INF 212
TURING MACHINES

Alan Turing

Alan Turing was one of the founding fathers of CS.

- His computer model –the Turing Machine– was inspiration/premonition of the electronic computer that came two decades later
- Was instrumental in cracking the Nazi Enigma cryptosystem in WWII
- Invented the "Turing Test" used in AI
- Legacy: The Turing Award.

A Thinking Machine

First Goal of Turing’s Machine: A model that can compute anything that a human can compute. Before invention of electronic computers the term “computer” referred to a *person* who’s line of work is to calculate numerical quantities.

As this is a philosophical endeavor, it can’t really be proved.

Turing’s Thesis: Any “algorithm” can be carried out by one of his machines

A Thinking Machine

Second Goal of Turing’s Machine: A model that’s so simple, that can actually prove interesting epistemological results. Eyed Hilbert’s 10th problem, as well as a computational analog of Gödel’s Incompleteness Theorem in Logic.

Philosophy notwithstanding, Turing’s programs for cracking the Enigma cryptosystem prove that he really was a true hacker! Turing’s machine is actually easily programmable, if you really get into it. Not practically useful, though…

A Thinking Machine

Imagine a super-organized, obsessive-compulsive human computer. The computer wants to avoid mistakes so everything written down is completely specified one letter/number at a time. The computer follows a finite set of rules which are referred to every time another symbol is written down. Rules are such that at any given time, only one rule is active so no ambiguity can arise. Each rule activates another rule depending on what letter/number is currently read, EG:

A Thinking Machine
EG **Successor** Program

Sample Rules:

If read 1, write 0, go right, repeat.

If read 0, write 1, HALT!

If read ,☐write 1, HALT!

Let’s see how they are carried out on a piece of paper that contains the *reverse* binary representation of 47:

A Thinking Machine
EG **Successor** Program

If read 1, write 0, go right, repeat.

If read 0, write 1, HALT!

If read □, write 1, HALT!

1	1	1	1	0	1				

A Thinking Machine
EG **Successor** Program

If read 1, write 0, go right, repeat.

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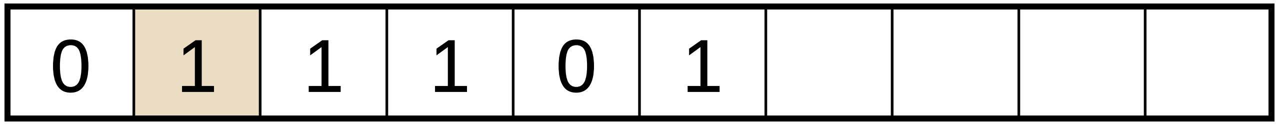

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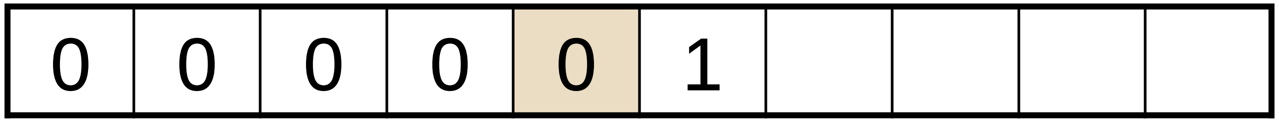

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A Thinking Machine
EG **Successor** Program

So the successor's output on 111101 was 000011 which is the reverse binary representation of 48.

Similarly, the successor of 127 should be 128:

A Thinking Machine
EG **Successor** Program

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If read □, write 1, HALT!

