Week1

> section 1.4 and 1.5

propositions:

predicates: a property that the subject of a statement can have.

using p(x) to annotate

once the variable x has been assigned, the statement P(x) becomes a proposition

when we have multiple variables, called P(x1,x2,x3...xn)

Quantification

- universal : "all" whether is always true
- existential : "some" one or more elements is true

P(x) for all valuables of x is in the domain

$$\forall x P(x)$$

true if all the domain of x makes P(x) true

Existential quantifier

$$\exists x P(x)$$

true if some of x make P(x) true

Restricted domain

$$\forall x < 0(x^2 > 0) == \forall x(x < 0 \longrightarrow x^2 > 0)$$

$$\exists z > 0(z^2 > 0) == \exists z(z > 0 \longrightarrow z^2 = 2)$$

> the restriction of an universal quantification is the same as conditional statement
>
> the restriction of an existential quantification is the same as conjunction

Precedence

annotation like

$$\exists$$
$$\forall$$

have a higher procedure than other annotations

Binding Variables

$$\forall x(x + y = z)$$

x: bound

y,x : free

scope: the expression that the variable is applied

$$the\ scope\ \ of\ \forall\ is\ (x+y=z)$$

Logical Equivalences

Statements involving predicates and quantifiers are *logically equivalent* if and only if they **have the same truth value** no matter which predicates are substituted into these statements and which **domain of discourse is used** for the variables in these propositional functions. We use the notation *S* ≡ *T* to indicate that two statements *S* and *T* involving predicates and quantifiers are logically equivalent.

$$\forall x(P(x) \wedge Q(x)) \equiv \forall x(P(x)) \wedge \forall Q(x)$$

$$De\ Morgan's\ Law$$
$$\neg\forall xP(x) \equiv \exists x\neg P(x)$$
$$\neg\exists xP(x) \equiv \forall x\neg P(x)$$

All logical laws

	Equivalence	Name
1	$p \wedge T \equiv p$	Identity laws
2	$p \vee F \equiv p$	
3	$p \vee T \equiv T$	Domination laws
4	$p \wedge F \equiv F$	
5	$p \vee p \equiv p$	Idempotent laws
6	$p \wedge p \equiv p$	
7	$\neg(\neg p) \equiv p$	Double negation law
8	$p \vee q \equiv q \vee p$	Commutative laws
9	$p \wedge q \equiv q \wedge p$	

	Equivalence	Name
10	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative laws
11	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	
12	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive laws
13	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	
14	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's laws
15	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	
16	$p \vee (p \wedge q) \equiv p$	Absorption laws
17	$p \wedge (p \vee q) \equiv p$	
18	$p \vee \neg p \equiv T$	Negation laws
19	$p \wedge \neg p \equiv F$	

20	$p \rightarrow q \equiv \neg p \vee q$
21	$p \rightarrow q \equiv \neg q \rightarrow \neg p$
22	$p \vee q \equiv \neg p \rightarrow q$
23	$p \wedge q \equiv \neg(p \rightarrow \neg q)$
24	$\neg(p \rightarrow q) \equiv p \wedge \neg q$
25	$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
26	$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
27	$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
28	$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

29	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
30	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
31	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
32	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Nested Quantifiers

$$\forall x \exists y (x + y = 0)$$

both x and y are bound

scope: (x+y=0)

When quantifiers are of the same quantity (all universal or all existential), the order does not matter.

But when they are mixed, the order becomes crucial.

Negating Nested Quantifiers

example

$$\begin{gathered} \neg \forall x \exists y (xy = 1) \\ \exists x \neg \exists y (xy = 1) \\ \exists x \forall y (xy\, ! = 1) \end{gathered}$$

Homework

prove that

$$\forall x (P(x) \wedge Q(x)) \;\equiv\; \forall x P(x) \wedge \forall x Q(x)$$

> the proof should use twice that is from left to right and from right to left.

> the proof is just using natural language but not using logical annotation