Data Structures and Algorithms 120

Lecture 7a: Linked Lists

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This Week

- Linked Lists
  - Simple linked lists
- Variants
  - Double-ended
  - Doubly-linked
  - Sorted linked lists
- Time complexity analysis
  - Big-O notation

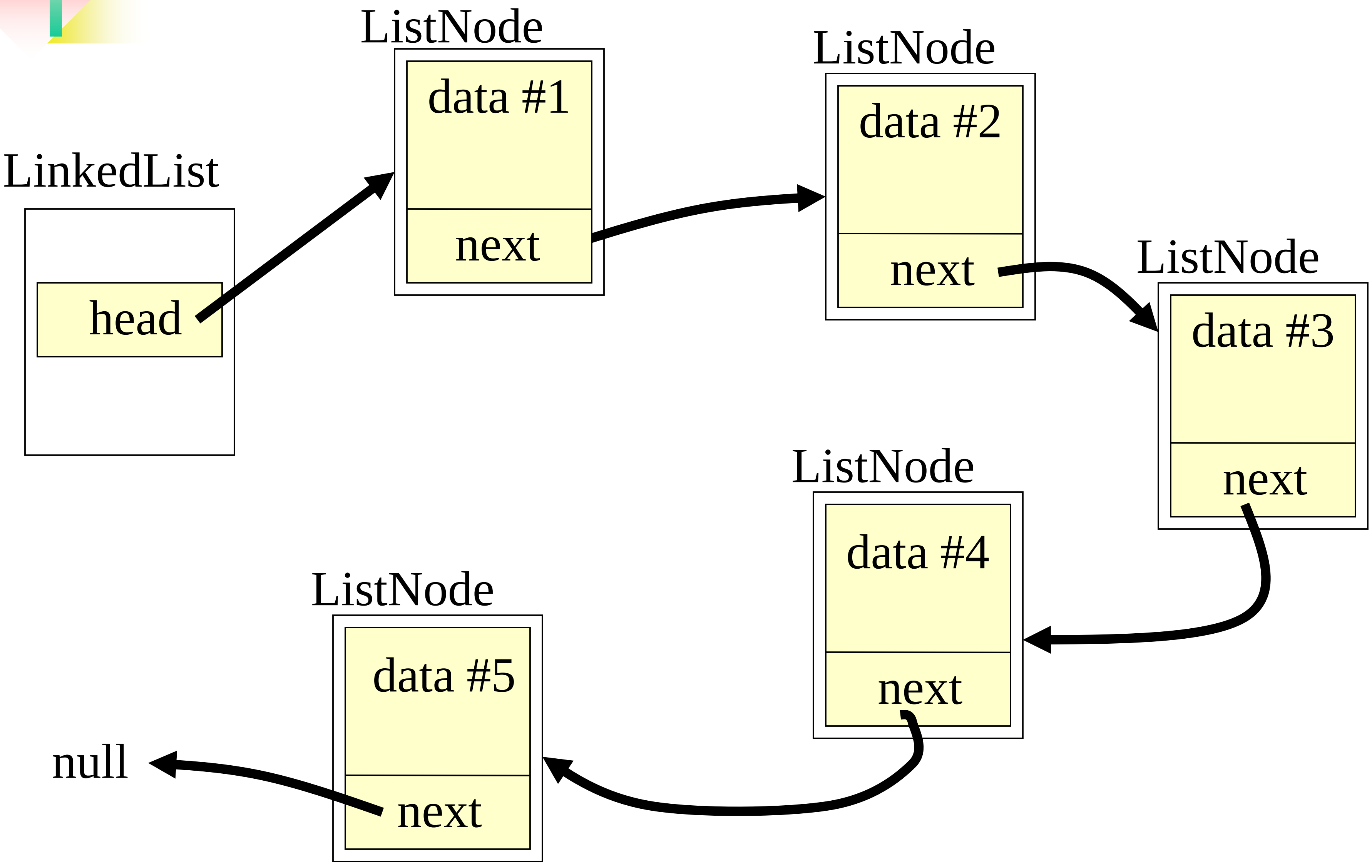
Arrays Aren't Everything

- As a data structure, arrays have their disadvantages
  - Fixed size – cannot grow/shrink to fit num elements
    - » If we don't know how many elements in advance, we'll need to overestimate the array's length, and thus waste space
  - Insertion – inserting an element between two existing elements will require shuffling later elements up by one
    - » Similar issues plague removal from the front/middle (*e.g.*, queues)
  - Contiguous – all elements must fit in a contiguous block
    - » This is sometimes a disadvantage – it's a bit inflexible
- We'd like a data structure that doesn't suffer from these
  - … but still does a similar job to arrays (storing list of data)
  - Fortunately, it's already been invented: the linked list

Linked Lists

- The idea behind a linked list is that each element stores a value *and* a reference to the *next* element
  - The list is essentially a chain of links
  - The list itself only stores a reference to the *first* element
    - » This is often called the 'head'
  - Since each element only points to the next, it is *singly-linked*
  - Since the list only points at the head, it is *single-ended*
- More complicated variants on the basic single-ended singly-linked list
  - Double-ended, doubly-linked, etc
  - We'll examine some of these variants later

Simple Linked List



Anatomy of a Linked List

- Each element is not just data but also a reference to the next element
  - Thus we need to create a composite of `value` + `next` ref
    - » Classes are ideally suited to this – make `value`, `next` class fields
      - – Typically called ListNode, or less commonly ListLink
    - » In pre-OO languages, use structs (C) or records (Pascal)
- Head is the only member field of LinkedList class
  - Points at the first ListNode in the list
- Last node's `next` points at null
  - Indicates the end of the linked list

ListNode Members

- ListNode only exists as a container for the data and the next pointer
- Thus it is really only a couple of member fields with associated getters/setters:
  - One member field for the data value, usually an Object so that the list can be general-purpose
  - A member `ListNode next` that is a reference to the next node in the list's chain
    - » This points at null if the ListNode is the last node in the chain

Traversing Linked Lists

– A linked list is just a chain of connected-but-independent nodes
  - Each node could be anywhere in memory
  - Only a node's predecessor knows where to find a given node (in memory)
  - So the only way to get to the fourth node is to first get to the third node
    - » which in turns requires us to have made it to the second node
    - » … and we can only get to that via the first node

Traversing Linked Lists

– Only the first node is available directly (via `head`)
  - Thus getting to a particular node requires that we traverse from the `head` (first) node, through all the `next` pointers until we reach the desired node
    - » This obviously can be pretty slow
  - Compare this to an array, which can get to *any* element in *one step*
    - » via a little bit of arithmetic – it works because the array elements are all stored in a contiguous block
    - » But linked lists aren't contiguous, so we have to do traversing
  - This is the main disadvantage of linked lists: access time

Linked List Methods

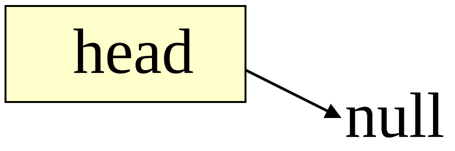
- isEmpty()
- insertFirst(), insertLast(), insertBefore(<dataType> valueToFind)
  - Insert a new item into the list
  - Should require a data item as import, NOT a ListNode
    - » ListNode is an internal detail of LinkedList's implementation
- removeFirst(), removeLast(), remove(<dataType> valueToFind)
  - Delete an item from the list
- peekFirst(), peekLast(), peek(<dataType> valueToFind)
  - Return the data value of an item in the list (not ListNode)
- find(<dataType> valueToFind)
  - Search whether a given data value exists in the list

