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CPTS 434

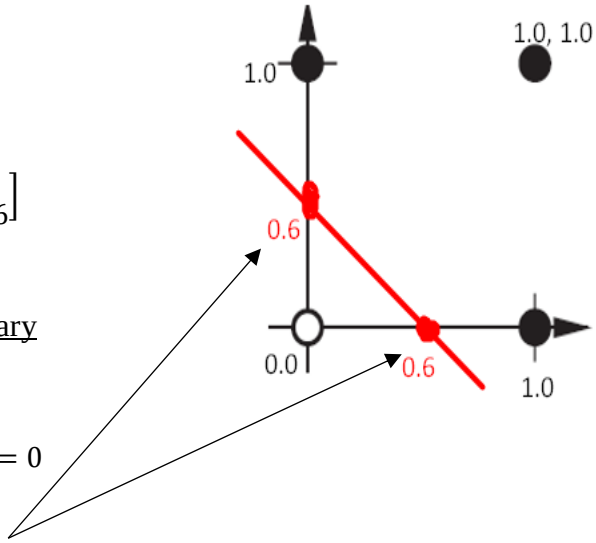
Introduction to Neural Network Design & Applications

10-15-2019

Homework #5

Part 1

1.



Logical OR

p_1	p_2	t
0	0	0
0	1	1
1	0	1
1	1	1

Bias

$w^T p + b = 0$

$-b = w^T p$

$-b = [1 \quad 1]\begin{bmatrix} 0 \\ 0.6 \end{bmatrix}$

$b = -0.6$

Decision Boundary

$w^T p + b = 0$

$[1 \quad 1]\begin{bmatrix} x \\ y \end{bmatrix} - 0.6 = 0$

$x + y = 0.6$

2.

Open-Circles Margin

$$d = \frac{|g(0,0)|}{||w||}$$

$$d = \frac{|[1 \quad 1]\begin{bmatrix} 0 \\ 0 \end{bmatrix} - 0.6|}{\sqrt{1^2 + 1^2}} = 0.42$$

Closed-Circles Margin

$$d = \frac{|g(0,0)|}{||w||}$$

$$d = \frac{|[1 \quad 1]\begin{bmatrix} 0 \\ 1 \end{bmatrix} - 0.6|}{\sqrt{1^2 + 1^2}} = 0.28$$

Part 2

1.

Step 1

$a = hardlim(w^T p_1 + b)$

$a = hardlim([0.5 \quad -1 \quad -0.5]\begin{bmatrix}-1\\1\\-1\end{bmatrix} + 0.5)$

$a = hardlim(-0.5)$

$a = 0$

$t_1 = 1 \; and \; a = 0, t_1 \neq a, therefore \; weights \; need \; to \; be \; adjusted$

$w_{new} = w_{old} + ep_1, we \; need \; to \; find \; e$

$e = t_1 - a$

$e = 1 - 0$

$e = 1$

$w_{new} = \begin{bmatrix}0.5\\-1\\-0.5\end{bmatrix} + \begin{bmatrix}-1\\1\\-1\end{bmatrix}$

$w_{new} = \begin{bmatrix}-0.5\\0\\-1.5\end{bmatrix}$

$b_{new} = b_{old} + e$

$b_{new} = 0.5 + 1$

$b_{new} = 1.5$

Step 2

$a = hardlim(w^T p_2 + b)$

$a = hardlim([-0.5 \quad 0 \quad -1.5]\begin{bmatrix}1\\1\\-1\end{bmatrix} + 1.5)$

$a = hardlim(2.5)$

$a = 1$

$t_2 = 0 \; and \; a = 1, t_2 \neq a, therefore \; weights \; need \; to \; be \; adjusted$

$w_{new} = w_{old} + ep_2, we \; need \; to \; find \; e$

$e = t_2 - a$

$e = 0 - 1$

$e = -1$

$w_{new} = \begin{bmatrix} -0.5 \\ 0 \\ -1.5 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$w_{new} = \begin{bmatrix} -1.5 \\ -1 \\ -0.5 \end{bmatrix}$

$b_{new} = b_{old} + e$

$b_{new} = 1.5 - 1$

$b_{new} = 0.5$

Step 3

$a = hardlim(w^T p_1 + b)$

$a = hardlim([-1.5 \quad -1 \quad -0.5]\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + 0.5)$

$a = hardlim(1.5)$

$a = 1$

$t_1 = 1 \; and \; a = 1, t_1 = a, therefore \; weights \; don't \; need \; to \; be \; adjusted$

Step 4

$a = hardlim(w^T p_2 + b)$

$a = hardlim([-1.5 \quad -1 \quad -0.5]\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + 0.5)$

$a = hardlim(-1.5)$

$a = 0$

$t_2 = 0 \; and \; a = 0, t_2 = a, therefore \; weights \; don't \; need \; to \; be \; adjusted$

Step 5

$No \; Adjustments \; were \; made \; for \; p_1 \; or \; p_2 \; therefore \; we \; are \; done$

Results

$w^T = [-1.5 \quad -1 \quad -0.5]$

$b = 0.5$

Decision Boundary

$w^T p + b = 0$

$$[-1.5 \quad -1 \quad -0.5]\begin{bmatrix} x \\ y \\ z \end{bmatrix} + 0.5 = 0$$

$$-1.5x - y - 0.5z + 0.5 = 0$$

2.

$Distance\ Calculation\ for\ p_1$

$$d_1 = \frac{|g(p_1)|}{||w||}$$

$$d_1 = \frac{|[-1.5 \quad -1 \quad -0.5]\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + 0.5|}{\sqrt{1.5^2 + -1^2 + -0.5^2}}$$

$d_1 = 0.80$

$Distance\ Calculation\ for\ p_2$

$$d_2 = \frac{|g(p_2)|}{||w||}$$

$$d_2 = \frac{|[-1.5 \quad -1 \quad -0.5]\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + 0.5|}{\sqrt{1.5^2 + -1^2 + -0.5^2}}$$

$d_2 = 0.80$