



BIREFRINGENT DIRAC FERMION IN ANISOTROPIC VELOCITY
MODULATED GRAPHENE JUNCTION

MR. EAKKARAT PATTRAWUTTHIWONG

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF SCIENCE (PHYSICS)
FACULTY OF SCIENCE
KING MONGKUT'S UNIVERSITY OF TECHNOLOGY THONBURI
2020

Electronic Transport of Dirac Fermion in Tilted Velocity Modulated Dirac
Material Junction

Mr. Eakkarat Pattrawutthiwong B.Sc. (Applied Physics)

A Thesis Submitted in Partial Fulfillment of the Requirements for
the Degree of Master of Science (Physics)

Faculty of Science

King Mongkut's University of Technology Thonburi

2020

Thesis Committee

..... Chairman of Thesis Committee
(Asst. Prof. Thana Sutthibutpong, Ph.D.)

..... Member and Thesis Advisor
(Asst. Prof. Watchara Liewrian, Ph.D.)

..... Member
(Lect. Tanapat Deesawan, Ph.D.)

..... Member
(Lect. Suwat Tangwancharoen, Ph.D.)

Copyright reserved

Thesis Title Electronic Transport of Dirac Fermion in Tilted Velocity
Modulated Dirac Material Junction

Thesis Credits 12

Candidate Mr. Eakkarat Patrawutthiwong

Thesis Advisor Asst. Prof. Watchara Liewrian

Program Master of Science

Field of Study Physics

Department Physics

Faculty Science

Academic Year 2020

Abstract

The ground state entanglement of the system, both in discrete-time and continuous-time cases, is quantified through the linear entropy. The result shows that the entanglement increases as the interaction between the particles increases in both time scales. It is also found that the strength of the harmonic potential affects the entanglement of the system. The different feature of the entanglement between continuous-time and discrete-time scales is that, for discrete-time entanglement, there is a cut-off condition. This condition implies that the system can never be in a maximally entangled state.

Keywords : Continuous-Time/ Discrete-Time/ Entanglement

ACKNOWLEDGEMENTS

This thesis is supported by my thesis advisor, Dr. Monsit Tanasittikosol for editing and supporting my thesis writing. Next, I would like to thank another advisor, Asst. Prof. Dr for invaluable teaching and giving the motivation in my thesis. Without their advices, this thesis would not have been completed.

CONTENTS

	PAGE
ABSTRACT IN ENGLISH	ii
ACKNOWLEDGMENTS	iii
CONTENTS	iv
LIST OF FIGURES	vi
CHAPTER	
1. INTRODUCTION	1
1.1 Background and motivation	1
2. THEORETICAL BACKGROUND	2
3. MODEL AND METHOD	3
3.1 Transmission probability of electron in tilted Dirac cone	3
4. RESULT AND DISCUSSION	4
4.1 Angular dependent of transmission probability	4
4.2 The tilted strength identification by means of the tunneling resonance properties	5
4.3 Oscillatory behavior of electron resonant tunneling	6
4.4 Revisit: The tilted strength identification by means of the tunneling resonance properties	7
4.5 Pseudo magnetic field	9

	PAGE
4.6 The key consequence of the mismatch effect	9
4.7 Transmission under the influence of pseudo magnetic field	11
5. CONCLUSION	13
REFERENCES	14
BIOGRAPHY	15

LIST OF FIGURES

FIGURE	PAGE
2.1 The device structure of field-effect transistor.	2
4.1 The same cup of coffee. Two times.	5
4.2 tp fixed angle	7
4.3 (a) The device structure for the measurement of resonant tunneling electron. Straight blue lines represent the transport region where both arms are 45° angled with the normal direction. Green and yellow region represents top gate and electrode respectively. (b) Angular-dependent transmission for different applied voltages, $U=180.2$ meV for solid line and $U=185.85$ meV for dashed-dotted line. These voltages satisfy the resonance condition at $\phi = \frac{\pi}{4}$	8
4.4 (a) The Fermi surfaces of Dirac cones in different regions. The solid line represents the Fermi surface of non-tilted Dirac cone in region I and the dashed line represents the Fermi surface of tilted Dirac cone $w_0 = 0.5$ in region II. (b) The transmission probabilities as a function of incident angle, where the solid (dashed) line is the case of the system with (without) the mismatch of Dirac cones. The Fermi energy $E_F = 80$ meV and gate potential is zero.	10
4.5 The polar plot of transmission probabilities as a function of incident angle of the system with tilted mismatch Dirac cone (solid line) and non-tilted system under the influence of delta magnetic field (dashed line). The Fermi energy $E_F = 83$ meV is the same for all plot, the gate potential $U = 285$ meV for (a) and (b), $U = 0$ for (c) and (d) . .	12