Inserting a ListNode

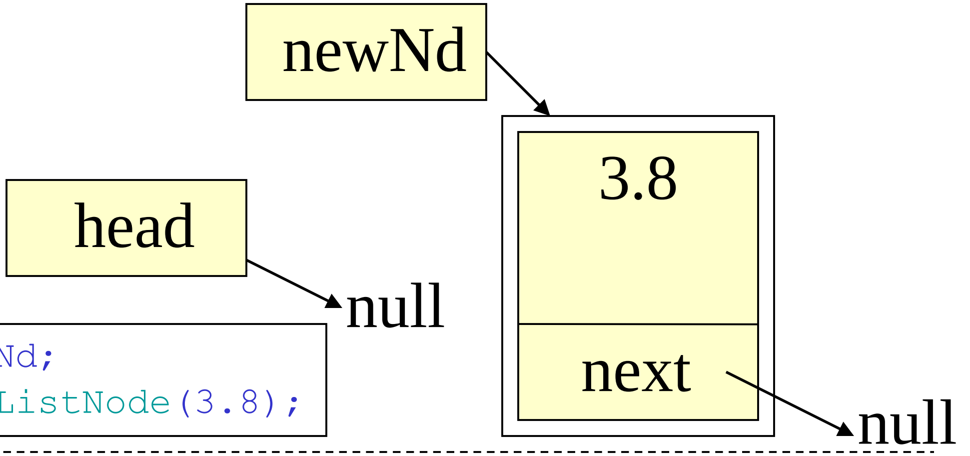
- There are four possible scenarios for node insertion:
  - The list is empty and we are inserting the first node
  - We are inserting before the head node
  - We are inserting after the tail of the list
  - We are inserting somewhere in the middle of the list
- In code, some of these cases turn out to be the same thing, but when designing a solution you should *always* be thinking through all the possibilities

Inserting First ListNode

– Initial: empty list
(ie: head points at null)

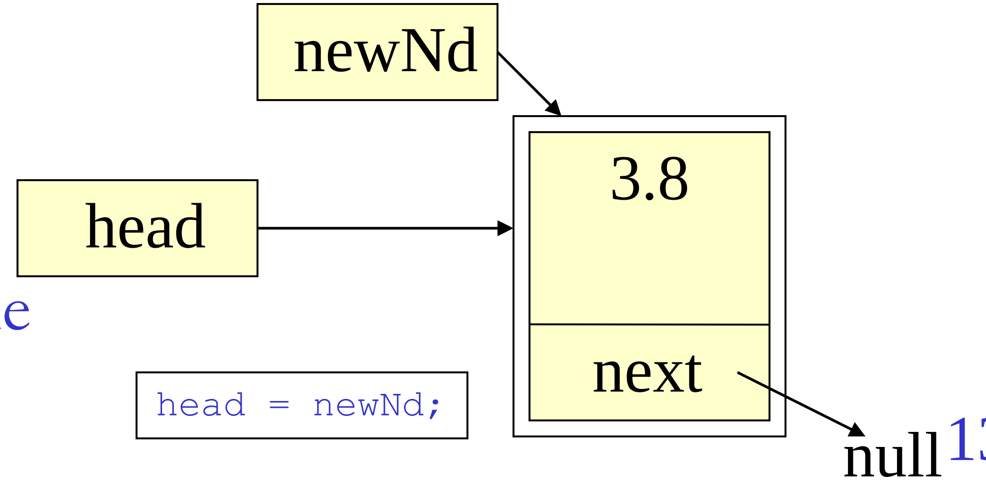


– Step 1: Create new ListNode
with (say) value of 3.8




```
ListNode newNd;
newNd = new ListNode(3.8);
```

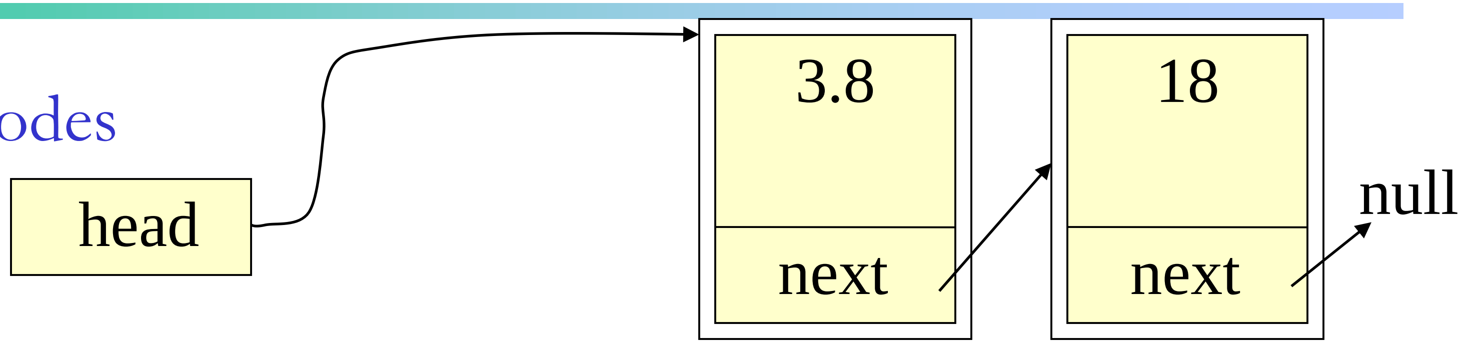
– Step 2: Point head at the
newNd to make it the first node




```
head = newNd;
```

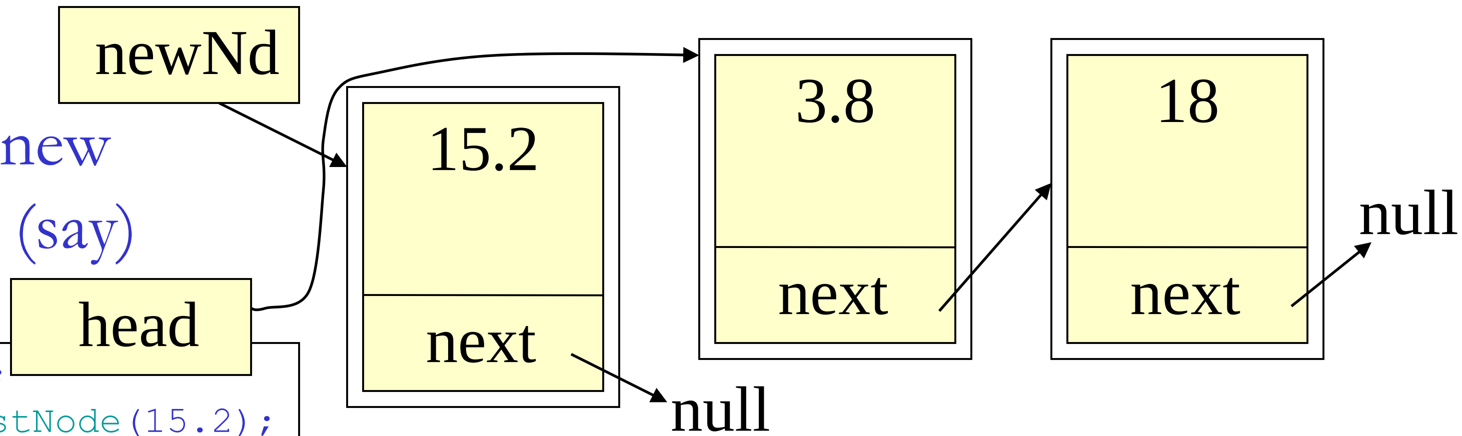
Inserting ListNode Before Head

– Initial: a few nodes in the list



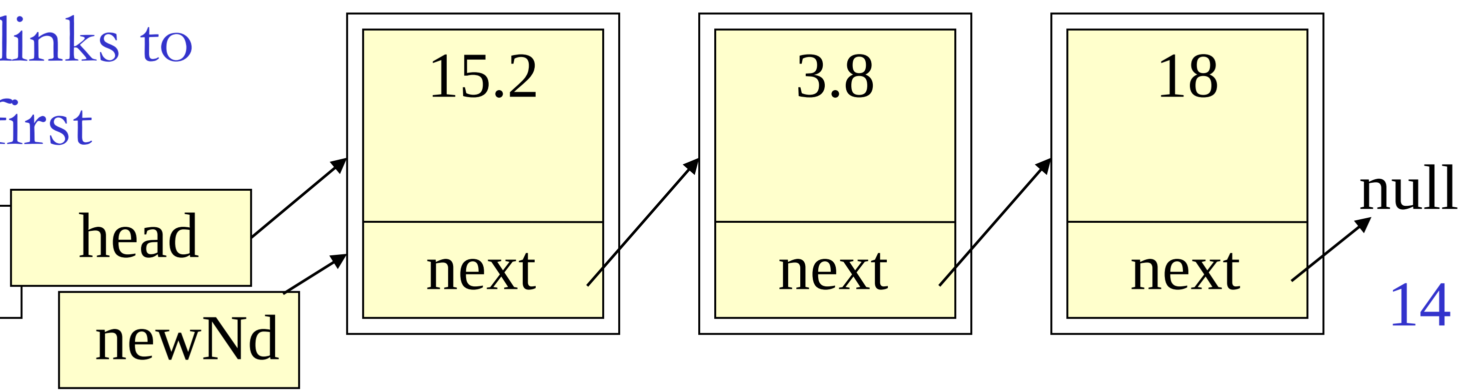
– Step 1: Create new ListNode with (say) value of 15.2

```
ListNode newNd;
newNd = new ListNode(15.2);
```



