STA219: Probability and Statistics for Engineering

Assignment 3

Note: The assignment can be answered in Chinese or English, either is fine. Please provide derivation and computation details, not just the final answer. Please submit a PDF file on BB.

1. (10 points) Suppose that the average household income in some country is 900 coins, and the standard deviation is 200 coins. Assuming the normal distribution of incomes:

   (1) Compute the proportion of "the middle class", i.e., whose income is between 600 and 1200 coins. (5 points)

   (2) The government of the country decides to issue food stamps to the poorest 3% of households. Below what income will families receive food stamps? (5 points)

2. (10 points) Let $X \sim N(\mu, \sigma^2)$. Suppose that the probability that the quadratic equation $y^2 + 4y + X = 0$ has no real roots (i.e., its discriminant is negative) is 0.5, please determine the value of μ.

3. (10 points) A survey shows that the English score (hundred-mark system) of students approximately follows a normal distribution $N(\mu, \sigma^2)$ with $\mu = 72$. If the number of students with more than 96 points accounts for 2.3% of the total students, what is the probability that the score is between 60 and 84 points?

4. (10 points) Suppose that the diameter of a disc follows a uniform distribution on (a, b), what is the expected area of this disc?

5. (10 points) Let $Z \sim N(0,1)$. Find $\mathrm{E}\big(\Phi(Z)\big)$ and $\mathrm{Var}\big(\Phi(Z)\big)$, where Φ is the CDF of Z.

6. (10 points) If $X \sim N(0,1)$, please derive the PDF of the following random variables:

   (1) $Y_1 = |X|$; (5 points)

   (2) $Y_2 = 2X^2 + 1$. (5 points)

7. (15 points) Suppose that random variable X follows an exponential distribution with parameter 2. Show that both $Y_1 = e^{-2X}$ and $Y_2 = 1 - e^{-2X}$ follow the uniform distribution on $(0,1)$.

8. (15 points) Suppose the joint PDF of a random vector (X, Y) is given by

$$f(x, y) = \begin{cases} ke^{-(3x+4y)}, & 0 < x, y < \infty \\ 0, & \text{otherwise} \end{cases}.$$

(1) Determine the constant k; (5 points)

(2) Find the joint CDF $F(x, y)$ of (X, Y); (5 points)

(3) Compute $P(X + Y \leq 1)$. (5 points)

9. (10 points) Suppose the joint PDF of a random vector (X, Y) is given by

$$f(x, y) = \begin{cases} e^{-y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases},$$

determine $f_X(x)$ and $f_Y(y)$, i.e., the marginal PDFs of X and Y, respectively.