Causal Inference with Panel Data

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Potential outcomes framework [5,15]

Suppose I have COVID-19. Would going to the hospital improve my health?

Defining *causal effect* of "go hospital"(D) on my "health"(Y):

- 2 potential outcomes: $Y_i(D = \text{``go"})$ or $Y_i(D = \text{``stay"})$
- Individual causal effect defined by the difference in potential outcomes:

$$Y_i(D = \text{"go"}) - Y_i(D = \text{"stay"})$$

Fundamental problem [13]:

- Cannot infer individual causal effects

Instead, we aim to recover the average treatment effect with multiple units

Average Treatment Effect [15]

Assume that:

- 1. Treatment assigned independently from outcome and pre-treatment attributes (ignorability, or "unconfoundedness"; "exogeneity"; "selection on observables"; "omitted variables")
- 2. Each unit has some chance to receive a treatment (overlap; *common support*)
- 3. Units receive same version of treatment (Stable Unit Treatment Value Assumption)
- 4. No spillover or carry-over effects across units and times
- 5. Treatment effect is linear

Under above assumptions, aim to recover following *causal estimands*:

Average Treatment Effect:

$$E[Y_i(go) - Y_i(stay)]$$

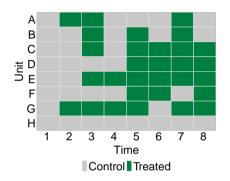
Average Treatment Effect on the Treated:

$$E[Y_i(go) - Y_i(stay) | \underbrace{D_i = "go"}_{treated group}]$$

Observational "panel" or "TSCS" data [7]

- Panel: repeated cross-section data (different units sampled over time)
- Time-series-cross-section: "same" units repeatedly observed across time periods
 → Balanced panel
- Today's methods generally apply to both cases

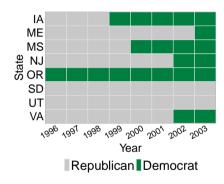
Mapping panel data onto grids



- Y-axis indicates unit (N)
- X-axis indicates time (T)
- Green cells are treated units
- Grey cells are control units

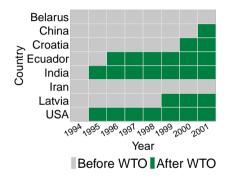
- Treatment is binary (either get treated or remain as control)
- Aim to recover average treatment effect or average treatment effect for the treated under various settings

Examples



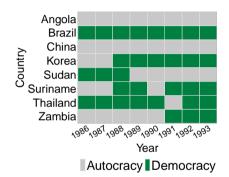
 Does a state governor's party ID affect policy performances?
 (Dynes & Holbein 2019) [11]

Examples



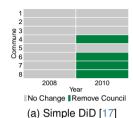
 Does WTO accession increase trade? (Rose 2004; Tomz, Goldstein & Rivers 2007) [18,20]

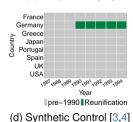
Examples

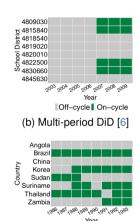


 Does a country's regime type matter for economic growth?
 (Acemoğlu et al. 2019) [1]

Panel/TSCS data with heterogeneous treatment paths

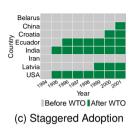




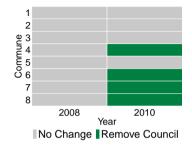


Autocracy Democracy

(e) Treatment Reversal [1]



Difference-in-Differences design



Malesky et al. (2014, APSR [17])

- Question: Does decentralization lead to better public service delivery?
- Treatment: Removal of elected councils in Vietnamese districts
- 30 outcome variables (public transport, TV broadcasting, post office, health care···)
- Two periods (2008, 2010), treatment assigned on April 25, 2009
- Two groups (communes that removed vs. maintained councils)

Identification under a DiD design

Assumptions:

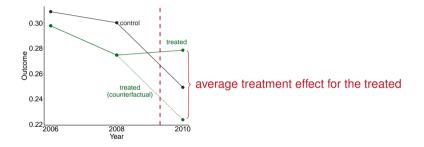
- Treatment is exogenous → not random (treatment carefully assigned)
- Same treatment for each unit → "Removal of councils"
- No spill-over or carry-over effect
- Linear treatment effect

