# Survey of Machine Learning Feature Selection Methods: Boruta, SVD, PCA, LASSO, RFE, ...

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Kansas City R Users Group
2018-09-08

https://github.com/EarlGlynn/kc-r-users-feature-selection

#### Continuation from last year ...



## Using R's Caret Package for Machine Learning

Earl F Glynn
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9 September 2017

https://github.com/EarlGlynn/kc-r-users-caret-2017

#### From last year ....

## Caret Machine Learning Examples

Group	Algorithms	Caret Model	FILE
Linear	Linear Discriminant Analysis	lda	lda
Methods	Linear Discriminant Analysis with	lda w/YeoJohnson	lda-YeoJohnson
	YeoJohnson preprocessing		
	LASSO, Ridge, and Elastic Net	glmnet	glmnet
	LASSO, Ridge, and Elastic Net	glmnet w/SMOTE	glmnet-SMOTE
	with Synthetic Minority Over-		
	Sampling Technique (SMOTE)		
Non-Linear	Neural Network	nnet	nnet
Methods	Support Vector Machine with Radial	svmRadial	svmRadial
	Basis Function Kernel		
	Naïve Bayes	nb	nb
	Naïve Bayes with Independent	nb w/ICA	nb-ica-
	Component Analysis (ICA)		transform
	k Nearest Neighbors	knn	knn
Trees and	J48	J48	J48
Rules	Classification and Regression Trees	rpart	rpart
Ensembles of	C5.0	C5.0	C50
Trees	Random Forest	rf	rf
	Random Forest with SMOTE	rf w/SMOTE	rf-SMOTE

https://github.com/EarlGlynn/kc-r-users-caret-2017

Files: Forensic-Glass-caret-FILE.Rmd and Forensic-Glass-caret-FILE.html

#### Survey of ...



- 1. Remove highly correlated variables.

  2. Run OLS and select significant features.

  3. Forward selection and backwards selection. 7.

  4. Random Forest feature importance.

  or re
- 5 Lasso.

BY CHRIS ALBON

or recursive

- 6. Boruta "All Relevant" Variables
- 7. SVD (Singular Value Decomposition)
- 8. PCA (Principal Component Analysis)

#### Small dataset used in examples:

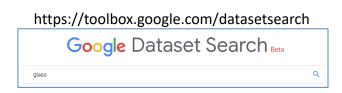
### Forensic Glass Dataset



https://sha.org/bottle/links.htm

#### fgl dataset in MASS package

- RI = refractive index
- Percentages by weight of oxides: Na, Mg, Al, Si, K, Ca, Ba, Fe
- Predict type from measured values above
  - window float glass (WinF)
  - window non-float glass (WinNF)
  - vehicle window glass (Veh)
  - containers (Con)
  - tableware (Tabl)
  - vehicle headlamps (Head)



Also available through <u>UCI Repository</u>.

Discussed in book <u>Data Mining and Business Analytics with R</u>.

## Forensic Glass Dataset

#### Forensic Glass Data

```
library (MASS)
                                  # fgl data
                                                        rawData <- fgl
                                                        dim(rawData)
                                                        [11 214 10
                                                        rawData
                                                                                                              %>%
                                                          kable("html", caption="Forensic Glass Data")
                                                                                                             %>%
                                                          kable styling(bootstrap options=c("striped", "bordered", "condensed"),
                                                                         position="left", font size=12,
                                                                         full width=FALSE)
                                                                                                              %>%
                                                          scroll box(height="200px")
                                                       Forensic Glass Data
                                                                               Si K Ca Ba Fe type
                                                         3.01 13.64 4.49 1.10 71.78 0.06 8.75 0.00 0.00 WinF
                                                        -0.39 | 13.89 | 3.60 | 1.36 | 72.73 | 0.48 | 7.83 | 0.00 | 0.00 | WinF
                                                        -1.82 | 13.53 | 3.55 | 1.54 | 72.99 | 0.39 | 7.78 | 0.00 | 0.00 | WinF
                                                        -0.34 | 13.21 | 3.69 | 1.29 | 72.61 | 0.57 | 8.22 | 0.00 | 0.00 | WinF
                                                         0 0 43 37 3 63 4 34 73 00 0 0 0 0 0 0 0 0 0 0 0 0
                                                        table(rawData$type)
                                                         WinF WinNF Veh Con Tabl Head
                                                              76 17 13 9 29 Class imbalance
```

