STA219: Probability and Statistics for Engineering

Assignment 7

Note: The assignment can be answered in Chinese or English, either is fine. Please provide derivation and computation details, not just the final answer. Please submit a PDF file on BB.

1. (10 points) A network provider investigates the load of its network. The number of concurrent users is recorded at ten locations (thousands of people),

   17.2 22.1 18.5 17.2 18.6 14.8 21.7 15.8 16.3 22.8

   (1) Compute the sample mean, sample variance, and sample standard deviation of the number of concurrent users. (5 points)

   (2) Compute the sample lower and upper quartile, and sample interquartile range. (5 points)

2. (15 points) Let X_1, X_2, X_3 be a simple random sample from the population $X \sim \mathrm{U}(0, \theta)$.

   (1) Show that $\hat{\theta}_1 = \frac{4}{3} X_{(3)}$ and $\hat{\theta}_2 = 4X_{(1)}$ are both unbiased estimators of θ. (10 points)

   (2) Which of these two estimators is more efficient? (5 points)

3. (10 points) The average white blood cell count per liter of blood in normal adult males is 7.3×10^9, with a standard deviation of 0.7×10^9. Using Chebyshev's inequality, estimate the lower bound for the probability that the white blood cell count per liter of blood is between 5.2×10^9 and 9.4×10^9.

4. (10 points) Let $X_1, \ldots, X_n$ be a simple random sample from the population X, and $\mathrm{E}(X) = \mu$, $\mathrm{Var}(X) < \infty$. Prove that

   $$\hat{\mu} = \frac{2}{n(n+1)} \sum_{k=1}^{n} k X_k$$

   is a consistent estimator of μ.

   (Hint 1: use the conclusion that asymptotic unbiasedness + vanishing variance $\Longrightarrow$ consistency; Hint 2: $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$.)

5. (15 points) Estimate the unknown parameter θ from a sample

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   drawn from a population X with the probability mass function

$$\begin{cases} P(X = 3) = \theta; \\ P(X = 7) = 1 - \theta. \end{cases}$$

(1) Derive the moment estimator $\hat{\theta}_1$ of θ, and calculate its estimated value based on the sample; (5 points)

(2) Calculate the expectation and variance of $\hat{\theta}_1$. Is $\hat{\theta}_1$ an unbiased estimator? (5 points)

(3) Calculate the maximum likelihood estimate of θ based on the sample. (5 points)

6. (15 points) A sample $(X_1, ..., X_{10})$ is drawn from a population with a PDF

$$f(x; \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \qquad 0 < x < \infty.$$

The sum of all 10 sample observed values equals 150.

(1) Derive the moment estimator $\hat{\theta}_1$ of θ, and calculate its estimated value based on the sample. (5 points)

(2) Derive the standard error of the moment estimator $\hat{\theta}_1$. (5 points)

(3) Derive the maximum likelihood estimator $\hat{\theta}_2$ of θ, and calculate its estimated value based on the sample. (5 points)

7. (15 points) Installation of a certain hardware takes random time with a standard deviation of 5 minutes.

(1) A computer technician installs this hardware on 64 different computers, with the average installation time of 42 minutes. Compute a 95% confidence interval for the population mean installation time. (10 points)

(2) Suppose that the installation time follows normal distribution, and population mean installation time is 40 minutes. A technician installs the hardware on your PC. What is the probability that the installation time will be within the interval computed in (1)? (5 points)

8. (10 points) Assuming that the height of a randomly selected woman aged 18 to 25 follows a normal distribution. We collected data from two regions, A and B. In **region A**, 40 women were sampled with a mean height of 1.64m and a standard deviation of 0.2m. In **region B**, 50 women were sampled with a mean height of 1.62m and a standard deviation of 0.4m. Estimate the difference in mean height between women from these two regions with 90% confidence.