Set 4 - Homework

Name:

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(1) Show that, for $\alpha \in \mathbb{Z}[i]$, we have $N(\alpha) = 1$ if and only if α is a unit.

(2) Prove the following theorem.

Theorem. *Let $\alpha, \beta \in \mathbb{Z}[i]$ be nonzero, and let $g \in \mathbb{Z}[i]$ be a greatest common divisor of α and β. Then $z \in \mathbb{Z}[i]$ can be written as $z = \alpha x + \beta y$ for some $x, y \in \mathbb{Z}[i]$ if and only if z is a multiple of g.*

(3) State and prove your conjecture from (Set 4, C1).