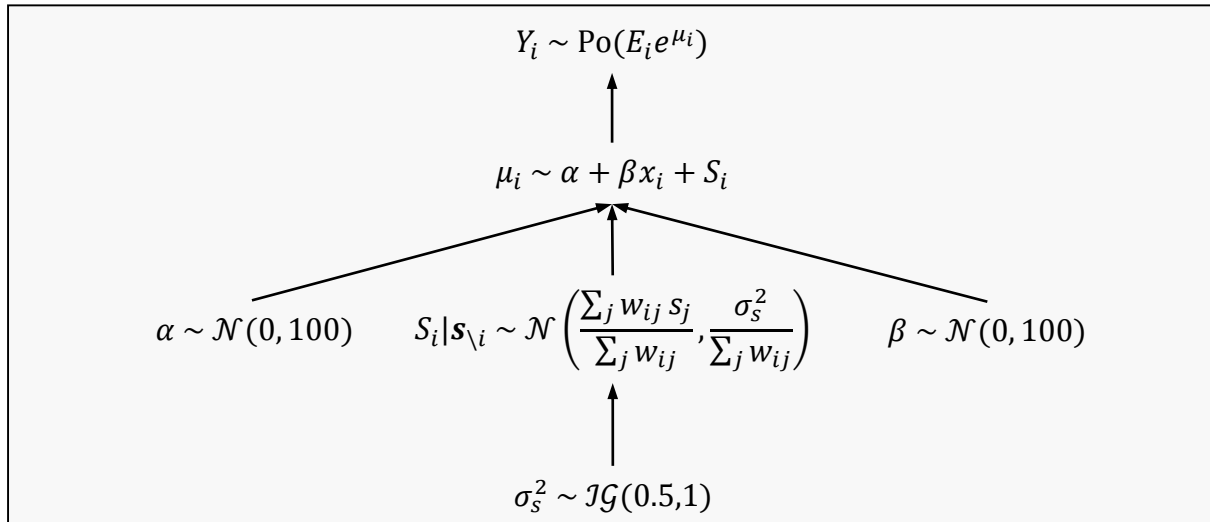


## ICAR Model (Besag 1974; Besag, York, Mollié 1991)



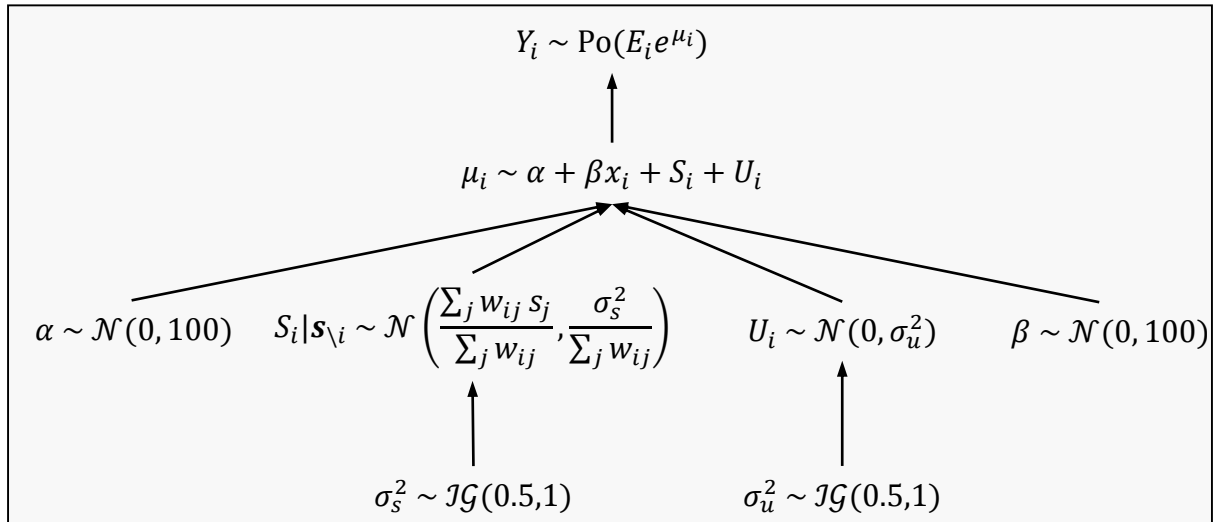
Concordance of parameter names, and values used in CARBayes to define the priors:

Model parameter		S.CARleroux argument	Value
Schematic	CARBayes		
$S, R$	$\phi$	--	
$\alpha, \beta$	$\beta^*$	prior.mean.beta	c(0, 0)
		prior.var.beta	c(100, 100)
$\sigma_s^2$	$\tau^2$	prior.tau2	c(0.5, 1)
(1)	$\rho^{**}$	fix.rho	TRUE
		rho	1
<b>W</b>	<b>W</b>	W	W

\* In the CARBayes package,  $\beta$  includes the intercept, so the prior for both  $\alpha$  and  $\beta$  must have the same *form* of distribution, but potentially different parameters.