CHAPTER 1 INTRODUCTION

1.1 Background and motivation

The electronic properties of material are defined by the ability of charge carriers to move throughout the crystal structures [1]

The ability of charge carriers to tunnel throughout the material is defines by the crystal, electronic structures of their host material.

CHAPTER 2 THEORETICAL BACKGROUND

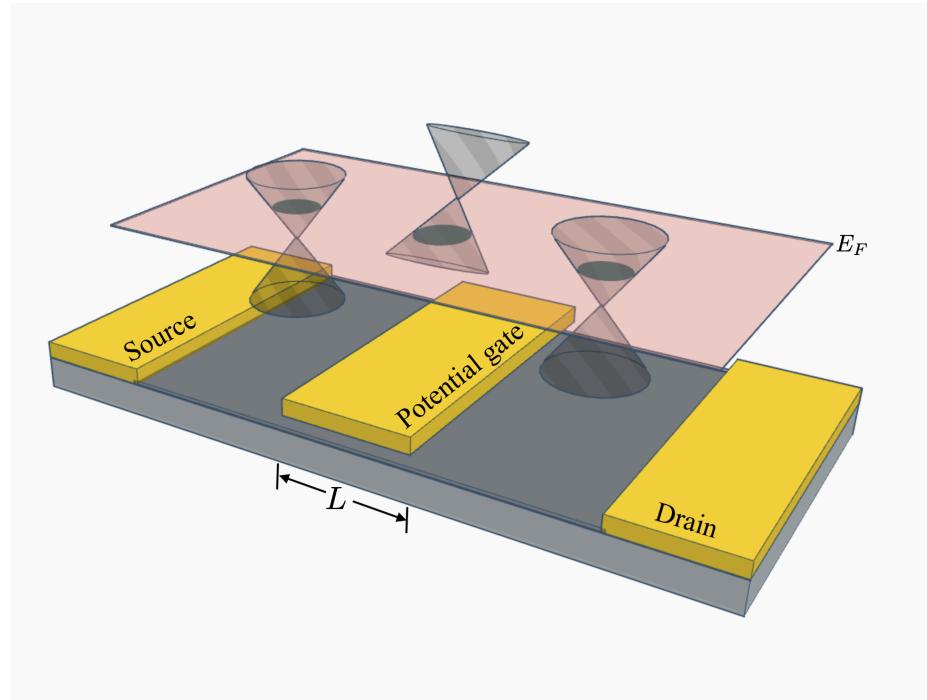


Figure 2.1 The device structure of field-effect transistor.

CHAPTER 3 MODEL AND METHOD

3.1 Transmission probability of electron in tilted Dirac cone

$$\psi_1 = \begin{cases} e^{ik_x x} + r e^{-ik_x x} & , x < 0 \\ s(e^{ik_x x} e^{i\phi} + r e^{-ik_x x} e^{-i\phi}) & , x < 0 \end{cases} \quad (3.1)$$

$$\psi_2 = \begin{cases} a e^{iq_x x} + b e^{-iq_x x} & , 0 \leq x < L \\ s'(a e^{iq_x x} e^{i\theta} - b e^{-iq_x x} e^{-i\theta}) & , 0 \leq x < L \end{cases} \quad (3.2)$$

$$\psi_3 = \begin{cases} t e^{ik_x x} & , x \geq L \\ s t e^{ik_x x} e^{i\phi} & , x \geq L \end{cases} \quad (3.3)$$

(3.4)

$$T = \frac{\cos^2 \theta \cos^2 \phi}{\cos^2(Lq_x) \cos^2 \theta \cos^2 \phi + \sin^2(Lq_x)(1 - ss' \sin \theta \sin \phi)^2} \quad (3.5)$$

CHAPTER 4 RESULT AND DISCUSSION

We start by investigating the tunneling properties of electron across tilted Dirac cone heterojunctions where we focus on how the effect of the gate potential and tilt affect the electron transmission. Then, we demonstrate a method to measure the tilted strength of the Dirac cone by identifying the tunneling behavior. Finally, we show that the transport behaviors of electron in tilted Dirac cone material are analogous to electron under the influence of magnetic field. We also show the derivation of magnetic field strength as a function of gate potential and tilted parameter.

