STA215 Ass2 　 123104011 王子恒

1.

(1) $P(\text{Fischer}) = 0.4 + 0.3\times0.4 + 0.3^2\times0.4 + \cdots + 0.3^9\times0.4$

$= 0.4\times\left(\frac{0.3(1-0.3^9)}{1-0.3} + 1\right)$

$= 0.57$

(2) $P(n) = \begin{cases} 0.7\times0.3^{n-1} & n\in[1,9] \\ 0.3^9 & n=10 \end{cases}$

2. (1) $P(X=1) = \frac{1}{5}$

$P(X=2) = \frac{4}{5}\times\frac{1}{4} = \frac{1}{5}$

$P(X=3) = \frac{4}{5}\times\frac{3}{4}\times\frac{1}{3} = \frac{1}{5}$

$P(X=4) = \frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2} = \frac{1}{5}$

$P(X=5) = \frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2} = \frac{1}{5}$

$\therefore P(X=k) = \frac{1}{5}$ 　 $k=1,2,3,4,5$

(2) $P(X=k) = \frac{1}{5}\times\left(\frac{4}{5}\right)^{k-1}$ 　 $k=1,2,3\cdots$

3. $P(X=0) = \frac{4}{5}$

$P(X=1) = \frac{1}{5}\times\frac{8}{9} = \frac{8}{45}$

$P(X=2) = \frac{1}{5}\times\frac{1}{9} = \frac{1}{45}$

$E(X) = 0\times\frac{4}{5} + 1\times\frac{8}{45} + 2\times\frac{1}{45} = \frac{2}{9}$

$E(X^2) = 0\times\frac{4}{5} + 1\times\frac{8}{45} + 4\times\frac{1}{45} = \frac{4}{15}$.

$Var(X) = E(X^2) - [E(X)]^2 = \frac{4}{15} - \frac{4}{81} = \frac{88}{405}$

4. $\left.\begin{aligned} E(X) &= \int_0^1 xf(x)\,dx = \int_0^1 bx^3+ax^2\,dx = \frac{1}{4}b+\frac{1}{3}a = \frac{2}{3} \\ \int_{-\infty}^{\infty} f(x)\,dx &= \int_0^1 ax+bx^2\,dx = \frac{1}{3}b+\frac{1}{2}a = 1 \end{aligned}\right\} \Rightarrow \begin{cases} a=2 \\ b=0. \end{cases}$

$E(X^2) = \int_0^1 x^2 f(x)\,dx = \int_0^1 bx^4+ax^3\,dx = \frac{1}{5}b+\frac{1}{4}a = \frac{1}{2}$

$Var(X) = E(X^2) - [E(X)]^2 = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$

5. $P_X(k) = \frac{\lambda^k}{k!} e^{-\lambda}$.

$P_X(k-1) = \frac{\lambda^{k}}{(k-1)!} e^{-\lambda}$

$P_X(k) - P_X(k-1) = e^{-\lambda} \frac{\lambda}{(k-1)!}\left(\frac{\lambda}{k} - 1\right)$.

When $\frac{\lambda}{k} - 1 \geq 0$, which means $k \leq \lambda \Rightarrow P(k) \geq P(k-1)$

When $\frac{\lambda}{k} - 1 \leq 0$, which means $k \geq \lambda \Rightarrow P(k) \leq P(k-1)$.

Thus $p_X(k)$ increases monotonically when $k \leq \lfloor \lambda \rfloor$.
and decreases monotonically when $k \geq \lceil \lambda \rceil$

6. Suppose when k peoples dead and the company loses money

$\Rightarrow$ $2000k > 12 \cdot 2500$

$k > 15$

Let X be the numper of deaths among the 2500 participants, then $X \sim \text{Binomial}(2500, 0.002)$. We would like to compute

$P(X > 15) = 1 - P(X \leq 15) = \sum_{k=0}^{15} \binom{2500}{k} 0.002^k \, 0.998^{2500-k}$

Use Possion $(2500 * 0.002) = \text{Possion}(5)$ to approximate Binomial$(2500, 0.002)$.

$P(X \leq 15) \approx \sum_{k=0}^{15} \frac{5^k}{k!} e^{-5} \approx 0.999930992$

$P(X > 15) \approx 0.000069008$

7. $\lambda = \frac{3}{60} = 0.05$ times per minutes

(1) suppose t is the time between jobs, then $t \sim \text{Exp}(\lambda)$

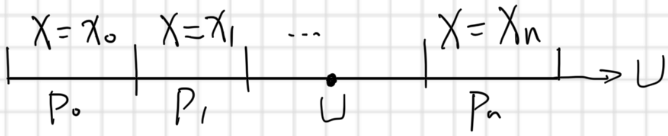
$E(t) = \frac{1}{\lambda} = 20$ miantes

(2) $P(t \leq 5) = 1 - e^{-0.05 \times 5} \approx 22.12\%$

8. Suppose we have a Geometric distribution $X \sim Geometric(p)$ and a Exponenential distribution $Y \sim Exp(\lambda)$

we define a Uniform random variabe U, it's distribution can draw as a line in $[0,1]$

① for the Geometric distribution, we can draw the X on the line



where U locate, we let $X = X_n$, thus we can built a X distribute in Geometric. Meanwhile, we can observe that the CDF of X $F(X)$ have $F(X-1) \leq U < F(x)$

From it we can solve the value of X

For Geometric distribution:

$F(x) = 1-(1-p)^x$

$$\Rightarrow 1-(1-p)^x > U \qquad\qquad 1-(1-p)^{x-1} \leq U$$

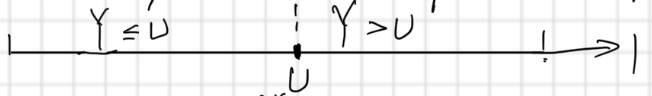
$$\Rightarrow (1-p)^x < 1-U \qquad\qquad x \leq \frac{\ln(1-U)}{\ln(1-p)} + 1$$

$$\Rightarrow x\ln(1-p) < \ln(1-U)$$

$$x > \frac{\ln(1-U)}{\ln(1-p)} \qquad\qquad \frac{\ln(1-U)}{\ln(1-p)} < X \leq \frac{\ln(1-U)}{\ln(1-p)} + 1$$

$$\therefore X = \left\lceil \frac{\ln(1-U)}{\ln(1-p)} \right\rceil$$

② similarily for the exponential function, but in this time $F(Y) = U$



$$\therefore F(Y) = 1-e^{-\lambda Y} = U$$

$$e^{-\lambda Y} = 1-U$$

$$-\lambda Y = \ln(1-U)$$

$$Y = -\frac{1}{\lambda}\ln(1-U).$$

It seems that X and Y are very similar, and with $-\lambda = \ln(1-p)$, we can say that the X is the ceiling of Y. In other word, Exponential distribution is the continuous analogue of the Geometric distribution. If we divide the discrete analogue into infinity parts, then we can get the Exp.