609 - Week 1 Homework

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```
## Warning: package 'ggplot2' was built under R version 3.1.3

## Warning: package 'dplyr' was built under R version 3.1.3

##
## Attaching package: 'dplyr'
##
## The following objects are masked from 'package:stats':
##
##     filter, lag
##
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

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10) Your grandparents have an annuity. The value of the annuity increases each month by an automatic deposit of 1% interest on the previous month's balance. Your grandparents withdraw $1000 at the beginning of each month for living expenses. Currently, they have $50,000 in the annuity. Model the annuity with a dynamical system. Will the annuity run out of money? When? Hint: What value will an have when the annuity is depleted?

$a_{n+1} = a_n + 0.01a_n - 1000$ $a_0 = 50000$

```
a <- 50000
rate <- 0.01
withdrawl <- 1000

model <- function(an, i, w)
  {
  a_plus <- an + (an * i) - w
  return (a_plus)
  }

years <- data.frame(month=c(0), value=c(a))
for(n in 1:100)
{
  a <- model(a, rate, withdrawl)

  years <- rbind(years, c(n, a))

  if(a < 0)
  {
    break
  }
```

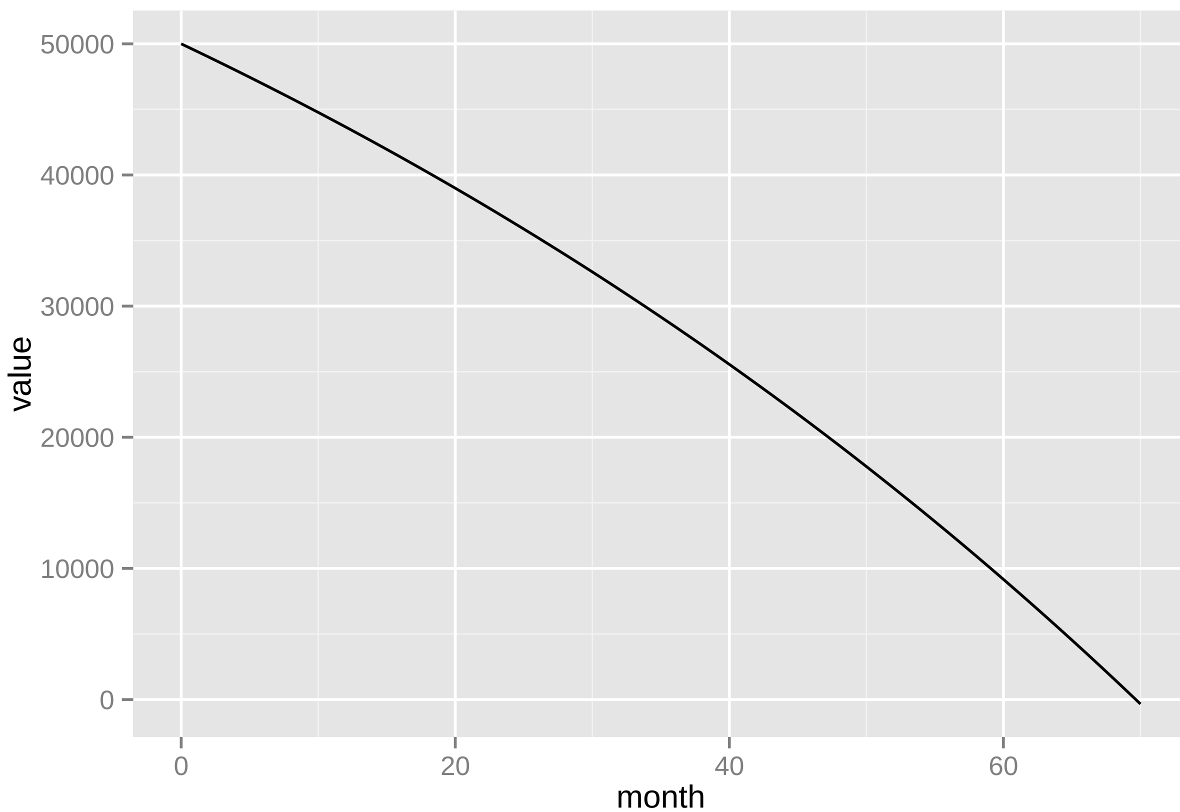
```
}

colnames(years) <- c("month", "value")

tail(years)
```

```
##    month     value
## 66    65 4531.6756
## 67    66 3576.9923
## 68    67 2612.7623
## 69    68 1638.8899
## 70    69  655.2788
## 71    70 -338.1684
```

```
p <- ggplot(years, aes(x=month, y=value)) + geom_line()
p
```



Annuity will run out at at Month 70.