The calculation of transmission probability is carried out using Eq. 3.5

4.1 Angular dependent of transmission probability

The transmission probabilities across the tilted Dirac cone heterojunctions under the variation of gate potential are presented in Fig. 4.1. The transmission profiles are symmetric in the case of $w_0 = 0$ regardless of the gate potential. When the tilted parameter is non-zero, the transmission profiles are shifted along the direction of the tilt and consequently become asymmetric, where the magnitude of the shift depends on the tilted strength of the Dirac cone. However, the present of the tilt barely affects the tunneling profiles when the applied gate potential is close to the Fermi energy as shown in Fig. 4.1b. This is because the Fermi surface is small and the allowed wavevector states are narrowed. Therefore, electron propagations other than the normal incident are backscattered.

Interestingly, when the applied gate voltage U is larger than the Fermi energy E_F , the transmission profiles exhibit peak tunneling as shown in Fig. 4.1c-d. These kind of tunnelings are called resonant tunneling, which occurred when the condition $q_x L = n\pi$, $n = 0, \pm 1, \dots$ in Eq. 3.5 is met.

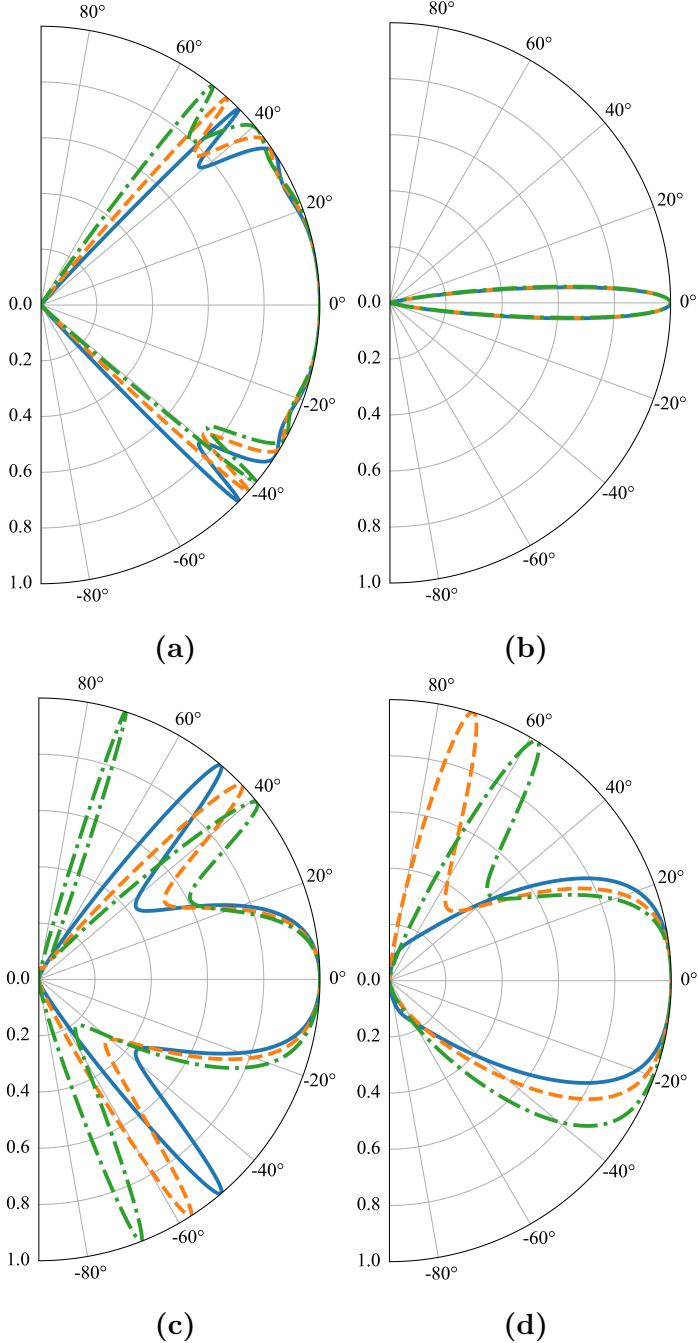


Figure 4.1 The same cup of coffee. Two times.

