# Computationally-Efficient Methods for Blind Adaptive Equalization

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# Outline

1. Introduction
2. Computationally-Efficient Methods
3. Proposed Selective Update Method
4. Simulation Results
5. Conclusions
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# Introduction

- Blind adaptive equalization is used in systems where the transmission of a training sequence is impractical
- Common blind algorithms include the reduced constellation algorithm (RCA), the constant modulus algorithm (CMA), and the multimodulus algorithm (MMA)
- Equalization can consume in excess of 80% of the total arithmetic computations needed to demodulate a transmitted symbol sequence into binary words, which has resulted in a number of computationally-efficient methods
- We present a survey of efficient methods for blind equalization and propose a new method that selectively updates the equalizer taps based on the equalizer output radius for QAM signal constellations

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# Computationally-Efficient Methods

- Adaptive filtering consists of two operations: convolution of the received symbol sequence with the tap coefficients and updating the tap coefficients
- For an adaptive FIR filter of length $M$, each of the previous operations require $4M$ multiplications for a total of $8M$ multiplications when the received signal is complex
- One method to improve computational efficiency is to simplify or reduce the amount of multiplications
- Our focus is the reduction of multiplications in the equalizer tap update and we consider the signed-error, dithered signed-error, quantized-error, block, and update decimation methods

