Set 6 - Homework

Name:

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(1) Given a finite group G and an element $a \in G$, consider the set $H_a = \{e, a, a^2, a^3, ...\}$. Show that the size of H_a is equal to the order of a.

(2) Find a formula for $\sum_{i=1}^{n} i^2$ for any integer $n \geq 1$, and prove it by using induction.

(3) Show that $(x^2 + x + 1)(x + 2) = x^3 + 2$ in $\mathbb{Z}/3[x]$. In which of the following number systems is the polynomial $f(x) = x^3 + 2$ irreducible (i.e. does not factor into a product of polynomials both of degree strictly less than 3)? Prove your claim. [Hint: if it did split, what would be the degrees of the factors? Use this to show the polynomial would have to have a root]

(1) $\mathbb{Q}[x]$

(2) $\mathbb{Z}/5[x]$

(3) $\mathbb{Z}/7[x]$