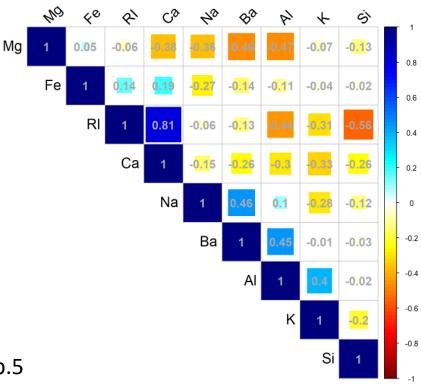
Files: Forensic-Glass-Boruta.Rmd and Forensic-Glass-Boruta.html

## 1. Remove Highly Correlated Variables

#### trainSet Correlation Matrix

Use only training set data or we'll have a data leak.

```
colorScale <- colorRampPalette(c("#7F0000", "red", "#FF7F00", "yellow", "white",
                                  "cyan", "#007FFF", "blue", "#00007F"))(100)
corMatrix <- cor(trainSet %>% dplyr::select(-type)) # Create correlation matrix
corrplot(corMatrix, type="upper", method="square", order="AOE",
         tl.col="black", tl.srt=45, tl.cex=1.5,
         addCoef.col="darkgrev", number.cex=1.25,
         col=colorScale)
mtext("Correlation Matrix (angular order of eigenvectors)", line=3)
```



Correlation Matrix (angular order of eigenvectors)

order="AOE" (angular order of eigenvalues) http://www.datavis.ca/papers/corrgram.pdf, p.5

Files: Forensic-Glass-Correlation.Rmd and Forensic-Glass-Correlation.html

## Remove Highly Correlated Variables

I often use a cutoff of 0.9 or above, but here 0.8 is used to trigger a removal.

#### Removing highly correlated pairs with caret

Some machine learning algorithms are impeded by highly correlated predictors. Caret's findCorrelation procedure can be used to remove one of the highly-correlated pairs.

```
HIGH_CORRELATION_CUTOFF <- 0.80

corHigh <- findCorrelation(corMatrix, HIGH_CORRELATION_CUTOFF)

if (length(corHigh) > 0)
{
    cat("Removing highly-correlated variable(s): ", names(trainSet)[corHigh])
    trainSet <- trainSet[, -corHigh]
    testSet <- testSet[, -corHigh]
}</pre>
```

```
Removing highly-correlated variable(s): Ca
```

Note: Remove constant variables, too.

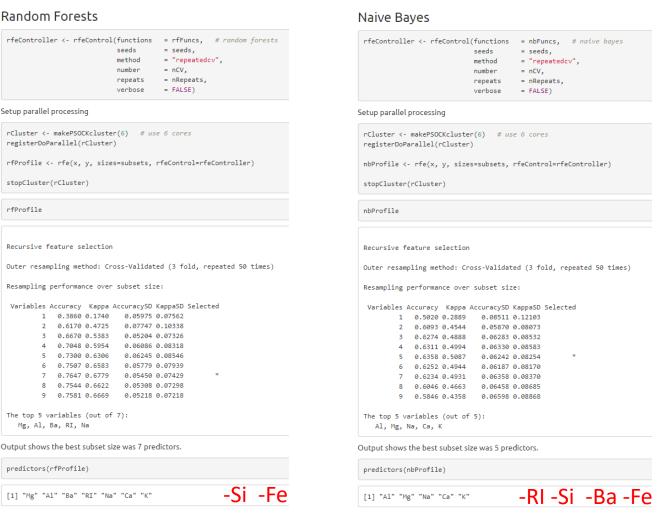
Files: Forensic-Glass-Correlation.Rmd and Forensic-Glass-Correlation.html

