Discrete Math Ass 5 12310401 王子恒.

1. Suppose that $R_1 \subseteq R_2$ and that R_2 is antisymmetric.
Let $(a,b) \in R_1$ and $(b,a) \in R_1$. Since these two pair is also in R_2, thus $a=b$, hence R_1 is antisymmetric.

2. (1) Yes. for every $a \in R$ $a-a=0$ is rational. then $(a,a) \in R$.

(2) Yes. for all $a,b \in R$ if $(a,b) \in R$, then $a-b$ is rational. Hence $b-a=-(a-b)$ is rational. Then $(b,a) \in R$.

(3) No. Counterexample: $(a,a-1) \in R$ for $a-(a-1)=1$, $(a-1,a) \in R$ for $a-1-a=-1$, but $a \neq a-1$.

(4) Yes for every $a,b,c \in R$ $(a,b) \in R$, $(b,c) \in R$, then $a-b=k_1$ is rational, $b-c=k_2$ is rational. And $a-c=k_1+k_2$ is also rational Thus $(a,c) \in R$.

3. (a) $2^{1+2+\cdots+n} = 2^{\frac{n(n+1)}{2}}$

(b)

aRb	bRa	aRa
1	0	0
0	1	1
0	0	

$N = 3^{C_n^2} \times 2^n = 2^n 3^{\frac{n(n-1)}{2}}$

(c) $2^{n(n-1)}$.

(d) $2^{\frac{n(n-1)}{2}}$

(e) $2^{n^2} - 2^{n^2-n+1}$

(f) $3^{C_n^2} = 3^{\frac{n(n-1)}{2}}$

(g) 1

4. First step we prove if R is symmetric then R^n is symmetric
By induction:

Base: R is symmetric

induction step: let $(a,c) \in R^{n+1} = R^n \circ R$, then there have a b that $(a,b) \in R$ and $(b,c) \in R^n$. since R^n and R are symmetric then $(b,a) \in R$ and $(c,b) \in R^n$. Thus $(c,a) \in R \circ R^n = R^{n+1}$

Thus R^{n+1} is symmetric

That we get R^n is symmetric.

And $(R^*)^{-1} = \left(\bigcup_{n=1}^{\infty} R^n\right)^{-1} = \bigcup_{n=1}^{\infty} (R^n)^{-1} = \bigcup_{n=1}^{\infty} R^n = R^*$

So R^* is symmetric

5. $\{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$

6. For every relation (a,b) in R, we know that (a,a) and (b,b) are in R for the reflexive.

Then we could find $(a,b) \circ (b,b)$ to form the (a,b) in R^2

Thus $(a,b) \in R^2$. Hence $R \subseteq R^2$

7. (1) $R \cap S$ is transitive.

Let $(a,b) \in R \cap S$ and $(b,c) \in R \cap S$

$$\Rightarrow \begin{cases} (a,b) \in R, & (b,c) \in R \\ (a,b) \in S, & (b,c) \in S. \end{cases}$$

since R is transitive, then $(a,c) \in R$

since S is transitive, then $(a,c) \in S$

Therefore $(a,c) \in R \cap S$. Hence $R \cap S$ is transitive

(2) $R = \{(1,2),(3,4)\}$ $S = \{(2,3),(4,5)\}$

$R \cup S = \{(1,3),(3,5)\}$

$\{(1,3),(3,5)\} \in R \cup S$ but $(1,5) \notin R \cup S$

(3) $R = \{ (1,4) \quad (2,5) \}$

$S = \{ (4,2) \quad (5,3) \}$

$R \circ S = \{ (1,2), (2,3) \}$

$(1,2)$
$(2,1) \quad (3,1)$
$(1,3)$

8. (1) $R = \{ (1,2), (1,3) \}$

transitive closure of the symmetric closure is

$\{ (1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2) \}$

symmetric closure of transitive closure is

$\{ (1,1), (2,2), (3,3), (1,2), (1,3) \}$

$\{ (1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1) \}$.

They are not equivalent.

(2) Suppose (a,b) is in the symmetric closure of the transitive closure. Then at least (a,b) or (b,a) is in the transitive closure. Hence there must have (a,b) or (b,a) in R

If the (a,b) is in R then we can form (a,b) in the transitive closure of the symmetric closure of R. If (b,a) is in R. then we form (a,b) by symmetric closure and have the (a,b) in the transitive closure of symmetric closure of R.

Hence (a,b) is in the transitive closure of the symmetric closure.

9. (1) reflexive : $m^2-m^2=0$ $\quad 3\mid 0$ $\quad$ thus $(m,m)\in R$

symmetric : if $(m,n)\in R, \Rightarrow 3\mid m^2-n^2$ then $3\mid n^2-m^2 \Rightarrow (n,m)\in R$

transitive : if $(m,n)\in R$, $(n,p)\in R \Rightarrow 3\mid m^2-n^2$, $3\mid n^2-p^2$

then $3\mid m^2-n^2+n^2-p^2 = 3\mid m^2-p^2 \Rightarrow (m,p)\in R$.

Thus R is an equivalance equation.