\*\* The ICAR model is a special case of the Leroux model when the spatial autocorrelation  $\rho$  is fixed at 1. Hence, the S.CARleroux R function is used.

## BYM Model (Besag, York, Mollié 1991)

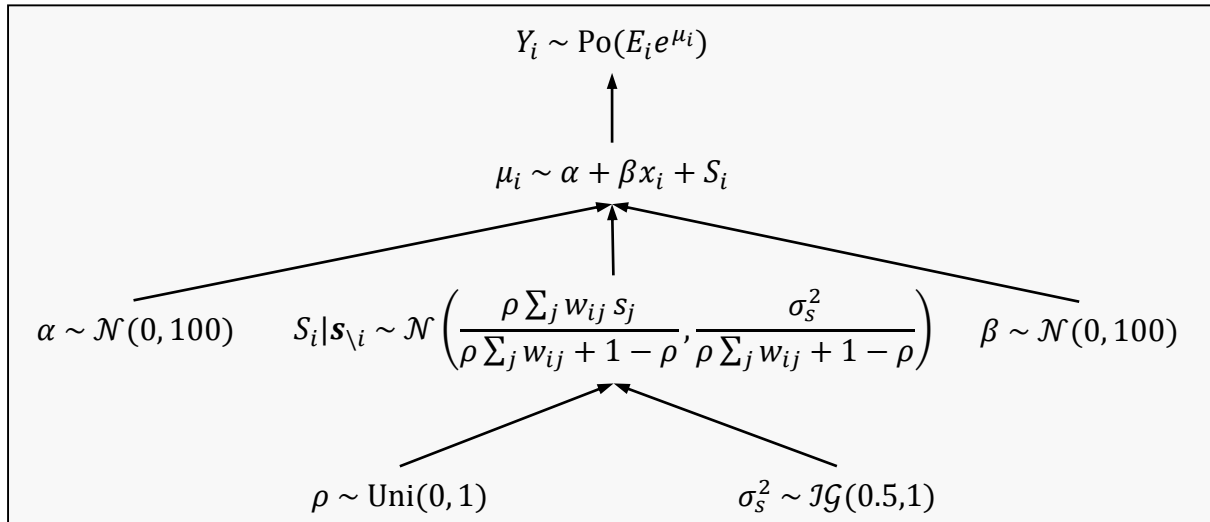


Concordance of parameter names, and values used in CARBayes to define the priors:

Model parameter		S . CARbym argument	Value
Schematic	CARBayes		
$R$	$\psi$	--	
$\alpha, \beta$	$\beta^*$	prior.mean.beta	c(0, 0)
		prior.var.beta	c(100, 100)
$\sigma_s^2$	$\tau^2$	prior.tau2	c(0.5, 1)
$\sigma_u^2$	$\sigma^2$	prior.sigma2	c(0.5, 1)
<b>W</b>	<b>W</b>	W	W

\* In the CARBayes package,  $\beta$  includes the intercept, so the prior for both  $\alpha$  and  $\beta$  must have the same *form* of distribution, but potentially different parameters.

## Leroux Model (Leroux et al. 2000)



Concordance of parameter names, and values used in CARBayes to define the priors:

Model parameter		S.CARleroux argument	Value
Schematic	CARBayes		
$S, R$	$\phi$	--	
$\alpha, \beta$	$\beta^*$	prior.mean.beta	c(0, 0)
		prior.var.beta	c(100, 100)
$\sigma_s^2$	$\tau^2$	prior.tau2	c(0.5, 1)
$\rho$	$\rho$	fix.rho	FALSE
		rho	NULL
<b>W</b>	<b>W</b>	W	W

\* In the CARBayes package,  $\beta$  includes the intercept, so the prior for both  $\alpha$  and  $\beta$  must have the same *form* of distribution, but potentially different parameters.

## Dissimilarity Model (Lee and Mitchell 2012)

This model is based on the Leroux model, except here  $\rho$  is fixed at 0.99 to ensure strong global spatial smoothing, while the weights are estimated rather than fixed. The weights are estimated according to the dissimilarity between areas, as determined by a non-negative dissimilarity metric  $\mathbf{Z}$  with elements

