Tarea corta #1
Ericka Melissa Araya Hidalgo
C20553

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① $T(n) = 8T(\frac{n}{2}) + \theta(1)$ $\quad T(1) = 1 \quad a^{\log_b c} = c^{\log_b a}$

Suponga que $n = 2^k$, remplazando tenemos:

$T(2^k) = 8T(2^{k-1}) + \theta(1)$ ya que $\frac{2^k}{2} = 2^k 2^{-1} = 2^{k-1}$

- $T(2^k) = 8[8T(2^{k-2}) + \theta(1)] + \theta(1)$

$T(2^k) = 8^2 T(2^{k-2}) + 8\theta(1) + \theta(1)$

- $T(2^k) = 8^2[8T(2^{k-3}) + \theta(1)] + 8\theta(1) + \theta(1)$

$T(2^k) = 8^3 T(2^{k-3}) + 8^2\theta(1) + 8\theta(1) + \theta(1)$

$T(2^k) = 8^i T(2^{k-i}) + \theta(1)[8^0 + 8^1 + \cdots + 8^{i-2} + 8^{i-1}]$

La recursión termina cuando $i = k$

$T(2^k) = 8^k T(2^{k-k}) + \theta(1)[\underbrace{8^0 + 8^1 + \cdots + 8^{i-2} + 8^{i-1}}]$

serie geométrica, cuya suma es $\frac{r^{n+1} - 1}{r - 1}$

Entonces:

$$T(2^k) = 8^k \underbrace{T(1)}_{1} + \theta(1)\left(\frac{8^k - 1}{8 - 1}\right)$$

$$T(2^k) = 8^k + \frac{\theta(1) \cdot 8^k - \theta 1}{7}$$

$$T(2^k) = 8^k + \frac{\theta(1)[8^k - 1]}{7}$$ Como $n = 2^k$, $k = \log n$

$$T(n) = 8^{\log n} + \frac{\theta(1)[8^{\log n} - 1]}{7}$$ $a^{\log_b c} = c^{\log_b a}$

$$T(n) = n^{\log 8} + \frac{\theta(1)[n^{\log 8} - 1]}{7} = n^3 + \frac{\theta(1)[n^3 - 1]}{1}$$

$$T(n) = \theta(n^3)$$

② $T(n) = K + n^2 + 5T(\frac{n}{3}) \quad T(1) = K$

Suponga que $n = 3^K$

$T(3^K) = K + n^2 + 5T(3^{K-1})$

• $T(3^K) = K + n^2 + 5[K + n^2 + 5T(3^{K-2})]$

$T(3^K) = K + n^2 + 5K + 5n^2 + 5^2 T(3^{K-2})$

• $T(3^K) = K + n^2 + 5K + 5n^2 + 5^2[K + n^2 + 5T(3^{K-3})]$

$T(3^K) = K + n^2 + 5K + 5n^2 + 5^2K + 5^2n^2 + 5^3 T(3^{K-3})$

$T(3^K) = 5^0K + 5^0n^2 + \dots + 5^{i-2}K + 5^{i-2}n^2 + 5^{i-1}K + 5^{i-1}n^2 + 5^i T(3^{K-i})$

La recursión termina cuando $i = K$

$$T(3^K) = 5^0K + 5^0n^2 + \dots + 5^{K-2}K + 5^{K-2}n^2 + 5^{K-1}K + 5^{K-1}n^2 + 5^K \underbrace{T(3^{K-K})}_{\underbrace{T(1)}_{K}}$$

$T(3^K) = (K + n^2)[5^0 + \dots + 5^{K-2} + 5^{K-1}] + 5^K K$, con $\frac{r^{n+1} - 1}{r - 1}$

$T(3^K) = (K + n^2)\left(\frac{5^K - 1}{5 - 1}\right) + 5^K K$ Como $n = 3^K$, $K = \log_3 n$ y $a^{\log_b c} = c^{\log_b a}$

$T(n) = (\log_3 n + n^2)\left(\frac{n^{\log_3 5} - 1}{4}\right) + (n^{\log_3 5} \cdot \log_3 n)$

$T(n) = O(n^{2 \cdot \log_3 5} \log_3 n)$.