1	1	1	1	1	1	1			

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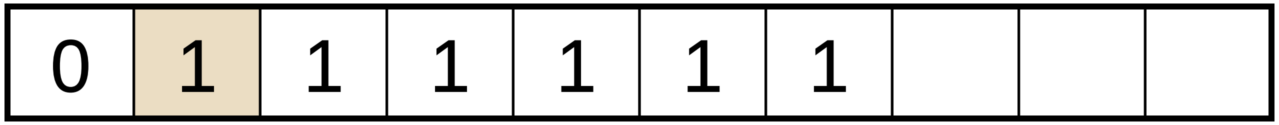

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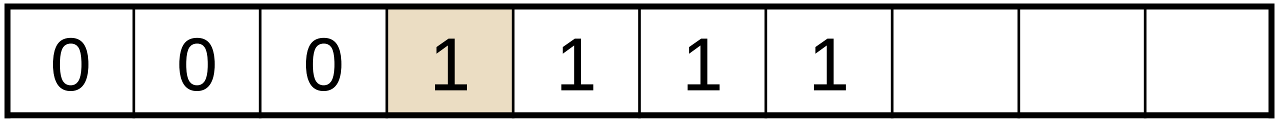

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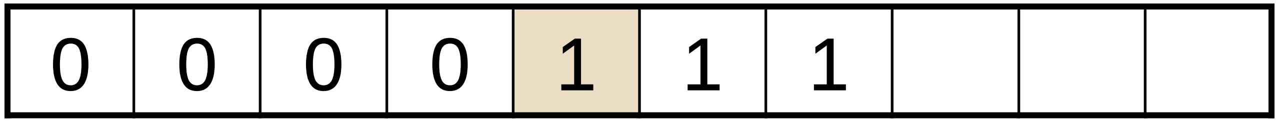

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EG **Successor** Program

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0	0	0	0	0	0	0	▨		

A Thinking Machine
EG **Successor** Program

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0	0	0	0	0	0	0	1		

A Thinking Machine

It was hard for the ancients to believe that *any* algorithm could be carried out on such a device. For us, it's much easier to believe, especially if you've programmed in assembly!

However, ancients did finally believe Turing when Church's lambda-calculus paradigm (on which lisp programming is based) proved equivalent!

Turing Machines

A Turing Machine (**TM**) is a device with a finite amount of *read-only* "*hard*" memory (states), and an unbounded[1] amount of read/write tape-memory. There is no separate input. Rather, the input is assumed to reside on the tape at the time when the TM starts running.

Just as with Automata, TM's can either be input/output machines (compare with Finite State Transducers), or yes/no decision machines. Start with yes/no machines.

Comparison with Previous Models

Device	Separate Input?	Read/Write Data Structure	Deterministic by default?
FA			
PDA			
TM			

Comparison with Previous Models

Device	Separate Input?	Read/Write Data Structure	Deterministic by default?
FA	Yes	None	Yes
PDA			
TM			

Comparison with Previous Models

Device	Separate Input?	Read/Write Data Structure	Deterministic by default?
FA	Yes	None	Yes
PDA	Yes	LIFO Stack	No
TM			

Comparison with Previous Models

Device	Separate Input?	Read/Write Data Structure	Deterministic by default?
FA	Yes	None	Yes
PDA	Yes	LIFO Stack	No
TM	No	1-way infinite tape. 1 cell access per step.	Yes (but will also allow crashes)

Turing Machine
Decision Machine Example

First example (adding 1 bit to reverse binary string) was basically something that a Finite Transducer could have achieved (except when there's overflow). Let's give an example from next step up in language hierarchy.

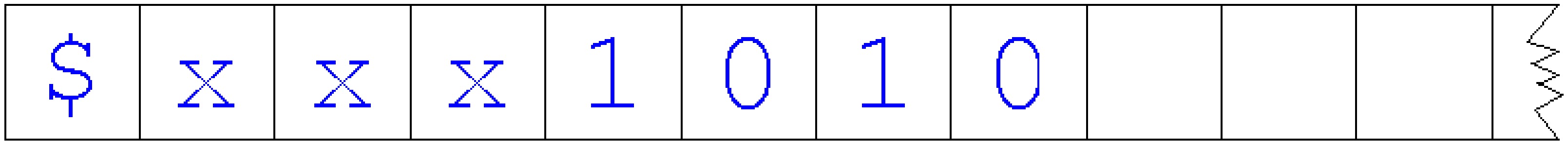
{bit-strings with same number of 0's as 1's}

–a context free language:

Turing Machine
Decision Machine Example

This is a "true" Turing machine as:

- Tape is semi-infinite (indicated by torn cell):