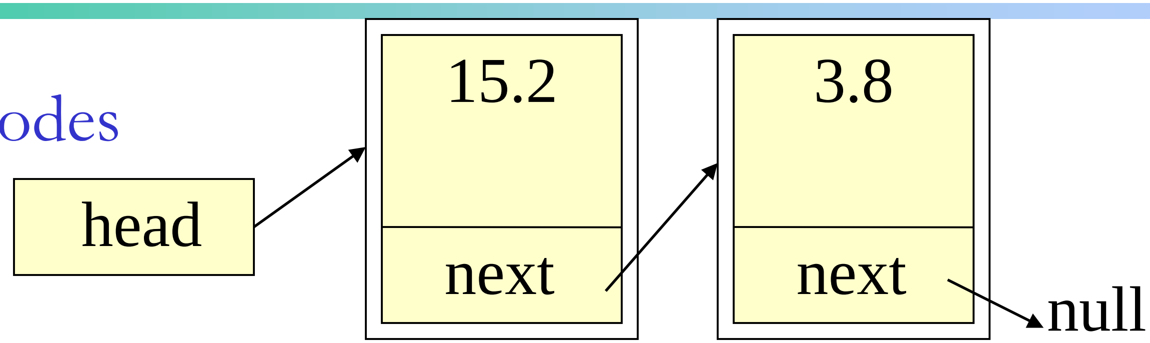
– Step 2&3: Fix links to make newNd first

```
newNd.next = head;
head = newNd;
```

