Simulation of the unstable rotation of a cuboid

Books in space

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What you should remember from last time...

- We looked at the rotation of a rigid body; a book
- We showed a clip of an experiment in space
- Rotation around two of the main axis is stable, rotation around the third is not

String particle simulation

- In the lecture on Molecular Dynamics we've seen how to simulate a vibrating string with particles
- Particles are connected by springs that create interacting forces
- From the forces we can calculate how to update the velocity of the points ($a = \frac{F}{m}$)
- From the velocity we can calculate how to update the position of points

Particle simulation

Model of a book

- We represent the book with 8 particles, located at the corners
- Each point has the same mass
- There is an interacting force (spring) between each pair of particles
- This force is proportional to the distance between the particles minus their initial distance

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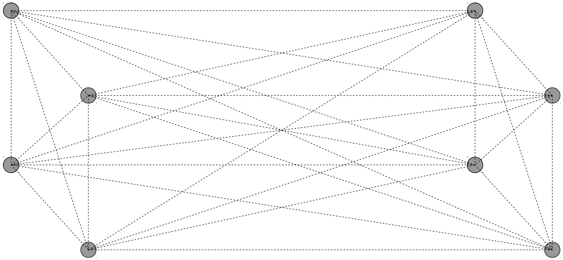


Particle simulation

Computations

- Calculate force on particle i:

$$F_i(t) = \sum_j (x_i(t) - x_j(t)) - (x_i(0) - x_j(0))$$

- Calculate velocity of particle i:

$$v_i(t) = v_i(t - \Delta t) + F_i(t) \times \Delta t$$

- Calculate position of particle i:

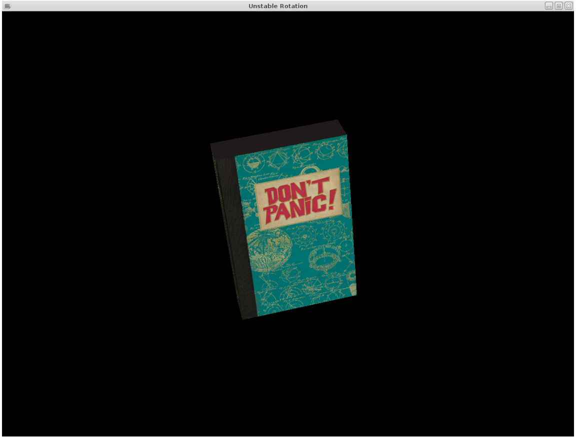
$$x_i(t) = x_i(t - \Delta t) + v_i(t) \times \Delta t$$

Using Euler integration, for n particles, in each time step we need to compute

- $\frac{(n-1)^2}{2}$ forces
- n velocities
- n positions

Particle simulation

Demonstration



Particle simulation

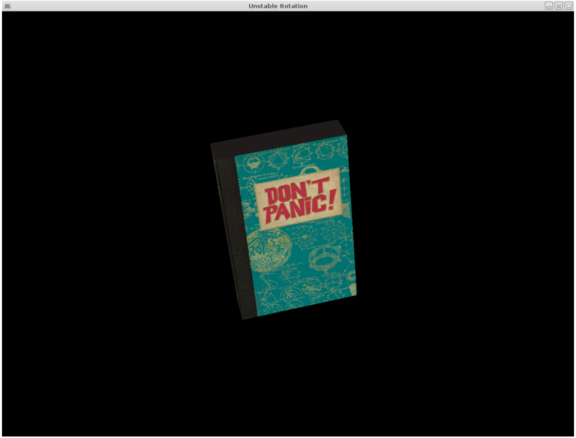
Observations

- The book seems to behave naturally
- Rotation is stable around two axis and unstable around the third
- After some time the simulation “explodes”

- Euler integration introduces a small error in each time step
- Eventually, the shape of the book changes
- Other integration techniques can reduce the error, but not eliminate it
- We can normalise the model using the conservation of energy

Particle simulation

Demonstration with normalisation



- Energy is conserved
- Angular momentum is eventually transformed into vibration

