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CPTS 453

Graph Theory

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Homework 2

1. Assume there exists a cycle G where,

G = v1, e1, v2, e2, v3, e3, ..., v1.

Vs and Es being vertices and edges of the cycle G respectively. The number of vertices and edges is n. Then assume there is a non-trivial trail T where,

T = v1, e1, v2, e2, v3, e3, …, vn.

Now we assume T is also closed so,

T = v1, e1, v2, e2, v3, e3, … vn, en, v1.

We can observe that,

G = v1, e1, v2, e2, v3, e3, ..., v1.

T = v1, e1, v2, e2, v3, e3, … vn, en, v1.

G = T

Since it was assumed G has n vertices and edges just like graph T does as well. Since T has an identical graph which is a cycle we can say that T is a cycle itself since T is a cycle, then it is the case that T contains a cycle. Thus we conclude that every non-trivial closed trail contains a cycle.

One could also notice that the definition of non-trivial closed trail is a simple cycle, and a simple cycle is a cycle so by definition they are equal which means it contains itself. Thus by definition a non-trivial closed trail is a simple cycle.

2.

A) Suppose V0 is the set $\{n$-tuples with an even number of 1s$\}$ and $V1$ is the set $\{n$-tuples with an odd number of 1s$\}$. If there exists an edge between the two vertices x and y then they differ by one coordinate position. Therefore if x has an even number of 1s then y has an odd number of 1s. **Which means x and y can't be in V0 and V1. So V0 and V1 formed together must be bipartite.**

B) Suppose V0 is the set $\{n$-tuples with an even number of 1s$\}$ and $V1$ is the set $\{n$-tuples with an odd number of 1s$\}$. Both sets are nonempty when n>1. Since the two sets were proven bipartite in problem 2A there exists no edges between a vertex of V0 and a vertex of V1 because if an edge did exist there then either 2 coordinates are different which either means that the number of 1s in the vertices are off by 0 or 2. **Therefore V0 and V1 are not connected so Qn,2 is not connected.**

C) If n>=2. Then in Qn,2 there would be a cycle of 3 edges which is an odd cycle. By definition of bipartite graphs, bipartite graphs on have even cycles **therefore the graph is not bipartite.**

D) An edge exists between two vertices of Qn,n when the two vertices differ in the n coordinates. Therefore every component that's connected in Qn,n has 2 vertices. **So the total number of vertices of Qn,n is $2n - 1$.**

3. Bipartite means that the graph has two disjoint sets. So we can break up the graph into two sets, set A and set B. The graph has n vertices and each set has half the vertices so set A has n/2 vertices and set B also has n/2 vertices. To be maximally bipartite each vertex in set A is connected to each vertex in set B and vice versa. So we have n/2 vertices connected to n/2 vertices giving us,

$\frac{n}{2} * \frac{n}{2} = \frac{n^2}{4}$ **Therefore the maximum number of edges in a connected simple bipartite graph with n vertices is $\frac{n^2}{4}$.**

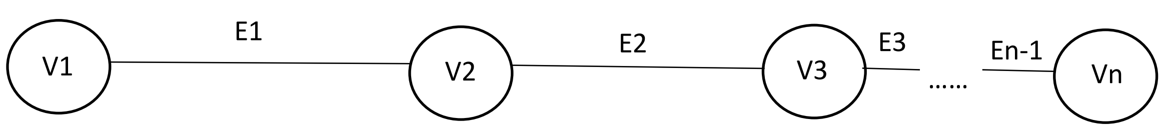
After drawing out several minimally connected bipartite graph the results were

Vertices	2	4	6	8	10	3
Edges	1	3	5	7	9	2

By looking at the table it is noticeable that the number of edges is always one less than the number of vertices giving us the function $n - 1$. **Therefore the minimum number of edges in a connected simple bipartite graph with n vertices is $n - 1$.**

4.

A)



This tree's longest path, L(T) = n – 1. If we place them in a line the number of edges will be one less than the number of vertices which guarantees us a path of length n – 1.

B)

A tree with n vertices having a maximum length of 2 can see be seen below, where besides the root (V1) all other vertices are leaves.