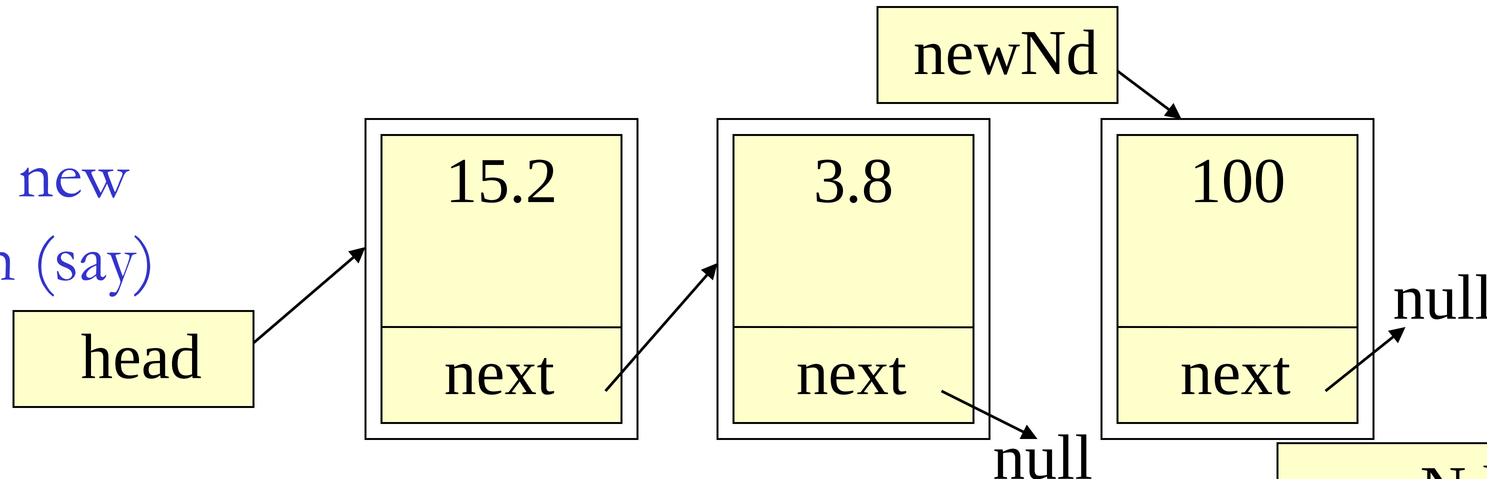


Inserting ListNode After Tail

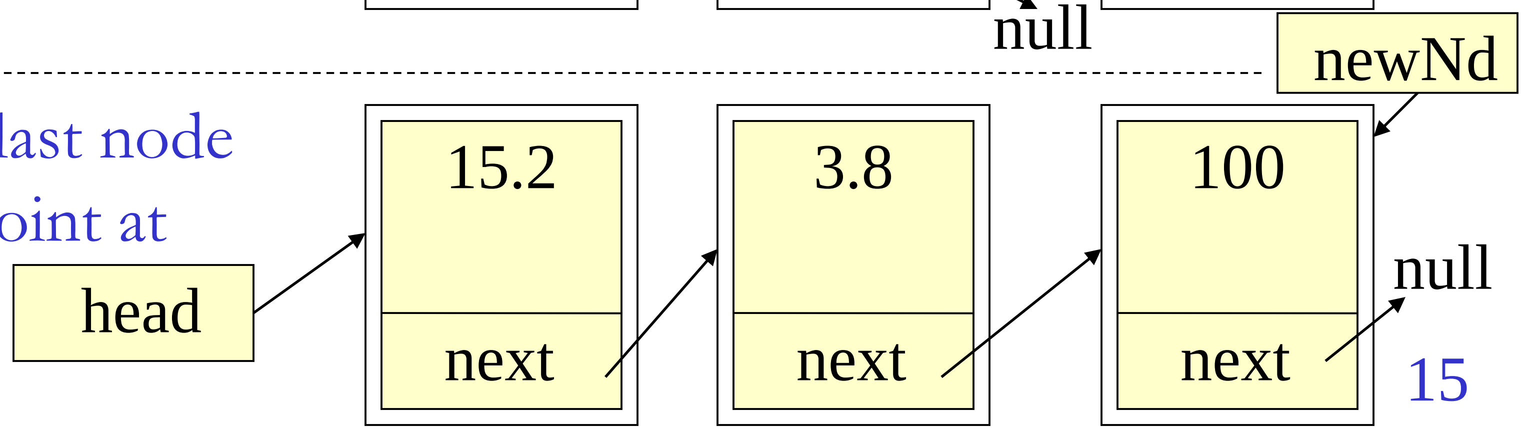
– Initial: a few nodes in the list



– Step 1: Create new ListNode with (say) value of 100

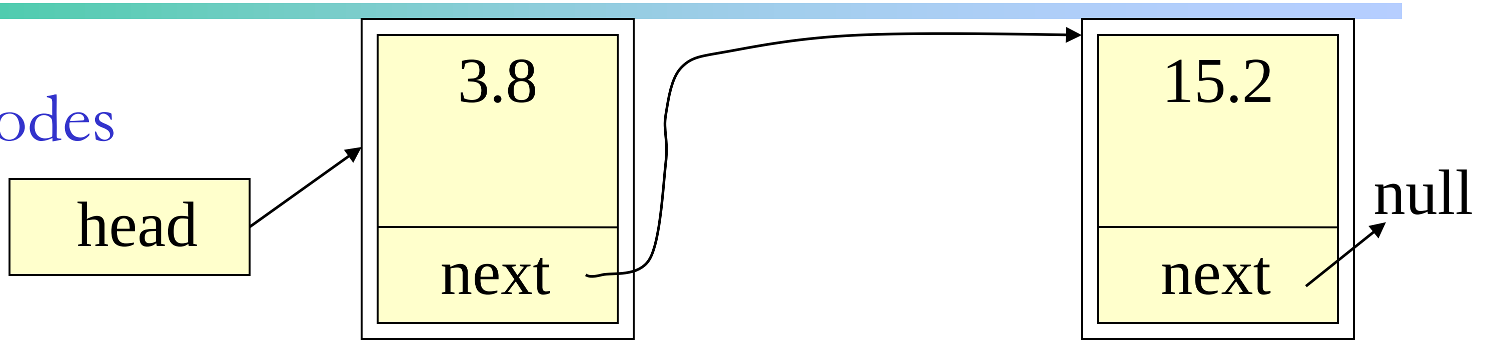


– Step 2: Make last node in the chain point at the newNd

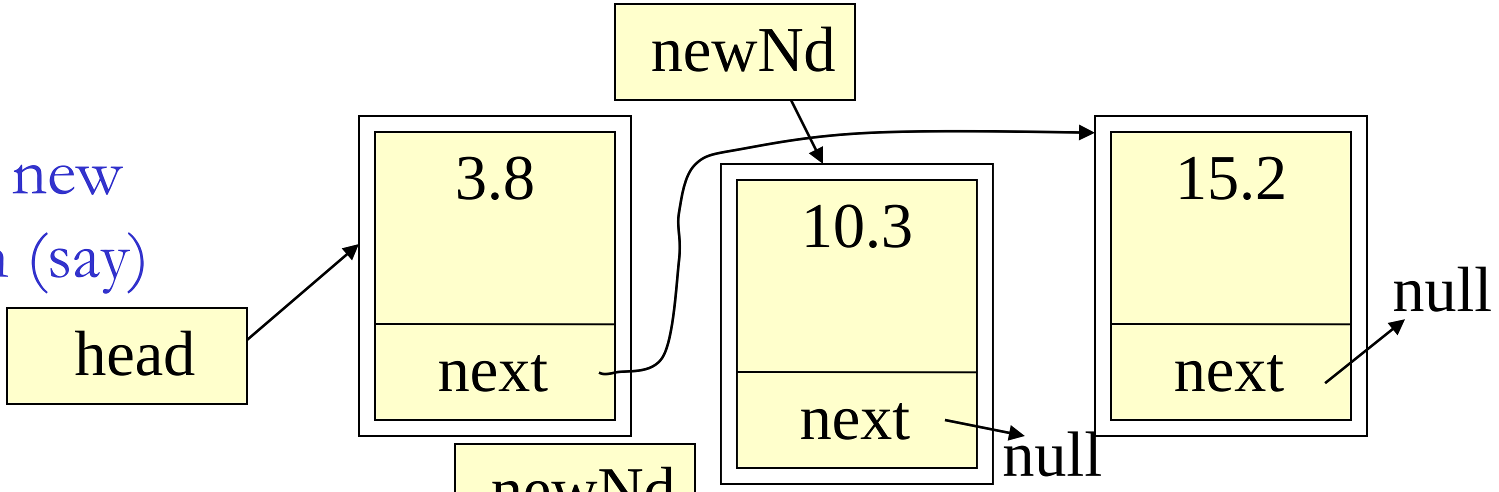


Inserting ListNode In The Middle

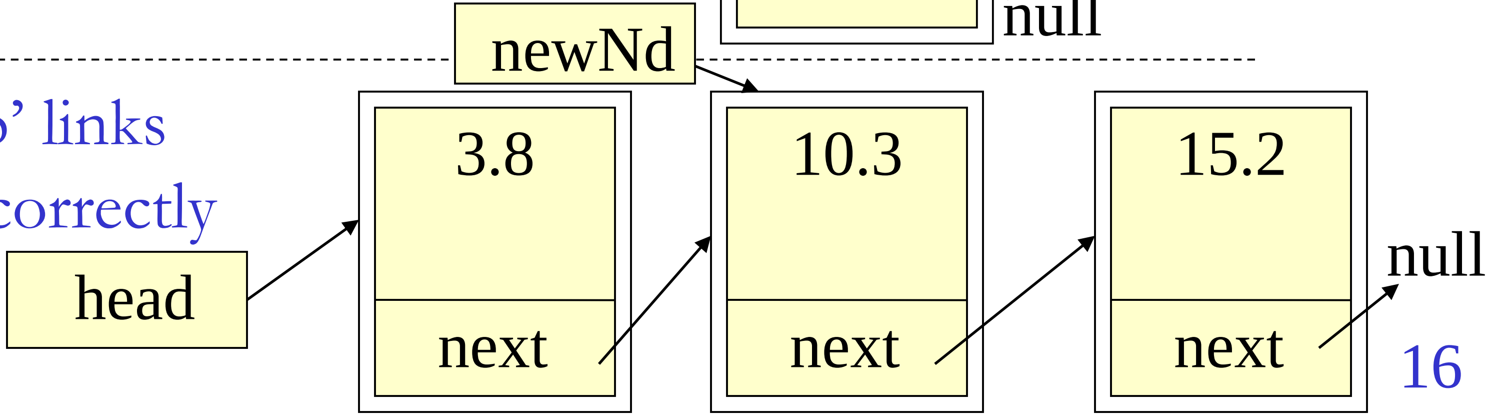
- Initial: a few nodes in the list



- Step 1: Create new ListNode with (say) value of 10.3



- Step 2: 'Fix up' links to make it fit correctly in the list

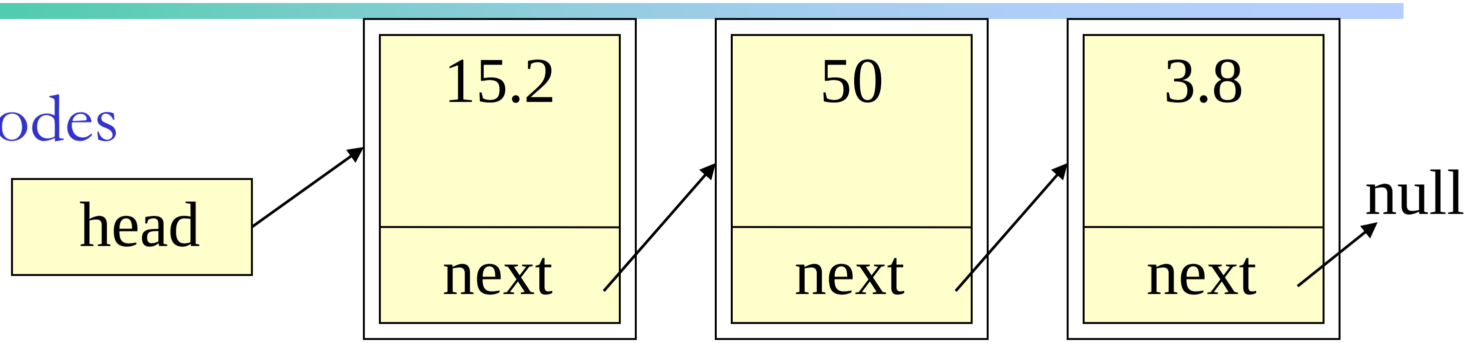


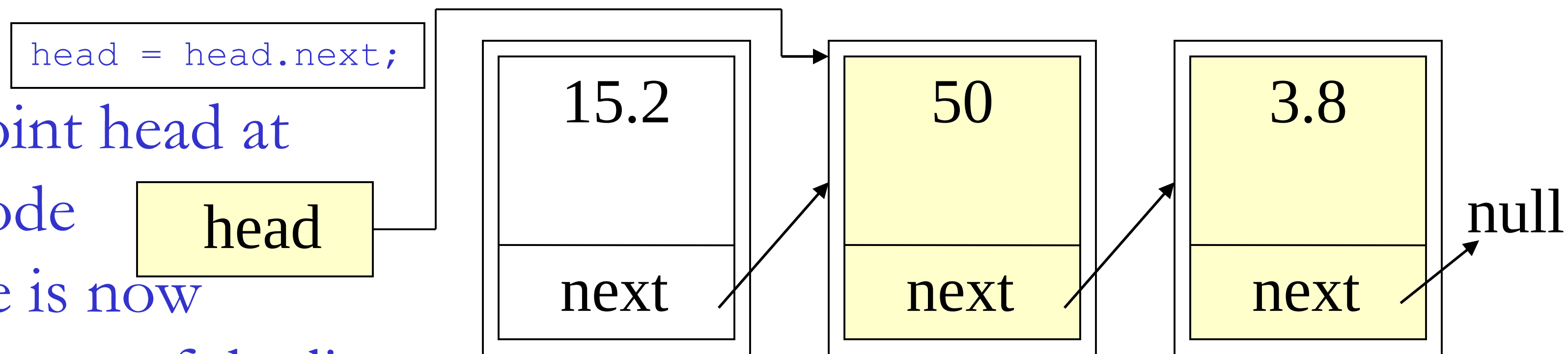
Removing a ListNode

- As with insertion, there are four cases:
  - Removing the head ListNode
  - Removing the tail ListNode
  - Removing a ListNode in the middle of the list
  - Removing the sole remaining ListNode in the list
- And as with insertion, some of these cases end up being the same in the code