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9)

The data in the accompanying table show the speed n (in increments of 5 mph) of an automobile and the associated distance an in feet required to stop it once the brakes are applied. For instance, n = 6 (representing 6 * 5 = 30 mph) requires a stopping distance of a6=47 ft.

Create figure in book.

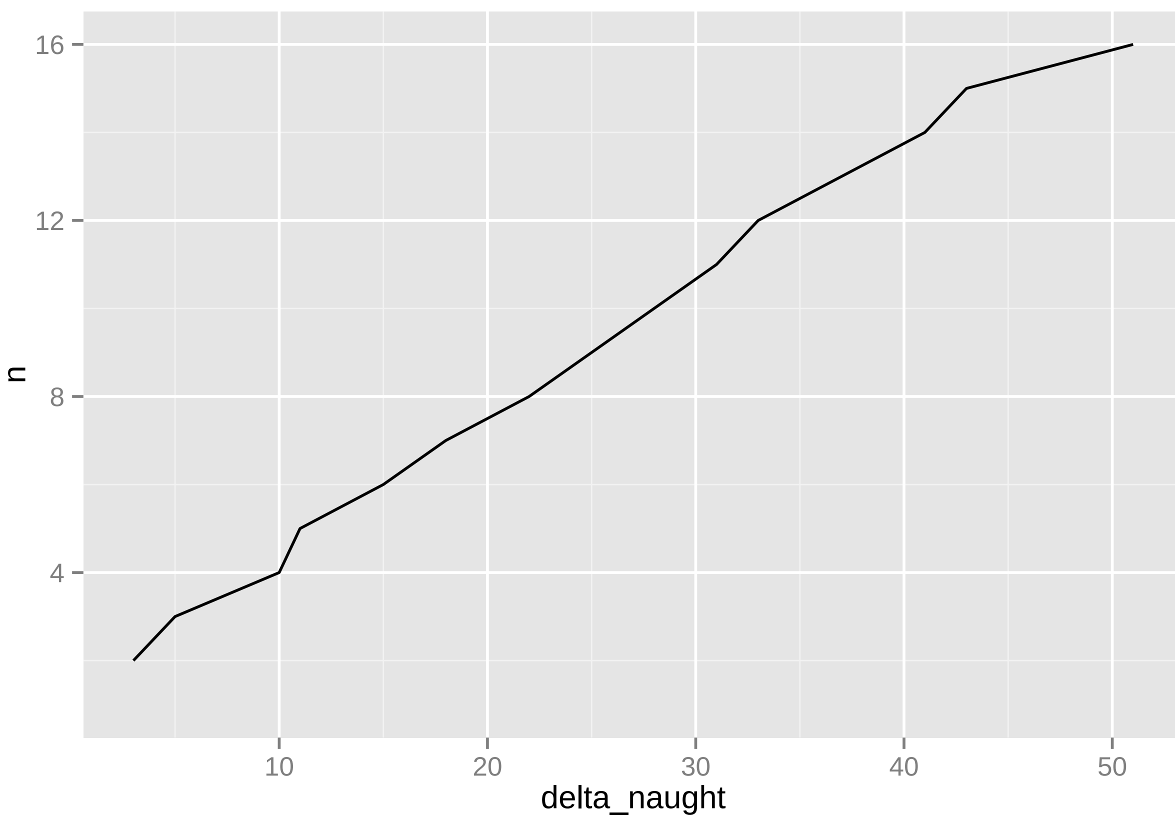
```r
n <- 1:16
a_naught <- c(3,6,11,21,32,47,65,87,112,140,171,204,241,282,325,376)
mph <- (n * 5)
```

a. Calculate and plot the change a n versus n. Does the graph reasonably approximate a linear relationship?

```r
delta_naught <- c()
d_naught <- NA
for(i in 1:length(a_naught))
{
  delta_naught[i] <- a_naught[i] - a_naught[i - 1]
}
data <- data.frame(n, mph, a_naught, delta_naught)

p1 <- ggplot(data, aes(x=delta_naught, y=n)) + geom_line()
p1
```

```
## Warning: Removed 1 rows containing missing values (geom_path).
```



Yes there is a reasonable approximate liner relationship.

b. Based on your conclusions in part a, find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against n. Discuss the appropriateness of the model.

find slope

```
change_delta <- max(data$delta_naught, na.rm=TRUE)
change_delta_n <- max(data$n)
slope <- change_delta/change_delta_n
```