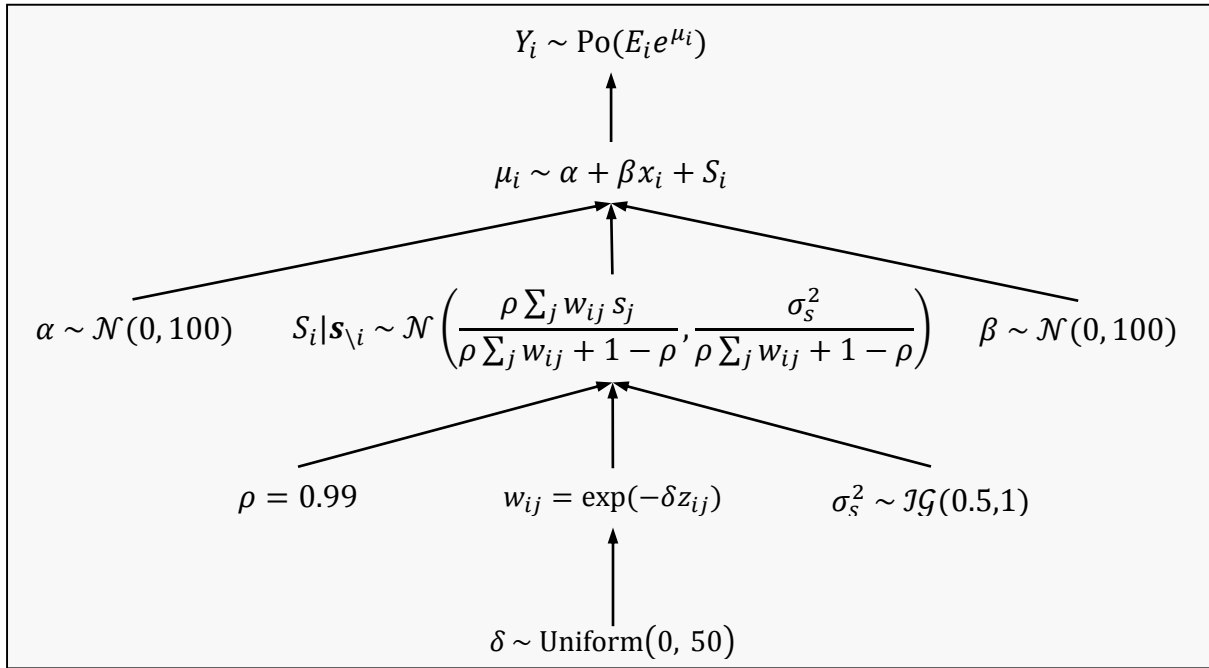
$$z_{ij} = \frac{|z_i - z_j|}{\sigma_z}$$

where  $\sigma_z$  is the standard deviation of  $|z_i - z_j|$  over all pairs of contiguous areas. This metric is then treated as a regression parameter, weighted by  $\delta$ . The metric can be based on geography (e.g. distance between area centroids), covariates (which would then be excluded from the stage 2 of the model), or residuals from another model. The weights can be either binary or non-binary. The non-binary formulation is given by:

$$w_{ij}(\delta, z_{ij}) = \exp(-\delta z_{ij})$$

$$\delta \sim \text{Uniform}(0, 50).$$

The larger the dissimilarity measure  $z_{ij}$ , the closer to zero the weights  $w_{ij}$  will be.



Concordance of parameter names, and values used in CARBayes to define the priors:

Model parameter		S.CARdissimilarity argument	Value
Schematic	CARBayes		
$S, R$	$\phi S$	--	
$\alpha, \beta$	$\beta^*$	prior.mean.beta	c(0, 0)
		prior.var.beta	c(100, 100)
$\sigma_s^2$	$\tau^2$	prior.tau2	c(0.5, 1)
$\rho$	$\rho$	fix.rho	TRUE
		rho	0.99
<b>W</b>	<b>W</b>	W	W
		W.binary	TRUE or FALSE

<b>Z</b>	<b>Z</b>	Z	list(Z = Z)
$\delta$	$\alpha$	--	

\* In the CARBayes package,  $\beta$  includes the intercept, so the prior for both  $\alpha$  and  $\beta$  must have the same *form* of distribution, but potentially different parameters.

## References

- Besag, J. 1974. Spatial interaction and the statistical analysis of lattice systems. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* **36** (2): 192-236.
- Besag, J., J. York, and A. Mollié. 1991. Bayesian image restoration with application in spatial statistics. *Annals of the Institute of Statistical Mathematics* **43** (1):1-20. doi: 10.1007/BF00116466.
- Leroux, B. G., X. Lei, and N. Breslow. 2000. "Estimation of disease rates in small areas: a new mixed model for spatial dependence". In *Statistical models in epidemiology, the environment and clinical trials*, edited by M. E. Halloran and D. Berry, pp. 179-191. The IMA Volumes in Mathematics and its Applications, vol 116. New York: Springer. doi: 10.1007/978-1-4612-1284-3\_4.
- Lee, D. and R. Mitchell. 2012. Boundary detection in disease mapping studies. *Biostatistics* **13** (3): 415-426. doi: 10.1093/biostatistics/kxr036.

See also the CARBayes paper and documentation:

Lee, D. 2013. CARBayes: An R package for Bayesian spatial modelling with conditional autoregressive priors. *Journal of Statistical Software* **55** (13): 1-24. URL: <http://www.jstatsoft.org/v55/i13/>.

<https://cran.r-project.org/web/packages/CARBayes/CARBayes.pdf>

<https://cran.r-project.org/web/packages/CARBayes/vignettes/CARBayes.pdf>