(2) if $m\equiv 0 \mod 3$ then $m^2\equiv 0 \mod 3$

if $m\equiv 1 \mod 3$ then $m^2\equiv 1 \mod 3$

if $m\equiv 2 \mod 3$ then $m^2\equiv 1 \mod 3$

$[0]_R$: $\{3k \mid k\in \mathbb{Z}\}$

$[1]_R$: $\{3k+1 \mid k\in \mathbb{Z}\}\cup\{3k+2 \mid k\in\mathbb{Z}\}$.

10. reflexive : $A\cup A\setminus(A\cap A)=\emptyset$ $\quad \emptyset\subseteq T$ Thus $(A,A)\in R$

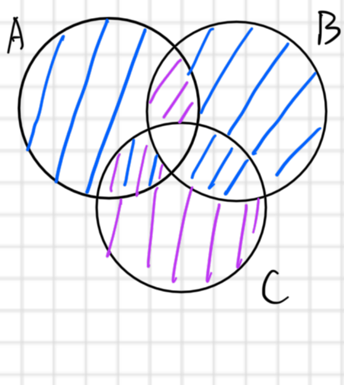
symmetric : if $(A,B)\in R$ then $A\cup B\setminus(A\cap B)\subseteq T$

By the symmetic of '$\cup$' and '$\cap$' $B\cup A\setminus(B\cap A)=A\cup B\setminus(A\cap B)\subseteq T$

Thus $(B,A)\in R$

transitive : If $(A,B)\in R$, $(B,C)\in R$, Then

$A\cup B\setminus A\cap B\subseteq T$, $\quad B\cup C\setminus B\cap C\subseteq T$



which means $AB'+A'B\subseteq T$, $BC'+B'C\subseteq T$

$\Rightarrow AB'C$, $\underline{\underline{AB'C'}}$, $\underline{\underline{A'BC}}$, $A'BC'$, $\underline{\underline{ABC'}}$, $AB'C$, $\underline{\underline{A'B'C}} \subseteq T$

Then $A\cup C\setminus A\cap C = AC'+A'C$

$= ABC'+AB'C'+A'BC+A'B'C$

$\subseteq T$

Then $(A,C)\in R$. Hence is is an equivalence relation

11. reflexive: $((a,b),(a,b)) \in R$ because $a+b=a+b$.

symmetric: if $((a,b),(c,d)) \in R$ then $a+d=b+c$, which means

$c+b=d+a$ Thus $((c,d),(a,b)) \in R$.

transitive: if $((a,b),(c,d)) \in R$, $((c,d),(e,f)) \in R$.

then $a+d=b+c$, $c+f=d+e$.

then $a+d+c+f=b+c+d+e$

$\Rightarrow a+f=b+e$ which means $((a,b),(e,f)) \in R$.

12. let $R = '\sim'$.

reflexive: $(x,x) \in R$ because $x = 2^0 \cdot x$

symmetric: if $(x,y) \in R$ then $x=2^k y$ or $y=2^k x$

if $x=2^k y$ then by the definition $(y,x) \in R$

if $y=2^k x$ by definition $(y,x) \in R$.

Thus $(y,x) \in R$.

transitive: if $(x,y) \in R$, $(y,z) \in R$ then

	$x=2^{k_1}y$	$y=2^{k_1}x$
$y=2^{k_2}z$	$x=2^{k_1+k_2}z$	$x=2^{k_2-k_1}z \begin{cases} k_2 \geq k_1 & x=2^{k_2-k_1}z \\ k_2 < k_1 & z=2^{k_1-k_2}z \end{cases}$
$z=2^{k_2}y$	$x=2^{k_1-k_2}z \begin{cases} k_1 \geq k_2, & x=2^{k_1-k_2}z \\ k_1 < k_2, & z=2^{k_2-k_1}z \end{cases}$	$z=2^{k_1+k_2}x$

Thus $(x,z) \in R$.

13.

	(a) $=$	(b) $\neq$	(c) $\geq$	(d) $\nmid$
reflexive	$a=a$ ✓	$a \neq a$ ✗	$a \geq a$ ✓	$a \nmid a$ ✗
antisymmetric	$a=b,\ b=a \Rightarrow a=b$ ✓		$a \geq b,\ b \geq a \Rightarrow a=b$ ✓	
transitive	$a=b,\ b=c \Rightarrow a=c$ ✓		$a \geq b,\ b \geq c \Rightarrow a \geq c$ ✓	
posets	✓	✗	✓	✗

14. (a) reflexive: $f \leq f$ ✓

symmetric: $f \propto g \Rightarrow f \leq g$ but $g \leq f$ may false ✗

$\propto$ not equivalence

(b) antisymmetric: $f \propto g \Rightarrow f = O(g) \Rightarrow f \leq g$

$g \propto f \Rightarrow f = O(g) \Rightarrow g \leq f$

$\Rightarrow f = g$ ✓

transitive: $f \propto g,\ g \propto h \quad f \leq g \leq h$ ✓.

