609 - Week 1 Homework

Ben Arancibia

September 1, 2015

Page 8

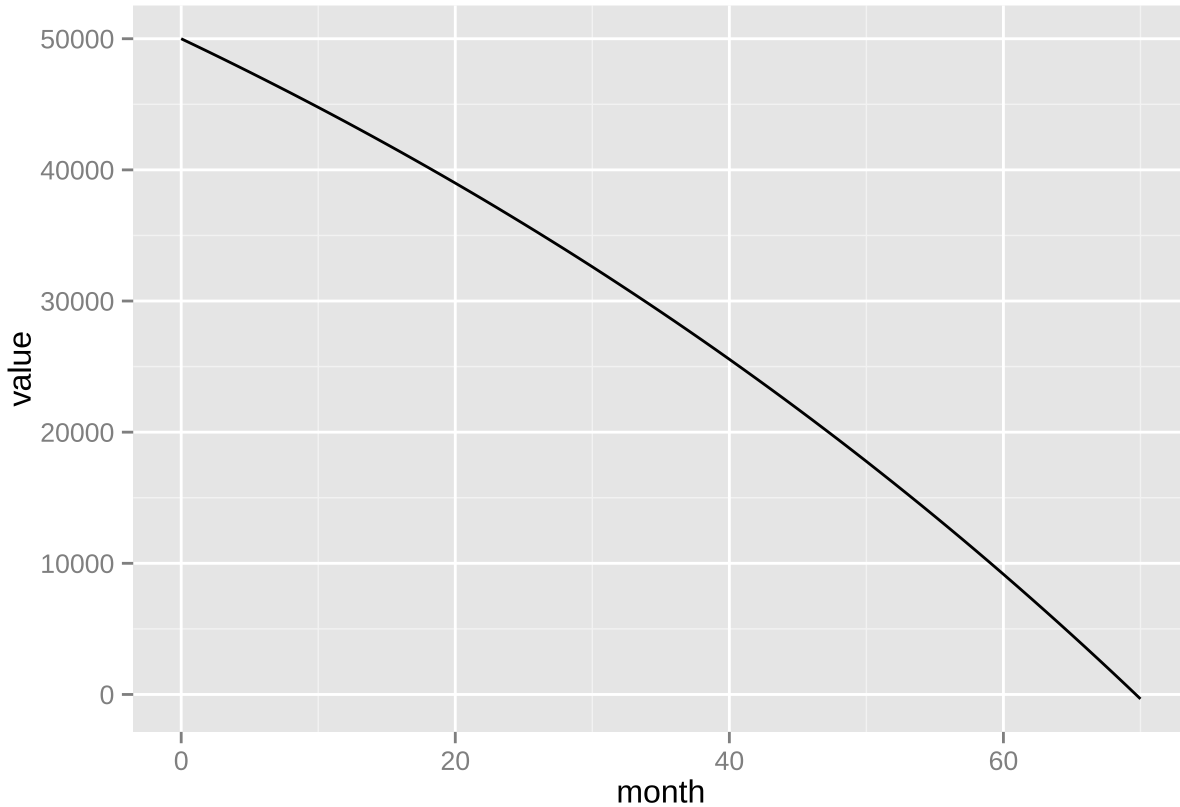
10) Your grandparents have an annuity. The value of the annuity increases each month by an automatic deposit of 1% interest on the previous month's balance. Your grandparents withdraw \$1000 at the beginning of each month for living expenses. Currently, they have \$50,000 in the annuity. Model the annuity with a dynamical system. Will the annuity run out of money? When? Hint: What value will an have when the annuity is depleted?