4.2 The tilted strength identification by means of the tunneling resonance properties

The resonant tunnelings are arisen if the given U, E_F and, w_0 satisfy the resonance condition. Modulating these parameters result in shifting of resonant tunneling angles as previously reported in section 4.1. In this section, we demonstrate that by measuring the asymmetric resonant tunneling angles, the tilted parameter can be

determined. Consider the resonance condition

$$\begin{aligned} L \sqrt{\left(\frac{E_F - U}{\hbar v_F} + w_0 k_y \right)^2 - k_y^2} &= n\pi \\ w_{0\pm} &= \frac{U - E_F}{\hbar v_F k \sin \phi} \mp \sqrt{1 + \left(\frac{n\pi}{kL \sin \phi} \right)^2} \end{aligned} \quad (4.1)$$

where subscript $+(-)$ satisfy the positive(negative) angle ϕ region. One can obtain the tilted parameter by applying the gate voltage and Fermi energy then measure the resonant tunneling angle, which can be experimentally observed by four-point probes technique [2]. To illustrate how to calculate for the tilted parameter, we substitute the configuration of dashed-dotted line in Fig. 4.1c to Eq. 4.1. We choose the resonance condition $n = 4$, which corresponds to the resonant tunneling angle $\phi = 72^\circ$. We find $w_0 = 0.1$.

However, this method is not at all practical since the variable n is unlikely observable. Also, the only way to manipulate the electron propagations is by tuning the voltages through the bottom and top gate. In section 4.4, we propose a more practical method to identify the tilted strength, which again, involve with the resonant tunneling behaviors.

4.3 Oscillatory behavior of electron resonant tunneling

To understand the behaviors of resonant tunneling under the effect of the tilt and gate potential, we plot the transmission probabilities at particular incident angle $\phi = 45^\circ$ shown in Fig. 4.2. We found that the resonant tunneling oscillate with the increasing of gate voltage where the voltage difference between each resonance condition is the same. Also The voltages required to satisfy the resonance conditions are increased with the tilted strength. These resonant tunnelings are shifted uniformly with the tilted parameter, which can be confirmed mathematically by taking the derivative to Eq. 4.1 with respect to U

$$\begin{aligned} \frac{dw_{0\pm}}{dU} &= \frac{d}{dU} \left(\frac{U - E_F}{\hbar v_F k \sin \phi} \mp \sqrt{1 + \left(\frac{n\pi}{kL \sin \phi} \right)^2} \right) \\ &= \frac{1}{\hbar v_F k \sin \phi}. \end{aligned} \quad (4.2)$$

Eq. 4.2 indicates that the tilted parameter is linearly proportional to gate voltage.