Additionally, we assume the parallel trends assumption

"Parallel Trends" assumption

Assume that:

Pre-treatment trends of the DV is same for treated & control groups



$$\mathbb{E}\left[\begin{array}{c|c} Y_{i1}(0) & - & Y_{i0}(0) \end{array} \middle| \underbrace{D_i = 1}\right] = \mathbb{E}\left[\begin{array}{c|c} Y_{i1}(0) & - & Y_{i0}(0) \end{array} \middle| \underbrace{D_i = 0}\right]$$
post-treat DV pre-treat DV control

Estimation

The DiD estimator:

(E[treated units' post-treat outcomes] — E[treated units' pre-treat outcomes])

(E[control units' post-treat outcomes] — E[control units' pre-treat outcomes])

Under the parallel trends assumption, we compute this DiD estimator via:

$$Y_{it} \sim \alpha + \theta D_i + \gamma T_t + \beta (D_i \times T_t)$$

In R, equivalent to:

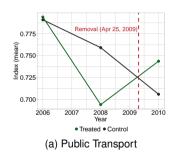
$$lm(y \sim treat + post + treat*post, data = data)$$

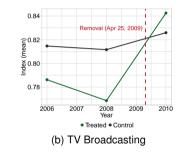
- treat $(D_i) \in \{0,1\}$ (treatment or control group)
- post $(T_t) \in \{0,1\}$ (pre- or post- treatment period)
- control variables can be added

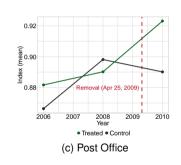
Coefficient for the treat*post (β) numerically equivalent to the DiD estimator (ATT)

Check the parallel trends assumption

Plot DV trends for both treated/control group:







- What to do when the parallel trends seem implausible?
 - ⇒ Use information from multiple "pre-treatment" period (ongoing works) [21]

Summary

Difference-in-Differences:

- Special case of panel/TSCS data settings
- Importance of the parallel trends assumption
- Estimation is straightforward when all assumptions are plausible
- Linear treatment effect

Statistical software on Difference-in-Differences design

- In R:
 - panelView (by Xu)
 - did (by Callaway & Sant'Anna)
 - DIDdesign (by Egami & Yamauchi)
- In STATA:
 - DID_MULTIPLEGT (by de Chaisemartin & D'Haultfœuille)
- caution! all under development (read documentation carefully)

Two-way Fixed Effects: Setup [14]

From two units and two time periods to

- multiple units
- multiple periods
- varying treatment adoptions (staggered adoption; treatment status on and off)

Two-way Fixed Effects (2FE) Model

- Two problems
 - treatment effects heterogeneity
 - unboserved time-varying confounders
- Practical solutions

Two-way Fixed Effects: Setup [14]

$$Y_{it} = \alpha_i + \gamma_t + \beta(D_i \times T_t) + \epsilon_{it}$$

- α_i is time-invariant unit specific fixed effects (e.g. culture)
- γ_t is unit-invariant time specific fixed effects (e.g. pandemic)
- $D_i = 1$ if the unit is ever treated
- $T_t = 1$ if it is the post-treatment time periods
- β is the estimate of interests
- Covariates can be easily added

Two-way Fixed Effects: Setup [14]

2FE model and DID

- 2FE is not equivalent to DID with multiple units and time periods
- 2FE is equivalent to DID only with two units and two time periods
- The connection between 2FE and DID will be illustrated with an example later

Two-way Fixed Effects: Setup [9]

Interpret 2FE using potential outcome framework

- Demean: compare Y_{it} with the average of other observations from the same group
- Estimated counterfactuals in 2FE model: $M_i + M_t M_o$
- M_i : mean of other observations from the same group
- *M_t*: mean of other observations from the same time period
- M_o: mean of other observations from neither the same group nor the same time period
- There is no observation that is both in the same group and the same time period

Two-way Fixed Effects: Setup [9]

Causal assumptions

- 1. No unobserved time-varying confounders (parallel trend)
- 2. Past outcomes do not affect current treatment status (exogeneity)
- 3. Linearity assumption (functional form)
 - Even when 1 and 2 are satisfied, violation of 3 results in inconsistent estimation of β

Note

- Past outcomes are allowed to affect current outcomes: need to cluster standard errors
- Past treatments are allowed to affect currect outcomes: add lags and leads
- The leads are often used to test for parallel trend
- Trade-off between fixed effects and lagged dependent variables: assumption 2

Problem: treatment effects heterogeneity across units or over time

Consequences

• Inconsistent estimation: β_{2fe} could be negative even when ATE for each unit is positive

Why?

