Abgabe - Übungsblatt 2

Einführung in die Computergraphik und Visualisierung

Svetlana Shishkovets Tran Linn Chi Viktor Lopatin

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Exercise 1

Point p can be embedded in a quaternion as
$p \mapsto q_p = (0, p) = xi + yj + zk$.

(a) Give a formula to determine the angle of rotation α.
The angle of rotation is an angle between $\vec{p_1}$ and $\vec{p_2}$, where $\vec{p} = p - O$ and O is the center of coordinates.
$\alpha = \arccos\left(\frac{\vec{p_1} \cdot \vec{p_2}}{\|\vec{p_1}\| \|\vec{p_2}\|}\right)$

(b) Give a formula to detemine the rotation axis $\mathbf{v}$.
Rotation axis $\mathbf{v}$ is perpendicular to $\vec{p_1}$ and $\vec{p_2}$ and goes through the coordinates center. Therefore, it is the normalized cross-product of the two vectors.
$\mathbf{v} = \frac{\vec{p_1} \times \vec{p_2}}{\|\vec{p_1} \times \vec{p_2}\|}$.

(c) Write down the quaternion $\mathbf{q}$ which performs the rotation with angle α around $\mathbf{v}$.
$\mathbf{q} = (\cos(\alpha/2), \mathbf{v_x} \sin(\alpha/2), \mathbf{v_y} \sin(\alpha/2), \mathbf{v_z} \sin(\alpha/2))$.

(d) Write down the relationship between p_1 and p_2 using quaternion multiplication.
$p_1 \mapsto q_{p1} = (0, p_1) = x_1 i + y_1 j + z_1 k$.
$p_2 \mapsto q_{p2} = (0, p_2) = x_2 i + y_2 j + z_2 k$.
$q_{p2} = \mathbf{q} q_{p1} \mathbf{q^{-1}}$.

Exercise 2