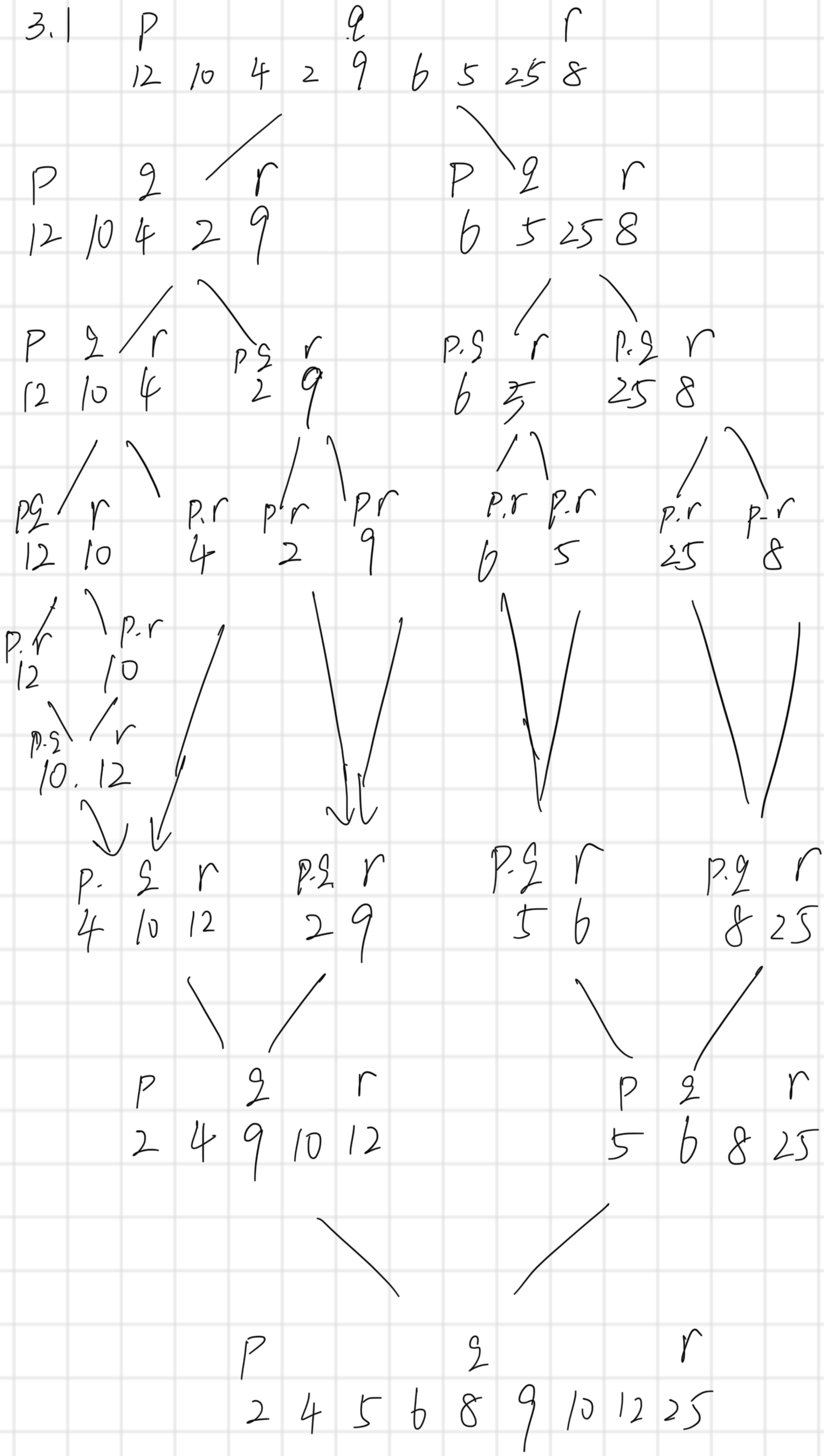
Exercise 3 12310401 王子恒

3.1



3.2 Base: when $n=1$

$T(1) = d + c \cdot \log 1 = d$. the base is true

Assume for $k \geq 1$, $n = 2^k$, we have

$$T(2^k) = d \cdot 2^k + c \cdot 2^k \log 2^k$$

we should show that the statement still halds for

$$T(2^{k+1}) = d \cdot 2^{k+1} + c \cdot 2^{k+1} \log 2^{k+1}$$

proof: $T(2^{k+1}) = 2\left(d \cdot 2^k + c \cdot 2^k \log 2^k\right) + c \cdot 2^{k+1}$

$$= d \cdot 2^{k+1} + c \cdot 2^{k+1} \log 2^k + c \cdot 2^{k+1}$$

$$= d \cdot 2^{k+1} + c \cdot 2^{k+1} \left(\log 2^k + \log 2\right)$$

$$= d \cdot 2^{k+1} + c \cdot 2^{k+1} \log 2^{k+1}$$

Thus, we have shown that the solution to the recurrence is $T(n) = dn + cn\log n$ for all $n = 2^k$, $(k \geq 0)$.

3.3 $n^{\log_4 2} = n^{\frac{1}{2}}$

1. $f(n)=1 = O(n^{\frac{1}{2}-\varepsilon})$ for any $0<\varepsilon<\frac{1}{2}$

$\therefore T(n) = \Theta(n^{\frac{1}{2}})$.

2. $f(n)=\sqrt{n} = \Theta(n^{\frac{1}{2}}\log^0 n)$ for $k=0$

$\therefore T(n) = \Theta(n^{\frac{1}{2}}\log^{k+1} n) = \Theta(n^{\frac{1}{2}}\log n)$.

3. $f(n) = \sqrt{n}\log^2 n = \Theta(n^{\frac{1}{2}}\log^2 n)$ for $k=2$

$\therefore T(n) = \Theta(n^{\frac{1}{2}}\log^{k+1} n) = \Theta(n^{\frac{1}{2}}\log^3 n)$.

4. $f(n) = n = \Omega(n^{\frac{1}{2}+\varepsilon})$ for $0<\varepsilon<\frac{1}{2})$

and $af(\frac{n}{b}) = 2f(\frac{n}{4}) = \frac{n}{2} \leq \frac{2}{3}n$ for $c=\frac{2}{3}$

$\therefore T(n) = \Theta(n)$

3.4

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BinarySearch (A, x, low, heigh):
    if low > high:                                    Θ(1)
        return -1                                     Θ(1)
    mid = low + (heigh - low)/2                       Θ(1)
    if A[mid] == x:                                   Θ(1)
        return mid                                    Θ(1)
    else if A[mid] > x:                               Θ(1)
        return BinarySearch(A, x, low, mid)           T(n/2)
    else:
        return BinarySearch(A, x, mid+1, high)        T(n/2)
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$$T(n) = T\left(\frac{n}{2}\right) + \Theta(1)$$

$$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$$

$$f(n) = \Theta(1) = \Theta(1 \cdot \log^k n) \text{ for } k = 0$$

$$\therefore T(n) = \Theta(n^0 \cdot \log^{k+1} n) = \Theta(\log n)$$