Simulation of the unstable rotation of a cuboid

Books in space

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Experiment in space

- http://www.youtube.com/watch?v=GgVpOorcKqc

Stable and unstable rotation

- The book can stably rotate around two of its axis
- Rotation around the third axis is not stable

Modelling the behaviour

We make a few simplifications

- The book is a cuboid
- The density is constant

Model

State vector

For a point on the body we keep track of the position, the orientation, the total linear momentum and the total angular momentum

$$S(t) = \begin{pmatrix} x(t) \\ R(t) \\ P(t) \\ L(t) \end{pmatrix}$$

Model

Updating the sate vector

To update we need the velocity, the angular speed, the force and the torque (moment of force)

$$\frac{d}{dt}S(t) = \begin{pmatrix} v(t) \\ \omega(t) * R(t) \\ F(t) \\ \tau(t) \end{pmatrix}$$

Model

More simplifications

When a book is rotating in space, the model is simple because

- there is no force, $F(t) = 0$
- there is no torque, $\tau(t) = 0$

Thus, we only need to calculate the velocity and angular speed

Model

Updating the state vector

Velocity is given by

$$v(t) = \omega(t)r(t)$$

Angular speed is given by

$$\omega(t) = I(t)^{-1}L(t)$$

$I(t)$ depends on $R(t)$ and the moment of inertia of the body I_{body}

$$I(t) = R(t)I_{body}R(t)^{T}$$

Moment of inertia

- The moment of inertia of an object is a measure of the resistance of an object to changes in the rotation
- For a cuboid the moment of inertia is given by

$$I_{body} = \frac{1}{12}\begin{pmatrix} M(b^2+c^2) & 0 & 0 \\ 0 & M(a^2+c^2) & 0 \\ 0 & 0 & M(a^2+b^2) \end{pmatrix}$$

where a, b and c are the dimensions of the cuboid

Methods to solve the calculations

- Euler intergration
- Midpoint method
- 4th order Runga Kutta

Implementation

We intend to implement this in C/C++ using OpenGL for the visualisation

Questions?