Abgabe - Übungsblatt [3]

Einführung in die Computergraphik und Visualisierung

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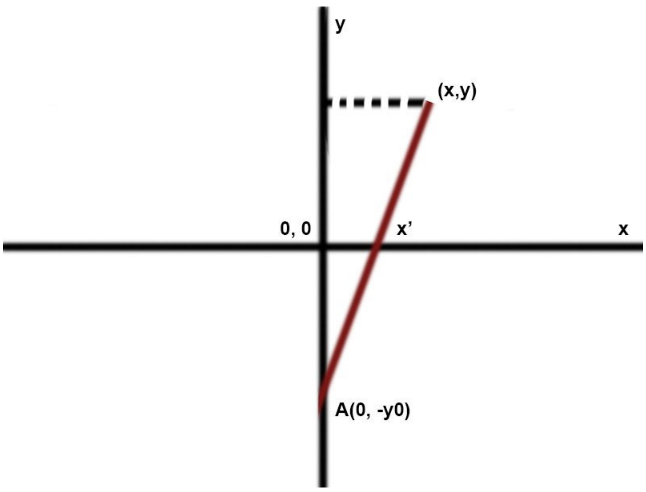
First Exercise

- (a) $p_1, p_2 \in P(\mathbb{R}^3)$
  Remember that $(a, b, c, d)^T = (\frac{a}{d}, \frac{b}{d}, \frac{c}{d})^T$ if $d \neq 0, \Rightarrow$
  $p_1 = (\frac{14}{2}, \frac{3}{2}, \frac{4}{2}) = (7, 1.5, 2)$,
  $p_2 = (\frac{12}{3}, \frac{0}{3}, \frac{3}{3}) = (6, 0, 1)$
- (b) Since we cannot divide by 0(otherwise we will get vector of infinite length), we cannot represent in $\mathbb{R}^3(a, b, c, 0)$

Second Exercise

We will use ideas, similar to the lecture.



$\triangle\ (A; (0; y); (x; y)) \sim \triangle\ (A; (0; 0); (0; x')) \Rightarrow$
$\frac{x'}{x} = \frac{y_0}{y+y_0} \Leftrightarrow x' = \frac{y_0 x}{y+y_0}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x \\ y \end{pmatrix} - \frac{y}{y+y_0} \begin{pmatrix} x \\ y+y_0 \end{pmatrix}$$

Let's represent with the a homogenious 3x3 matrix:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} y_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & y_0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{y_0} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

We can decompose it into two mappings: a perspective transformation followed by a parallel projection:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{y_0} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{y_0} & 1 \end{pmatrix}$$

Where left matrix is parallel projection and right perspective transformation.

Third Exercise

Quternions q and $-q$ represent the same rotation, and it can be show by reproducing the calculation of the rotation axis and angle.
For $-q$ the rotation of point p will be written as $p' = q^{-1}pq$.
Thus the rotation axis has to be transformed into itself:
$(\cos\theta - r\sin\theta)r(\cos\theta + r\sin\theta = \cos\theta^2 r - \sin\theta^2 r^3 = r$
And the angle is calculated as:
$\langle R(q), q\rangle = \langle(\cos\theta - r\sin\theta)q(\cos\theta + r\sin\theta), q\rangle =$
$= \langle\cos\theta^2 q + \sin\theta\cos\theta r \times q \sin\theta\cos\theta q \times r - \sin\theta^2 q, q\rangle =$
$= \cos\theta^2 - \sin\theta^2 = \cos 2\theta$
Therefore, as the axis and the angle are the same, this inverse quaternion represents the same rotation.