STA 219 Ass 8 123|0401 王子恒.

1. (1). when $\mu = 2$

$$\frac{\bar{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}} > Z_\alpha.$$

$$\bar{x} > Z_\alpha \sqrt{\frac{\sigma^2}{n}} + \mu = 2.6$$

$$\Rightarrow Z_\alpha = 2.68.$$

$$\alpha = 0.0037$$

when $\mu = 3$

$$\frac{\bar{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}} \le Z_\beta$$

$$\bar{x} \le Z_\beta \sqrt{\frac{\sigma^2}{n}} + \mu = 2.6.$$

$$\Rightarrow Z_\beta = -1.78.$$

$$\beta = 0.0375$$

(2)
$$\frac{\bar{x}-\mu}{\sqrt{\frac{\sigma^2}{n}}} \le Z_{0.01}$$

$$\bar{x} \le Z_{0.01} \sqrt{\frac{\sigma^2}{n}} + \mu = 2.6.$$

$$\Rightarrow n = 34$$

2. (1) $H_0: \mu = 8$ $H_1: \mu < 8.$

$$RR: T = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} < -Z_{0.05}$$

$$\bar{x} < 7.67.$$

Because $\bar{x} = 6.5 < 7.67$, then it reject the statement in the newspaper.

(2)
$$T = \frac{\bar{x}-\mu}{\frac{S_x}{\sqrt{n}}}$$

$$t_{obs} = \frac{6.5-8}{\frac{2}{\sqrt{100}}} = -7.5$$

$$\text{p-value} = P(T < -7.5 \mid \mu = 8) = \Phi(-7.5) < \Phi(-1.65) = 0.05$$

So we won't reject H_0.

3. $T \overset{approx}{\sim} N\left(\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}, 1\right) = N\left(-\frac{\sqrt{n}}{4}, 1\right)$.

$$\text{power} = P\left(T < -1.65 \,\middle|\, \bar{x} = 7.5\right)$$

$$= P\left(\underbrace{(T - (-\tfrac{\sqrt{n}}{4}))}_{\sim N(0,1)} < -1.65 \cdot 1 - (-\tfrac{\sqrt{n}}{4}) \,\middle|\, \bar{x} = 7.5\right).$$

$$= \Phi\left(-1.65 + \frac{\sqrt{n}}{4}\right) \geq 0.9$$

$$\frac{\sqrt{n}}{4} - 1.65 \geq 1.28$$

$$n \geq 138.$$

4. (1) Let X be binary random variables suggesting whether a randomly support part from Town A and B. Then $X \sim Bernoulli(P_A)$, $Y \sim Bernoulli(P_B)$.

$H_0: P_A - P_B = 0$, $H_1: P_A - P_B \neq 0$.

$$\bar{X} - \bar{Y} \overset{approx}{\sim} X\left(P_A - P_B, \frac{P_A(1-P_A)}{n} + \frac{P_B(1-P_B)}{n}\right).$$

$$T = \frac{\hat{P_A} - \hat{P_B} - 0}{\sqrt{\frac{\hat{P_A}(1-\hat{P_A}) + \hat{P_B}(1-\hat{P_B})}{n}}} \overset{approx}{\sim} N(0,1) \text{ under } H_0.$$

since H_1 is two-sided. RR: $\{X : |T| > z_{\frac{\alpha}{2}}\}$.

$\Rightarrow \{X : |T| > 2.32\}$.

$$t_{obs} = \frac{0.45 - 0.35}{\sqrt{\frac{0.45(0.55)\ 0.35(0.65)}{400}}} = 2.90 > 2.32.$$

since we have sufficient evidence to reject H_0 at $\alpha = 0.02$. Then there is a significant difference between the support rates in town A and town B.

(2) $X \sim N\left(P_A, \frac{P_A(1-P_A)}{n}\right)$ $\bar{Y} \sim N\left(P_B, \frac{P_B(1-P_B)}{n}\right)$

$1-\alpha = 98\%$ $\frac{\alpha}{2} = 0.01$

The 98% CI of P_A is

$$\left(\hat{P_A} - Z_{0.01} \times \sqrt{\frac{\hat{P_A}(1-\hat{P_A})}{n}}, \hat{P_A} + Z_{0.01} \times \sqrt{\frac{\hat{P_A}(1-\hat{P_A})}{n}}\right)$$

$= (0.392, 0.508)$

Similarily. 98% CI of is $\left(\hat{P_B} - Z_{0.01} \times \sqrt{\frac{\hat{P_B}(1-\hat{P_B})}{n}}, \hat{P_B} + \underbrace{Z_{0.01} \times \sqrt{\frac{\hat{P_B}(1-\hat{P_B})}{n}}}_{0.055}\right)$

$= (0.295, 0.405)$

Siilarily. 98% CI of $P_A - P_B$ is

$(0.025, 0.165)$.

(3) No, we can't. Because the interval of the P_A, P_B depends on the center mean value. And it affect the overlap area. It can't decide the significance level α.

(4) Yes if the 98% CI of $P_A - P_B$ contain 0, then we can say that they have no significantly different, and the level is the Reject area, which just is the 0.02.

5. we transfer $H_0: \mu_1 - \mu_2 = 0 \quad H_1: \mu_1 - \mu_2 > 0$

$$\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \frac{16}{n} + \frac{16}{m}\right)$$

$$T = \frac{X - Y - 0}{\sqrt{\frac{16}{n} + \frac{16}{m}}} \overset{approx}{\sim} N(0, 1)$$

$$RR: \{\mathbf{X}: T > Z_{0.05}\} \rightarrow \{\mathbf{X}: T > 1.65\}.$$

(2) $P(T \leq 1.65 \mid H_1 \text{ is true})$.

$$H_1: T \sim N\left(\frac{2}{\sqrt{\frac{16}{10} + \frac{16}{11}}}, 1\right) = (1.14, 1)$$

$$P(T \leq 1.65) = P\left(\frac{T - 1.14}{1} \leq \frac{1.65 - 1.14}{1}\right) = \Phi(0.51) = 0.695$$

(3) power $= P(T > 1.65 \mid H_1 \text{ is true})$.

$$H_1: T \sim N\left(\frac{2}{\sqrt{\frac{16}{n} + \frac{16}{m}}}, 1\right)$$

$$P(T > 1.65) = P\left(T - \frac{2}{\sqrt{\frac{16}{n} + \frac{16}{m}}} > 1.65 - \frac{2}{\sqrt{\frac{16}{n} + \frac{16}{m}}}\right)$$

$$= 1 - \Phi\left(1.65 - \frac{2}{\sqrt{\frac{16}{n} + \frac{16}{m}}}\right) \geq 0.9$$

$$\Phi\left(1.65 - \frac{2}{\sqrt{\frac{16}{n} + \frac{16}{m}}}\right) \leq 0.1$$

$$1.65 - \frac{2}{\sqrt{\frac{16}{n} + \frac{16}{m}}} \leq -1.28.$$

$$\left\{\begin{aligned} &\frac{2}{\sqrt{\frac{16}{n} + \frac{16}{m}}} \geq 2.93. \\ &\sqrt{\frac{16}{n} + \frac{16}{m}} \leq 0.683 \\ &\frac{16}{n} + \frac{16}{m} \leq 0.466. \\ &\frac{1}{n} + \frac{1}{m} \leq 0.029 \\ &0.029(m + n) \geq 2 + \frac{n}{m} + \frac{m}{n} \\ &\qquad \geq 4 \\ &m + n \geq 138. \end{aligned}\right.$$