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CPTS 434

Introduction to Neural Networks Design & Applications

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### Homework 7

1.

$$f_1 = \exp(-|X - [1,1]|/2)$$

$$f_2 = \exp(-|X - [0,0]|/2)$$

X	$f_1$	$f_2$
(1,1)	1	0.1353
(0,1)	0.3678	0.3678
(0,0)	0.1353	1
(1,0)	0.3678	0.3678

Decision boundary is  $f_1 + f_2 = 0.94$

Use  $w^T = [1, 1]$  to calculate bias and margins

$$w^T f + b = 0$$

$$w^T f = -b$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = -b$$

$$(1 * f_1) + (1 * f_2) = -b$$

$$f_1 + f_2 = -b$$

$$f_1 + f_2 = 0.94$$

$$b = -0.94$$

$$r = 0$$

$$d = \frac{|g(1, 0.1353)|}{||w||}$$

$$d = \frac{|\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0.1353 \end{bmatrix} - 0.94|}{\sqrt{1^2 + 1^2}}$$

$$d = \frac{0.1953}{\sqrt{2}}$$

$$d = 0.1381$$

$$\underline{r = 1}$$

$$d = \frac{|g(0.3678, 0.3678)|}{||w||}$$

$$d = \frac{|[1 \quad 1] \begin{bmatrix} 0.3678 \\ 0.3678 \end{bmatrix} - 0.94|}{\sqrt{1^2 + 1^2}}$$

$$d = \frac{0.2044}{\sqrt{2}}$$

$$\mathbf{d} = \mathbf{0.1445}$$

2.

Coordinates of features  $z_1 = \text{sigmoid}(x_1 - x_2 - 0.5)$

in the hidden layer  $z_2 = \text{sigmoid}(-x_1 + x_2 - 0.5)$

$x_1$	$x_2$	arg1	$z_1$	arg2	$z_2$	r
0	0	-0.5	0.38	-0.5	0.38	0
0	1	-1.5	0.18	0.5	0.62	1
1	0	0.5	0.62	-1.5	0.18	1
1	1	-0.5	0.38	-0.5	0.38	0

Bias

$$\frac{|g(0.38, 0.38)|}{||w||} = \frac{|g(0.18, 0.62)|}{||w||}$$

$$|g(0.38, 0.38)| = |g(0.18, 0.62)|$$

$$\left| [1 \quad 1] \begin{bmatrix} 0.38 \\ 0.38 \end{bmatrix} + b \right| = \left| [1 \quad 1] \begin{bmatrix} 0.18 \\ 0.62 \end{bmatrix} + b \right|$$

$$|0.76 + b| = |0.8 + b|$$

$$\mathbf{b} = -\mathbf{0.78}$$

Decision Boundary

$$w^T z + b = 0$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - 0.78 = b$$

$$(1 * z_1) + (1 * z_2) - 0.78 = 0$$

$$\mathbf{z_1 + z_2 = 0.78}$$

Verification

$$\underline{r = 0}$$

$$d = \frac{|g(0.38, 0.38)|}{||w||}$$

$$d = \frac{\left| \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0.38 \\ 0.38 \end{bmatrix} + b \right|}{\sqrt{1^2 + 1^2}}$$

$$d = \frac{|0.76 - 0.78|}{\sqrt{2}}$$

$$d = \frac{0.2}{\sqrt{2}}$$

$$\mathbf{d = 0.014}$$

$$\underline{r = 1}$$

$$d = \frac{|g(0.18, 0.62)|}{||w||}$$

$$d = \frac{\left| \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0.18 \\ 0.62 \end{bmatrix} + b \right|}{\sqrt{1^2 + 1^2}}$$

$$d = \frac{|0.8 - 0.78|}{\sqrt{2}}$$

$$d = \frac{0.2}{\sqrt{2}}$$

$$\mathbf{d = 0.014}$$

**They are indeed, equal**