Problem: treatment effects heterogeneity across units or over time

2FE model:
$$Y_{it} = \alpha_i + \gamma_t + \beta D_i \times T_t + \epsilon_{it}$$

Decompose β_{2fe}

- β_{2fe} is a weighted average of treatment effects of each treated unit in each time period
- The weights can be negative

Problem: treatment effects heterogeneity across units or over time

An example: two units and three time periods with staggered treatment adoption

Data generating process:

```
# Treatment assignments
DT <- list(c(0,0,1),c(0,1,1))
# Unit and time specific treatment effects
beta <- list(c(0,0,1),c(0,1,4))
# time invariant unit specific effects
alpha <- list(1.7,2.6)
# unit invariant time specific effects
gamma <- c(0,0,0)
# Data generating process for unit 1 and unit 2
Y_1 <- alpha[[1]] + gamma + beta[[1]]*DT[[1]]
Y_2 <- alpha[[2]] + gamma + beta[[2]]*DT[[2]]</pre>
```

Problem: treatment effects heterogeneity across units or over time

An example: two units and three time periods with staggered treatment adoption

```
## id time DT Y

## 1 Maryland 1 0 1.7

## 2 Maryland 2 0 1.7

## 3 Maryland 3 1 2.7

## 4 California 1 0 2.6

## 5 California 2 1 3.6

## 6 California 3 1 6.6
```

Problem: treatment effects heterogeneity across units or over time

An example: two units and three time periods with staggered treatment adoption

Estimate β_{fe} using two-way fixed effects model

```
# 2FE model
m <- lm(data = d_example, Y ~ DT + as.factor(id) + as.factor(time))
beta_fe <- coef(m)[2]
beta_fe
## DT
## -0.5</pre>
```

Why is β_{fe} negative when $\beta_{MD,t=3}=1$, $\beta_{CA,t=2}=1$, and $\beta_{CA,t=3}=4$ are all positive?

Problem: treatment effects heterogeneity across units or over time

An example: two units and three time periods with staggered treatment adoption

```
# calculate the weights
m_weight <- lm(data=d_example,DT ~ as.factor(id) + as.factor(time))</pre>
d_example <- d_example %>% mutate(W = m_weight$residuals)
d_example
            id time DT Y
##
## 1
     Maryland 1 0 1.7 0.1666667
## 2 Maryland 2 0 1.7 -0.3333333
      Maryland 3 1 2.7 0.1666667
## 3
## 4 California 1 0 2.6 -0.1666667
## 5 California 2 1 3.6 0.3333333
## 6 California 3 1 6.6 -0.1666667
```

Problem: treatment effects heterogeneity across units or over time

An example: two units and three time periods with staggered treatment adoption

$$\omega_{MD,t=3} = \frac{1}{6}, \, \omega_{CA,t=2} = \frac{1}{3}, \, \omega_{CA,t=3} = -\frac{1}{6}$$

After normalization,
$$\omega_{MD,t=3}=\frac{1}{2},\,\omega_{CA,t=2}=1,\,\omega_{CA,t=3}=-\frac{1}{2}$$

$$\beta_{\text{fe}} = \frac{1}{2} \times 1 + 1 \times 1 - \frac{1}{2} \times 4 = -0.5$$

Why β_{fe} is negative: heterogenous treatment effects and negative weights

Problem: treatment effects heterogeneity across units or over time

An example: two units and three time periods with staggered treatment adoption

The connection between β_{fe} and DID estimator

Two DID estimators:
$$\beta_{fe} = \frac{DID_1 + DID_2}{2}$$

$$DID_1 = (Y_{MD,t=3} - Y_{MD,t=2}) - (Y_{CA,t=3} - Y_{CA,t=2})$$

$$DID_2 = (Y_{CA,t=2} - Y_{CA,t=1}) - (Y_{MD,t=2} - Y_{MD,t=1})$$

Problem: treatment effects heterogeneity across units or over time

An example: two units and three time periods with staggered treatment adoption

The connection between β_{fe} and DID estimator

$$DID_1 = (2.7 - 1.7) - (6.6 - 3.6) = -2$$

$$DID_2 = (3.6 - 2.6) - (1.7 - 1.7) = 1$$

$$eta_{ ext{fe}} = rac{-2+1}{2} = -0.5$$

Why DID_1 is negative: treated observations (CA in time 2 and 3) are used as controls