Removing Head ListNode

– Initial: a few nodes in the list

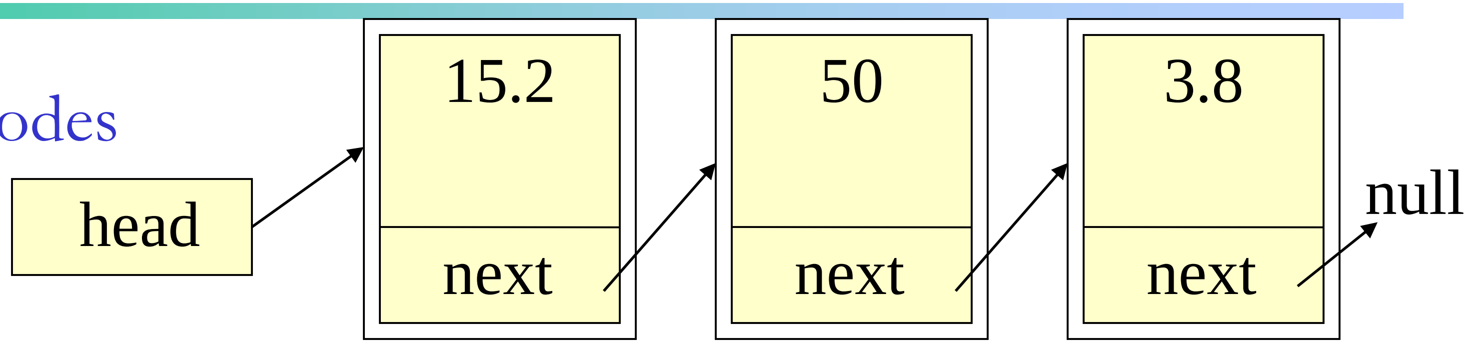




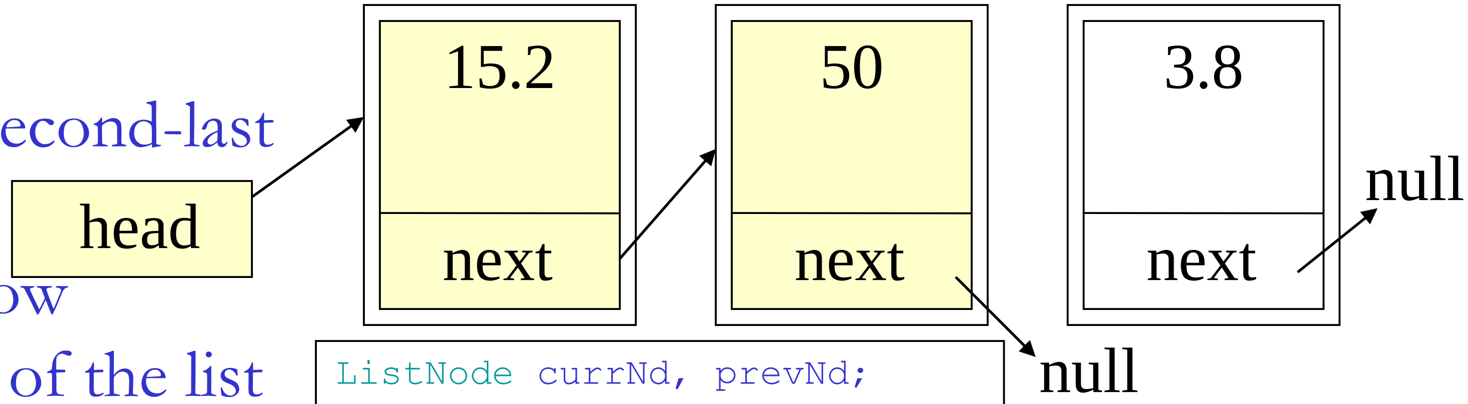
– Step 1: Point head at second node
First node is now no longer part of the list and its memory usage will be garbage-collected by Java (in C/C++, you would have to explicitly free it)
  - The fact that Node 15.2 still points at the second node is not relevant: nothing points *at* Node 15.2, therefore it will be garbage-collected

Removing Tail ListNode

– Initial: a few nodes in the list



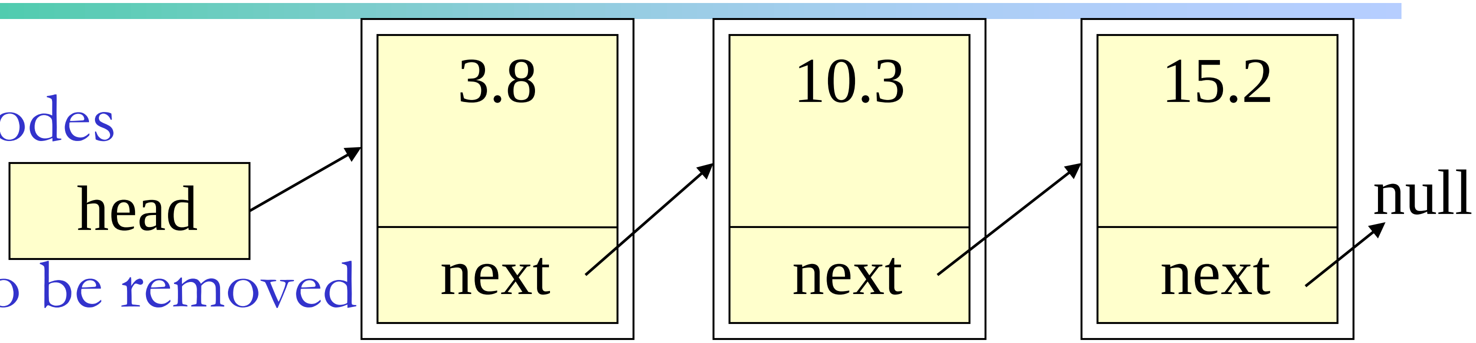
– Step 1: Point second-last node at null. Last node is now no longer part of the list



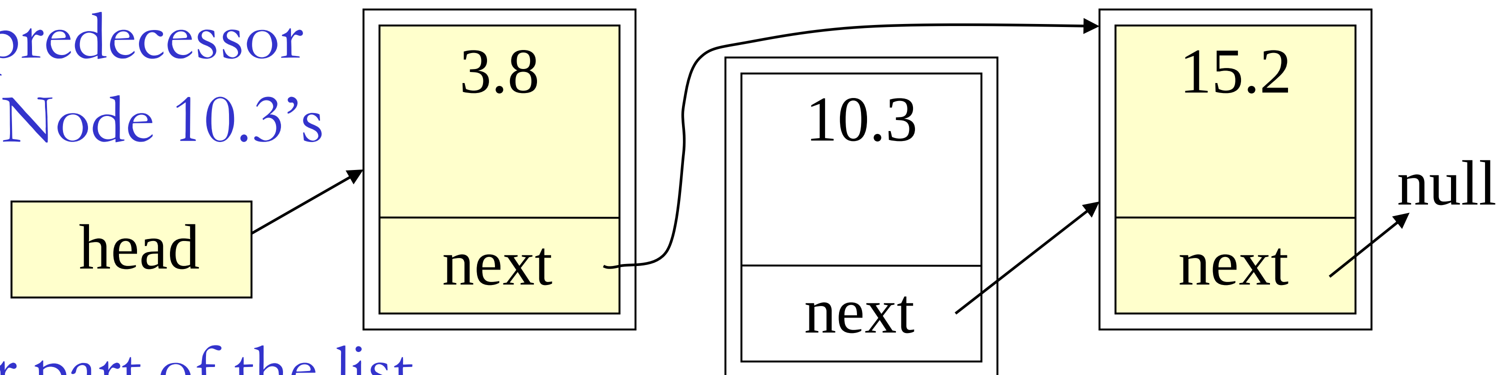

```
ListNode currNd, prevNd;
currNd = head;
prevNd = null;

// Traverse to last node
while (currNd.next != null) {
   prevNd = currNd;
   currNd = currNd.next;
}
prevNd.next = null;
```

Removing ListNode In The Middle

– Initial: a few nodes in the list; Node 10.3 is to be removed

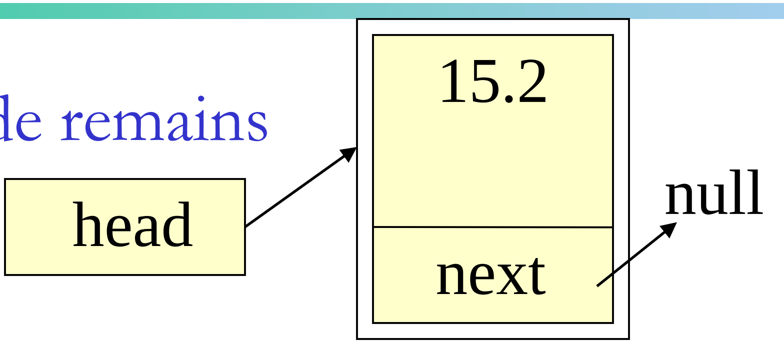


– Step 1: Have predecessor node point at Node 10.3's `next` node. Node 10.3 is now no longer part of the list