# Adaptive FIR Filter Structure

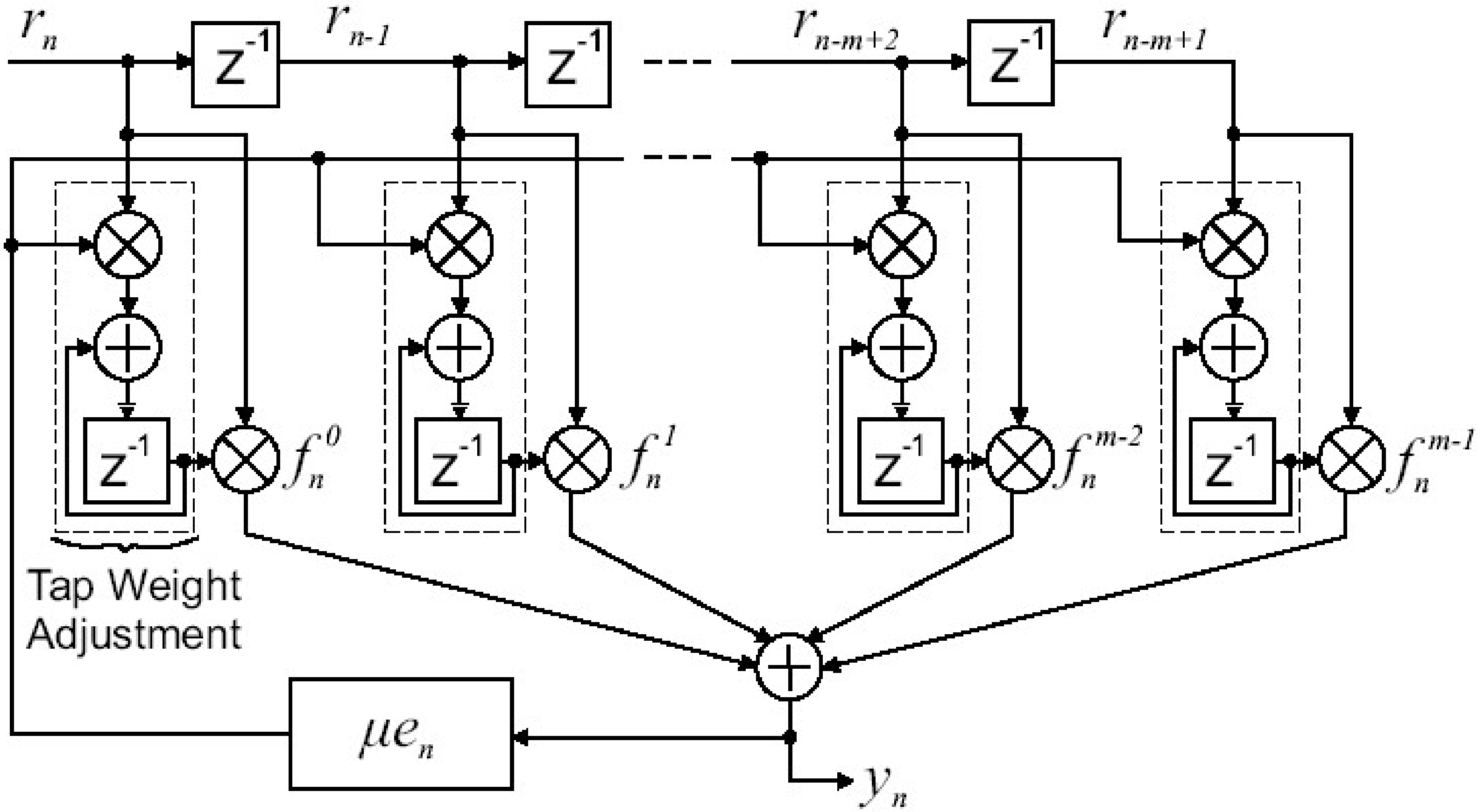


Figure 1: Adaptive FIR filter for real input samples

# Signed-Error Method

- Only the sign of the respective error signal is retained
- When coupled with a power-of-two stepsize, a multiply-free fixed-point equalizer tap update can be realized reducing the total multiplications by a factor of two
- The general signed-error tap update algorithm is:

$$\mathbf{f}_{n+1} = \mathbf{f}_n + \mu \cdot \mathrm{csgn}(e_n)\mathbf{r}_n^{\star}$$

- Signed-error algorithms are straight forward to implement and have been proposed for RCA and CMA, and can be extended to MMA

# Dithered Signed-Error Method

- The convergence of signed-error CMA is not robust and is known to diverge
- This can be overcome by the application of a controlled noise or dither signal, which improves robustness
- The general dithered signed-error tap update algorithm is:

$$\mathbf{f}_{n+1} = \mathbf{f}_n + \mu \cdot \alpha \mathrm{csgn}(e_n + \alpha d_n)\mathbf{r}_n^\star$$

- Where *α* is a positive constant and $d_n$ is an independent identically distributed (i.i.d.) dithering process uniformly distributed over (-1,1]

# Quantized-Error Method

- The error signal of the respective algorithm is quantized using a nonlinear power-of-two quantizer
- When coupled with a power-of-two stepsize the equalizer tap update becomes shift and add operations
- The general quantized-error tap update algorithm is:

$$\mathbf{f}_{n+1} = \mathbf{f}_n + \mu \cdot \mathrm{Q}\{e_n\}\mathbf{r}_n^{\star}$$

- Where

$$\mathrm{Q}\{x\} = \begin{cases} \mathrm{sgn}(x), & |x| \geq 1 \\ 2^{\lfloor \log_2 |x| \rfloor}\mathrm{sgn}(x), & 2^{-B+2} \leq |x| < 1 \\ \tau sgn(x), & |x| < 2^{-B+2} \end{cases}$$

- And τ is set to either 0 or $2^{-B+1}$ and *B* is the data word length

# Block Method

- A block of equalizer input samples and instantaneous error samples are used to update the tap coefficients once every *L* input samples, where *L* is the block length
- The general block tap update algorithm is:

$$\mathbf{f}_{(n+1)L} = \mathbf{f}_{nL} + \mu \cdot \sum_{k=0}^{L-1} e_{nL+k} \cdot \mathbf{r}^{\star}_{nL+k}$$

- Estimates the gradient over *L* iterations, which allows a larger stepsize to be applied since the variance of a block of gradient updates is less than that for individual updates
- Can be implemented in frequency domain to increase rate of convergence
- Have been proposed for CMA and can be extended to MMA

# Update Decimation Method

- The equalizer taps are updated once every *k* iterations, where *k* is a positive integer greater than one
- It is expected that update-decimated algorithms would obtain similar steady-state mean-squared error (MSE) with 1/*k* times the computations, while taking *k* times the time-to-convergence

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# Selective Update Method

- The square decision region of an estimated symbol point in a QAM constellation is divided in two by a circular boundary, $C_b$, which corresponds to radius $R_b$
- Equalizer taps are updated only if $R_n > R_b$, where $R_n$ is the distance from the estimated symbol to the equalizer output defined as:

$$R_n = \left|\hat{s}_n - Y_n\right|$$

- The general selective update tap update algorithm is:

$$\mathbf{f}_{n+1} = \mathbf{f}_n + \mu \cdot \psi(Y_n, R_n)\mathbf{r}_n^\star$$

- where

$$\psi(Y_n, R_n) = \begin{cases} e_n, & R_n > R_b \\ 0, & R_n \leq R_b \end{cases}$$