- Input is prepared at beginning of tape
- No intrinsic way to detect left tape end
  - similar to empty stack detection problem for PDA's
  - similar trick used –introduce $ as the end symbol
- All rules must include a move direction (R/L)
- Situations that can't happen aren't dealt with (technically under-deterministic)

Turing Machine Decision Machine Example

{bit-strings with same number of 0's as 1's}:

Pseudocode:

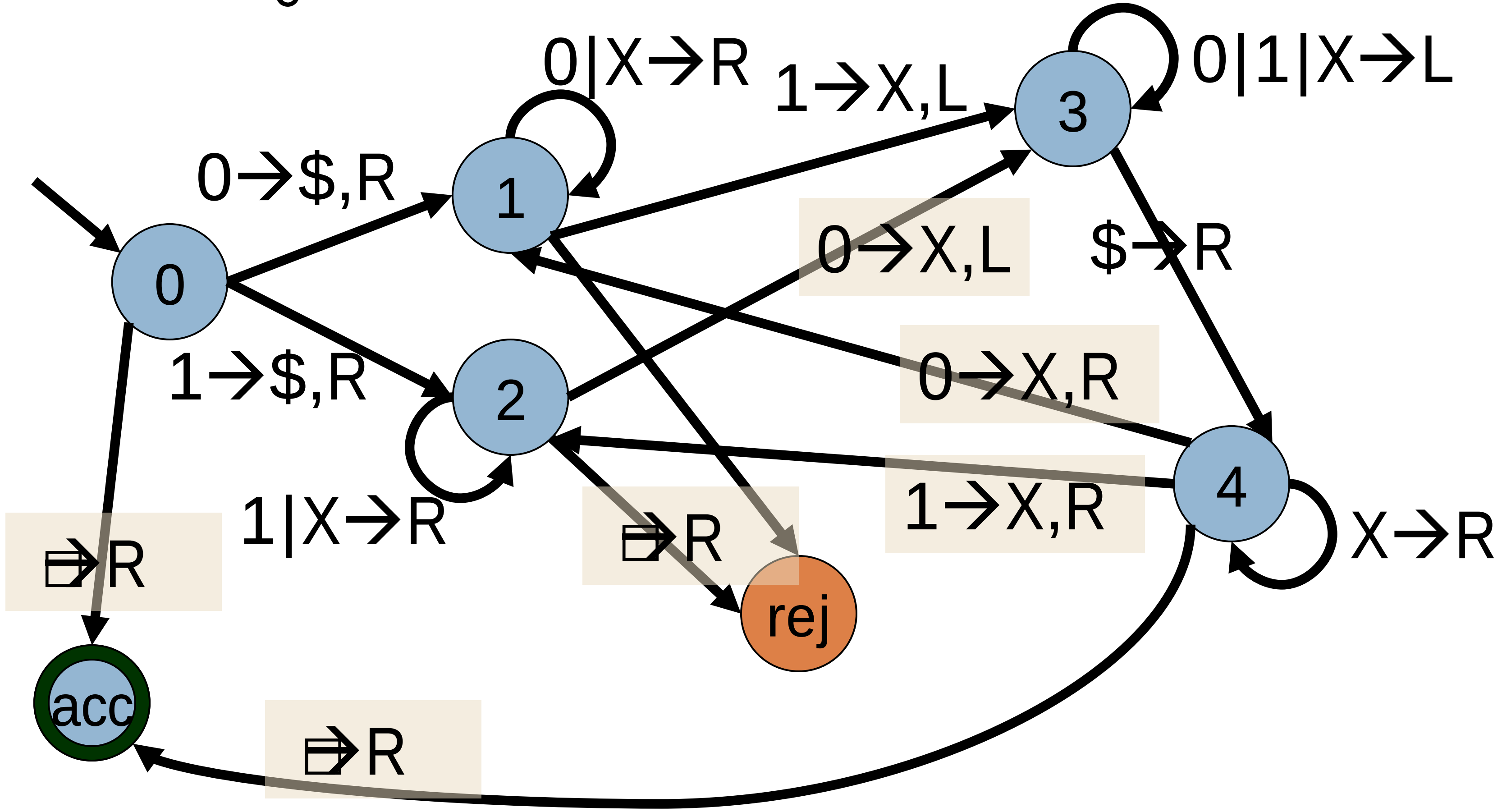
```
while (there is a 0 and a 1)
  cross these out
if (everything crossed out)
  accept
else
  reject
```