## 2. Run OLS and select significant features

OLS = Ordinary Least Square, but approach also applies to NLS = Non-linear Least Squares, e.g.,
Levenberg-Marquardt non-linear curve fitting

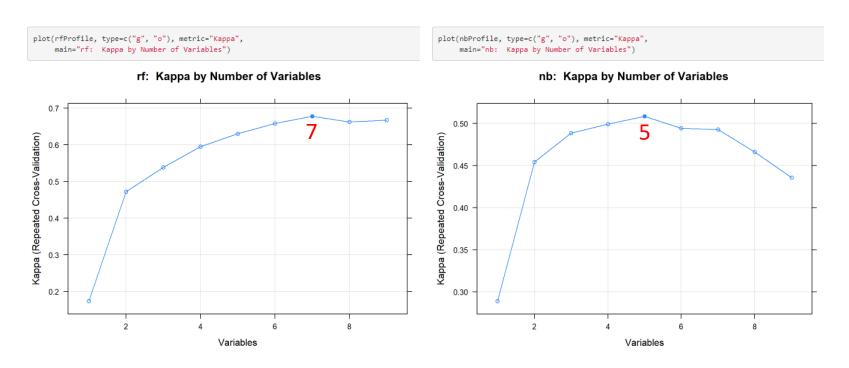
library(minpack.lm) # nls.lm

## 3. Caret's Recursive Feature Extraction (RFE)



Files: Forensic-Glass-caret-RFE.Rmd and Forensic-Glass-caret-RFE.html

## Caret's Recursive Feature Extraction (RFE)



Kappa is generally a better metric than Accuracy with imbalanced classification.

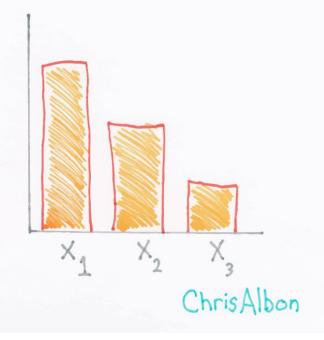
Files: Forensic-Glass-RFE.Rmd and Forensic-Glass-RFE.html

## 4. Feature Importance



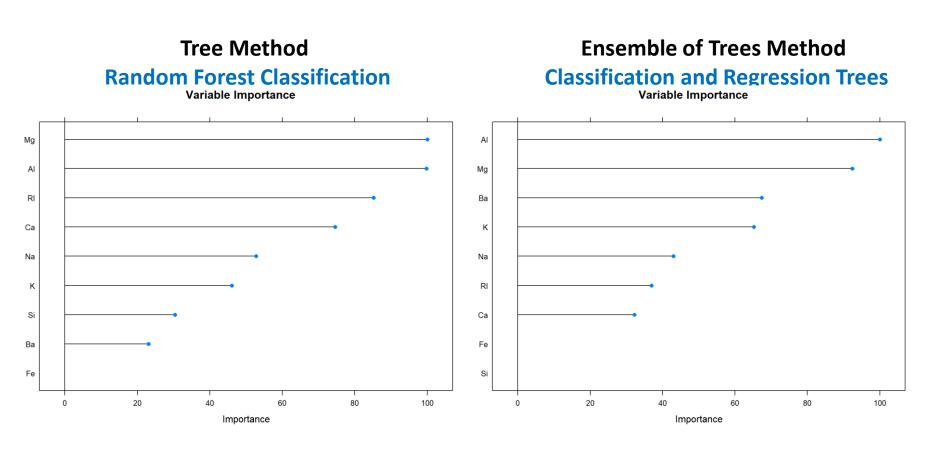
Decision trees make splits that maximize the decrease in impurity.

By calculating the mean decrease in impurity for each feature across all trees we can know that feature's importance.



