ICAR Model (Besag 1974; Besag, York, Mollié 1991)

$$Y_i \sim \mathrm{Po}(E_i e^{\mu_i})$$

$$\mu_i \sim \alpha + \beta x_i + S_i$$

$$\alpha \sim \mathcal{N}(0, 100) \qquad S_i|\boldsymbol{s}_{\setminus i} \sim \mathcal{N}\left(\frac{\sum_j w_{ij}\, s_j}{\sum_j w_{ij}}, \frac{\sigma_s^2}{\sum_j w_{ij}}\right) \qquad \beta \sim \mathcal{N}(0, 100)$$

$$\sigma_s^2 \sim \mathcal{IG}(0.5, 1)$$

Concordance of parameter names, and values used in CARBayes to define the priors:

Model parameter		`S.CARleroux` argument	Value
Schematic	CARBayes		
S, R	ϕ	`--`	
α, β	β *	`prior.mean.beta`	`c(0, 0)`
		`prior.var.beta`	`c(100, 100)`
σ_s^2	τ^2	`prior.tau2`	`c(0.5, 1)`
(1)	ρ **	`fix.rho`	`TRUE`
		`rho`	`1`
W	**W**	`W`	`W`

* In the CARBayes package, β includes the intercept, so the prior for both α and β must have the same *form* of distribution, but potentially different parameters.

** The ICAR model is a special case of the Leroux model when the spatial autocorrelation ρ is fixed at 1. Hence, the `S.CARleroux` R function is used.

BYM Model (Besag, York, Mollié 1991)

$$Y_i \sim \mathrm{Po}(E_i e^{\mu_i})$$

$$\mu_i \sim \alpha + \beta x_i + S_i + U_i$$

$$\alpha \sim \mathcal{N}(0, 100) \qquad S_i | \boldsymbol{s}_{\setminus i} \sim \mathcal{N}\left(\frac{\sum_j w_{ij}\, s_j}{\sum_j w_{ij}}, \frac{\sigma_s^2}{\sum_j w_{ij}}\right) \qquad U_i \sim \mathcal{N}(0, \sigma_u^2) \qquad \beta \sim \mathcal{N}(0, 100)$$

$$\sigma_s^2 \sim \mathcal{IG}(0.5,1) \qquad \sigma_u^2 \sim \mathcal{IG}(0.5,1)$$

Concordance of parameter names, and values used in CARBayes to define the priors:

Model parameter			
Schematic	CARBayes	`S.CARbym` argument	Value
R	ψ	`--`	
α, β	β*	`prior.mean.beta`	`c(0, 0)`
		`prior.var.beta`	`c(100, 100)`
σ_s^2	τ^2	`prior.tau2`	`c(0.5, 1)`
σ_u^2	σ^2	`prior.sigma2`	`c(0.5, 1)`
W	**W**	`W`	`W`

* In the CARBayes package, β includes the intercept, so the prior for both α and β must have the same *form* of distribution, but potentially different parameters.

Leroux Model (Leroux et al. 2000)

$$Y_i \sim \mathrm{Po}(E_i e^{\mu_i})$$

$$\mu_i \sim \alpha + \beta x_i + S_i$$

$$\alpha \sim \mathcal{N}(0, 100) \qquad S_i | \boldsymbol{s}_{\backslash i} \sim \mathcal{N}\left(\frac{\rho \sum_j w_{ij} s_j}{\rho \sum_j w_{ij} + 1 - \rho}, \frac{\sigma_s^2}{\rho \sum_j w_{ij} + 1 - \rho}\right) \qquad \beta \sim \mathcal{N}(0, 100)$$

$$\rho \sim \mathrm{Uni}(0, 1) \qquad \sigma_s^2 \sim \mathcal{IG}(0.5,1)$$

Concordance of parameter names, and values used in CARBayes to define the priors:

Model parameter		`S.CARleroux` argument	Value
Schematic	CARBayes		
S, R	ϕ	`--`	
α, β	β*	`prior.mean.beta`	`c(0, 0)`
		`prior.var.beta`	`c(100, 100)`
σ_s^2	τ^2	`prior.tau2`	`c(0.5, 1)`
ρ	ρ	`fix.rho`	`FALSE`
		`rho`	`NULL`
$\mathbf{W}$	$\mathbf{W}$	`W`	`W`

* In the CARBayes package, β includes the intercept, so the prior for both α and β must have the same *form* of distribution, but potentially different parameters.

Dissimilarity Model (Lee and Mitchell 2012)

This model is based on the Leroux model, except here ρ is fixed at 0.99 to ensure strong global spatial smoothing, while the weights are estimated rather than fixed. The weights are estimated according to the dissimilarity between areas, as determined by a non-negative dissimilarity metric **Z** with elements

$$z_{ij} = \frac{|z_i - z_j|}{\sigma_z}$$

where σ_z is the standard deviation of $|z_i - z_j|$ over all pairs of contiguous areas. This metric is then treated as a regression parameter, weighted by δ. The metric can be based on geography (e.g. distance between area centroids), covariates (which would then be excluded from the stage 2 of the model), or residuals from another model. The weights can be either binary or non-binary. The non-binary formulation is given by:

$$w_{ij}(\delta, z_{ij}) = \exp(-\delta z_{ij})$$
$$\delta \sim \text{Uniform}(0, 50).$$

The larger the dissimilarity measure z_{ij}, the closer to zero the weights w_{ij} will be.

$$Y_i \sim \text{Po}(E_i e^{\mu_i})$$

$$\mu_i \sim \alpha + \beta x_i + S_i$$

$$\alpha \sim \mathcal{N}(0, 100) \qquad S_i | \boldsymbol{s}_{\setminus i} \sim \mathcal{N}\left(\frac{\rho \sum_j w_{ij} s_j}{\rho \sum_j w_{ij} + 1 - \rho}, \frac{\sigma_s^2}{\rho \sum_j w_{ij} + 1 - \rho}\right) \qquad \beta \sim \mathcal{N}(0, 100)$$

$$\rho = 0.99 \qquad w_{ij} = \exp(-\delta z_{ij}) \qquad \sigma_s^2 \sim \mathcal{IG}(0.5, 1)$$

$$\delta \sim \text{Uniform}(0,\ 50)$$

Concordance of parameter names, and values used in CARBayes to define the priors:

Model parameter		`S.CARdissimilarity` argument	Value
Schematic	CARBayes		
S, R	$\phi\ S$	--	
α, β	$\beta*$	`prior.mean.beta`	`c(0, 0)`
		`prior.var.beta`	`c(100, 100)`
σ_s^2	τ^2	`prior.tau2`	`c(0.5, 1)`
ρ	ρ	`fix.rho`	`TRUE`
		`rho`	`0.99`
W	**W**	`W`	`W`
		`W.binary`	`TRUE` or `FALSE`

Z	**Z**	`Z`	`list(Z = Z)`
δ	α	`--`	

* In the CARBayes package, β includes the intercept, so the prior for both α and β must have the same *form* of distribution, but potentially different parameters.

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See also the CARBayes paper and documentation:

Lee, D. 2013. CARBayes: An R package for Bayesian spatial modelling with conditional autoregressive priors. *Journal of Statistical Software* **55** (13): 1-24. URL: http://www.jstatsoft.org/v55/i13/.

https://cran.r-project.org/web/packages/CARBayes/CARBayes.pdf

https://cran.r-project.org/web/packages/CARBayes/vignettes/CARBayes.pdf