③ $T(n) = 2T(\frac{n}{2} + 17) + n \quad T(18) = 1$

Suponga que $n = 2^k$

$T(2^k) = 2T(2^{k-1} + 17) + n$

- $T(2^k) = 2[2T(2^{k-2} + 17) + n] + n$

$T(2^k) = 2^2 T(2^{k-2} + 17) + 2n + n$

- $T(2^k) = 2^2 [2T(2^{k-3} + 17) + n] + 2n + n$

$T(2^k) = 2^3 T(2^{k-3} + 17) + 2^2 n + 2n + n$

$T(2^k) = 2^i T(2^{k-i} + 17) + 2^0 n + 2^1 n + \cdots + 2^{i-2} n + 2^{i-1} n$

La recursión termina cuando $i = k$

$$T(2^k) = 2^k \underbrace{T(2^{k-k} + 17)}_{\underbrace{T(18)}_{1}} + 2^0 n + 2^1 n + \cdots + 2^{k-2} n + 2^{k-1} n$$

$T(2^k) = 2^k + n[2^0 + 2^1 + \cdots + 2^{k-2} + 2^{k-1}]$, con $\frac{r^{n+1} - 1}{r - 1}$

$T(2^k) = 2^k + n\left(\frac{2^k - 1}{2 - 1}\right)$ $\quad$ Como $n = 2^k$

$T(n) = n + n\left(\frac{n-1}{2-1}\right) = n^2$

$T(n) = O(n^2)$

④ $T(n) = 4T(\frac{n}{2}) + \Theta(n) \quad T(1) = 1$

Suponga $n = 2^k$

$T(2^k) = 4T(2^{k-1}) + \Theta(n)$

$T(2^k) = 4[4T(2^{k-2}) + \Theta(n)] + \Theta(n)$

$T(2^k) = 4^2T(2^{k-2}) + 4\Theta(n) + \Theta(n)$

$T(2^k) = 4^2[4T(2^{k-3}) + \Theta(n)] + 4\Theta(n) + \Theta(n)$

$T(2^k) = 4^3T(2^{k-3}) + 4^2\Theta(n) + 4\Theta(n) + \Theta(n)$

$T(2^k) = 4^iT(2^{k-i}) + 4^0\Theta(n) + 4^1\Theta(n) + \cdots + 4^{i-2}\Theta(n) + 4^{i-1}\Theta(n)$

Recursión termina cuando $i = k$

$T(2^k) = 4^k \underbrace{T(2^{k-k})}_{\underbrace{T(1)}_{1}} + 4^0\Theta(n) + 4^1\Theta(n) + \cdots + 4^{k-2}\Theta(n) + 4^{k-1}\Theta(n)$

$T(2^k) = 4^k + \Theta(n)[4^0 + 4^1 + \cdots + 4^{k-2} + 4^{k-1}]$, con $\frac{r^{n+1}-1}{r-1}$

$T(2^k) = 4^k + \Theta(n)\left(\frac{4^k-1}{4-1}\right)$, $n = 2^k$ y $k = \log n$

$T(n) = 4^{\log n} + \Theta(n)\left(\frac{4^{\log n}-1}{3}\right)$

$T(n) = n^{\log 4} + \Theta(n)\left(\frac{n^{\log 4}-1}{3}\right) = n^2 + \Theta(n)\left(\frac{n^2-1}{3}\right)$

$T(n) = \Theta(n^2)$

⑤ $T(n) = 3T(\frac{n}{4}) + \Theta(n^2)$, $T(1) = 1$

Suponga que $n = 4^k$

$T(4^k) = 3T(4^{k-1}) + \Theta(n^2)$

• $T(4^k) = 3[3T(4^{k-2}) + \Theta(n^2)] + \Theta(n^2)$

$T(4^k) = 3^2 T(4^{k-2}) + 3\Theta(n^2) + \Theta(n^2)$

• $T(4^k) = 3^2[3T(4^{k-3}) + \Theta(n^2) + 3\Theta(n^2) + \Theta(n^2)$

$T(4^k) = 3^3 T(4^{k-3}) + 3^2\Theta(n^2) + 3\Theta(n^2) + \Theta(n^2)$

$T(4^k) = 3^i T(4^{k-i}) + 3^0\Theta(n^2) + 3^1\Theta(n^2) + \cdots + 3^{i-2}\Theta(n^2) + 3^{i-1}\Theta(n^2)$

La recursión termina cuando $i = k$

$$T(4^k) = 3^k \underbrace{T(4^{k-k})}_{\underbrace{T(1)}_{1}} + \Theta(n^2)[3^0 + 3^1 + \cdots + 3^{k-2} + 3^{k-1}]$$

$T(4^k) = 3^k + \Theta(n^2)[3^0 + 3^1 + \cdots + 3^{k-2} + 3^{k-1}]$, con $\frac{r^{n+1}-1}{r-1}$

$T(4^k) = 3^k + \Theta(n^2)\left(\frac{3^k - 1}{r-1}\right)$, $n = 4^k$ y $3^k = n^{\log_4 3}$

$T(n) = n^{\log_4 3} + \Theta(n^2)\left(\frac{n^{\log_4 3} - 1}{2}\right)$

$T(n) = \Theta(n^{2\log_4 3})$