Tutorial01

Q1

use a truth table to prove its tautology

$$(p \to (q \lor r) \leftrightarrow (p \land \neg q) \to r)$$

$$(p \to (q \to r)) \leftrightarrow ((p \land q) \to r)$$

try to use the truth table, it's easy anyway.

Q2

1. **Show that (**$p \to r$**) ∨ (**$q \to r$**)and (**$p \land q$**)** $\to r$ **are logically equivalent**

$$\begin{gathered}(p \land q) \to r \\ = \neg(p \land q) \lor r \;||\; rule(20) \\ = (\neg p \lor \neg q) \lor r \;||\; De\ Morgan's\ laws \\ = (\neg p \lor r) \lor (\neg q \lor r) \\ = (p \to r) \lor (q \to r)\end{gathered}$$

2. **Show that ¬**$p \to$ **(**$q \to r$**) and** $q \to$ **(**$p \lor r$**) are logically equivalent**

$$\begin{gathered}\neg p \to (q \to r) \\ = p \lor (\neg q \lor r) \;||\; rule(20) \\ = \neg q \lor (p \lor r) \;||\; commutative\ laws \\ = q \to (p \lor r) \;||\; rule(20)\end{gathered}$$

3. **Show that (p→ q) → r and p→ (q → r) are not equivalent**

$$\begin{gathered}(p \to q) \to r \\ = \neg(\neg p \lor q) \lor r \;||\; rule(20) \\ = (p \land \neg q) \lor r \;||\; De\ Morgan's\ law\end{gathered}$$

$$\begin{gathered}p \to (q \to r) \\ = \neg p \lor (\neg q \lor r) \;||\; rule(20) \\ = (\neg p \lor \neg q) \lor r \;||\; commutative\ law\end{gathered}$$

in this case:

$$(p \land q) \lor r \;\; != (\neg p \lor \neg q) \lor r$$

Q3

Let $P\ x$, $Q\ x$, and $R\ x$ be the statements “x is a professor,” “x is ignorant,” and “x is vain,” respectively. Express each of these statements using quantifiers; logical connectives; and P x , $Q\ x$, and $R\ x$, where the domain consists of all people.
a) No professors are ignorant.

$$\forall x \neg(P(x) \wedge Q(x))$$

b) All ignorant people are vain.

$$\forall x Q(x) \rightarrow R(x)$$

c) No professors are vain.

$$\neg \exists x P(x) \wedge R(x)$$

d) Does (c) follow from (a) and (b)?

$$no, for\ \ not\ \ all\ \ vain\ \ people\ \ are\ \ ignorant$$

Q4

Use quantifiers to express the statement that “There is a woman who has taken a flight on every airline in the world.”

P(x) a person is a women

Q(x) a person take a flight on every airline in the world

$$\exists x P(x) \wedge Q(x)$$

Use quantifiers to express the statement that “There does not exist a woman who has taken a flight on every airline in the world.”

$$\neg \exists x P(x) \wedge Q(x)$$

Q5

Suppose that the domain of $Q\ x, y, z$ consists of triples x, y, z , where x = 0, 1, or 2, y = 0 or 1 , and z = 0 or 1 . Write out these propositions using disjunctions and conjunctions.
a) ∀y Q(0, y, 0)

$$Q(0,0,0) \wedge Q(0,1,0)$$

b) $\exists x\ Q\ (x$, 1,1)

$$Q(0,1,1) \vee Q(1,1,1) \vee Q(2,1,1)$$

c) ∃z¬ Q(0,0, z)

$$?$$

d) $\exists x \neg\ Q(x$, 0,1)

$$?$$

Q6

Use rules of inference to show that if $\forall x\,(P(x) \to (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.