Simulation of the unstable rotation of a cuboid

Books in space

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What you should remember from last time...

- Rotation of a rigid body; a book
- Clip of an experiment in space
- Rotation around two of the main axes is stable, rotation around the third is not

String particle simulation

- Lecture on Molecular Dynamics: simulation of vibrating string with particles
- Springs create interacting forces between particles
- From forces, calculate update velocity of points ($a = \frac{F}{m}$)
- From velocity, calculate update position of points

Particle simulation

Model of a book

- Eight particles, located at the corners
- Each point has the same mass
- Interacting force (spring) between each pair of particles
- Force is proportional to distance between the particles minus initial distance

Particle simulation

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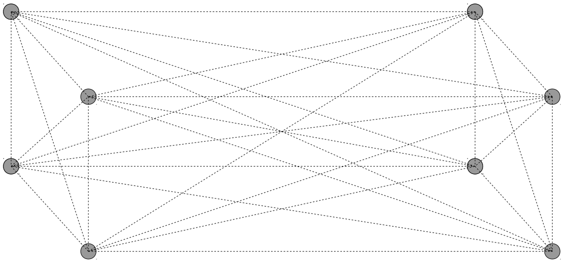


- Calculate force on particle i:

$$F_i(t) = \sum_{j \neq i} k \left(\frac{\|x_i(0) - x_j(0)\|}{\|x_i(t) - x_j(t)\|} - 1 \right) \left(x_i(t) - x_j(t) \right)$$

- Calculate velocity of particle i:

$$v_i(t + \Delta t) = v_i(t) + \frac{1}{m} F_i(t)\, \Delta t$$

- Calculate position of particle i:

$$x_i(t + \Delta t) = x_i(t) + v_i(t)\, \Delta t$$

Particle simulation

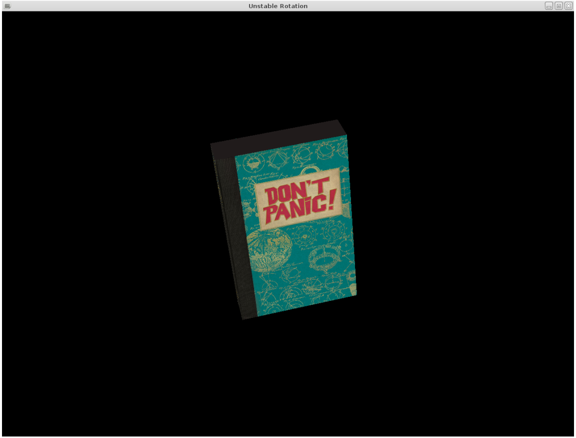
Computations

Using Euler integration, for n particles, in each time step we need to compute

- $\frac{n(n-1)}{2}$ forces
- n velocities
- n positions

Particle simulation

Demonstration



Particle simulation

Observations

- Book behaves naturally
- Rotation is stable around two axis and unstable around the third
- After some time the simulation “explodes”

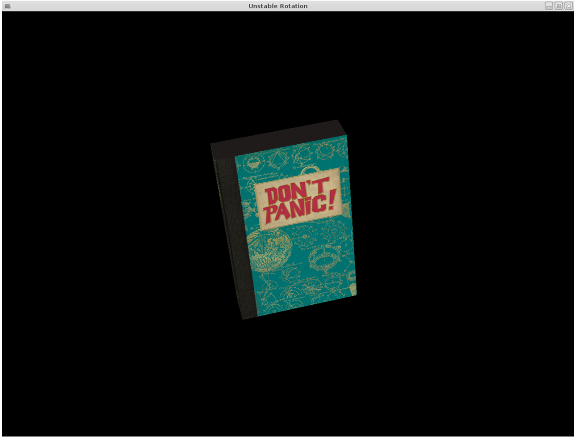
Particle simulation

Normalisation

- Euler integration introduces a small error in each time step
- Shape of book changes eventually
- Other techniques can reduce, but not eliminate the error
- Normalise the model using the conservation of energy

Particle simulation

Demonstration with normalisation



- Energy is conserved
- Angular momentum is transformed into vibration

A different approach

- Different approach as introduced last time
- Calculate angular velocity to update the orientation

Angular velocity based simulation

Computations

- Angular velocity calculated from angular momentum and moment of inertia

$$I(t) = R(t)^T \; I_{body} \; R(t)$$
$$I(t)^{-1} = R(t) \; I_{body}^{-1} \; R(t)^T$$
$$\omega(t) = I(t)^{-1} \; L$$