$\propto$ is a partial ordering

(c) For any function from N^+ to R either $f \leq g$ or $g \leq f$

Thus $\propto$ is a total ordering

15. (1) reflexive: $((a,b,c),(a,b,c)) \in R$ for $2^a 3^b 5^c \leq 2^a 3^b 5^c$

antisymmetric: if $((a_1,b_1,c_1),(a_2,b_2,c_2)) \in R$ and

$((a_2,b_2,c_2),(a_1,b_1,c_1)) \in R$

Then $2^{a_1} 3^{b_1} 5^{c_1} \leq 2^{a_2} 3^{b_2} 5^{c_2}$, $2^{a_2} 3^{b_2} 5^{c_2} \leq 2^{a_1} 3^{b_1} 5^{c_1}$

by the Fundamental Theorem of Arithmetic

$a_1 = a_2,\ b_1 = b_2,\ c_1 = c_2$

transitive $\quad$ if $((a_1,b_1,c_1),(a_2,b_2,c_2)) \in R$ and

$((a_2,b_2,c_2),(a_3,b_3,c_3)) \in R$

then $2^{a_1}3^{b_1}5^{c_1} \le 2^{a_2}3^{b_2}5^{c_2} \le 2^{a_3}3^{b_3}5^{c_3}$

then $((a_1,b_1,c_1),(a_3,b_3,c_3)) \in R$

(2) comparable: $(1,0,0)$, $(0,0,1)$

incomparable: not exist

(3) $(5,0,1) \Rightarrow 2^5 \cdot 3^0 \cdot 5^1 = 160.$

$(1,1,2) \Rightarrow 2^2 \cdot 3^1 \cdot 5^2 = 150.$

least upper bound: $(5,0,1)$

greatest lower bound $(1,1,2)$.

(4) minimal: $(0,0,0)$

maximal: not exist

16. reflexive: $(a,b) \preceq (a,b)$ for $(a,b)=(a,b)$

antisymmetric: if $(a,b) \preceq (c,d)$ and $(c,d) \preceq (a,b)$

then $\begin{cases} (a,b)=(c,d) \text{ or } a^2+b^2<c^2+d^2 & ① \\ (a,b)=(c,d) \text{ or } c^2+d^2<a^2+b^2 & ② \end{cases}$

if $a^2+b^2 \le c^2+d^2$, then from ② we get $(a,b)=(c,d)$

if $a^2+b^2 > c^2+d^2$, then from ① we get $(a,b)=(c,d)$

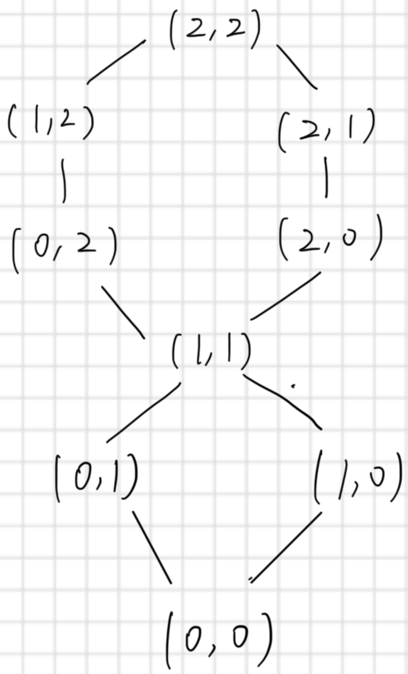
Therefore $(a,b)=(c,d)$

transitive: If $(a,b) \preceq (c,d)$ and $(c,d) \preceq (e,f)$

then	$(a,b)=(c,d)$	$a^2+b^2<c^2+d^2$
$(c,d)=(e,f)$	$(a,b)=(c,d)=(e,f)$	$a^2+b^2<c^2+d^2=e^2+f^2$
$c^2+d^2<e^2+f^2$	$a^2+b^2=c^2+d^2<e^2+f^2$	$a^2+b^2<c^2+d^2<e^2+f^2$

Thus $(a,b)=(e,f)$ or $a^2+b^2<e^2+f^2$, Hence $(a,b) \preceq (e,f)$

$B = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$.



17. (a) $R = P(N)$:

By contradiction: if R has maximal element, the element is A

suppose M is the maximum number in A, then we have $B = A \cup \{M+1\} \in R$

and $A \subseteq B$, contradict

Thus R has no maximal element.

(b) Disprove:

Contradiction: Suppose nonempty R has no minimal element

If A is an element of R, then there exist no element

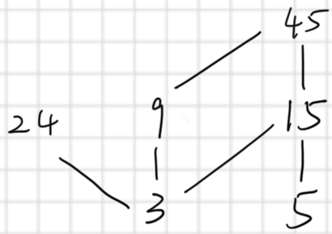
that $x \subseteq A$, but $A \subseteq A$, contradict.

Thus R not exist.

(c) Disprove: From (b) we know R with no minimal element

is not exist. Thus T also not exist.

18.



(1) $\{24, 45\}$

(2) $\{3, 5\}$

(3) No

(4) No

(5) $\{15, 45\}$

(6) 15

(7) $\{3, 5, 15\}$

(8) 15