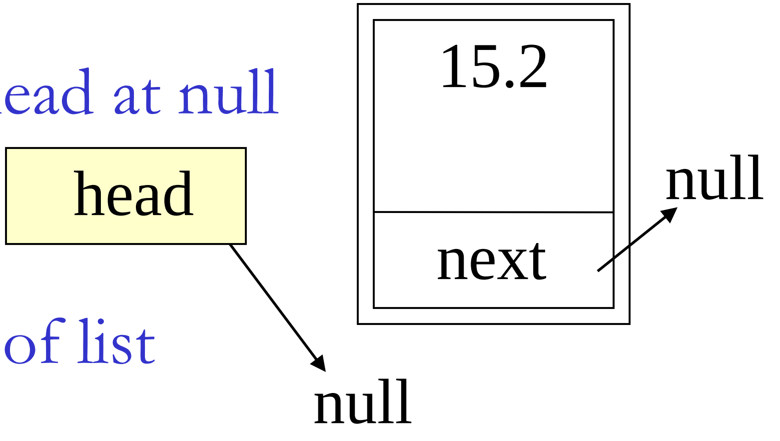
Removing Last Remaining ListNode

- Initial: one node remains in the list



- Step 1: Point head at null

Node is now no longer part of list




```
// Same as removing first node
head = head.next;
```

ListNode - Pseudo-code

```
Class DSAListNode
Class fields : value (Object), next (DSAListNode)    // Could make these public members
                                                     // and avoid the need for getters/setters since
                                                     // ListNode is only used INSIDE LinkedList

Alternate constructor IMPORT inValue (Object)        // Only need one constructor
  value ← inValue                                    // No need for validation
  next ← NULL

ACCESSOR getValue IMPORT none EXPORT value

MUTATOR setValue IMPORT inValue (Object) EXPORT none
  value ← inValue

ACCESSOR getNext IMPORT none EXPORT next

MUTATOR setNext IMPORT newNext (DSAListNode) EXPORT none
  next ← newNext
```

Simple LinkedList - Pseudo-code

```
Class DSALinkedList
Class fields : head (DSAListNode)

Default constructor
  head ← NULL

MUTATOR insertFirst IMPORT newValue (Object) EXPORT none
  newNd ← allocate DSAListNode(newValue)
  IF isEmpty() THEN
    head ← newNd
  ELSE
    newNd.setNext(head)
    head ← newNd
  ENDIF

MUTATOR insertLast IMPORT newValue (Object) EXPORT none
  newNd ← allocate DSAListNode(newValue)
  IF isEmpty() THEN
    head ← newNd
  ELSE
    currNd ← head
    WHILE currNd.getNext() <> NULL DO          // Traverse to last node
      currNd ← currNd.getNext()
    ENDWHILE
    currNd.setNext(newNd)
  ENDIF
```

<continued next slide>

Simple LinkedList - Pseudo-code

```
ACCESSOR isEmpty IMPORT none EXPORT empty (boolean)
 empty ← (head = NULL)


ACCESSOR peekFirst IMPORT none EXPORT nodeValue (Object)
  IF isEmpty() THEN
    abort
  ELSE
    nodeValue ← head.getValue()
  ENDIF

ACCESSOR peekLast IMPORT none EXPORT nodeValue (Object)
  IF isEmpty() THEN
    abort
  ELSE
    currNd ← head
    WHILE currNd.getNext() <> NULL DO                   // Traverse to last node
      currNd ← currNd.getNext()
    ENDWHILE
    nodeValue ← currNd.getValue()
  ENDIF
```

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Simple LinkedList - Pseudo-code

```
MUTATOR removeFirst IMPORT none EXPORT nodeValue (Object)
  IF isEmpty() THEN
    abort
  ELSE
    nodeValue ← head.getValue()
    head ← head.getNext()
  ENDIF

MUTATOR removeLast IMPORT none EXPORT nodeValue (Object)
  IF isEmpty() THEN
    abort
  ELSEIF head.getNext() = NULL THEN
    nodeValue ← head.getValue()
    head ← NULL
  ELSE
    prevNd ← NULL
    currNd ← head
    WHILE currNd.getNext() <> NULL DO          // Traverse to last node
      prevNd ← currNd                          // We need to get the second-last node
      currNd ← currNd.getNext()                // in order to 'drop' the last node from the list
    ENDWHILE
    prevNd.setNext(NULL)                       // Remove currNd from list
    nodeValue ← currNd.getValue()              // Return value of node that is being removed
  ENDIF
```

Private Inner classes - Java

```
public class DSALinkedList
{
   private class DSAListNode
   {    // Private inner class
      private Object m_value;
      private DSAListNode m_next;
      //could make the classfields public as they can
      //only be seen inside DSALinkedList
      public DSAListNode(Object inValue)
      {
         m_value = inValue;
         m_next = null;
      }
      // normal accessors and mutators if required
   }              // end private inner class DSAListNode
// class DSALinkedList continues
```

Simple LinkedList / ListNode

- NOTE WELL:
  - In the practicals, you are supposed to implement a *double-ended* linked list by adjusting this pseudocode
    - » (this pseudocode is just a single-ended linked list)
    - » We will be discussing double-ended lists in a moment

Linked Lists vs Arrays

- Advantages of linked lists over arrays
  - ☑ Can grow and shrink with data - no space wasted
  - ☑ No limits on size (besides memory available)
  - ☑ Easy to insert items at any part of the list
    - » Arrays are only easy to add items to the end
- Disadvantages
  - ☒ Time to access any given element: N/2 steps (on average)
    - » Where N is the number of nodes in the list
    - » Array is direct access: only takes 1 step
  - ☒ `next` pointer is an overhead in terms of memory usage
    - » Adds 1 word (memory address) extra per element in addition to actual data

Order of Complexity

– lets analyse LinkedList find() in Big-O notation

- Best case: the head is the node to find
  - » This is one step, so O(1)
- Worst case: last node is the match, which is N node hops
  - » O(N) in Big-O notation
  - » Each hop involves multiple CPU instructions, but we aren't concerned with these details, so we don't talk about O(5N)
- Average case: On average, we must go halfway: N/2 hops
  - » O(N) – again, the constant multiplying factor of ½ is irrelevant
- We are mostly interested in the average and worst cases