- Change in orientation calculated from angular velocity

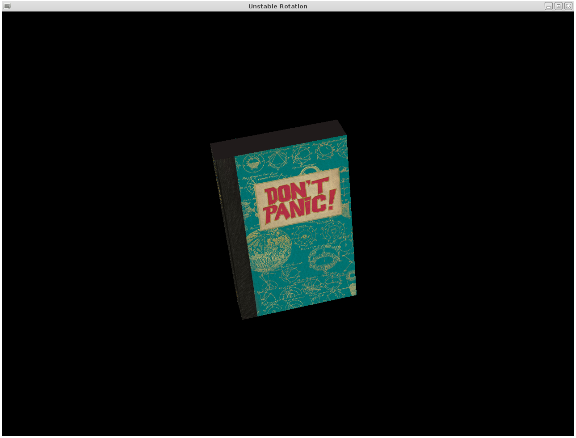
$$R(t + \Delta t) = R(t) + \omega^*(t) \; R(t) \; \Delta t$$

Using Euler integration, in each time step we need to do

- 5 matrix multiplications
- 1 vector to tensor calculation
- 1 matrix inverse

Angular velocity based simulation

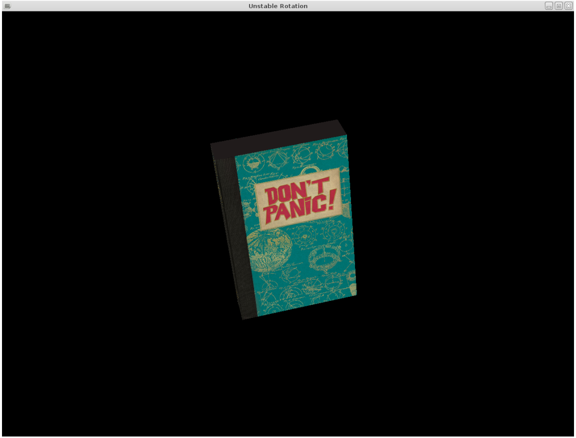
Demonstration



- Euler integration introduces a small error in each time step
- Book changes shape
- Orientation matrix normalised by singular value decomposition (SVD)

Angular velocity based simulation

Demonstration with normalisation

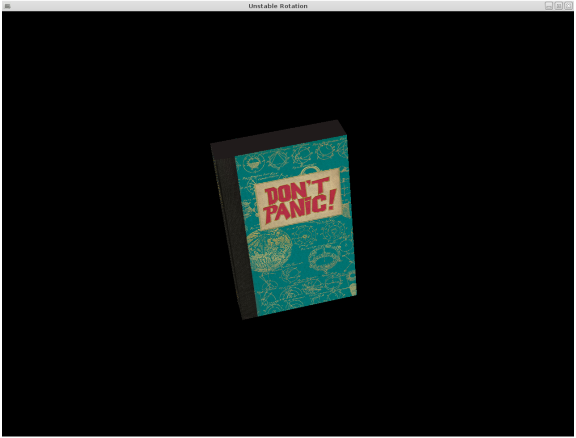


Quaternions

- Orientation has 3 degrees of freedom, but a 3×3 matrix has 9
- Quaternions have 4 degrees of freedom
- Again, normalisation to prevent shape from changing

Angular velocity based simulation

Demonstration with quaternions



Conclusion

- (Un)stable rotation of a book can be simulated with particles
- Simulation eventually produces “unnatural” results
- Other approach preserves angular momentum
- Simulation is more robust
- Only works when moment of inertia is known

Questions?

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Questions

about particle simulation

Q: What happens if you change the spring constant in calculating the forces?

A: For a high spring constant the shape and size of the book is contained better, but the book quickly goes to the vibrating state. For a low spring constant it takes longer to reach the vibrating state, but the shape and size is distorted more.

Q: Why don't you normalise the distance between particles?

A: If we do that, how will we calculate the forces?

Questions

about particle simulation

Q: Can you improve this model?

A: There are several extensions one can think of. For example, damping may produce better results.

Questions

about quaternions

Q: What are quaternions?

A: They're an extension to complex numbers of the form $q = a + b \cdot i + c \cdot j + d \cdot k$ that can be used to describe a rotation in 3d space. For a point at position $p(0, x, y, z)$ the position after rotation is $p'(0, x', y', z') = q \cdot p \cdot \overline{q}$.

Questions

about quaternions vs. matrix method

Q: Which method is faster?

A: In theory, quaternions produce better results and can be normalised less frequently. However, in this case the simulation is fast enough to do a normalisation in every time step.