$a_{n+1} = a_n + 0.01a_n - 1000$ $a_0 = 50000$

```r
a <- 50000
rate <- 0.01
withdrawl <- 1000

model <- function(an, i, w)
  {
  a_plus <- an + (an * i) - w
  return (a_plus)
  }

years <- data.frame(month=c(0), value=c(a))
for(n in 1:100)
{
  a <- model(a, rate, withdrawl)

  years <- rbind(years, c(n, a))

  if(a < 0)
  {
    break
  }
}

colnames(years) <- c("month", "value")

tail(years)
```

```
##    month     value
## 66    65 4531.6756
## 67    66 3576.9923
## 68    67 2612.7623
## 69    68 1638.8899
## 70    69  655.2788
## 71    70 -338.1684
```

```
p <- ggplot(years, aes(x=month, y=value)) + geom_line()
p
```



Annuity will run out at at Month 70.

Page 17

9)

The data in the accompanying table show the speed n (in increments of 5 mph) of an automobile and the associated distance an in feet required to stop it once the brakes are applied. For instance, n = 6 (representing 6 * 5 = 30 mph) requires a stopping distance of a6=47 ft.

Create figure in book.

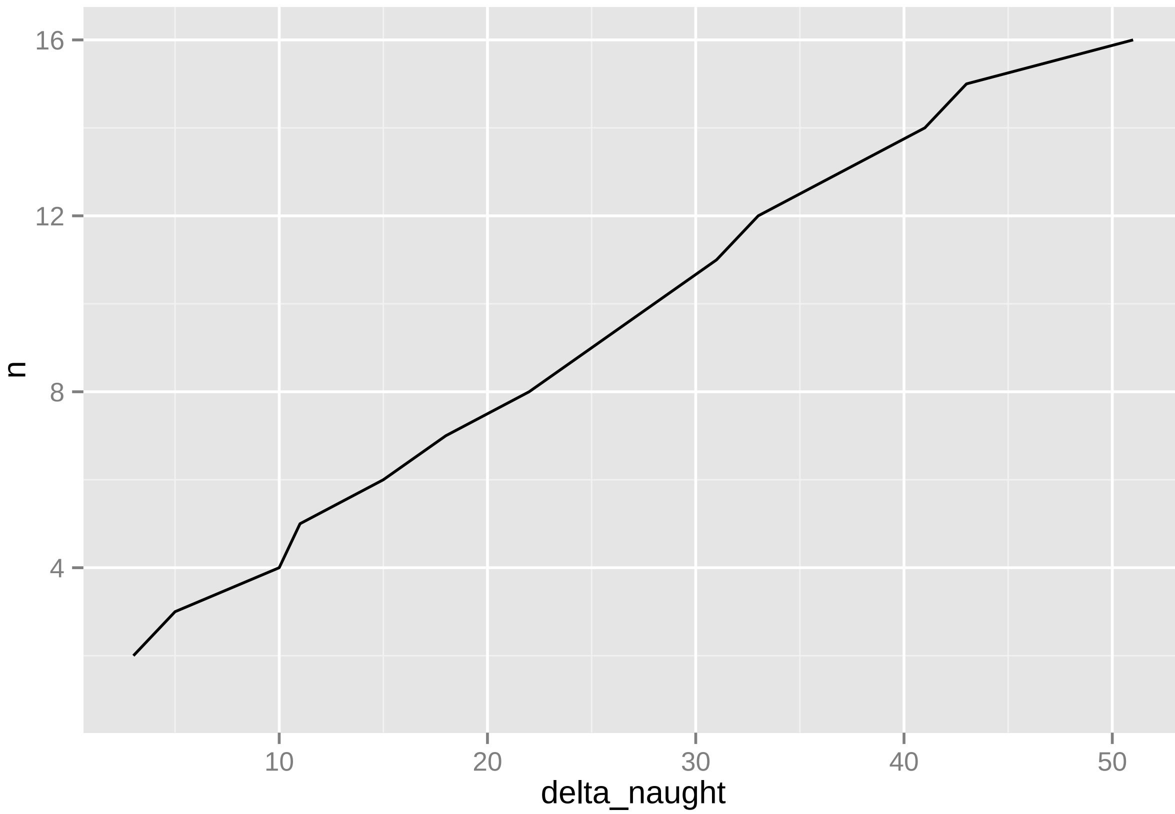
```
n <- 1:16
a_naught <- c(3,6,11,21,32,47,65,87,112,140,171,204,241,282,325,376)
mph <- (n * 5)
```

