Tests for the relativistic Boris pusher with RR

K. Krylov[1], S. Rykovanov[2]

[1] *National Research Nuclear University "MEPhI"*

[2] *Helmholtz Institute Jena*

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Equation

Assumptions:

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Equation to solve:[1][2]

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}_L - K\mathbf{v},$$

$$\mathbf{F}_L = -(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad K \equiv \varepsilon_{\mathrm{rad}} \gamma^2 [\mathbf{F}_L^2 - (\mathbf{v} \cdot \mathbf{F}_L)^2],$$

$$\varepsilon_{\mathrm{rad}} \equiv \frac{4\pi}{3} \frac{r_{\mathrm{e}}}{\lambda} \approx 1.18 \cdot 10^{-8} \text{ for } \lambda = 1\mu\text{m}$$

[1] Tamburini et al. 2010.

[2] units: $m_e = e = c = 1$, time measured in $1/\omega$

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- Step with RR:[4] $\mathbf{p}^{(n+1/2)} = \mathbf{p}_{\mathrm{L}}^{(n+1/2)} + \mathbf{F}_{\mathrm{R}}^{(n)} \Delta t$

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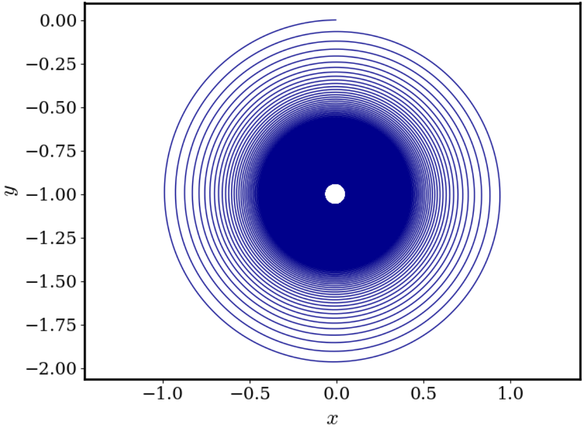
- Step with RR:[4] $\mathbf{p}^{(n+1/2)} = \mathbf{p}_{\mathrm{L}}^{(n+1/2)} + \mathbf{F}_{\mathrm{R}}^{(n)}\Delta t$
- Estimate $\mathbf{p}^{(n)} \approx \dfrac{\mathbf{p}_{\mathrm{L}}^{(n+1/2)} + \mathbf{p}^{(n-1/2)}}{2}$,

  $$\mathbf{v}^{(n)} \approx \frac{\mathbf{p}^{(n)}}{\gamma^{(n)}}, \quad \gamma^{(n)} = \sqrt{1 + (\mathbf{p}^{(n)})^2}$$

[3] Birdsall and Langdon 2004.

[4] Tamburini et al. 2010.

Tests: 1. Constant magnetic field



$dt = 0.005$

time $= 200 \cdot 2\pi$

$p_{0x} = -100$

$B_z = 100$

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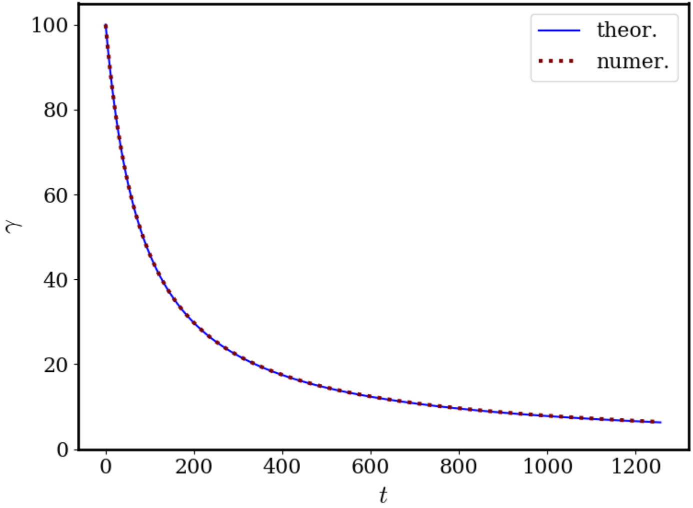

