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CPTS 453

Graph Theory

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Homework 4

1.

Forest of 7 trees G = $\{T_1, T_2, T_3, T_4, T_5, T_6, T_7\}$

Each T in the G has 5 vertices, $T_k = \{V_1, V_2, V_3, V_4, V_5\}$

There are no cycles in a forest therefore the $\mathbf{dim(C(G)) = 0}$

We know that $\mathrm{dim(B(G)) + dim(C(G)) = |E(G)|}$

We already know $\mathrm{dim(C(G)) = 0}$, so $\mathrm{dim(B(G)) + 0 = |E(G)|}$

Since each T in G is a tree then we know that $\mathrm{|E(T)| = |V(T)| - 1}$

$$|E(G)| = V(T_1) - 1 + V(T_2) - 1 + V(T_3) - 1 + V(T_4) - 1 + V(T_5) - 1 + V(T_6) - 1 + V(T_7) - 1$$

$|E(G)| = |V(G) - 7|$

We already know each tree has 5 vertices so, $V(G) = 5$

$|E(G)| = |5 - 7|$

$|E(G)| = 2$

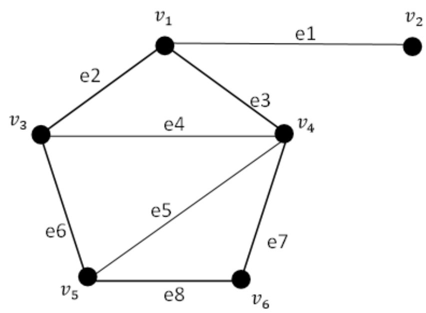
Recall, $\mathrm{dim(B(G)) + 0 = |E(G)|}$

Therefore

$\mathrm{dim(B(G)) + 0 = 2}$

$\mathbf{dim(B(G)) = 2}$

2.



A)

Basis of $B(G) = \{v_2\}$

So $\dim(B(G)) = 1$

B)

We know that $\dim(B(G)) + \dim(C(G)) = |E(G)|$

$1 + \dim(C(G)) = 6$

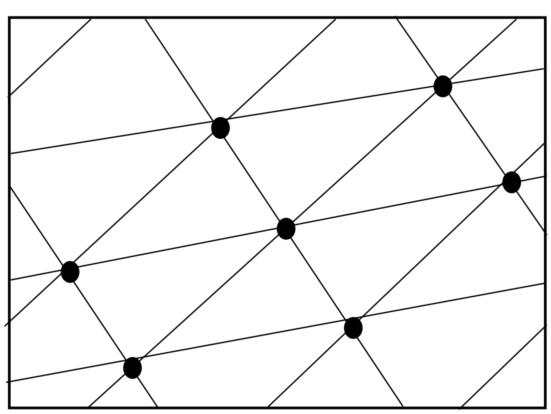
$\dim(C(G)) = 5$

Basis of $C(G) = \{v_1, v_3, v_4, v_5, v_6\}$

C)

No nonzero vector exists

3.



4.

K_8 Has 8 vertices and 28 edges.

We know the equation $n - m + f = 0$

The length of each face is $Length(f) \geq 3$

Every edge in K_8 belongs to 2 faces.

Since we know that there are 28 edges then we know $f = 28 * 2 = 56$

If we plug our numbers back into the equation $n - m + f = 0$ we get,

$8 - 28 + 56 = 0$

$-20 + 56 = 0$

$36 = 0$

Which is not true therefore K_8 cannot be drawn on a torus

5.

A)

$n - m + f = 2 - 2\chi$

1 face so, f = 1

6 edges so, m = 6

1 distinct vertex so, n = 1

We plug it in and we get

$1 - 6 + 1 = 2 - 2\chi$

$-4 = 2 - 2\chi$

$-6 = -2\chi$

$6 = 2\chi$

$\boldsymbol{\chi = 3}$

B)

No, Nothing changes there is still 1 distinct vertex so the equation wouldn't change.

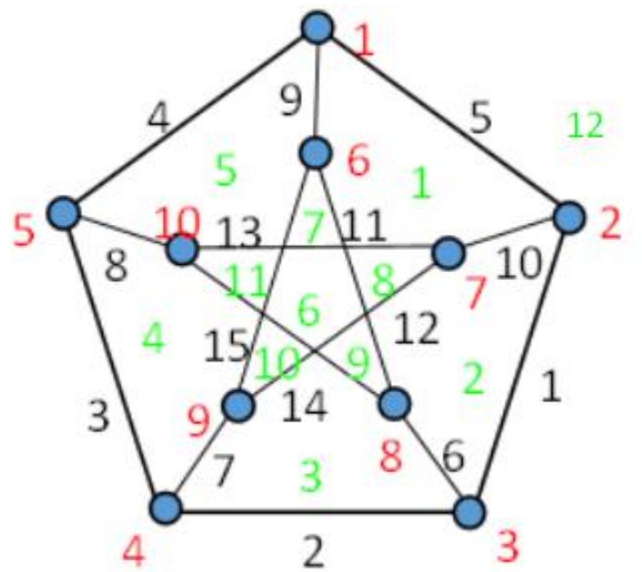
6.

Peterson Graph

Number of Edges = 15

Number of Vertices = 10

Number of Faces = 12



Euler's Formula for a planar graph is $n + f - e = 2$

$15 + 12 - 10 \neq 2$

Therefore the Peterson graph is not planar.