## 4. Feature Importance

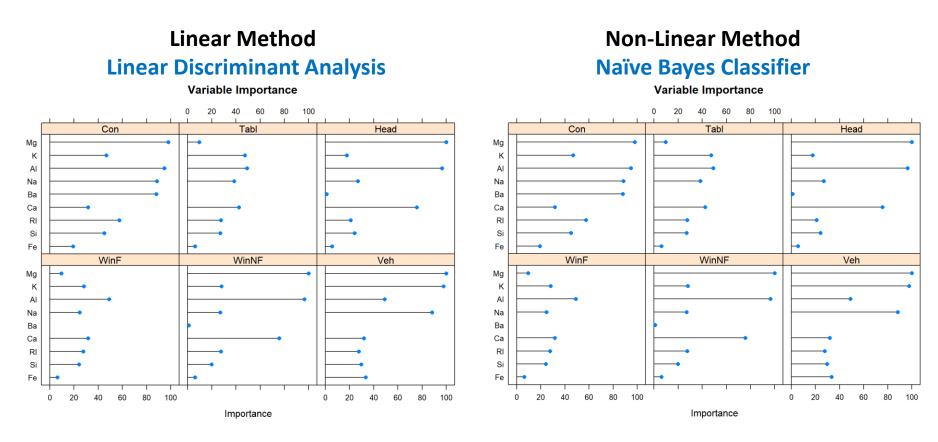
Use feature importance from one or more machine learning models to select features. Study consistency and differences among models. Single Predictors here.



2017 Files: Forensic-Glass-caret-rf.html and Forensic-Glass-caret-rpart.html

## Feature Importance

Use feature importance from one or more machine learning models to select features. Study consistency and differences among models. Separate Predictors, Very Similar.

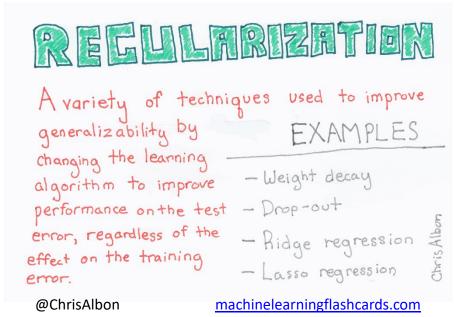


2017 Files: Forensic-Glass-caret-Ida.html and Forensic-Glass-caret-nb.html

## 5. glmnet

Ridge, Lasso and Elastic-Net Regularized Generalized Linear Models ( $\alpha$ =0 for Ridge,  $\alpha$ =1 for Lasso)

- Math is a bit complicated:
  - https://quantmacro.wordpress.com/2016/04/26/fitting-elastic-net-model-in-r/https://web.stanford.edu/~hastie/glmnet/glmnet\_alpha.html
- Idea is to force some coefficients to zero to exclude from model.



## Feature Selection glmnet

#### glmnet (lasso and elastic-net regularization)

```
tuneGrid <- expand.grid(alpha = seq(0.25, 0.75, by=0.05), # alpha 1 for Lasso, 0 for Ridge
                       lambda = c(0.05, 0.005, 0.0005)) # strength of penalty on coefficients
set.seed(29)
CVfolds <- 5 # 5-fold cross validation (not enough data for 10 fold here)
CVrepeats <- 10 # repeat 10 times
# Used createMultiFolds to study
indexFolds <- createMultiFolds(trainSet$type, CVfolds, CVrepeats) # for repeated CV
trainControlParms <- trainControl(method = "repeatedcv", # repeated cross validation
                                 number = CVfolds,
                                 repeats = CVrepeats,
                                 index = indexFolds,
                                 classProbs = TRUE,  # Estimate class probabilities
                                 summarvFunction = defaultSummarv)
fit <- train(type ~ ., data=trainSet,
            preProcess = c("center", "scale"),
            method = "glmnet",
            metric = "Kappa",
            tuneGrid = tuneGrid,
            trControl = trainControlParms)
```

Forensic-Glass-caret-glmnet.Rmd and Forensic-Glass-caret-glmnet.html

## glmnet

Each class has a separate set of coefficients.

