STA219 Assignment1 12310401 王子恒

1、 A: problem with MB B: problem with HD

$P(A) = 0.4$ $P(B) = 0.3$ $P(AB) = 0.15$

$P(A\cup B) = P(A) + P(B) - P(AB) = 0.55$

$P(\bar{A}\cap\bar{B}) = 1 - P(A\cup B) = 0.45$

2、(1) A: know Java B: know Python

$P(A) = 0.7$ $P(B) = 0.6$ $P(AB) = 0.5$

(1) $P(A\cup B) = P(A) + P(B) - P(AB) = 0.8$

$P(\bar{A}\cap\bar{B}) = 1 - P(A\cup B) = 0.2$

(2) $P(A\bar{B}) = P(A) - P(AB) = 0.2.$

(3) $P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{5}{6}$

3. Assume I have n distinct elements and randomly selecting k elements with replacement

(1) permutation: for every position I can choose n terms. so the total is n^k

(2) combination: it is like between k number items place n-1 board to distinguish their category, to make every space can place only one board, we add every categories one item, so we actually choose n-1 spaces from n+k-1 spaces.

$\underbrace{0 0 | 0 | 0 0 | 0 0 | 0 0 \cdots | 0 0}_{k}$ which means $C_{n+k-1}^{n-1} = \dfrac{(n+k-1)!}{k!\,(n-1)!}$

4. (1) $P = \dfrac{C_n^k}{C_{2n}^{2k}}$

(2) choose one shoe from every 2k pair shoes

$\rightarrow \dfrac{C_n^{2k}\cdot 2^{2k}}{C_{2n}^{2k}}$

(3) $\dfrac{C_n^1 \cdot C_{n-1}^{2(k-1)} \cdot 2^{2k-2}}{C_{2n}^{2k}}$

5. Assume a_i is the number of the situation that i couples don't have paired couple

so we have $a_n = (n-1)(a_{n-1} + a_{n-2})$

for $n=1$ obviously $a_1 = 0$

$n=2$ $a_2 = 1$

so we can get $a_3 = 2.$ $a_4 = 9$

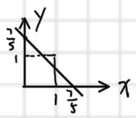
$P = 1 - \dfrac{9}{A_4^4} = \dfrac{5}{8}$

6. suppose the two number is x and y

then $x+y < \frac{7}{5}$

$\Rightarrow \quad y < -x + \frac{7}{5}$

$$P = \frac{1 - \frac{1}{2} \times \frac{3}{5} \times \frac{3}{5}}{1 \times 1} = \frac{41}{50}$$



7. A: have the disease and have symptom

B: test say have the disease

$P(A) = 0.001$

$P(B|A) = \frac{P(AB)}{P(A)} = 0.95 \rightarrow P(AB) = 0.95 \times 0.001$

$P(B|\bar{A}) = \frac{P(\bar{A}B)}{P(\bar{A})} = 0.001 \rightarrow P(\bar{A}B) = 0.001 \times (1 - 0.001)$

$P(B) = P(AB) + P(\bar{A}B)$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.95 \times 0.001}{0.95 \times 0.001 + 0.001 \times (1 - 0.001)} = 0.4874$$

compare with the answer in 1.13 $P = 0.0094$. the probability is much larger.

8. The first is correct, because we have the order of the children, so the 'GB' is not the case

we have only {BB, BG} the probability is $\frac{1}{2}$

9. A: 1 B: 2 C: 3 D: 4 E: 5 work properly

$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = P(\bar{D}) = P(\bar{E}) = 0.3$

$P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - 0.3 \times 0.3 \times 0.3 = 0.973$

$P(D \cup E) = 1 - P(\bar{D} \wedge \bar{E}) = 1 - 0.3 \times 0.3 = 0.9$

$P = P(A \cup B \cup C) \cdot P(D \cup E) = 0.88543$