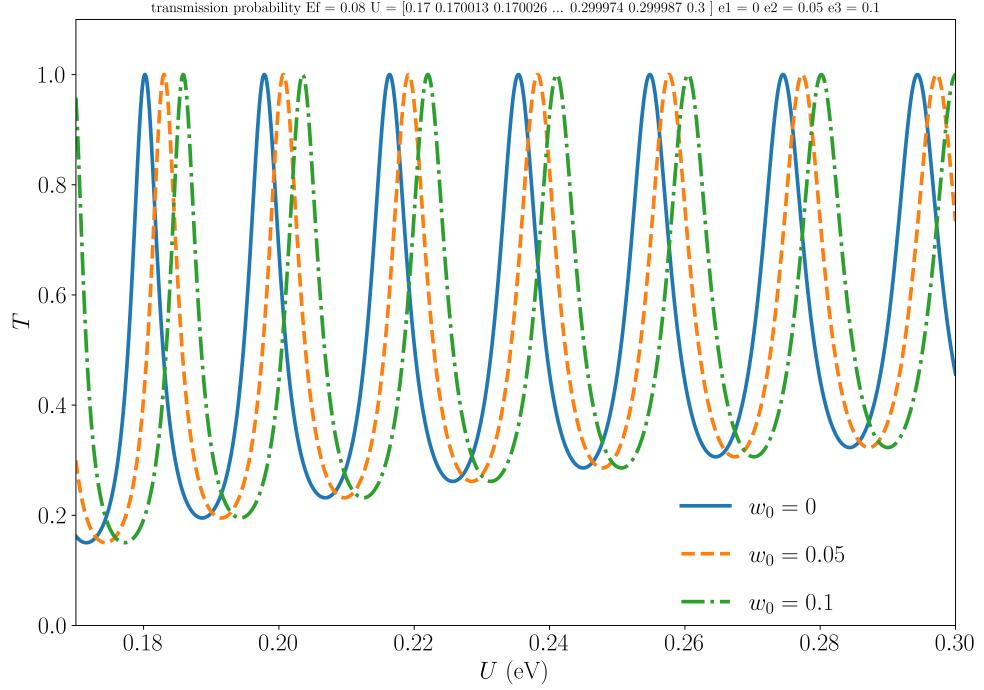


Figure 4.2 tp fixed angle

4.4 Revisit: The tilted strength identification by means of the tunneling resonance properties

In section 4.2, we have demonstrated a method to identify the character of Dirac cone using the resonant tunneling properties. The method can be applied to any resonance condition given that we have known its corresponding resonant tunneling angle. However, it is still not a practical method since the parameter n is unobservable. Also, most device configurations have fixed angled probes to observe the resonant tunneling current. In this section, we revisit this study again and eliminate the drawbacks mentioned above.

We have discussed earlier in section 4.3 that the relation of w_0 and U are linear with the slope of $1/\hbar v_F k \sin \phi$. For this reason, we can identify the tilted strength by measuring the difference between two voltages satisfied the same resonance con-

dition.

$$\Delta w = \frac{\Delta U}{\hbar v_F k_y} \quad (4.3)$$

To apply Eq. 4.3 to determine the tilted parameter, the voltages satisfied the same resonance condition have to be first determined, which can be achieved using the device structure depicted in Fig. 4.3a. We have to tune U until the resonant tunneling occurs at $\phi = \pi/4$, where the electron current can be measured as shown in Fig. 4.3b. The voltage at which the resonant tunneling occurs at $\phi = \pi/4$ for the case of non-tilted Dirac cone can be easily obtained since it is the system found in pristine graphene. Substituting these gate voltages to Eq. 4.3 we obtain $w_0 = 0.1$