Fit coefficients for best  $\lambda$ :

coef(fit\$finalModel, s = fit\$bestTune\$lambda)

```
10 x 1 sparse Matrix of class "dgCMatrix"
(Intercept) 1.207375322
           0.250065503
Na
           -0.224390621
Mg
            1.630701058
Δ1
           -1.402125023
Si
            0.004345775
K
Ca
            0.277709942
Ba
Fe
$WinNF
10 x 1 sparse Matrix of class "dgCMatrix"
(Intercept) 2.0909299
RI
Na
           -0.7154609
Mg
            0.1743042
A1
Si
            -0.9109666
K
            0.1107151
Ca
            -0.1779174
Ba
Fe
            0.2913854
10 x 1 sparse Matrix of class "dgCMatrix"
(Intercept) 0.16121177
RI
           -1.02662928
Na
Mg
            0.56240805
A1
           -1.36706675
Si
           -1.29460588
K
           -0.48120821
Ca
            0.05261939
Ba
Fe
            0.14586035
```

```
10 x 1 sparse Matrix of class "dgCMatrix"
(Intercept) -1.5639444
RТ
            -0.6234961
Mq
            -1.3894046
            1.4634459
Si
            0.1103138
K
            1.0771763
Ca
            0.5982293
            -0.5683051
            0.4363922
10 x 1 sparse Matrix of class "dqCMatrix"
(Intercept) -1.1941202
            1.4674086
Mg
            -0.1975055
A1
Si
            0.4315638
K
            -1.7011573
            0.1962890
            -0.6153015
            -0.6018880
10 x 1 sparse Matrix of class "dgCMatrix"
(Intercept) -0.70145244
            0.86756205
            1.23281956
Na
            -0.78050324
Al
            1.29083825
Si
            0.02129146
K
Ca
            -0.82978126
Ba
            1.29411893
            -0.65922012
```

glmnet creates a separate model for each class.

Some coefficients are driven to zero by the method, which effectively excludes predictor from model.

glmnet can be used as a prediction model, or as a tool to select relevant variables for other models.

### 6. Boruta "All Relevant" Variables

#### Define train and test datasets

```
set.seed(71)

trainSetIndices <- createDataPartition(rawData$type, p=0.50, list=FALSE)

trainSet <- rawData[ trainSetIndices, ]

testSet <- rawData[-trainSetIndices, ]

dim(trainSet)

## [1] 109 10</pre>
```

#### Boruta 'All Relevant' Variables

Setup parallel processing



Boruta was a demon/god in Slavic mythology, who lived in trees. https://www.pinterest.com/pin/497577458807361039/

### Boruta "All Relevant" Variables

```
Boruta performed 48 iterations in 1.187219 secs.
8 attributes confirmed important: Al, Ba, Ca, K, Mg and 3 more;
1 attributes confirmed unimportant: Fe;
```

#### **Basic Idea of Boruta Algorithm**

Perform shuffling of predictors' values and join them with the original predictors and then build random forest on the merged dataset. Then make comparison of original variables with the randomised variables to measure variable importance. Only variables having higher importance than that of the randomised variables are considered important.

https://www.listendata.com/2017/05/feature-selection-boruta-package.html https://www.datacamp.com/community/tutorials/feature-selection-R-boruta

