Homework 4

Submit answers for problems 1–5 only. Please indicate your discussion section on your solutions.

1. In this problem we generalize the optimality condition on page 4-9 of the slides. We consider an optimization problem

$$\text{minimize} \quad f(x) + g(x)$$

where $f : \mathbf{R}^n \to \mathbf{R}$ is differentiable and $g : \mathbf{R}^n \to \mathbf{R}$ is convex. We do not assume that f is convex or that its domain is a convex set. (However, recall our convention that the domain of a differentiable function is an open set.) The problem on page 4-9 is a special case with g the indicator function of a convex set. (The indicator function of a set C is the function with domain C, and function value zero on C.)

The generalization of the optimality criterion on page 4-9 is

$$\hat{x} \in \mathbf{dom}\, f \cap \mathbf{dom}\, g, \quad \nabla f(\hat{x})^T (y - \hat{x}) + g(y) - g(\hat{x}) \geq 0 \text{ for all } y \in \mathbf{dom}\, g. \tag{1}$$

(a) Show that (1) is a necessary condition for $\hat{x}$ to be locally optimal.

(b) Assume f is convex. Show that (1) is also sufficient for $\hat{x}$ to be optimal.

(c) Take $g(x) = \|x\|_1$. Show that (1) reduces to the following: $\hat{x} \in \mathbf{dom}\, f$ and for each $i = 1, \ldots, n$,

$$\frac{\partial f(\hat{x})}{\partial x_i} = -1 \quad \text{if } \hat{x}_i > 0, \qquad \left| \frac{\partial f(\hat{x})}{\partial x_i} \right| \leq 1 \quad \text{if } \hat{x}_i = 0, \qquad \frac{\partial f(\hat{x})}{\partial x_i} = 1 \quad \text{if } \hat{x}_i < 0.$$

2. Exercise T4.8 (e).

3. Exercise T4.13.

4. Exercise T4.25.

5. Exercise A7.9.

Problems 6–8 are additional practice problems and will not be graded.

6. Exercise T4.11 (e).

7. Exercise A3.5.

8. Exercise T4.27.