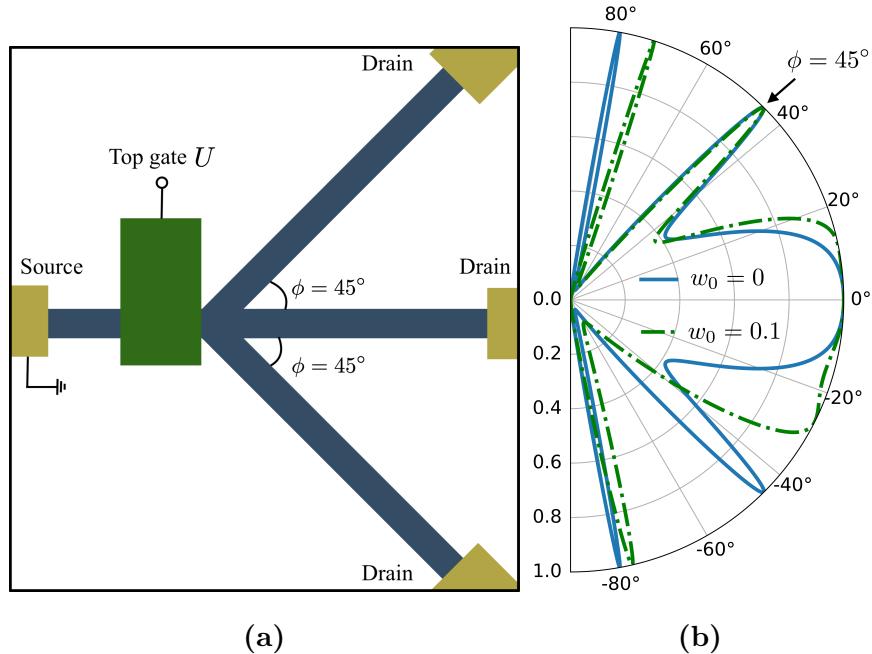


Figure 4.3 (a) The device structure for the measurement of resonant tunneling electron. Straight blue lines represent the transport region where both arms are 45° angled with the normal direction. Green and yellow region represents top gate and electrode respectively. (b) Angular-dependent transmission for different applied voltages, $U=180.2$ meV for solid line and $U=185.85$ meV for dashed-dotted line. These voltages satisfy the resonance condition at $\phi = \frac{\pi}{4}$.

4.5 Pseudo magnetic field

In section 4.1, we have shown that the tunneling behavior of electron across the tilted Dirac cone exhibits asymmetric transmission. Previously, the transmission of this kind can be achieved by applying the magnetic barrier to the system [3, 4]. In this section, we demonstrate that the similar transmission profile can also be achieved in the tilted Dirac cone system without the magnetic barrier. Consider the x-component wavevector inside the barrier region $q_x = \sqrt{q^2 - k_y^2}$, which can be rearranged into the form

$$q_x \approx \sqrt{q'^2 - (k_y - q'w_0)^2} \quad (4.4)$$

where $q' = \frac{E_F - U}{\eta \hbar v_F}$. Notice that the y-component wavevector in Eq. 4.4 is shifted by the tilted Dirac cone similar to the wavevector shift by the effect of magnetic vector potential. Based on this analogy, we can derive the equivalent pseudo magnetic field

$$\begin{aligned} -q'w_0 &= \frac{\xi}{l_B} \\ -\left(\frac{E_F - U}{\eta \hbar v_F}\right) w_0 &= \xi \sqrt{\frac{|e|B}{\hbar}} \\ B &= \left(\frac{\varepsilon w_0}{v_F}\right)^2 \frac{1}{\xi \gamma \hbar |e|} \end{aligned} \quad (4.5)$$

where $\varepsilon = E_F - U$ is effective Fermi energy. $\xi = \pm 1$ is in fact the direction of magnetic field, but since these fields are induced by the tilted Dirac cone, it can be considered as the direction of the tilt. The positive(negative) sign mean that the Dirac cone tilted to the left(right) side with respect to normal direction. $\gamma = \pm 1$ indicate the carrier type in Fermi energy level.

4.6 The key consequence of the mismatch effect

We have found in section 4.5 that both the potential and tilt are the source of pseudo-magnetic field effect, where the strength and direction of the field can be controlled by tuning the effective Fermi energy. This effect actually been proposed in Weyl semimetal n-p-n junction with tilted Weyl cones [5]. However, the coupling of the tilt and top gate potential barrier is required to preserve the effect. In contrast to our work, where the top gate potential can be set to zero, and pseudo-magnetic field effect still occurred according to Eq. 4.5. Which is the consequence of the

existence of the mismatch between Dirac cones with different tilted parameters. To show how the mismatch plays an important role in mimicking the pseudo-magnetic field effect without the requirement of potential barrier. We sketch the Fermi surface of the first and second regions, as shown in Fig. 4.4a, where top gate potential is set to zero. Hence, electrons in both regions occupy in the same Fermi energy level. If the Dirac cones are tilted homogeneously, the Fermi surface of both regions are completely overlapped. Consequently, the electron can symmetrically propagate in all directions as shown in Fig. 4.4b. When the inhomogeneity is introduced, where only the Dirac cone in the second region is tilted while the first region is vertical non-tilted. The Fermi surfaces are partially overlapping as indicated by shaded area, which lead to asymmetric tunneling.