Files: Forensic-Glass-Boruta.Rmd and Forensic-Glass-Boruta.html

## Boruta "All Relevant" Variables

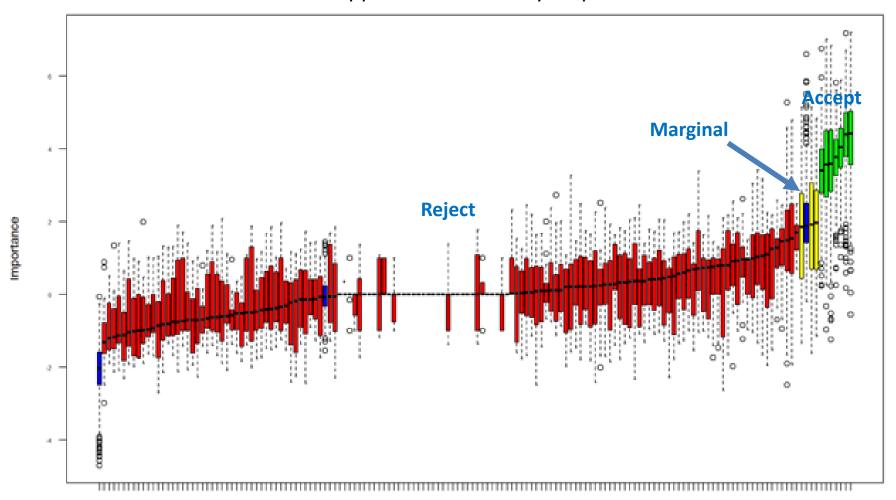
plot(BorutaModel, las=2, cex.axis=0.75, main="Boruta Importance")
grid()

## **Boruta Importance** 0.25 0.20 0.15 Accept Importance 0.05 Reject -0.00 Attributes

Files: Forensic-Glass-Boruta.Rmd and Forensic-Glass-Boruta.html

### Boruta "All Relevant" Variables

Sometimes Boruta approach can be very helpful with selections



### Boruta "All Relevant" Variables

```
stats <- attStats(BorutaModel)
statsOrdered <- stats[order(stats$medianImp, decreasing=TRUE), ]</pre>
```

#### Boruta Features

Feature	meanImp	medianImp	minImp	maxImp	normHits	decision
Mg	0.2127641	0.2125817	0.1883694	0.2417506	1.0000000	Confirmed
Al	0.1406346	0.1412464	0.1204132	0.1553545	1.0000000	Confirmed
K	0.1400205	0.1408165	0.1181545	0.1625252	1.0000000	Confirmed
Na	0.1260519	0.1267271	0.1045717	0.1443913	1.0000000	Confirmed
Ca	0.0953332	0.0965700	0.0795888	0.1080054	1.0000000	Confirmed
RI	0.0860814	0.0849388	0.0722158	0.1030754	1.0000000	Confirmed
Si	0.0676871	0.0681237	0.0576441	0.0776154	1.0000000	Confirmed
Ba	0.0630710	0.0633130	0.0504127	0.0762487	1.0000000	Confirmed
Fe	0.0113589	0.0108034	0.0033247	0.0246047	0.2708333	Rejected

## Boruta "All Relevant" Variables

Importance History

plotImpHistory(BorutaModel, main="Importance History")
grid()

10

20

Classifier run

0.20

0.15

0.10

0.05

0.00

Importance

30

40

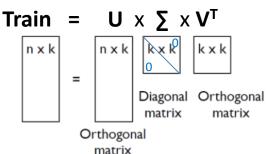
## 7. Singular Value Decomposition

- Dataset is a matrix of 214 rows of 9 predictors.
- Original matrix can be written as a product of three matrices with svd.
- Diagonal of middle matrix contains the singular values (eigenvalues if centered data).
- Right-most matrix contains the right singular vectors, which can give information about feature importance.

#### Train Matrix = $U \times \sum \times V^T$

In R:

```
svd1 <- svd(scale(trainData))
svd1$u %*% diag(svd1$d) %*% t(svd1$v)</pre>
```



U = left singular vectors = 214-by-9 unitary matrix. svd1\$u in R.

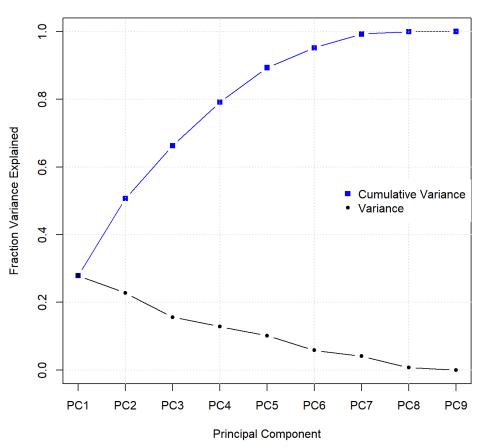
 $\Sigma$  = singular values matrix = 9-by-9 diagonal matrix. diag(svd1\$d) in R.