a. Calculate and plot the change a n versus n. Does the graph reasonably approximate a linear relationship?

```
delta_naught <- c()
d_naught <- NA
for(i in 1:length(a_naught))
{
  delta_naught[i] <- a_naught[i] - a_naught[i - 1]
}
data <- data.frame(n, mph, a_naught, delta_naught)
```

```r
p1 <- ggplot(data, aes(x=delta_naught, y=n)) + geom_line()
p1
```

```
## Warning: Removed 1 rows containing missing values (geom_path).
```



Yes there is a reasonable approximate liner relationship.

b. Based on your conclusions in part a, find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against n. Discuss the appropriateness of the model.

find slope

```r
change_delta <- max(data$delta_naught, na.rm=TRUE)
change_delta_n <- max(data$n)
slope <- change_delta/change_delta_n
```

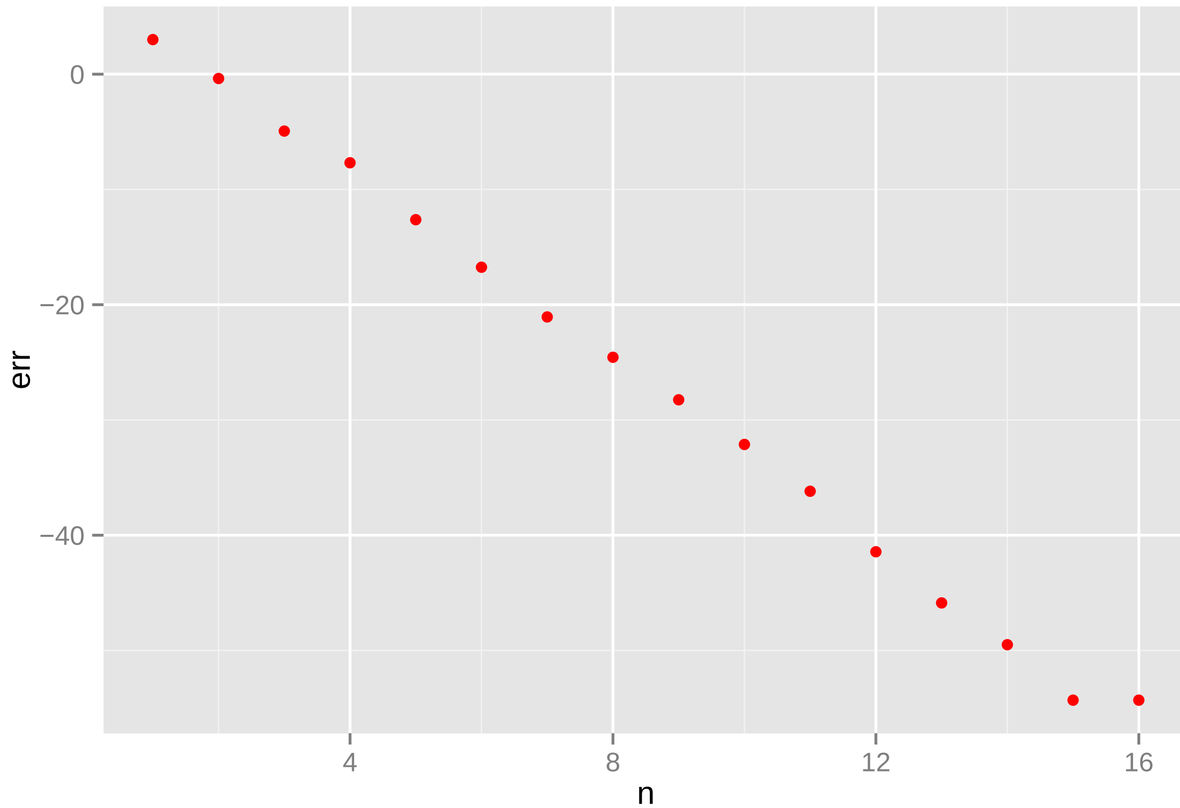
Difference equation model

```r
a_naught <- c(3,6,11,21,32,47,65,87,112,140,171,204,241,282,325,376)
model <- function(n, a, slope)
{
    an <- slope * n + a
    return(an)
}
```

```r
m <- c()
m[1] <- 0
for(i in 2:length(a_naught))
{
  m[i] <- model(i, m[i-1], slope)
}

data1 <- cbind(data, m)
data1
```

```
##     n mph a_naught delta_naught        m
## 1   1   5        3           NA   0.0000
## 2   2  10        6            3   6.3750
## 3   3  15       11            5  15.9375
## 4   4  20       21           10  28.6875
## 5   5  25       32           11  44.6250
## 6   6  30       47           15  63.7500
## 7   7  35       65           18  86.0625
## 8   8  40       87           22 111.5625
## 9   9  45      112           25 140.2500
## 10 10  50      140           28 172.1250
## 11 11  55      171           31 207.1875
## 12 12  60      204           33 245.4375
## 13 13  65      241           37 286.8750
## 14 14  70      282           41 331.5000
## 15 15  75      325           43 379.3125
## 16 16  80      376           51 430.3125
```

```r
data1$err <- data1$a_n - data1$m
data1err <- ggplot(data=data1, aes(x=n)) + geom_point(color="red", aes(y=err))
data1err
```



