IS 609 Week 11 Homework

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November 5, 2015

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In Problems 5-8, find and classify the rest points of the given autonomous system.

6)

dx/dt = -(y-1)

dy/dt = x-2

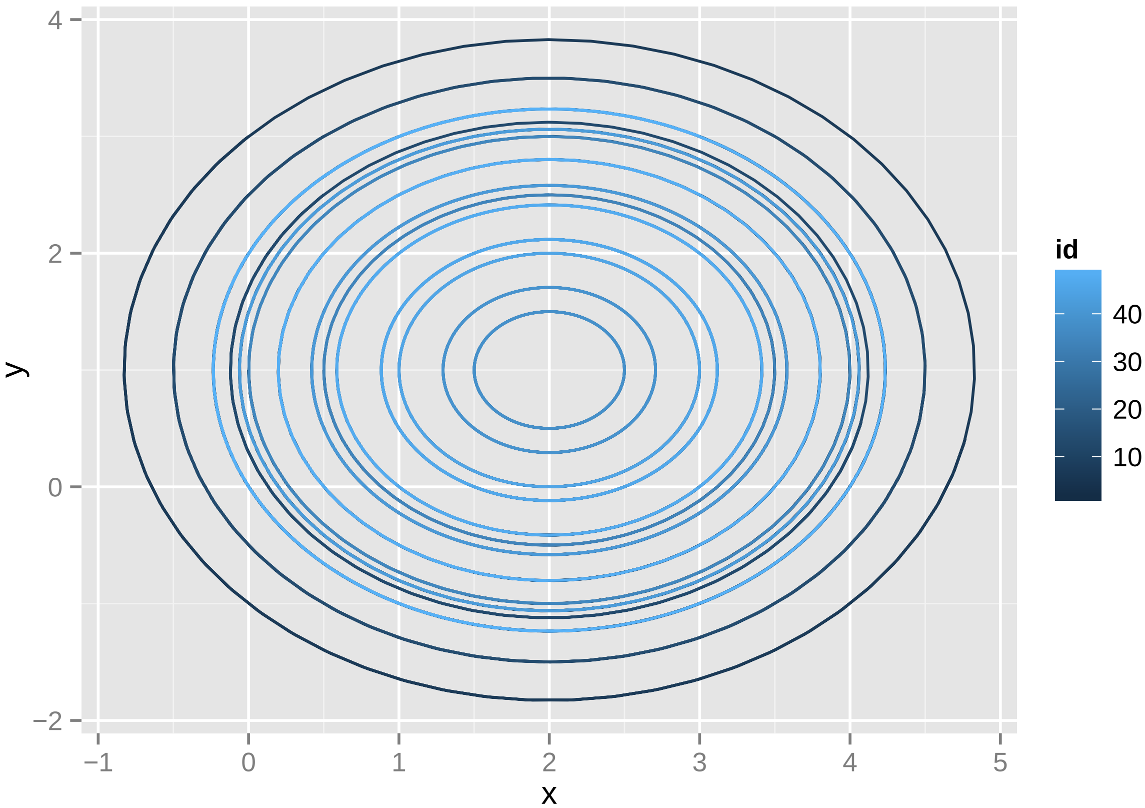
The rest point is (2,1). Use deSolve package to help us visualize the various trajectories.

```
##
## Attaching package: 'deSolve'
##
## The following object is masked from 'package:graphics':
##
##     matplot
```

```
##    id         x          y
## 1   1 0.0000000  0.0000000
## 2   1 0.1098251 -0.1946710
## 3   1 0.2385362 -0.3774052
## 4   1 0.3848472 -0.5463769
## 5   1 0.5472964 -0.6998977
## 6   1 0.7242604 -0.8364336
## 7   1 0.9139712 -0.9546206
## 8   1 1.1145333 -1.0532776
## 9   1 1.3239427 -1.1314189
## 10  1 1.5401070 -1.1882638
```

Based on the visualization, (2,1) is stable rest point.

```
g1 <- ggplot(data=dfRk) +
  geom_path(aes(x=x, y=y, colour=id, group=id))
g1
```



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7) Show that the two trajectories leading to m/n, a/b shown in Figure 12.8 are unique.

a)

dy/dx=(m-nx)y/(a-b)x]

Dividing two autonomous differential equations with each other produces the derivative dy/dx

dy/dx=dy/dt dx/dr =(m-nx)y/(a-b)x

b)

dy/dx=(m-nx)y/(a-b)x

(a-b) dy/y=(m-nx) dx/x

a/y - b/y dy = m/x - n dx

Integrate both sides:

$$a \ln y - b \ln y = m \ln x - nx + K$$

$$a \ln y - b \ln y - m \ln x + nx = K$$

Simplifying, we get:

$$y^a e^{-by} = Kx^m e^{-nx}$$

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1) Apply the first and second derivative tests to the function f(y) = y^a / e^by to show that y=a/b is a unique critical point that yields the relative maximum f(a/b) Show also that f(y) approaches zero as y tends to infinity.

First derivative: f(y) = y$^{a/e}$by

f(y) = y^a * e^-by

product rule:

f'(y) = y^a * -b e^-by + ay^a-1 * e^-by

Sove for y

f'(y) = y^a * -b e^-by + ay^a-1 * e^-by = 0

f'(y) = e^-by (-by^a + ay^a-1) = 0

2 zeroes of the first derivative.

e^-by = 0 and -by^a + ay^a-1 = 0

Solve for y from equation -by^a + ay^(a-1) = 0:

-by^a + ay^a-1 = 0

ay^(a-1) = by^a

a = by$^{a/y}$(a-1) = by

(a)/(b) = y