The diagonal terms are the singular values, usually listed in decreasing order.

 $V^T$  = transpose of V, where V = right singular vectors = 9x9 unitary matrix. svd1\$v in R.

## Singular Value Decomposition





Singular values are usually in decreasing order. 1<sup>st</sup> one largest.

Square of n<sup>th</sup> singular value proportional to variance associated with n<sup>th</sup> singular vector.

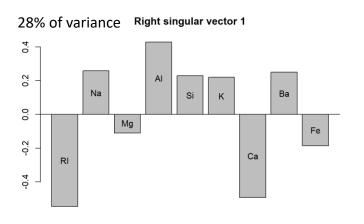
Often few terms explain large % of total variance.

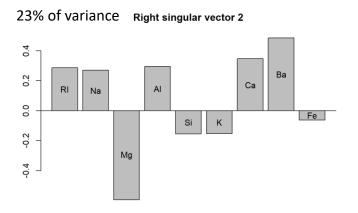
#### Cumulative variance explained:

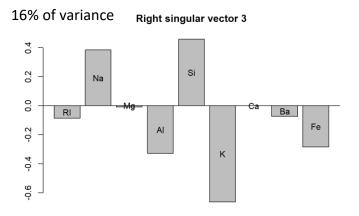
```
[1] 0.2790 0.5068 0.6629 0.7915
0.8931 0.9517 0.9927 0.9998
1.0000
```

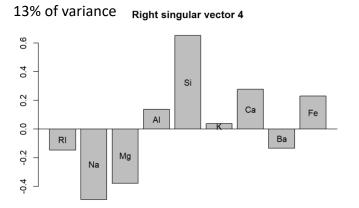
Files: Forensic-Glass-SVD.Rmd and Forensic-Glass-SVD.html

## Singular Value Decomposition





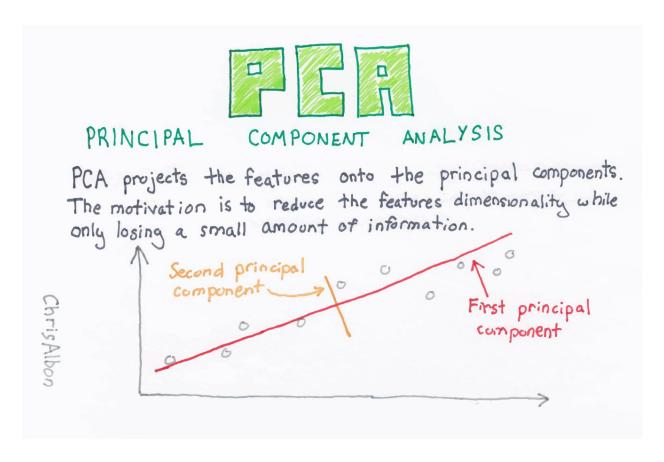




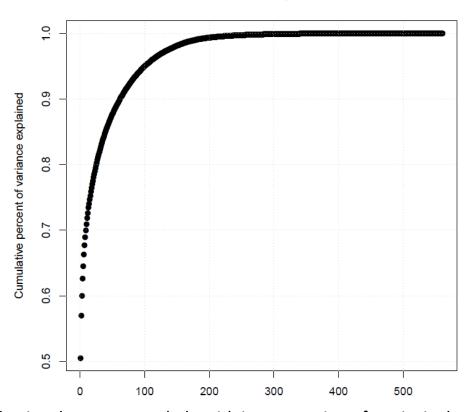
Pick variables with large contributions (+ or -): 1: RI, Ca, Al; 2: Mg, Ba; 3: K, Si

## 8. Principal Component Analysis

PCA often computed from SVD



## Feature Selection SVD / PCA Samsung



Right singular vector can help with interpretation of a principal component. Here, 9 of 500+ PCs explained over 70% of total variance.