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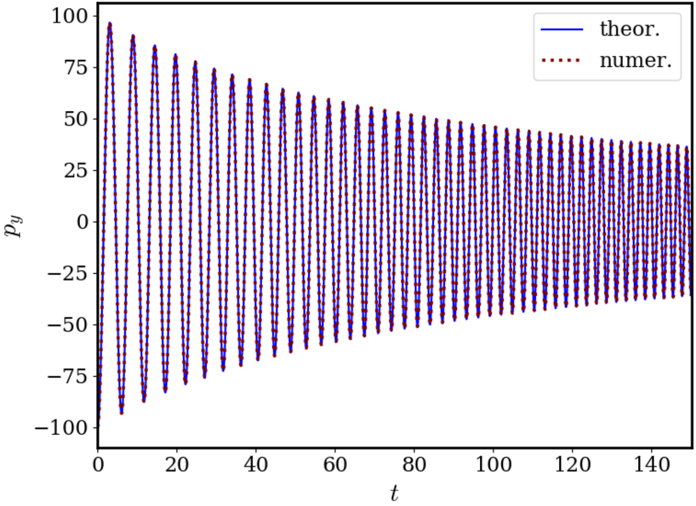
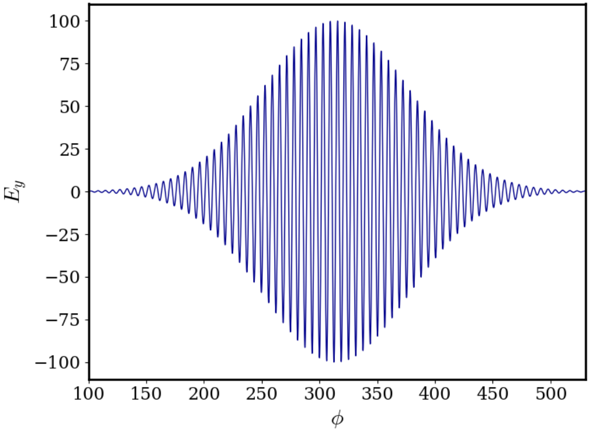
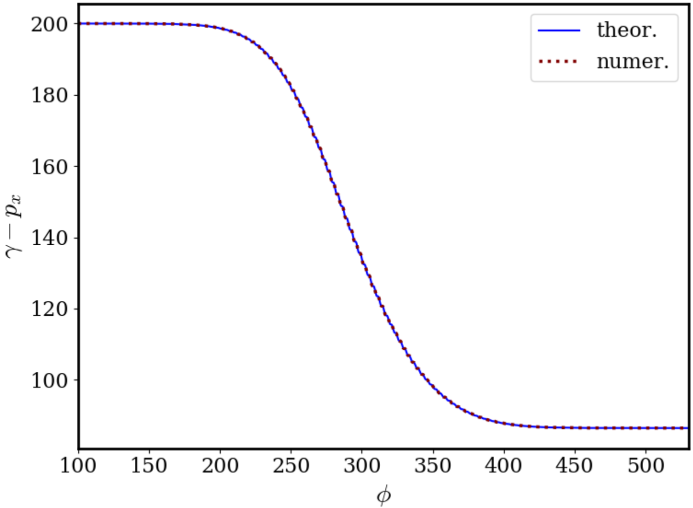


Tests: 2. Linearly polarized gaussian plane wave



$$dt = 0.005$$
$$p_{0x} = -100$$
$$\sigma = 10 \cdot 2\pi$$
$$a_0 = 100$$

Tests: 2. Linearly polarized gaussian plane wave



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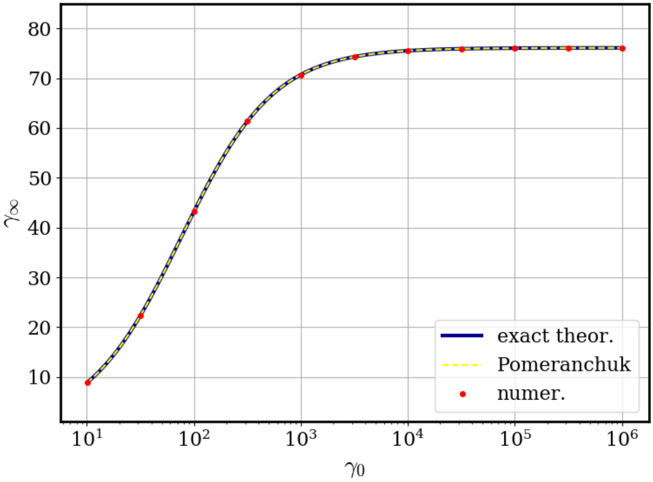
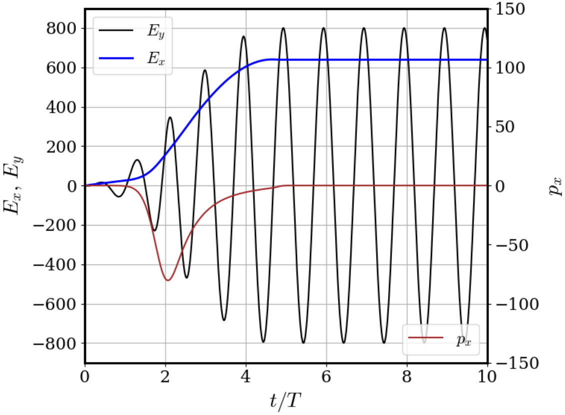
Pomeranchuk solved it in 1939 for arbitrary fields and $\gamma \gg 1$ [5]



[5] Landau and Lifshitz 1994, § 76

Tests: 3. Zel'dovich problem[6]

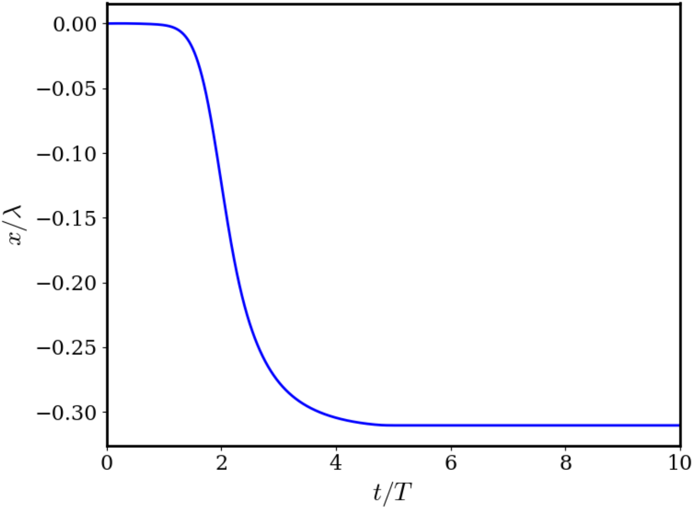


$$a_0 v_\perp \sin\varphi = W$$
$$p_\perp = a_0 \cos\varphi$$
$$a_0 v_\perp \sin\varphi = E_x$$

$$dt = 0.002$$
$$a_0 = 800$$

[6] Zel'dovich 1975.

Tests: 3. Zel'dovich problem



Thank you for your attention!

References

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