12310401 王子恒

Q2.1

a) $3n^2+5n-2=\Theta(n^2)$ since for all $n \geq n_0$

$c_1 n^2 \leq 3n^2+5n-2 \leq c_2 n^2$

when saying $c_1=1$, $c_2=10$, $n_0=5$

b) $42=\Theta(1)$ since for all $n \geq n_0$

$c_1 \leq 42 \leq c_2$

when saying $c_1=41$, $c_2=43$, $n_0=1$

c) $4n^2(1+\log n)-2n^2=\Theta(n^2\log n)$ since for all $n \geq n_0$

$c_1 n^2\log n \leq 4n^2(1+\log n)-2n^2 \leq c_2 n^2 \log n$

when saying $c_1=1$, $c_2=100$, $n_0=10$

Q2.2

$f(n)$	$g(n)$	O	o	Ω	ω	Θ
$\log n$	$\sqrt{n}$	yes	yes	no	no	no
n	$\sqrt{n}$	no	no	yes	yes	no
n	$n\log n$	yes	yes	no	no	no
n^2	$n^2+(\log n)^3$	yes	no	yes	no	yes
2^n	n^3	no	no	yes	yes	no
$2^{n/2}$	2^n	yes	yes	no	no	no
$\log_2 n$	$\log_{10} n$	yes	no	yes	no	yes

Q2.3

A:

$1+3n(n-2) = 3n^2-6n+1 = \Theta(n)$

as $c_1 n^2 \leq 3n^2-6n+1 \leq c_2 n^2$ for all $n \geq n_0$

when saying $c_1 = 1$, $c_2 = 5$, $n_0 = 6$.

B:

$1+n+\frac{n}{2}+\frac{n}{2} = 2n+1 = \Theta(n)$

as $c_1 n \leq 2n+1 \leq c_2 n$ for all $n \geq n_0$

when saying $c_1 = 1$ $c_2 = 4$ $n_0 = 1$

C:

$1+\frac{(n+1)n}{2}+n+1 = \frac{1}{2}n^2+\frac{3}{2}n+2 = \Theta(n^2)$

as $c_1 n^2 \leq \frac{1}{2}n^2+\frac{3}{2}n+2 \leq c_2 n^2$ for all $n \geq n_0$

when saying $c_1 = \frac{1}{4}$, $c_2 = 8$ $n_0 = 8$.

Q2.4

1. True because $O(\sqrt{n}) \subseteq O(n)$

2. False select $n = o(n^2)$ thus $2n = \omega(n)$ is false
because $\lim_{n \to \infty} \frac{2n}{n} = 2 \neq 0$

3. True because
$n \log n \leq 3n \log n + O(n) \leq 3n \log n + n \leq 10n \log n$
when $n \geq 10$

The $O(n)$ describe an upper bound but "at least" describe A's lower bound without upper bound, which means A has no lower or upper bound from the sentence that is meaningless.

Q2.5

$$\Theta(n) \cdot \Theta(n) \cdot \left(\Theta(1) + \Theta(n)\right) = \Theta(n) \cdot \Theta(n) \cdot \Theta(n)$$
$$= \Theta(n^3)$$