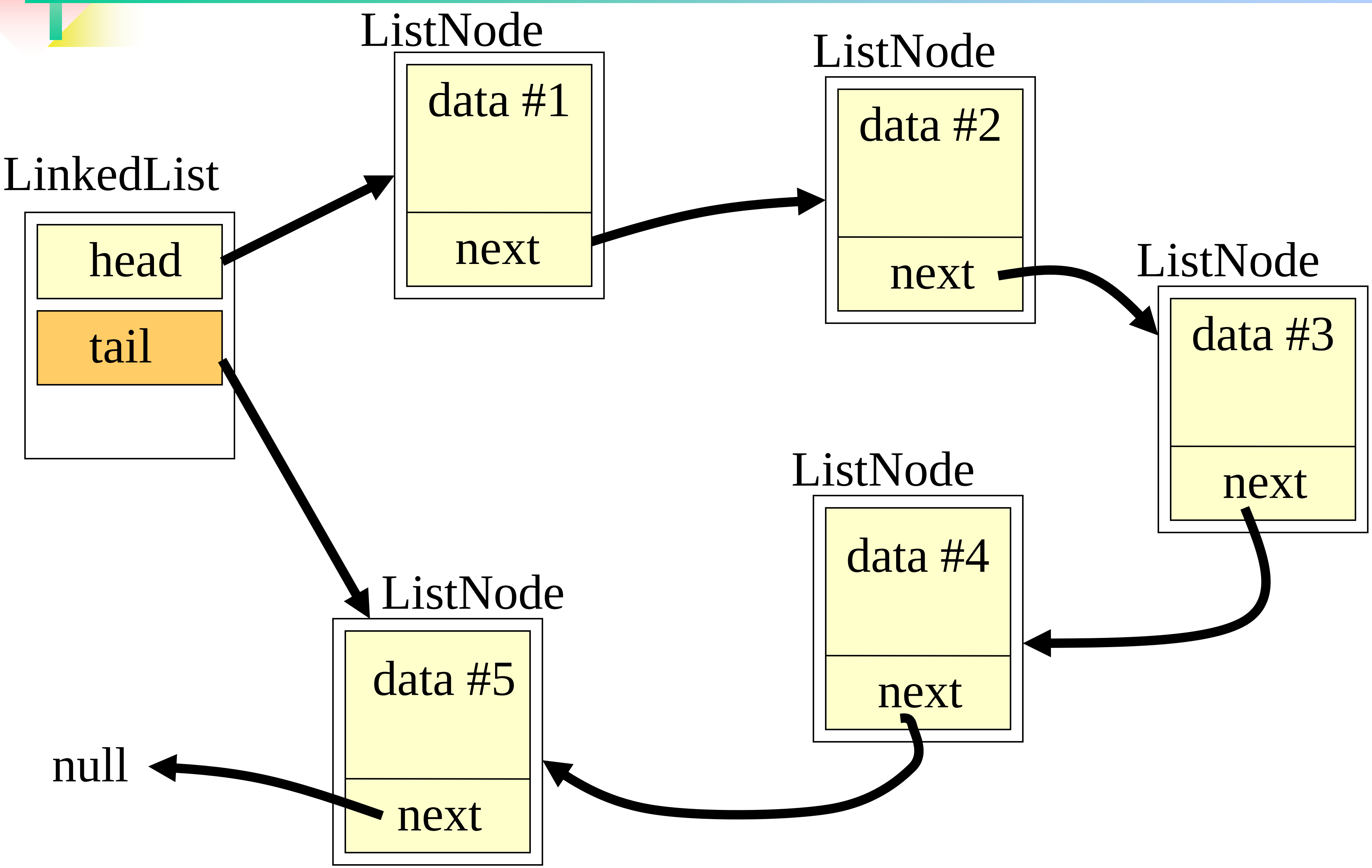
Double-Ended Linked Lists

- We often want to work at the start *and* end of a list
  - *e.g.*, queue adds to end, takes from front
- Simple list is slowest when doing anything at the end
  - Takes O(N) to reach the end – we'd like to do better!
- Solution: add another pointer to point at the last node
  - Call it `tail`
  - Put it with `head` as a member field of the LinkedList
  - That way we can access both head and tail in O(1)
- No disadvantage to this
  - Only adding one pointer to the whole linked list
  - Some code gets more complex, but other code gets easier

Double-Ended vs Single-Ended Lists

– Adding a tail pointer changes how many of the LinkedList methods work
  - insertLast() obviously becomes as easy/fast as insertFirst()
    - » Same with peekLast()
  - insertFirst() and removeFirst() now have to potentially set tail as well in the case of a one-node list
    - » *i.e.*, must maintain tail as well as head everywhere in the code
  - Counter-intuitively, removeLast() isn't helped much
    - » We need to get at the *second*-last node to do removeLast()!

