

P(X>=3) = (20 | 3) (480 | 17) / (500 | 20)

P(X>=3) = 0.322

5.28

10 rolls fair die, X = number of fives rolled, Y = number of even numbers, Z = number of odd numbers

a) COV(X, Y)

-nP(x)P(y) -1/6 , 3/6 =-5/6

b) Cov(X, Z) -nP(X)P(Z)-*1 #but if a 5 is rolled that is both odd number and a 5 1/6, 3/6,* -1

=5/6

5.32

Let X be the number of days when there is no homework. N = 42, p = 1/30, condition is prime number (1/15) 10 prime numbers

(1/30) * 42 (N) * 10 (prime numbers)

Number days expect to have no homework is 14

5.34

This is a hypergeometric distribution

Let X be 3 winners. N = 30 and six are winners

6/30 0.2*10 E[X]=2

P(X=3) = (10 | 3) (20 | 17) / (30 | 6)

P(X=3) = 0.23

5.36

```
coupon <- function(n){
  i <- 0
  j <- 1
  a = vector(length=n)
  a[j] <- sample(1:n,1)
  remaining <- c(1:n)[-which(c(1:n) == a[j])]
  repeat
    {
      pick <- sample(remaining,1)
      i <- (i+1)
      if(any(pick == a))
        {
        next
        }
      else
        {
          j <- (j+1)
          a[j] <- pick
          }
      if(j == n)
        {
        break
        }
      }
  return(i)
  }
```

```
mean(replicate(100000,coupon(10)))
```

```
## [1] 25.44211
```

```r
sd(replicate(100000,coupon(10)))
```

```
## [1] 9.958189
```

5.39

```r
wins <- loss <- 0
successprob <- 0.25
series <- function(){
  gamecount <- 0
  while (wins < 4 && loss < 4){
    w <- rbinom(1,1,successprob)
    wins <- wins + w
    loss <- loss + 1 -w

  }
  wins + loss
}

mean(replicate(100000, series()))
```

```
## [1] 5.16393
```

```r
sd(replicate(100000, series()))
```

```
## [1] 1.021464
```