Week02 Inference rules

> section 1.6

Argument

An **argument** in propositional logic is a sequence of propositions. All but the final proposition in the argument are called **premises** and the final proposition is called the **conclusion**. An argument is valid if the truth of all its premises implies that the conclusion is true.

> argument form is true + premises are all true ==> conclusion is true

Inference Rules for Propositional Logic

Modus Ponens

$$\begin{array}{rl} & p \rightarrow q \\ & p \\ \hline \therefore & q \end{array}$$

The tautology $(p \wedge (p \rightarrow q)) \rightarrow q$ is the basis of the inference rule modus ponens.

Modus Tollens

$$\begin{array}{rl} & p \rightarrow q \\ & \neg q \\ \hline \therefore & \neg p \end{array}$$

Hypothetical syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Disjunctive syllogism

$$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$$

Addition

$$\begin{array}{l} p \\ \hline \therefore p \lor q \end{array}$$

Simplification

$$\begin{array}{l} p \land q \\ \hline \therefore p \end{array}$$

Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Resolution

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

exercise

Prove r, assuming the following:

$$t$$
$$\neg(\neg q \vee \neg p) \wedge q$$
$$p \vee \neg q$$
$$\neg(\neg q \wedge \neg p) \wedge (t \rightarrow (\neg(p \wedge \neg r) \wedge (p \rightarrow r)))$$

$$\begin{gathered} \neg(\neg q \vee \neg p) \wedge q \\ = (q \wedge p) \wedge q \\ p \; is \; true \\ \\ \neg(\neg q \wedge \neg p) \; is \; true \\ \\ that \; is \; to \; prove \; p \rightarrow r \; is \; true \\ r \; is \; true \\ Q.E.D \end{gathered}$$

Presenting... Give control Stop presenting

Steps	Reason
1. $\neg(\neg q \wedge \neg p) \wedge (t \to (\neg(p \wedge \neg r) \wedge (p \to r)))$	Premise
2. $\neg(\neg q \wedge \neg p)$	Simplification from
3. $q \vee p$	De Morgan's
4. $p \vee \neg q$	Premise.
5. p	Resolution from ③ ④
6. t	Premise.
7. $t \to (\neg(p \wedge \neg r) \wedge (p \to r))$	Simplification from 1.
8. $\neg(p \wedge \neg r) \wedge (p \to r)$	Modus ponens.
9. $\neg(p \wedge \neg r)$	Simplification ⑧
10. $\neg p \vee r$	negation
11. p	Disjunctive syllogism
12 r	

Rules of Inference for Quantified Statements

Universal instantiation

$$\begin{array}{l} \forall x\, P(x) \\ \hline \therefore P(c) \end{array}$$

Universal Generalization

$$\begin{array}{l} P(c) \text{ for an arbitrary } c \\ \hline \therefore \forall x\, P(x) \end{array}$$

Existential Instantiation

$$\begin{array}{l} \exists x\, P(x) \\ \hline \therefore P(c) \text{ for some element } c \end{array}$$

Existential Generalization

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x\, P(x)}$$

exercise1

Show that the premises

- Everyone in this discrete mathematics class has taken a course in computer science.
- Marla is a student in this class.

imply the conclusion

- Marla has taken a course in computer science.

D(x) one is in discrete class

C(x) x has taken a course in CS

$$\forall x(D(x) \rightarrow C(x))$$
$$D(Marla)\ is\ true$$
$$C(Marla)\ is\ true$$

exercise2

Show that the premises

- A student in this class has not read the book.
- Everyone in this class passed the first exam.

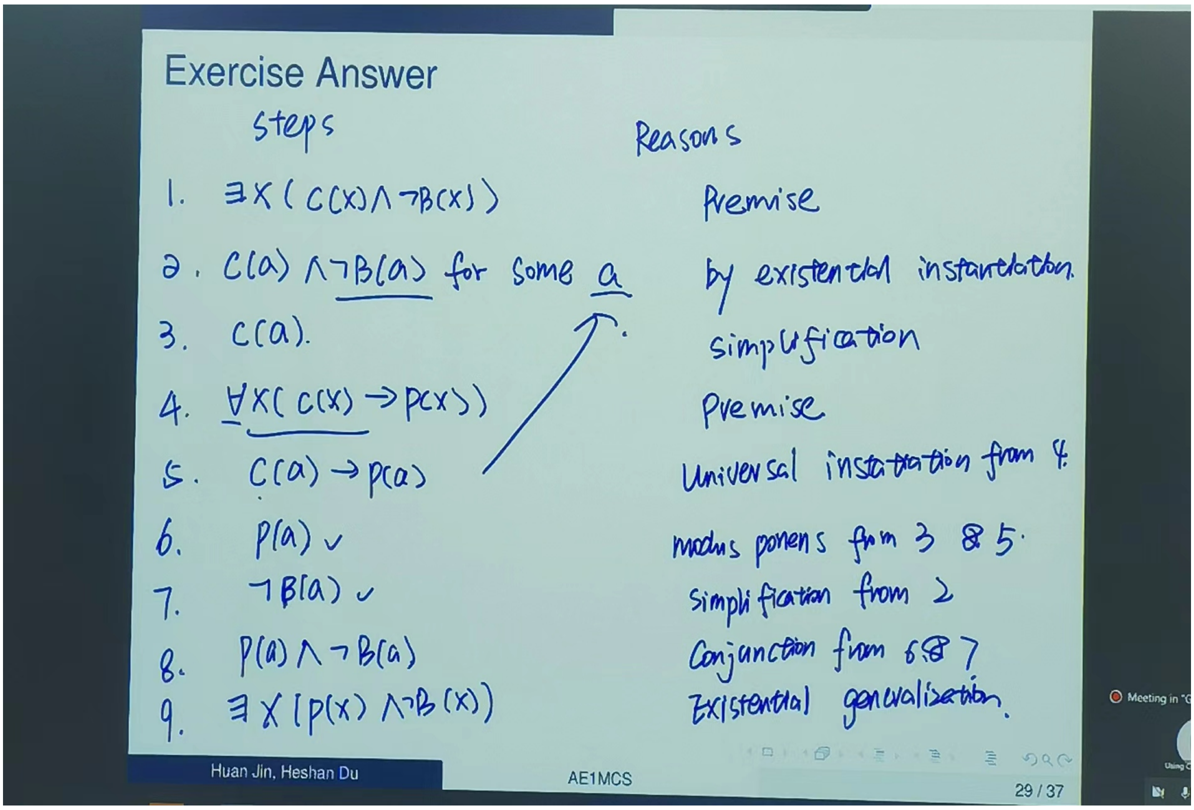
imply the conclusion

- Someone who passed the first exam has not read the book.

C(x) x is in the class

B(x) x has read the book

P(x) x has pass the exam


Exercise Answer
Steps
Reasons
1. ∃X (C(X) ∧ ¬B(X))
Premise
2. C(a) ∧ ¬B(a) for some a
by existential instantiation
3. C(a).
simplification
4. ∀X(C(X) → P(X))
Premise
5. C(a) → P(a)
Universal instantiation from 4.
6. P(a) ✓
modus ponens from 3 & 5.
7. ¬B(a) ✓
simplification from 2
8. P(a) ∧ ¬B(a)
Conjunction from 6 & 7
9. ∃X (P(X) ∧ ¬B(X))
Existential generalisation.
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Meeting in "G

exercise3