

STA219: Probability and Statistics for Engineering

Assignment 2

Note: The assignment can be answered in Chinese or English, either is fine. Please provide derivation and computation details, not just the final answer. Please submit a PDF file on BB.

1. (15 points) Fischer and Spassky play a chess match in which the first player to win a game wins the match. After 10 successive draws (平局), the match is declared drawn. Each game is won by Fischer with probability 0.4, is won by Spassky with probability 0.3, and is a draw with probability 0.3, independent of previous games.
 - (1) What is the probability that Fischer wins the match? (5 points)
 - (2) What is the PMF of the duration of the match? (10 points)
2. (15 points) You just rented a large house and the realtor gave you 5 keys, one for each of the 5 doors of the house. Unfortunately, all keys look identical, so to open the front door, you try them at random. Find the PMF of the number of trials you will need to open the door, under the following alternative assumptions:
 - (1) After an unsuccessful trial, you mark the corresponding key, so that you never try it again. (10 points)
 - (2) At each trial you are equally likely to choose any key. (5 points)
3. (10 points) Suppose there are 10 electric elements of the same type, among which 2 are unqualified. You randomly select one from these 10 elements. If it is unqualified, you throw it away and select one again from the remaining 9 elements; if the second try is still unqualified, you select another one from the remaining 8 elements. Let X be the number of unqualified elements you select before you get the qualified one. What is the variance of X ?
4. (15 points) Suppose the PDF of a continuous random variable X is given by
$$f(x) = \begin{cases} ax + bx^2, & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$
If the expectation of X is $E(X) = 2/3$, please calculate $\text{Var}(X)$.

5. (10 points) Let X be a Poisson random variable with parameter λ . Show that the PMF $p_x(k)$ increases monotonically with k up to the point where k reaches the largest integer not exceeding λ , and after that point decreases monotonically with k .
6. (10 points) Continued with Example 3.8 in the slides of Chapter 2 – Part 1, calculate the probability that the insurance company loses money in this life insurance, i.e., the profit is less than \$0.
7. (10 points) Jobs are sent to a printer independently at a constant rate of 3 jobs per hour.
 - (1) What is the expected time between jobs? (5 points)
 - (2) Suppose that a job just arrived, what is the probability that the next job is sent within 5 minutes? (5 points)
8. (15 points) Continued with Page 41 of Chapter 2 – Part 2, please write down the relationship between the geometric distribution and the exponential distribution with detailed derivation.