A different approach

- In our previous presentation we mentioned a different approach
- In that model we calculate the angular velocity to update the orientation of the book

Calculating the orientation

- The angular velocity is calculated from the angular momentum and the moment of inertia
- The change in orientation is calculated from the angular velocity

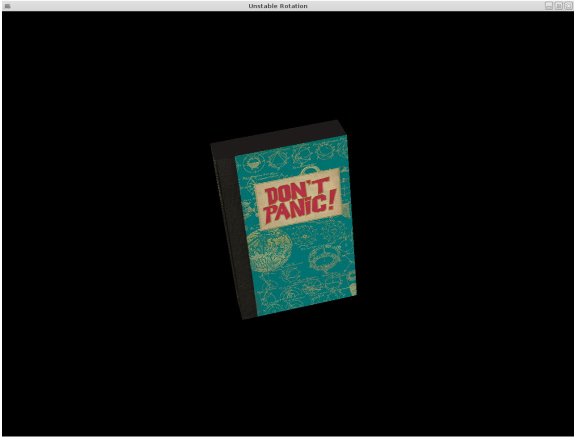
$$I(t) = R(t - \Delta t) \times I_{body} \times R(t - \Delta t)^T$$

$$\omega(t) = I(t)^{-1} \times L$$

$$R(t) = R(t - \Delta t) + R(t - \Delta t) \times \omega^*(t) \times \Delta t$$

Angular velocity based simulation

Demonstration



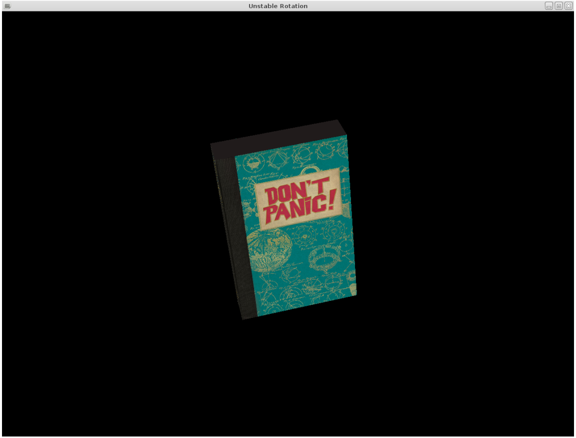
Calculating the orientation

Normalisation

- Again, Euler integration introduces a small error in each time step
- Book changes shape
- The orientation can be normalised by singular value decomposition (SVD) of the matrix that describes the orientation

Angular velocity based simulation

Demonstration with normalisation

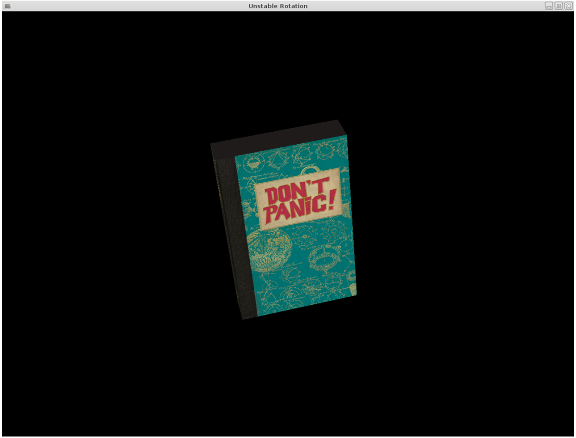


Quaternions

- The orientation has 3 degrees of freedom, but in the model it's represented by a 3×3 matrix (9 degrees of freedom)
- Representing the orientation with quaternions reduces the degrees of freedom to 4
- Again, we want normalisation to prevent the shape from changing

Angular velocity based simulation

Demonstration with quaternions



Conclusion

- The (un)stable rotation of a book can be simulated with particles
- Eventually this simulation produces “unnatural” results
- In the other approach angular momentum is conserved
- This simulation is more robust
- But this only works when the moment of inertia of an object is known

Questions?