Stochastic integration

Quiz, 6 questions

1 point

1.
Let $I(f) = \int_0^1 t^2 dW_t$. Find the mean of $I(f)$:

- 1
- $1/2$
- $1/4$
- none of above
- 0

1 point

2.
Let $I(f) = \int_0^1 t^2 dW_t$. Find the variance of $I(f)$:

- $1/2$
- none of above
- $1/4$
- 1
- $5/4$

1 point

3.
Let N_t be a Poisson process. Find the mean, covariance function and variance of $I(f) = \int_0^t N_s ds$:

- none of above
- $\mathbb{E}[I(f)] = \frac{\lambda t^2}{2}, Var(I(f)) = \frac{\lambda t^3}{3}, K(t,s) = \lambda(\frac{t^3}{6} + \frac{st^2}{2})$
- $\mathbb{E}[I(f)] = \lambda t, Var(I(f)) = (\lambda t)^2, K(t,s) = 0$
- $\mathbb{E}[I(f)] = \lambda t, Var(I(f)) = \lambda t, K(t,s) = 0$
- $\mathbb{E}[I(f)] = \frac{\lambda t^2}{2}, Var(I(f)) = \frac{\lambda t^2}{3}, K(t,s) = \lambda(\frac{t^3}{6} + \frac{st^3}{2})$

1 point

4.

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Let $X_t = \begin{cases} \xi_1, & t \in [0,1), \\ \xi_2, & t \in [1,2), \\ \xi_3, & t \geq 2, \end{cases}$

where ξ_1, ξ_2, ξ_3 - i.i.d. random variables having exponential distribution with parameter λ.

Also let $f(t) = \begin{cases} 2, & t \in [0,1), \\ 5, & t \in [1,2), \\ 0, & t \geq 2. \end{cases}$

Find mean and variance of $I(f) = \int_0^T X_t dt$:

- $\mathbb{E}[I(f)] = \begin{cases} \frac{T}{\lambda}, & 1 > T, \\ \frac{1}{\lambda} + \frac{T-1}{\lambda}, & 1 \leq T < 2, \\ \frac{1}{\lambda} + \frac{1}{\lambda} + \frac{T-2}{\lambda}, & T \geq 2, \end{cases}$

  $Var(f(t)) = \begin{cases} \frac{T^2}{\lambda^2}, & 1 > T, \\ \frac{1}{\lambda^2} + \frac{(T-1)^2}{\lambda^2}, & 1 \leq T < 2, \\ \frac{1}{\lambda^2} + \frac{1}{\lambda^2} + \frac{(T-2)^2}{\lambda^2}, & T \geq 2, \end{cases}$

- $\mathbb{E}[I(f)] = \frac{T}{\lambda}, Var(I(f)) = \frac{T^2}{\lambda^2} \; \forall T$
- none of above
- $\mathbb{E}[I(f)] = Var(I(f)) = \begin{cases} \frac{T}{\lambda}, & 1 > T, \\ \frac{1}{\lambda} + \frac{T-1}{\lambda}, & 1 \leq T < 2, \\ \frac{1}{\lambda} + \frac{1}{\lambda} + \frac{T-2}{\lambda}, & T \geq 2, \end{cases}$

1 point

5.

Compute the stochastic integral $\int_0^T W_t^2 dW_t$, where W_t is a Brownian motion:

- $\frac{1}{3} W_T^3$
- none of above
- $\frac{1}{3} W_T^3 - \frac{1}{2} W_T^2 + \frac{1}{2} T$
- $\frac{1}{2} W_T^2 - \frac{1}{2} T$

1 point

6.

Compute the variance of the stochastic integral $\int_0^T W_t dW_t$, where W_t is a Brownian motion:

- W_T^2
- none of above
- $\frac{T^2}{2}$
- T^2

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