As values are increase, the error increases.

Page 34

13) Consider the spreading of a rumor through a company of 1000 employees, all working in the same building. We assume that the spreading of a rumor is similar to the spreading of a contagious disease (see Example 3, Section 1.2) in that the number of people hearing the rumor each day is proportional to the product of the number who have heard the rumor previously and the number who have not heard the rumor. This is given by:

$rn + 1 = rn + krn(.1000 - n)$

where k is a parameter that depends on how fast the rumor spreads and n is the number of days. Assume $k = 0.001$ and further assume that four people initially have heardthe rumor. How soon will all 1000 employees have heard the rumor?

```
model <- function(k, rn, n)
{
  rn1 <- rn + (k * rn * (1000 - n))
  return (rn1)
}

rnx <- c()
rnx[1] <- 4
k <- 0.001
for(n in 1:100)
{
  rnx[n + 1] <- model(k, rnx[n], n)
  if(rnx[n+1] > 1000)
```

```
  {
    break
  }
}

rnx
```

```
## [1]    4.00000    7.99600   15.97601   31.90409   63.68056  127.04272
## [7]  253.32318  504.87309 1005.70720
```

Page 55

6) An economist is interested in the variation of the price of a single product. It is observed that a high price for the product in the market attracts more suppliers. However, increasing the quantity of the product supplied tends to drive the price down. Overtime, there is an interaction between price and supply. The economist has proposed the following model, where Pn represents the price of the product at year n, and Q n represents the quantity. Find the equilibrium values for this system.

a. Does the model make sense intuitively? What is the significance of the constants 100 and 500. Explain the significance of the signs of the constant -0.1 and 0.2.

Yes these make sense because they zero out the second term in each equation and produce X(n+1)=X_n0. Increasing quantity of the product supplied tends to drive the price down and a high price for the product in the market attracts more suppliers. The constants 100 and 500 are values where the price and quantity respectively change their effect on the others outcome. The signs of the constants -0.1 and 0.2 represent the economic behaviour of more suppliers decreaing prices, and higher prices increases suppliers.

