Week3 Set

Reading

Chapter 2, Section 2.1. Sets
Chapter 2, Section 2.2. Set Operations

Definition

A *set* is an unordered collection of objects, called *elements* or *members* of the set. A set is said to contain its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

Describe a Set

1. List all the members of a set (if it is possible):
   e.g. $\{a, b, c\}$, $\{1, a\}$, $\{1, 2, 3, ..., 99\}$ (positive integers < 100)
2. Use *set builder* notation: characterize all elements in a set by stating the property or properties they must have.
   - $O = \{x \mid x \text{ is an odd positive integer less than } 10\}$
   - or $O = \{x \in \mathbb{Z}^+ \mid x \text{ is odd and } x < 10\}$

Important Sets

N = the set of natural numbers
Z = the set of integers
Z+ = the set of positive integers
Q = the set of rational numbers
R = the set of real numbers
R+ or R>0 = the set of positive real numbers

Equal Sets

Two sets are equal if and only if they have the same elements

two sets are the same if and only if

$$\forall x\,(x \in A \leftrightarrow x \in B)$$

Empty Set and Singleton Set

Empty set: a set that has no element. It is denoted by $\emptyset$ or $\{\}$.
Singleton set: a set that has only one element.

Venn Diagram

The universal set U, which contains all the objects under consideration, is represented by a rectangle.

Subsets

The set A is a subset of B if and only if every element of A is also an element of B

$$A \subseteq B \text{ iff } \forall x\,(x \in A \rightarrow x \in B)$$

Proper Subset

A is a *proper subset* of B ($A \subset B$) if and only if

$$\forall x\,(x \in A \rightarrow x \in B) \wedge \exists x\,(x \in B \wedge x \notin A)$$

Equal Sets

$$A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A.$$

The Size of a Set (cardinality of Set)

If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a **finite set** and that n is the **size of the set**

Power Sets

Given a set S, the *power set* of S is the set of all subsets of the set S. The power set of S is denoted by $\mathcal{P}(S)$.

Ordered n-tuples

Definition

The *ordered n-tuple* $(a_1, a_2, ..., a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element,..., and a_n as its nth element.

- We say that two ordered n-tuples are equal if and only if each corresponding pair of their elements is equal.
- Ordered 2-tuples are called *ordered pairs*.

Cartesian products

Let A and B be sets. The *Cartesian product* of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence, $A \times B = \{(a, b) \mid a \in A \land b \in B\}$.

> AXB != BXA

$$A^n = \{(a_1, a_2, ..., a_n) \mid a_i \in A \textit{ for } i = 1, 2, ..., n\}$$

Using Set Notation with Quantifiers

$$\exists x \in S\ P(x) \equiv \exists x\ (x \in S \land P(x))$$

$$\forall x \in S\ P(x) \equiv \forall x\ (x \in S \rightarrow P(x))$$

Truth Sets and Quantifiers

$\forall x\ P(x)$ is true over the domain U if and only if the truth set of P is the set U.

$\exists x\ P(x)$ is true over the domain U if and only if the truth set of P is nonempty.

Set Operations

Union

$$A \cup B = \{x \mid x \in A \lor x \in B\}$$

Intersection

$$A \cap B = \{x \mid x \in A \land x \in B\}$$

Difference

$$A - B = \{x \mid x \in A \land x \notin B\}$$

Complement

$$\overline{A} = \{x \in U \mid x \notin A\}$$

Difference and Complement

$$A - B = A \cap \overline{B}$$

Set Identities

	Identity	Name
1	$A \cap U = A$	Identity laws
2	$A \cup \emptyset = A$	
3	$A \cup U = U$	Domination laws
4	$A \cap \emptyset = \emptyset$	
5	$A \cup A = A$	Idempotent laws
6	$A \cap A = A$	
7	$\overline{(\overline{A})} = A$	Complementation law
8	$A \cup B = B \cup A$	Commutative laws
9	$A \cap B = B \cap A$	

	Identity	Name
10	$A \cup (B \cup C) = (A \cup B) \cup C$	Associative laws
11	$A \cap (B \cap C) = (A \cap B) \cap C$	
12	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
13	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	
14	$\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws
15	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	
16	$A \cup (A \cap B) = A$	Absorption laws
17	$A \cap (A \cup B) = A$	
18	$A \cup \overline{A} = U$	Complement laws
19	$A \cap \overline{A} = \emptyset$	

Generalized Unions and Intersections

Definition

The *union* of a collection of sets is the set that contains those elements that are members of *at least one* set in the collection.

Definition

The *intersection* of a collection of sets is the set that contains those elements that are members of *all* the sets in the collection.

Geometric Progression (Sequences)

A *geometric progression* is a sequence of the form

$$a, ar, ar^2, ..., ar^n, ...$$

where the initial term a and the common ratio r are real numbers.

Arithmetic Progression (Sequences)

An *arithmetic progression* is a sequence of the form

$$a, a+d, a+2d, ..., a+nd, ...$$

where the initial term a and the common difference d are real numbers.

Recurrence Relation

A *recurrence relation* for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms of the sequence, namely, $a_0, a_1, ..., a_{n-1}$, for all integers n with $n \geq n_0$, where n_0 is a nonnegative integer. A sequence is called a *solution* of a recurrence relation if its terms satisfy the recurrence relation.

e.g. $a_0 = 1$. $a_{n+1} = a_n + 1$ for $n = 0, 1, 2, ...$