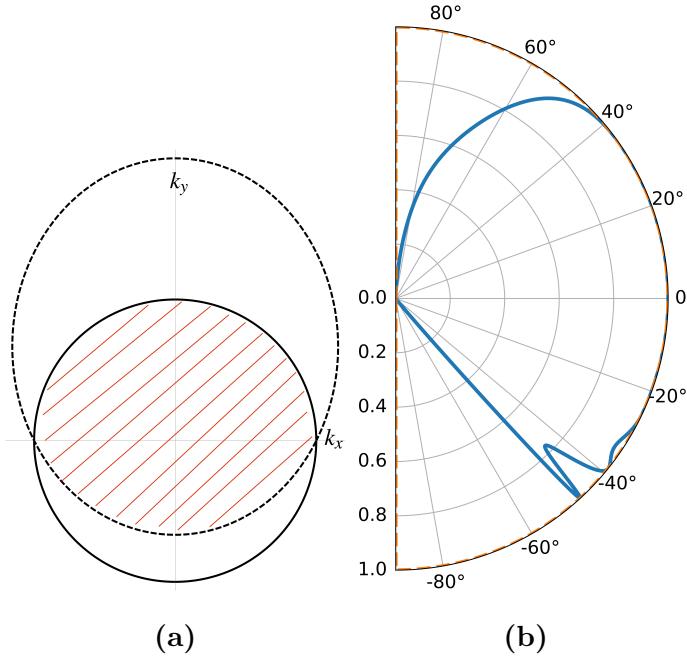


Figure 4.4 (a) The Fermi surfaces of Dirac cones in different regions. The solid line represents the Fermi surface of non-tilted Dirac cone in region I and the dashed line represents the Fermi surface of tilted Dirac cone $w_0 = 0.5$ in region II. (b) The transmission probabilities as a function of incident angle, where the solid (dashed) line is the case of the system with (without) the mismatch of Dirac cones. The Fermi energy $E_F = 80$ meV and gate potential is zero.

4.7 Transmission under the influence of pseudo magnetic field

To illustrate how pseudo magnetic field affects the tunneling behaviors compared to their real counterpart, the transmissions under the effect of pseudo and real magnetic fields are plotted as shown in Fig.4.5. First consider the case $U \gg E_F$, we found that when the tilted parameter is small, the transmission profiles between the two systems are almost identical, as shown in Fig. 4.5a, which confirms the existence of pseudo-MVP. However, the pseudo-MVP is valid only if the strength of the tilt is small. Otherwise, the transmission profiles would become distinguishable, as shown in Fig. 4.5b. The same applies in the case of potential barrier $U = 0$, where the pseudo-MVP occurs when w_0 is small and becomes weak as w_0 increase as shown in Fig. 4.5c-d.