Problem: treatment effects heterogeneity across units or over time

Summary

- 2FE estimator is a weighted average of treatment effects of each treated unit in each time period
- The weights can be negative
- β_{fe} cannot be consistenly estimated if treatment effects are heterogenous
- Units adopting the treatment earlier are more likely to receive some negative weights

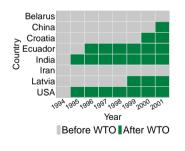
Problem: treatment effects heterogeneity across units or over time

Solutions

- ullet First, compare the treated only with untreated observations (e.g. define $\emph{DID}_1=0$)
- Second, calculate unit and time specific treatment effects
- Third, aggregate individual effects in a way without negative weights

Problem: treatment effects heterogeneity across units or over time

Solutions: matching based approach



Estimate $\beta_{Croatia,2000}$

- Find control units from Belarus, China, and Iran
- Estimate DID_{2000,pre2000}

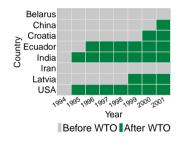
Estimate $\beta_{Croatia,2001}$

- Find control units from Belarus, Iran
- Estimate DID_{2001,pre2000}

Aggregate $DID_{2000,pre2000}$ and $DID_{2001,pre2000}$

Problem: treatment effects heterogeneity across units or over time

Solutions: model based approach (R package: fect)



 Use all untreated observations (the gray ones), and fit the following 2FE model

$$Y_{it} = \alpha_i + \gamma_t + \epsilon_{it}$$

- Use the estimated â and ŷ to imputate counterfactuals for treated observations (the green ones): Y(0)_{it} = â_i + ŷ_t
- Calculate treatment effects for unit i at time period t: $\beta_{it} = Y_{it}^{observed} Y(\hat{0})_{it}$
- Aggregate β_{it}

Problem: treatment effects heterogeneity across units or over time

Solutions: summary

- Only use untreated observations as control units
- Calculate unit and time specific treatment effects, and then aggregate
- Flexibility in choosing control units
- Flexibility in estimating counterfactuals
- Flexibility in aggregating results

Problem: unobserved time-varying confounders (violation of parallel trend)

Solutions

- Test for (no) pre-trend, allowing heterogenous effects (otherwise it is problematic)
- Directly model unobserved time-varying confounders
- Both need long pre-treatment periods

Problem: unobserved time-varying confounders (violation of parallel trend)

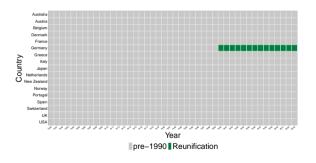
Solutions: test for (no) pre-trend, allowing heterogenous effects

- The trend between treated and control units before the adoption of treatment
- If parallel pre-treatment, more likely to be parallel post-treatment
- $\beta_{it} = Y_{it}^{observed} Y_{it}(0)$
- Plot β_{it} from t = -m, -m+1, ..., -2, -1, 0, 1, 2, ..., m-1, m
- Whether the effects in the pre-treatment periods exhibit upward or downward trend
- Visualize
- Formally test:
 - jointly zero
 - equivalent test (flip the null hypothesis)
 - placebo test (move treatment adoption backward)

Two-way Fixed Effects: Problems [2,3,4,10]

Problem: unobserved time-varying confounders (violation of parallel trend)

Solutions: model unobserved time-varying confounders (synthetic control)



- $Y_{Germany} = 0.2 Y_{Italy} + 0.3 Y_{France} + 0.5 Y_{Austria}$
- No control alone follows a parallel trend with Germany
- But a weighted combination of Italy, France, and Austria can be seen as counterfactual outcomes to Germany
- Require a relatively long pre-treatment period

Two-way Fixed Effects: Problems

Problem: unobserved time-varying confounders (violation of parallel trend)

