Abgabe - Übungsblatt 2

Einführung in die Computergraphik und Visualisierung

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Exercise 1

Point p can be embedded in a quaternion as
$p \mapsto q_p = (0, p) = xi + yj + zk$.

(a) Give a formula to determine the angle of rotation α.
The angle of rotation is an angle between $\vec{p_1}$ and $\vec{p_2}$, where $\vec{p} = p - O$ and O is the center of coordinates.
$\alpha = \arccos\left(\frac{\vec{p_1} \cdot \vec{p_2}}{\|\vec{p_1}\|\|\vec{p_2}\|}\right)$

(b) Give a formula to detemine the rotation axis $\mathbf{v}$.
Rotation axis $\mathbf{v}$ is perpendicular to $\vec{p_1}$ and $\vec{p_2}$ and goes through the coordinates center. Therefore, it is the normalized cross-product of the two vectors.
$\mathbf{v} = \frac{\vec{p_1} \times \vec{p_2}}{\|\vec{p_1} \times \vec{p_2}\|}$.

(c) Write down the quaternion $\mathbf{q}$ which performs the rotation with angle α around $\mathbf{v}$.
$\mathbf{q} = (\cos(\alpha/2), \mathbf{v_x} \sin(\alpha/2), \mathbf{v_y} \sin(\alpha/2), \mathbf{v_z} \sin(\alpha/2))$.

(d) Write down the relationship between p_1 and p_2 using quaternion multiplication.
$p_1 \mapsto q_{p1} = (0, p_1) = x_1 i + y_1 j + z_1 k$.
$p_2 \mapsto q_{p2} = (0, p_2) = x_2 i + y_2 j + z_2 k$.
$q_{p2} = \mathbf{q} q_{p1} \mathbf{q^{-1}}$.

Exercise 2

Composite transformation can be created as a multiplication of every transformation matrix in the corresponding order.

(a) Derive a matrix M_1 which first rotates the point α degrees (α is given in radians) around the axis $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$ and then performs a translation with an offset of $\begin{pmatrix} t_1 & t_2 & t_3 \end{pmatrix}^T$.

$$M_1 = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & t_1\cos\alpha - t_2\sin\alpha \\ \sin\alpha & \cos\alpha & 0 & t_1\sin\alpha + t_2\cos\alpha \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Derive a matrix M_1 which performs a translation with an offset of $\begin{pmatrix} t_1 & t_2 & t_3 \end{pmatrix}^T$ and then rotates the point α degrees (α is given in radians) around the axis $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$.

$$M_2 = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & t_1 \\ \sin\alpha & \cos\alpha & 0 & t_2 \\ 0 & 0 & 1 & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(c) In which way affects the order of operations the respective final transformation matrix in this case?
The importance of operations order can be shown on this picture:

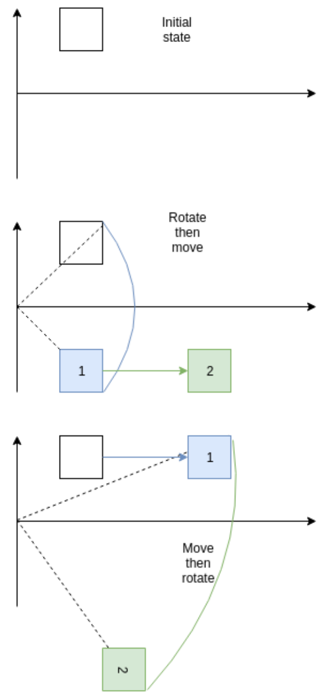

Abbildung 1: Visualistion of transformations in different order

The order of transformations affects the final matrix, because matrix multiplication is not commutative. That is why in the second case(rotation first) the final matrix is just a composition of two (everything gets multiplied by 1), and in the first case the final matrix gets a bit more complicated and the translation part of the matrix changes its x and y values. If for example the rotation was around y axis, then $x-$ and $z-$translations would be affected.