Difference equation model

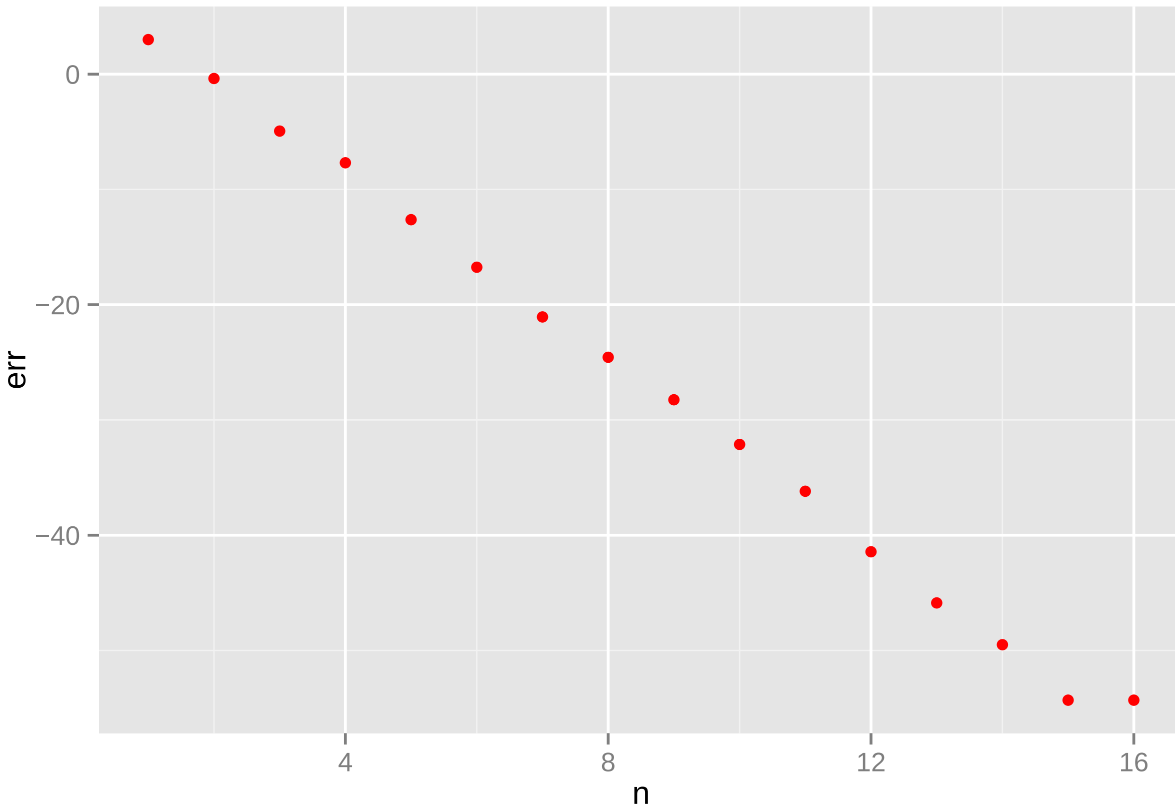
```
a_naught <- c(3,6,11,21,32,47,65,87,112,140,171,204,241,282,325,376)
model <- function(n, a, slope)
{
    an <- slope * n + a
    return(an)
}

m <- c()
m[1] <- 0
for(i in 2:length(a_naught))
{
  m[i] <- model(i, m[i-1], slope)
}

data1 <- cbind(data, m)
data1
```

```
##     n mph a_naught delta_naught        m
## 1   1   5        3           NA   0.0000
## 2   2  10        6            3   6.3750
## 3   3  15       11            5  15.9375
## 4   4  20       21           10  28.6875
## 5   5  25       32           11  44.6250
## 6   6  30       47           15  63.7500
## 7   7  35       65           18  86.0625
## 8   8  40       87           22 111.5625
## 9   9  45      112           25 140.2500
## 10 10  50      140           28 172.1250
## 11 11  55      171           31 207.1875
## 12 12  60      204           33 245.4375
## 13 13  65      241           37 286.8750
## 14 14  70      282           41 331.5000
## 15 15  75      325           43 379.3125
## 16 16  80      376           51 430.3125
```

```
data1$err <- data1$a_n - data1$m
data1err <- ggplot(data=data1, aes(x=n)) + geom_point(color="red", aes(y=err))
data1err
```



As values are increase, the error increases.

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13) Consider the spreading of a rumor through a company of 1000 employees, all working in the same building. We assume that the spreading of a rumor is similar to the spreading of a contagious disease (see Example 3, Section 1.2) in that the number of people hearing the rumor each day is proportional to the product of the number who have heard the rumor previously and the number who have not heard the rumor. This is given by:

$rn + 1 = rn + krn(.1000 - n)$

where k is a parameter that depends on how fast the rumor spreads and n is the number of days. Assume $k = 0.001$ and further assume that four people initially have heardthe rumor. How soon will all 1000 employees have heard the rumor?

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6)