Gaussian processes

Quiz, 6 questions

1 point

1.
Consider the condition from the Kolmogorov continuity theorem: $\mathbb{E}\left[|X_t - X_s|^\alpha\right] \leq K|t-s|^{1+\beta}, \qquad \forall\, t, s > 0.$

For which parameters α, K and β this condition holds, if X_t is a Brownian motion?

- [ ] none of above
- [ ] $\alpha = 3$, $K = 3$ and $\beta = 2$
- [ ] $\alpha = 4$, $K = 3$ and $\beta = 1$
- [ ] $\alpha = 4$, $K = 2$ and $\beta = 3$

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1 point

2.
Choose the right statements about the Brownian motion W_t:

- [ ] W_t has symmetric distribution for any $t > 0$
- [ ] W_t has continuous trajectories
- [ ] $W_0 = 0$ almost surely
- [ ] $W_t - W_s \sim N(0, t-s)$
- [ ] W_t has independent increments

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1 point

3.
Let $X_t = e^{W_t}$. Find mathematical expectation, variance and covariance of this process.

- [ ] none of above
- [ ] $\mathbb{E}\left[X_t\right] = e^{2t} - e^t$, $Var\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{t/2}$, $Cov(X_t, X_s) = e^{\frac{3s+2t}{2}}$
- [ ] $\mathbb{E}\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{t/2}$, $Var\left[X_t\right] = e^{2t} - e^t$, $Cov(X_t, X_s) = e^{\frac{3s+t}{2}}$
- [ ] $\mathbb{E}\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{t/2}$, $Var\left[X_t\right] = e^{2t} - e^t$, $Cov(X_t, X_s) = e^{\frac{3s+2t}{2}}$

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1 point

4.
Let $W(1)$ and $W(2)$ be Brownian motions with variances 1 and 2 respectively. Find $\mathbb{P}\{W(1) + W(2) > 2\}$.

- [ ] $\Phi\left(\frac{2}{\sqrt{5}}\right)$~~, where Φ is a normal distribution function~~
- [ ] none of above

- [ ] $\Phi\left(\frac{1}{2}\right)$, where Φ is a normal distribution function

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- [ ] 0
- [ ] $\frac{1}{2}$

1 point

5.

Which properties hold for a covariance function $K(t,s)$?

- [ ] K is positive definite, that is, $\sum_{j,k} u_j u_k K(t_j,t_k) > 0, \quad \forall\, t_1,...,t_n \in \mathbb{R}_+,\ \forall u_1,...,u_n \in \mathbb{R}$, $(u_1,...u_n) \neq (0,...0)$
- [ ] None of above
- [ ] K is positive semidefinite, that is, $\sum_{j,k} u_j u_k K(t_j,t_k) \geq 0, \quad \forall\, t_1,...,t_n \in \mathbb{R}_+,\ \forall u_1,...,u_n \in \mathbb{R}$
- [ ] K is symmetric, that is, $K(t,s) = K(s,t), \quad \forall\, t,s \in \mathbb{R}_+$

1 point

6.

Let W_t be a Brownian motion. Which one of the following processes are also Brownian motions?

- [ ] $W_{t+s} - W_s$ with some fixed $s > 0$
- [ ] $tW_{1/t}, t > 0$, and $W_0 = 0$
- [ ] $-W_t$
- [ ] aW_{t/a^2} with some fixed $a \neq 0$.

- [ ] I, **Mark R. Lytell**, understand that submitting work that isn't my own may result in permanent failure of this course or deactivation of my Coursera account.

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