Gaussian processes

Quiz, 6 questions

1 point

1.
Consider the condition from the Kolmogorov continuity theorem: $\mathbb{E}\left[|X_t - X_s|^\alpha\right] \le K|t-s|^{1+\beta}, \qquad \forall\, t, s > 0.$

For which parameters α, K and β this condition holds, if X_t is a Brownian motion?

- [ ] $\alpha = 3, K = 3$ and $\beta = 2$
- [x] $\alpha = 4, K = 3$ and $\beta = 1$
- [ ] $\alpha = 4, K = 2$ and $\beta = 3$
- [ ] none of above

---

1 point

2.
Choose the right statements about the Brownian motion W_t:

- [x] W_t has symmetric distribution for any $t > 0$
- [x] $W_t - W_s \sim N(0, t-s)$
- [x] W_t has continuous trajectories
- [x] $W_0 = 0$ almost surely
- [x] W_t has independent increments

---

1 point

3.
Let $X_t = e^{W_t}$. Find mathematical expectation, variance and covariance of this process.

- [ ] none of above
- [ ] $\mathbb{E}\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{t/2}, Var\left[X_t\right] = e^{2t} - e^t, Cov(X_t, X_s) = e^{\frac{3s+2t}{2}} - e^{\frac{s+t}{2}}$
- [ ] $\mathbb{E}\left[X_t\right] = e^{2t} - e^t, Var\left[X_t\right] = e^{t/2}, Cov(X_t, X_s) = e^{\frac{3s+2t}{2}} - e^{\frac{s-t}{2}}$
- [x] $\mathbb{E}\left[X_t\right] = \mathbb{E}\left[e^{W_t}\right] = e^{t/2}, Var\left[X_t\right] = e^{2t} - e^t, Cov(X_t, X_s) = e^{\frac{3s+t}{2}} - e^{\frac{s+t}{2}}$

---

1 point

4.
Let $W(1)$ and $W(2)$ be Brownian motions with variances 1 and 2 respectively. Find $\mathbb{P}\{W(1) + W(2) > 2\}$.

- [ ] $\frac{1}{2}$
- [ ] $\Phi\left(\frac{1}{2}\right)$, where Φ is a normal distribution function

- [x] ~~$1 - \Phi\left(\frac{2}{\sqrt{5}}\right)$, where Φ is a normal distribution function~~

Gaussian processes

Quiz, 6 questions

- [ ] ~~$\Phi\left(\frac{2}{\sqrt{5}}\right)$, where Φ is a normal distribution function~~
- [ ] none of above

1 point

5.
Which properties hold for a covariance function $K(t, s)$?

- [ ] None of above
- [x] K is symmetric, that is, $K(t, s) = K(s, t), \quad \forall\, t, s \in \mathbb{R}_+$
- [ ] K is positive definite, that is, $\sum_{j,k} u_j u_k K(t_j, t_k) > 0, \quad \forall\, t_1, ..., t_n \in \mathbb{R}_+,\ \forall u_1, ..., u_n \in \mathbb{R}$, $(u_1, ...u_n) \neq (0, ...0)$
- [x] K is positive semidefinite, that is, $\sum_{j,k} u_j u_k K(t_j, t_k) \geq 0, \quad \forall\, t_1, ..., t_n \in \mathbb{R}_+,\ \forall u_1, ..., u_n \in \mathbb{R}$

1 point

6.
Let W_t be a Brownian motion. Which one of the following processes are also Brownian motions?

- [x] $-W_t$
- [x] aW_{t/a^2} with some fixed $a \neq 0$.
- [x] $tW_{1/t}, t > 0$, and $W_0 = 0$
- [x] $W_{t+s} - W_s$ with some fixed $s > 0$

- [ ] I, **Mark R. Lytell**, understand that submitting work that isn't my own may result in permanent failure of this course or deactivation of my Coursera account.

Learn more about Coursera's Honor Code

Submit Quiz