Final Exam

Quiz, 8 questions

1 point

1.
Let N_t be a counting process of a renewal process $S_n = S_{n-1} + \xi_n$ such that the i.i.d. random variables $\xi_1, \xi_2, \dots$ have a probability density function

$$p_\xi(x) = \begin{cases} \frac{1}{3}e^{-x}(x+2), & x \geq 0 \\ 0, & x < 0. \end{cases}$$

Find the mean of X_t:

- [x] $-\frac{4}{25} + \frac{3}{5}x + \frac{4}{25}e^{-(5/3)x}$
- [ ] $-\frac{2}{25} + \frac{3}{5}x + \frac{4}{25}e^{-(5/3)x}$
- [ ] $-\frac{2}{25} + \frac{6}{5}x + \frac{2}{25}e^{5/3x}$
- [ ] none of above

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1 point

2.
Purchases in a shop are modelled with non-homogeneous Poisson process: $30t^{5/4}$ purchases are made on average during t hours after the opening of the shop. Find the probability that the interval between k and $k+1$ purchases will be more than 2 minutes, but less than 4 minutes, given that the purchase number k was in the time moment s:

- [x] $e^{-30(s+1/30)^{5/4}+30s^{5/4}} - e^{-30(s+1/15)^{5/4}+30s^{5/4}}$
- [ ] $e^{-30(s+1/30)^{5/4}+30s^{5/4}} - e^{-30(s+1/15)^{5/4}+30s^{5/4}} - 2$
- [ ] $1 - e^{-30(s+1/30)^{5/4}+30s^{5/4}} - e^{-30(s+1/15)^{5/4}+30s^{5/4}}$
- [ ] none of above
- [ ] $e^{-30(s+1/30)^{5/4}+30s^{5/4}} - e^{-30(s+1/15)^{5/4}+30s^{5/4}} - 1$

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3.

Find stationary distribution of Markov chain with the following 1-step transition matrix P:

$$P = \begin{pmatrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 0 & 1/2 \\ 0 & 1/2 & 0 & 1/2 \end{pmatrix}$$

- [ ] $(\frac{2}{17}\ \frac{2}{17}\ \frac{6}{17}\ \frac{7}{17})$
- [ ] none of above
- [x] $(\frac{6}{17}\ \frac{7}{17}\ \frac{2}{17}\ \frac{2}{17})$
- [ ] $(\frac{2}{17}\ \frac{6}{17}\ \frac{2}{17}\ \frac{7}{17})$
- [ ] $(\frac{7}{17}\ \frac{6}{17}\ \frac{2}{17}\ \frac{2}{17})$

1 point

4.

Let $X_t = e^{2W_t}$. Find mathematical expectation, variance and covariance of this process.

- [ ] $\mathbb{E}\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{2t}, Var\left[X_t\right] = e^{8t} - e^{4t}, Cov(X_t, X_s) = e^{4s+2t}$
- [ ] none of above
- [ ] $\mathbb{E}\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{t}, Var\left[X_t\right] = e^{4t} - e^{2t}, Cov(X_t, X_s) = e^{3s+2t}$
- [x] $\mathbb{E}\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{2t}, Var\left[X_t\right] = e^{8t} - e^{4t}, Cov(X_t, X_s) = e^{6s+2t}$
- [ ] $\mathbb{E}\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{t}, Var\left[X_t\right] = e^{6t} - e^{2t}, Cov(X_t, X_s) = e^{3s+2t}$

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5.

Let Y_t be a stochastic process which is defined as follows:

$\mathbb{E}\left[Y_t\right] = \alpha + \beta t$, $cov(Y_t, Y_{t+h}) = e^{-\lambda h}$, $\forall t > 0, h \geq 0$,

where $\lambda > 0$ and α, β are some constants. Find the mathemarical expectation and the covatiance function of process $X_t = Y_{t+1} - Y_t$:

- [ ] $\mathbb{E}\left[X_t\right] = \beta(t+1)$, $Cov(X_t, X_s) = e^{-\lambda h}$
- [ ] none of above
- [ ] $\mathbb{E}\left[X_t\right] = \beta t$, $Cov(X_t, X_s) = e^{-\lambda h}$
- [ ] $\mathbb{E}\left[X_t\right] = \beta$, $Cov(X_t, X_s) = e^{-\lambda h}$
- [x] $\mathbb{E}\left[X_t\right] = \beta$, $Cov(X_t, X_s) = e^{-\lambda|h|}$

1 point

6.

Let $X_t = \varepsilon_t + \xi \cos(\pi t/24)$, $t = 1, 2, ...$ and $\varepsilon_1, \varepsilon_2, ...$ be a sequence of i.i.d. random variables. Is the process X_t stationary and ergodic?

- [ ] X_t is weakly stationary and non-ergodic
- [ ] none of above
- [x] X_t is weakly stationary and ergodic

1 point

7.

Compute the stochastic integral $\int_0^T 3W_t^2 dW_t$, where W_t is a Brownian motion:

- [ ] $\frac{3}{2}W_T^2 - \frac{3}{2}T$
- [ ] none of above
- [ ] $W_T^3 - \frac{3}{2}W_T^2 + \frac{3}{2}T$

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- [ ] W_T^3
- [x] ~~$\frac{1}{3}W_T^3 - \frac{1}{2}W_T^2 + \frac{1}{2}T$~~

1 point

8.

Let $X_t = bt + N_t$, where N_t is a Compound Poisson Process with intensity λ and $b \in \mathbb{R}$. Find the Lévy triplet of this process.

- [ ] none of above
- [ ] $(\lambda, \lambda, 0)$
- [x] (b, λ, ν), where $\nu(B) = \lambda \mathbb{I}\{1 \in B\}$ for any Borel set B
- [ ] $(b, \lambda^2, 0)$
- [ ] $(b + \lambda, 0, \nu)$, where $\nu(B) = \lambda \mathbb{P}\{\xi_1 \in B\}$ for any Borel set B.

- [ ] I, **Mark R. Lytell**, understand that submitting work that isn't my own may result in permanent failure of this course or deactivation of my Coursera account.

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