TM Example Instructions Set

```
0.   if read □, go right (dummy move), ACCEPT
     if read 0, write $, go right, goto 1  // $ detects start of tape
     if read 1, write $, go right, goto 2
1.   if read □, go right, REJECT
     if read 0 or X, go right, repeat (= goto 1) // look for a 1
     if read 1, write X, go left, goto 3
2.   if read □, go right, REJECT
     if read 1 or X, go right, repeat                    // look for a 0
     if read 0, write X, go left, goto 3
3.   if read $, go right, goto 4                // look for start of tape
     else, go left, repeat
4.   if read 0, write X, go right, goto 1       // similar to step 0
     if read 1, write X, go right, goto 2
     if read X, go right, repeat
     if read □, go right, ACCEPT
```

TM Example
State Diagram

These instructions can be expressed by a familiar looking flow diagram:



TM Transition Notation

An edge from the state *p* to the state *q* labeled by …

- *a*→*b*,D means if in state *p* and tape head reading *a*, replace *a* by *b* and move in the direction D, and into state *q*
- *a*→D means if in state *p* and tape head reading *a*, don't change *a* and move in the direction D, and into state *q*
- *a*|*b*|…|*z* → … means that given that the tape head is reading any of the pipe separated symbols, take same action on any of the symbols

TM Configuration Notation

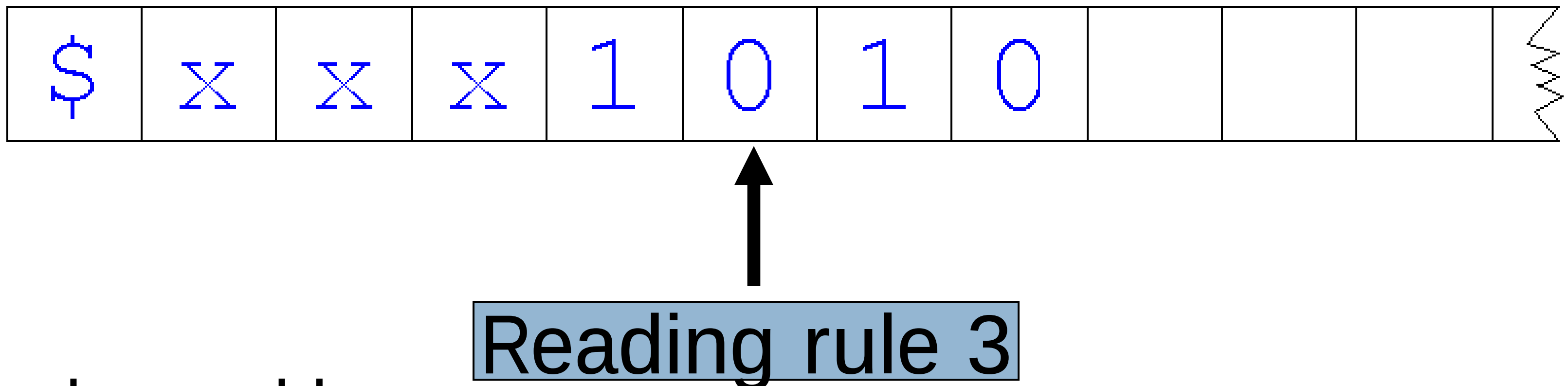
A TM’s next action is completely determined by current state and symbol read, so can predict all of future actions if know:

1. current state
2. current tape contents
3. current position of TM’s reading “head”

Handy notation lists all of these in a single string. A symbol representing current state, is sandwiched between content of tape to left of head, and content of tape to right (including tape head). The part of tape which is blank ad-infinitum is ignored.

TM Configuration Notation

For example



Is denoted by:

$\$xxx1q_3010$