Assignment 02 - Interpolation and Curve fitting

ESO208 - Computational methods in engineering | 2024-25 I
Instructor: Balaji Devaraju [`dbalaji`]
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1. Given the following data points:

x	1	2	3	5	7	8
$f(x)$	3	6	19	99	291	444

   i. Using Newton's interpolating polynomials of orders 1 through 4, calculate the value of $f(4)$. Analyze the results to determine the likely order of the polynomial that was originally used to generate the data provided in the table.

   ii. Repeat the calculations using Lagrange polynomials of orders 1 through 5. Consider adding an additional data point, such as $(9, 510)$. Reconstruct the polynomial with this new point included, and analyze how its addition affects the polynomial, particularly in terms of its degree and overall shape.

   iii. Estimate the value of f(4) using linear, quadratic, and natural cubic splines.

2. Given the following data, use least-squares regression to fit the models:

x	5	10	15	20	25
y	10	18	25	30	33

   i. A straight line.

   ii. A parabola.

   iii. A power equation.

   iv. A saturation-growth-rate equation.

$$y = \frac{a * x}{b + x}$$

   where a, b are the model parameters.

   Plot the data along with all the curves. Is any one of the curves superior? If so, justify.

3. Use Müller's method to find the positive real root of the following functions:

   i. $f(x) = x^3 + x^2 - 4x - 4$

   ii. $f(x) = x^3 - 0.5x^2 + 4x - 2$

Bonus Question

4. Given the following data:

x	1	50	100	200	250	300	350
$F(x)$	5.3	6.1	7.5	9.3	11.2	7.3	6

   i. Fit a polynomial curve using least squares regression analysis.

ii. Compute a least squares quadratic polynomial approximation using Legendre polynomials of 3 order as the basis functions.

iii. Using the true absolute error, determine which polynomial provides a better approximation of the function at $x = 50$, $x = 200$, and $x = 300$. Justify your conclusion with analytical reasoning.

Programming Problems

5. Develop a Python program to implement Bairstow's method and determine the real and complex roots for the following functions:

   i. $f(x) = 0.7x^3 - 4x^2 + 6.2x - 2$

   ii. $f(x) = -3.704x^3 + 16.3x^2 - 21.97x + 9.34$

   iii. $f(x) = x^4 - 2x^3 + 6x^2 - 2x + 5$

6. During a soil consolidation test, settlement data were recorded at specific time intervals. The recorded data is available through the provided link Soil Consolidation Data (CSV) . To gain a more comprehensive understanding of the settlement behavior over time, it is necessary to estimate the settlement at additional time points that were not originally measured:

   i. Estimate the settlement at 6 and 23 days using Lagranges Interpolation.

   ii. Compare the results obtained from Newton's Divided Difference method with those derived from Lagranges Interpolation.

   iii. Discuss the advantages and limitations of each interpolation method in the context of soil consolidation data.

   iv. Create a cubic spline interpolation of the data and plot the settlement curve.

7. Stress-strain data obtained from a triaxial test is given in the provided link. Stress-strain data (CSV) The following tasks are required:

   i. Fit the stress-strain data using the hyperbolic model given by the equation:

   $$y = \frac{x}{a + bx}$$

   where y is the stress, x is the strain, and a and b are model parameters.

   ii. Use polynomial regression to fit a higher-order curve to the data. Employ the matrix approach to obtain the polynomial coefficients.

   iii. For both models (hyperbolic and polynomial), plot the stress-strain data along with the fitted equations.

   iv. Compute the standard error of the estimate and the correlation coefficient for each model. Analyze the results to determine the goodness of fit.