Elijah Andrushenko

CPTS 453

Graph Theory

09-25-2019

Homework #3

1.

$2|E| = \sum_{v \in V} \deg(v) \leq K$

Then the maximum amount of leaves is K-1, So using contradiction we can show

$$\sum_{v \in V} \deg(v) = \sum_{v \in leaf} 1 + \sum_{v \in nonleaf} \deg(v) \leq (k-1) + K(n - K + 1)$$
$$= 2K - K^2 + Kn - 1$$

By definition the sum of all degrees in a tree is 2|E| = 2(n - 1) = 2n – 2 which leads to an inequality in regards to n.

2.A)

n = number of nonparents

Out of p parents, there exists one parent of degree $q, p - 1$ parents have a degree of $q + 1$.

Using the handshaking lemma, $q + (p-1)(q+1) + n$ = the number of edges

The number of edges = p + n– 1.

$$q(p-1)(q+1) + n = 2p + 2n - 1$$

$$q + pq - p - q - 1 + n = 2p + 2n - 1$$

$$\boldsymbol{n = (q-1)(p+1)}$$

B)

$$|V_{k+1}| \leq q|V_k| \leq q * q|v_{k-1}| \leq q * q * q|v_{k-2}| \leq \cdots \leq q^{k+1}|v_0|$$

$$|v_{k+1}| \leq q^{k+1}$$

$$|V_k| \leq q^k$$

C) For a binary tree with n vertices the minimum height will be $\log_2(n)+1$.

For a binary tree with 10^9 vertices the minimum height will be $\log_2(10^9)+1$.

For a 5-ary tree with 10^9 vertices the minimum height will be $\log_5(10^9)+1$.

$\log_5(10^9)+1$ = **13**

3.

The minimum possible height of 100 non parents in a full binary tree or 100! (100 factorial)?

If 100 just divide by 2 until you get to, or below 1.

(1) 100 / 2 = 50

(2) 50 / 2 = 25

(3) 25 / 2 = 12.5

(4) 12.5 / 2 = 6.25

(5) 6.25 / 2 = 3.125

(6) 3.125 / 2 = 1.5625

(7) 1.5625 / 2 = 0.78125

So the minimum possible height would be 7-1, we subtract one since the height of the root is 0.

The minimum height is 6 for a full-binary tree with 100 non-parent nodes.

If its 100 factorial then we have

100! = 9.332622e+157

(500) 9.332622e+157 / 2^500 = 28510564.9228

(520) 28510564.9228 / 2^20 = 27.1897935131

(523) 27.1897935131 / 2^3 = 3.39872418913

(524) 3.39872418913 / 2 = 1.69936209457

(525) 1.69936209457 / 2 = 0.84968104728

The minimum height is 524 for a full-binary tree with 100! non-parent nodes.

4.

[000]-[001]-[101]-[111]

[000]-[010]-[011]-[111]

[000]-[100]-[110]-[111]