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CPTS 453

Graph Theory

08/28/2019

Homework 1

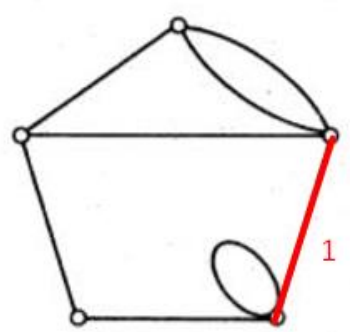
1.

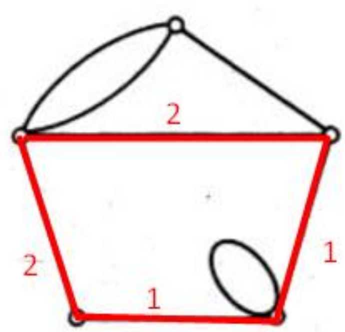
The equation to calculate the maximum number of edges in a simple graph is $\frac{n\,(n-1)}{2}$. So we have $n = 20$ which gives us $\frac{20\,(20-1)}{2} = \frac{380}{2} = 190$. **Therefore the maximum number of edges in a simple graph with 20 vertices is 190.**

Assuming the graph is connected then the equation to calculate the minimum number of edges in a simple graph is $n - 1$. Since we have 20 vertices n = 20, which gives us $20 - 1 = 19$. **Therefore the minimum number of edges in a simple graph with 20 vertices is 0 and if it is connected then the minimum number of edges is 19.**

2.

The two graphs are not isomorphic. Both graphs have two vertices with a degree of 4, but in graph G the shortest walk from one vertex of degree 4 to the other vertex of degree 4 has a length of 1. In graph H the shortest walk from one vertex of degree 4 to the other vertex of degree 4 has a length of 2. Therefore since the lengths of the two walks between the two degree 4 vertices are different in each graph, these two graphs cannot be isomorphic.





3.

A)

Incidence Matrix M

	14	15	16	24	25	26	34	35	36
1	1	1	1	0	0	0	0	0	0
2	0	0	0	1	1	1	0	0	0
3	0	0	0	0	0	0	1	1	1
4	1	0	0	1	0	0	1	0	0
5	0	1	0	0	1	0	0	1	0
6	0	0	1	0	0	1	0	0	1

B)

Adjacency Matrix A

	1	2	3	4	5	6
1	0	0	0	1	1	1
2	0	0	0	1	1	1
3	0	0	0	1	1	1
4	1	1	1	0	0	0
5	1	1	1	0	0	0
6	1	1	1	0	0	0

C)

Source for doing matrix multiplication

https://www.dcode.fr/matrix-power

Adjacency Matrix A^6

	1	2	3	4	5	6
1	243	**243**	243	0	0	0
2	**243**	243	243	0	0	0
3	243	243	243	0	0	0
4	0	0	0	243	243	243
5	0	0	0	243	243	243
6	0	0	0	243	243	243

The number of walks of length 6 between vertices 1 and 2 is 243.

D)

We calculate the Adjacency Matrix A^{10000}

The number of walks of length 10,000 between vertex 1 and vertex 4 is:

543783395114208624767752243060384905604044151194179331311606775397275243604773026249338521028261610322441734840774526514446852766639240128412715971512270416715106411838874146367514120961451889551988654833733805914942517666345775814181

5475250411845780622462535937873312408538957620672168268531058019969840153889500
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7395638109639118192284819091398520739337779248542834156891019808999704734180255
7725329399947495758727895099796309275098928813417666481595231394558441173901477
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E)

This is the Adjacency Matrix A^5

	1	2	3	4	5	6
1	**0**	0	0	81	81	81
2	0	**0**	0	81	81	81
3	0	0	**0**	81	81	81
4	81	81	81	**0**	0	0
5	81	81	81	0	**0**	0
6	81	81	81	0	0	**0**

This matrix shows all number of possible walks from one vertex to another. Since $A_{1,1}$ $A_{2,2}$ $A_{3,3}$ $A_{4,4}$ $A_{5,5}$ and $A_{6,6}$ all have 0 then **it must be the case that there does not exist a closed walk of length 5 for this graph.**

F)

They are not isomorphic, since $K_{3,3}$ is bipartite and the graph shown is not bipartite. One graph can’t be bipartite and the other not be bipartite and be isomorphic at the same time.