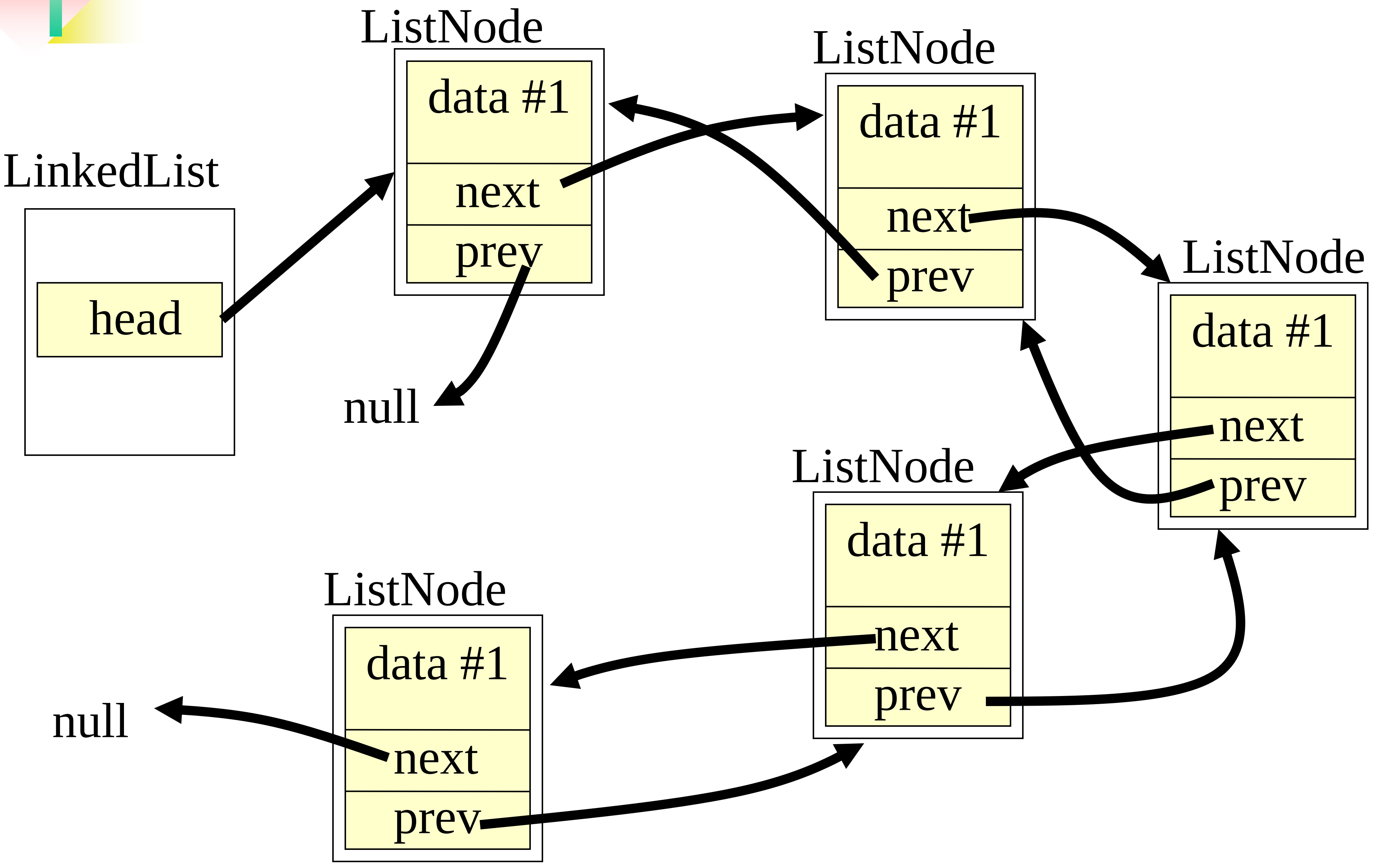
Double-Ended Linked List



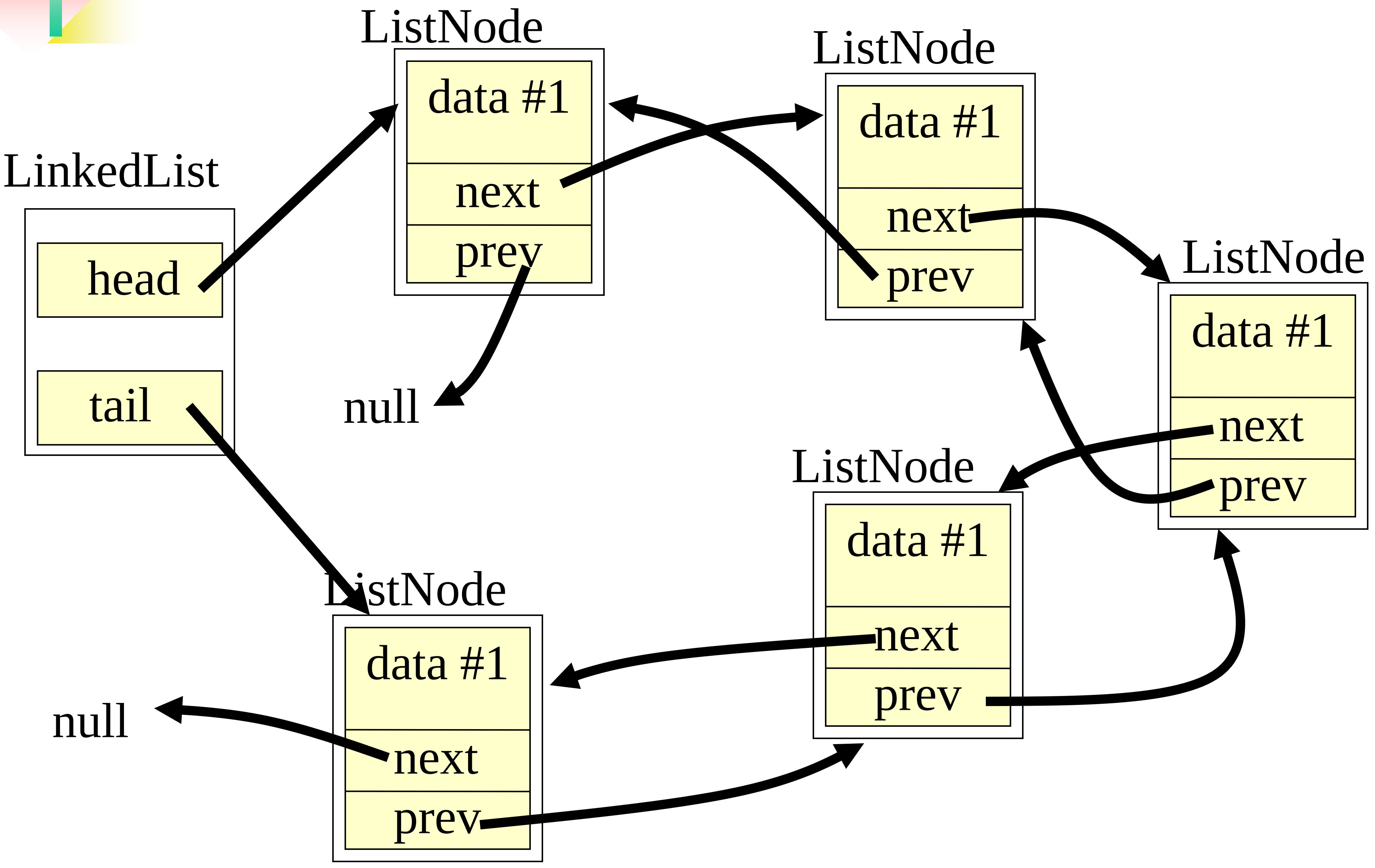
Doubly-Linked Lists

- Sometimes it's useful to be able to traverse both forwards *and backwards* through a list
  - *e.g.*, undo method in an application: in order to allow redo, must be able to go back and forth one node at a time
- Doubly-linked lists add another pointer to each ListNode: `prev`
  - Issue: Now overheads are *two* references per ListNode
    - » That's 2 words extra *per node* (8 bytes in a 32-bit machine; 16 bytes for 64-bit)
    - » Given that ints are only 4 bytes, it can easily *treble* space usage!
  - Benefit: removeLast() is easy if double-ended as well

Doubly-Linked List



Double-Ended, Doubly-Linked List



Sorted Linked Lists

- Since it's easy to insert new elements at any position in a linked list, it can also be convenient to insert them in sorted order
  - When adding a new element, find where it would fit (in sorted order) in the list and insert it there
- This is a variant on the Insertion Sort algorithm
  - A Doubly-Linked List makes this simple
    - » no moving elements – just search backwards then update links!

Saving Linked Lists to file

- We need to serialize our list.
  - we must put all the data together into a single block
- Only a few data structures are organised in blocks
  - Pretty much only arrays, and even then you need to add more info to define what the array's data type is
  - Each node of our linked list can be located in a completely different area of RAM
  - Remember, pointers cannot be made persistent
    - » You cannot save a pointer and expect it to be useful after re-loading it since the pointer says where the data used to be in the old run of the application!
    - » When the application runs again, it won't place objects in the same place – every time new is called, it returns a different address

Saving Linked Lists to file

- And it is not just our Linked List structure we have to worry about
  - our generic Object class for storing the data does not have file I/O methods!
- Create a new Abstract class that forces save/load methods
  - All of our data must then inherit from this class
- Or, simply use Java's object serialization!

Time Complexity Comparison

– Comparison of data structure speeds has many facets
  - Insertion, deletion, accessing and finding
    - » Access = getting the value at a known index
    - » Finding = searching through the data set to find the value
  - Best case, worst case, average case
    - » Usually corresponding to front, rear and middle
      - – … but not necessarily in that order!

Complexity: Array vs Simple LinkedList

Array vs List		Insert	Delete	Access	Find
Array	At front	O(N)	O(N)	**O(1)**	O(1)
	At end	O(1)	O(1)	**O(1)**	O(N)
	In middle	O(N)	O(N)	**O(1)**	O(N)
Linked List (single-ended)	At front	O(1)	O(1)	O(1)	O(1)
	At end	O(N)	O(N)	O(N)	O(N)
	In middle	O(N)	O(N)	O(N)	O(N)

NOTE: Although linked lists are fast to insert/delete, it still takes O(N) to traverse the list and make it to the place to insert/delete

Complexity: List Variants vs Simple List

List variants vs single-ended		Insert	Delete	Access	Find
Linked List (double-ended)	At front	O(1)	O(1)	O(1)	O(1)
	At end	O(1)	O(N)	O(1)	O(N)
	In middle	O(N)	O(N)	O(N)	O(N)
Linked List (doubly-linked) Same as singly-linked single-ended!	At front	O(1)	O(1)	O(1)	O(1)
	At end	O(N)	O(N)	O(N)	O(N)
	In middle	O(N)	O(N)	O(N)	O(N)

Complexity: Sorted List vs Simple List

Sorted list vs single-ended		Insert	Delete	Access	Find
Linked List (sorted)	At front	O(1)	O(1)	O(1)	O(1)
Same as singly-linked single-ended!	At end	O(N)	O(N)	O(N)	O(N)
	In middle	O(N)	O(N)	O(N)	O(N)

– Sorting will help only a little bit for find():
  - If the value *isn't* in the list ,we only need to search up to the point the number *should* be at, and then we can abort
    » Other lists are forced to search through all elements

– But it doesn't resolve poor scaling with larger N

Notes on Comparisons

- Arrays are unbeatable at O(1) for access time
  - » This makes them indispensable. Why? Because you often only add/remove an element *once*, but access an element *many times*
  - » But if you only need to work on the ends, linked lists are better
- Single-ended lists are only good at working with one end (the head), double-ended lists are good at working with both ends (head and tail)
  - » Otherwise, you might as well be using an array, UNLESS you need the ability to dynamically grow and shrink efficiently
- Doubly-*linked* lists don't help access/find time
  - » How can you know it will be faster to go *backwards*?
  - » Their real strength is application-specific back-and-forthing

Traversal

- Iterating over elements in a data set is a common task
  - *e.g.*, to calculate the average of an array of ages:

```
public double calcAverageAge(double[] ages) {
   double sum = 0.0;
   for (int ii = 0; ii < ages.length; ii++) {
      sum += ages[ii];
   }
   return sum / (double)ages.length;
}
```

» An array's O(1) time to access *any* element makes this efficient

- We could do the same with a LinkedList, but only if it provided some way to get at each element
  - Solution 1: Provide a peek(int index) method
  - Solution 2: Provide an *iterator* object

Inefficient LinkList Traversal - Indexing

– The problem with Solution 1 is that it is inefficient
  - peek(int index) must start at the beginning of the list every time it is called
    - » *i.e.*, it must traverse through the list from the beginning until it reaches the desired index
  - Thus a for loop to visit N elements will cause peek() to make N *traversals* of the list:

```
for (int ii = 0; ii < numAges; ii++)
    sum += ages.peek(ii);
```

    - » Steps: $1 + 2 + 3 + 4 + \ldots + N = N(N+1)/2$ steps $\approx N^2$
  - Complexity: $O(N^2)$ (!!)
    - » Truly awful scalability for a simple 'access each element' task

Efficient LinkList Traversal - Iterators

- Solution 2 seeks to solve this inefficiency problem
  - Keep a reference to the *last visited* ('current') node
    - » Sometimes called a cursor, since it indicates where you are
  - Every time the next node is requested, it only needs to make one hop forward
- Naïve approach: make `curr` a member of LinkedList
  - Issue: What if *two* loops access the list at the same time?
    - » Commonly happens with nested for loops
  - But there's only *one* `curr` node in the LinkedList
    - » Can't share it among many for loops
  - You can examine a proper solution to this in the generics slide set

Not the End

– There's still a second half to go.