STA219: Probability and Statistics for Engineering

Assignment 1

Note: The assignment can be answered in Chinese or English, either is fine. Please provide derivation and computation details, not just the final answer. Please submit a PDF file on BB.

1. (5 points) Suppose that after 10 years of service, 40% of computers have problems with motherboards (MB), 30% have problems with hard drives (HD), and 15% have problems with both MB and HD. What is the probability that a 10-year old computer still has fully functioning MB and HD?

2. (15 points) Among programmers of a certain firm, 70% know Java, 60% know Python, and 50% know both languages. If we randomly sample a programmer from the firm, compute the probability of the following events:

   (1) The probability that he/she does not know Python and does not know Java. (5 points)

   (2) The probability that he/she knows Java but not Python. (5 points)

   (3) The probability that he/she knows Java given that he/she knows Python. (5 points)

3. (10 points) Please derive the permutation and combination under the case of random selecting with replacement.

4. (15 points) Randomly pick $2k$ shoes from n pairs of shoes (left and right shoes are distinguishable) of different sizes ($2k < n$). Calculate the probability of the following events:

   (1) Exactly k pairs are formed among the $2k$ shoes picked. (5 points)

   (2) No pair is formed among the $2k$ shoes picked. (5 points)

   (3) Exactly one pair is formed among the $2k$ shoes picked. (5 points)

5. (10 points) Suppose that 4 male-female couples are at a party and that the males and females are randomly paired for a dance. Compute the probability that at least one couple is paired together.

6. (10 points) Take two numbers randomly from the interval $[0,1]$, try to find the probability of the event that the sum of these two numbers is less than 7/5.

7. (10 points) Continued with Example 1.13 in the slides of Chapter 1 used in class, if the rare disease occurs in 1 person in 1,000 for those who have a certain symptom, and other known conditions are the same. If you have the symptom and the test says you have the disease, what is the probability that you actually have the disease? How is the probability compared to that in Example 1.13?

8. (15 points) A family has two children. The gender of the two children are independent and a child being a boy (B) or a girl (G) with equal probability. You randomly meet one of the two children with equal probability and he is a boy, what is the probability that the other child is also a boy? There are two contradictory opinions:

   (1) Since the gender of the two children are independent, the probability that the other child is also a boy does not depend on whether the child you met is a boy or a girl, i.e., $1/2$.

   (2) The original sample space consists of 4 equally likely outcomes $\{GG, BB, BG, GB\}$. Since you have already met a boy, the sample space becomes $\{BB, BG, GB\}$. Among the remaining three outcomes, only BB suggests that the other child is also a boy, thus the probability is $1/3$.

   Which of the opinions is correct? Please provide detailed reason/derivation on why the other opinion is wrong.

9. (10 points) In the system in the figure below, each of the 5 component fails with probability 0.3 independently of other components. Compute the system's reliability, i.e., the probability that the system works properly.