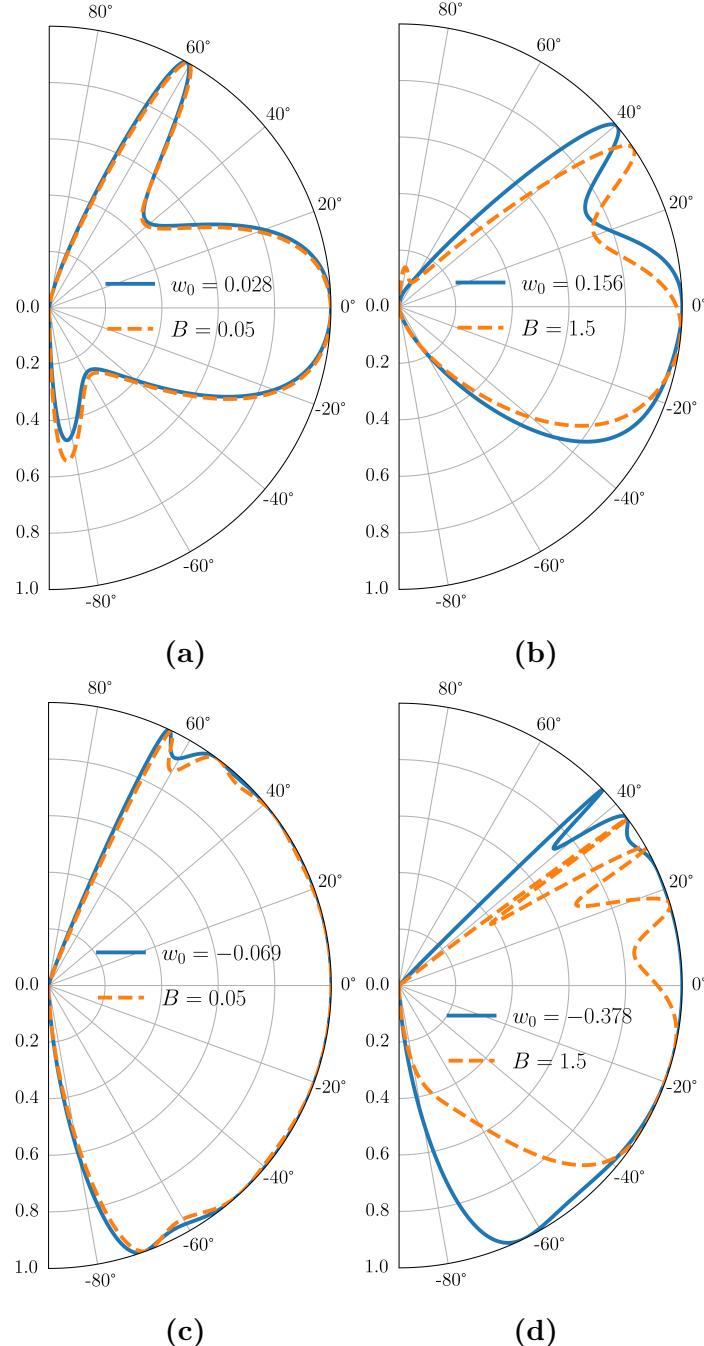


Figure 4.5 The polar plot of transmission probabilities as a function of incident angle of the system with tilted mismatch Dirac cone (solid line) and non-tilted system under the influence of delta magnetic field (dashed line). The Fermi energy $E_F = 83$ meV is the same for all plot, the gate potential $U = 285$ meV for (a) and (b), $U = 0$ for (c) and (d)

CHAPTER 5 CONCLUSION

The transmission probabilities of electron across tilted Dirac cone undergo the effect of pseudo magnetic field

REFERENCES

1. K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, “Two-dimensional gas of massless Dirac fermions in graphene,” **Nature**, vol. 438, no. 7065, pp. 197–200, nov 2005.
2. Atikur Rahman, Janice Wynn Guikema, Nora M. Hassan, and Nina Marković, “Angle-dependent transmission in graphene heterojunctions,” **Applied Physics Letters**, vol. 106, no. 1, 2015.
3. M. Ramezani Masir, P. Vasilopoulos, A. Matulis, and F. M. Peeters, “Direction-dependent tunneling through nanostructured magnetic barriers in graphene,” **Physical Review B - Condensed Matter and Materials Physics**, vol. 77, no. 23, pp. 1–11, 2008.
4. M. Ramezani Masir, P. Vasilopoulos, and F. M. Peeters, “Fabry-Pérot resonances in graphene microstructures: Influence of a magnetic field,” **Physical Review B - Condensed Matter and Materials Physics**, vol. 82, no. 11, pp. 1–12, 2010.
5. Can Yesilyurt, Zhuo Bin Siu, Seng Ghee Tan, Gengchiau Liang, Shengyuan A. Yang, and Mansoor B.A. Jalil, “Anomalous tunneling characteristic of Weyl semimetals with tilted energy dispersion,” **Applied Physics Letters**, vol. 111, no. 6, 2017.

CURRICULUM VITAE

NAME Mr. Eakkarat Pattrawutthiwong

DATE OF BIRTH 26 April 1995

EDUCATIONAL RECORD

HIGH SCHOOL	High School Graduation Kaen Nakhon Wittayalai School, 2014
BACHELOR'S DEGREE	Bachelor of Science (Applied Physics) King Mongkut's University of Technology Thonburi, 2018
MASTER'S DEGREE	Master of Science (Physics) King Mongkut's University of Technology Thonburi, 2020
PUBLICATION	Soodchomshom, B., Niyomsoot, K., Pattrawutthiwong, E, "Switching effects and spin-valley Andreev resonant peak shifting in silicene superconductor," Physica E: Low-dimensional Systems and Nanostructures , vol 97, pp. 375-383, 2018