Could use 9 PCs in machine learning that explain over 70% of variance instead of 500+ predictors.

## Principal Component Analysis

#### Setup

NOTE: place tidyverse after MASS below to avoid dplyr::select function clashes with MASS::select.

```
library(MASS) # fgl data
library(tidyverse) # place after MASS to avoid select conflict
library(caret) # preProcess, predict
library(rgl) # par3d, plot3d, movie3d, rglwidget
library(RColorBrewer) # brewer.pal
```

#### Principal Component Analysis

Let's compute the values for the first 4 principal components using caret's pca pre-processing.

"pca" requires "center" and "scale".

These four PCs account for nearly 80% of variance.

```
nPCAcomponents <- 4
transformSetup <- preProcess(rawData, method=c("center", "scale", "pca"), pcaComp=nPCAcomponents)
pcaScores <- predict(transformSetup, rawData)</pre>
```

```
pcaScores %>% head(10)
```

```
PC1 PC2 PC3 PC4

1 -1.148446843 -0.5282491 0.3712253 -1.72485681

2 0.572794160 -0.7580105 0.5554059 -0.75845396

3 0.937960515 -0.9276609 0.5536094 -0.20577184

4 0.141750924 -0.9594279 0.1168507 -0.41475157

5 0.350271021 -1.0886966 0.4839440 -0.06894065
```

## Principal Component Analysis

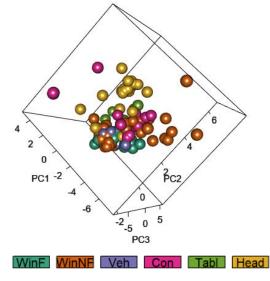
- caret's preProcess gives the same PCAscores as computed with SVD.
- Each PC is a weighted linear combination of all variables.
- PCs are orthogonal.
- PCs can be used as variables in other machine learning algorithms.
- Machine learning algorithm using PCs as predictors are limited by the amount of variance explained by the original variables in the given number of PCs.

## Interactive Exploratory Analysis Principal Component Analysis

Use **plot3d** in **rgl** package to create interactive 3D scatterplot of any three PCs

The first 3 PCs account for about 66% of variance in data.

Other sets of 3 PCs could be dislayed alternatively in 3D space, such as PC2, PC3, PC4.



Chrome browser works best to display above figure.

Drag mouse over figure to rotate. Use mouse wheel to zoom in and out.

## Exploratory Analysis Principal Component Analysis

## magick from ImageMagick needed to create animated GIF of 3D PCA scatterplot

#### **Animated GIF**

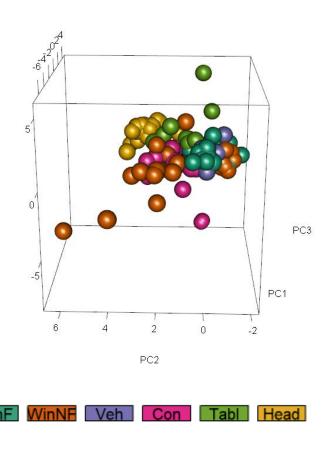
Create the animated GIF using magick from ImageMagick – this takes some time. Display below using HTML.

150 PNG images will be computed for 15 sec duration \* 10 frames/second.

Here's the HTML needed in the R Markdown document to embed the GIF into the HTML file created with knitr.

```
<div id="PCA">
  <img src="ForensicGlass-PCA.gif" alt="">
  </div>
```

## Exploratory Analysis Principal Component Analysis



## Take Home

- Variety of ways to select features for machine learning models.
- Explore several methods.
- Boruta often easy to apply without much data preparation. Many methods require preprocessing, like centering and scaling.
- Experiment only with training data to avoid "data leaks" and overfitting.