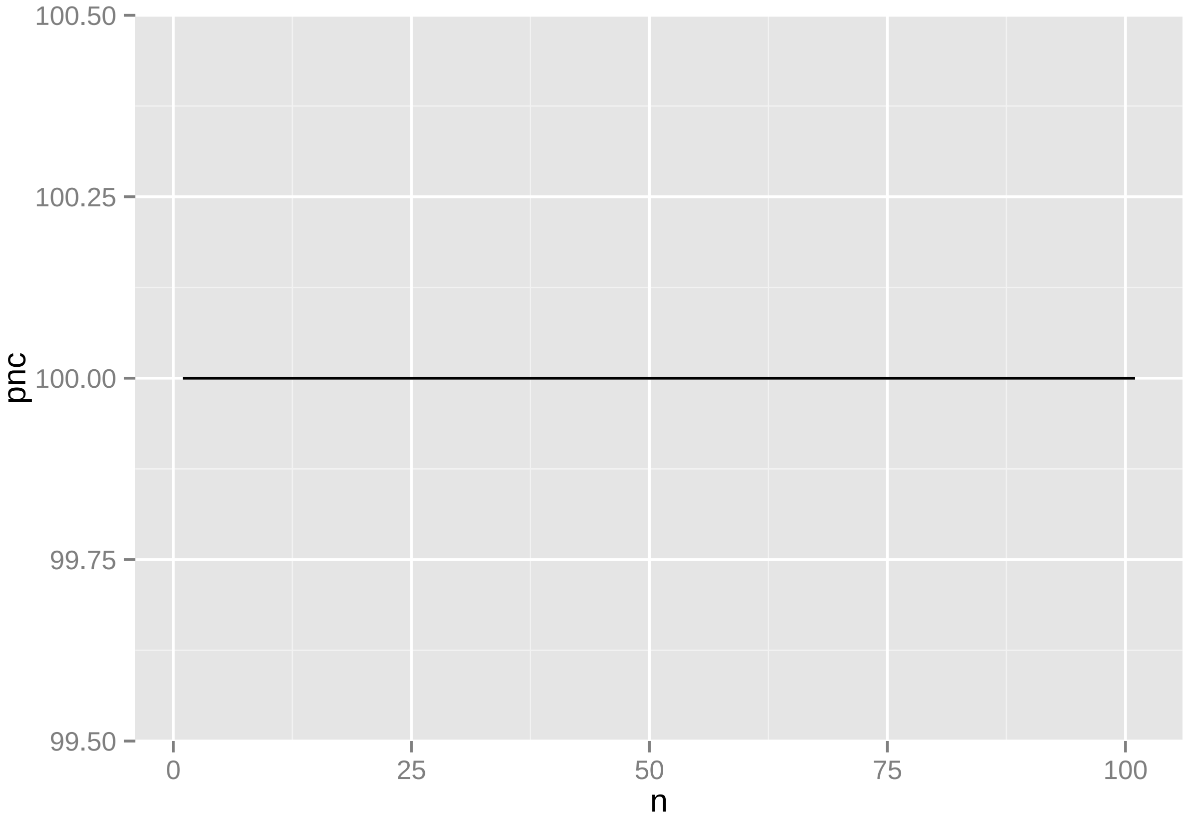
b. Test the initial conditions in the following table and predict the long-term behavior.

```
model <- function(p, q){
  px <- p - (0.1 * (q - 500))
  return (px)
}

model1 <- function(p, q){
  qx <- q + (0.2 * (p - 100))
  return (qx)
}

execModelLoop <- function(pnaught, qnaught, maxN, caseId)
{
  pnc <- c()
  pnc[1] <- pnaught

  qnc <- c()
  qnc[1] <- qnaught
  for(n in 1:maxN)
  {
    pnc[n + 1] <- model(pnc[n], qnc[n])
    qnc[n + 1] <- model1(pnc[n], qnc[n])
  }
```

```r
df <- data.frame(case=rep_len(caseId, maxN+1), n=1:(maxN+1), pnc, qnc)
  return (df)
}

maxIterations <- 100

CaseA <- execModelLoop(100, 500, maxIterations, "result")
head(CaseA)
```

```
##     case n pnc qnc
## 1 result 1 100 500
## 2 result 2 100 500
## 3 result 3 100 500
## 4 result 4 100 500
## 5 result 5 100 500
## 6 result 6 100 500
```

```r
f <- ggplot(CaseA, aes(x=n, y=pnc))+geom_line()
f
```



The conditions show that the equalibrium conditions are in a stable values and do not shift from the initial price.