Solutions: model unobserved time-varying confounders (synthetic control)

$$Y_{Germany} = 0.2 Y_{Italy} + 0.3 Y_{France} + 0.5 Y_{Austria}$$

How to understand the weights

- The outcome trajectories of some countries are more similar than others to Germany
 - DID: equal weights are assigned to each control unit
 - countries with very different trends are assigned 0 as weight
- Germany is correlated with other countries in some way
 - estimate the correlations using pre-treatment observations
 - assume that the correlations hold in post-treatment periods
 - use estimated correlations to impute counterfactuals for Germany at each post-treatment period

R package: Synth

Two-way Fixed Effects: Problems

Problem: unobserved time-varying confounders (violation of parallel trend)

Solutions: model unobserved time-varying confounders (synthetic control)

With multiple treated units

- Repeat the estimation for each treated unit and aggregate the effects
- Use R package fect with interactive fixed effects model (a factor model)

Note

- Parallel trend is not guaranteed, unless it mimics the true data generating process
- But this is easy to check (pre-trend)

Two-way Fixed Effects: Summary

Two problems

- Treatment effects heterogeneity
- unobserved time-varying confounders (violation of parallel trend)

Solutions

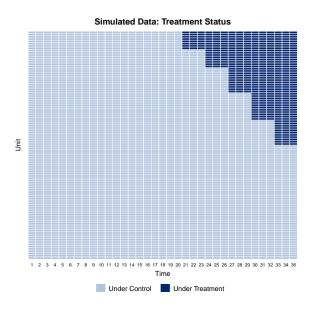
Other notes

- Number of N and T
- Statistical inference (standard errors): randomized inference or bootstrap

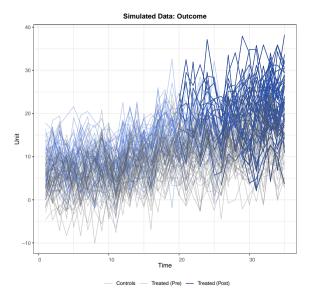
Installation and user instruction: http://yiqingxu.org/software/fect/fect.html

Reference: Licheng Liu, Ye Wang, Yiqing Xu (2019). "A Practical Guide to Counterfactual Estimators for Causal Inference with Time-Series Cross-Sectional Data." Available at SSRN: https://papers.ssrn.com/abstract=3555463.

```
# load required packages
library(fect)
library(panelView)
# load the data
data(fect)
# wiew the data
panelView(Y ~ D, data = simdata1, index = c("id", "time"),
          by.timing = TRUE,
          axis.lab = "time", xlab = "Time", ylab ="Unit",
          show.id = c(1:100).
          background = "white",
          main = "Simulated Data: Treatment Status")
```



```
# view the outcome
panelView(Y ~ D, data = simdata1, index = c("id","time"),
   axis.lab = "time", xlab = "Time", ylab = "Unit", show.id = c(1:100),
   theme.bw = TRUE, type = "outcome", main = "Simulated Data: Outcome")
```



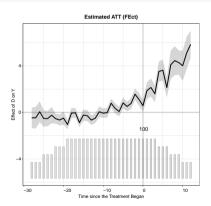
```
# ATE: equally weight each observation
out.fect$est.avg

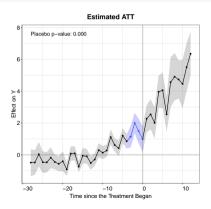
## ATT.avg S.E. CI.lower CI.upper p.value
## [1,] 3.489078 0.3885647 2.727505 4.25065 0

# ATE: equally weight each unit
out.fect$est.avg.unit

## ATT.avg.unit S.E. CI.lower CI.upper p.value
## [1,] 2.928551 0.3678433 2.207591 3.649511 1.776357e-15
```

```
# plot the estimated ATT
plot(out.fect, main = "Estimated ATT (FEct)", ylab = "Effect of D on Y",
    cex.main = 0.8, cex.lab = 0.8, cex.axis = 0.8)
```





Not parallel, what can we do

- Interactive fixed effects model
 - A factor model
 - Helpful only when the data follows a factor structure
 - Need a cross validation process to choose the number of factors

Match each treated unit with similar control units from the never treated

Thank you!

Slides and other materials available from:

https://github.com/gsa-gvpt/gvpt-methods

Follow-up Questions:

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