# Selective Update Method

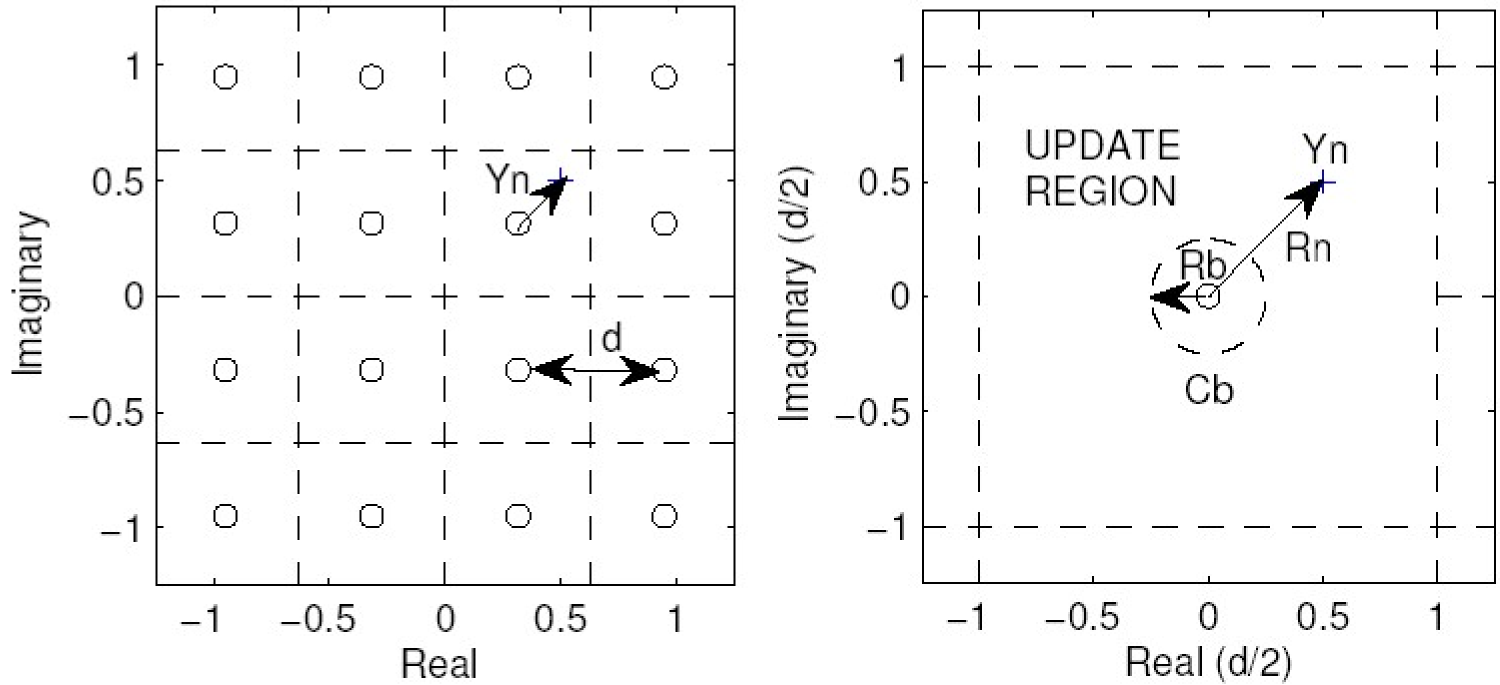


Figure 2: Decision regions for symbol estimates in 16-QAM (left) and decision regions for the selective update method (right).

# Selective Update Method

- The outer region corresponds to adaptation phases with high MSE, while the inner region corresponds to adaptation phases with low MSE
- Initially, the MSE will be high and the outer region will be selected most of the time, allowing the transient response of the base algorithm to remain unchanged
- In slow time-varying channels, once the MSE has been reduced, the inner region will be selected most of the time, which will result in a drastic reduction of tap updates
- If the channel experiences sudden changes, the MSE will increase and the process will repeat

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# Simulation Parameters

- Discussed and proposed methods applied to CMA & MMA
- Simulations are in a 35dB SNR environment for 16-QAM using SPIB microwave channels (#1,2,4-6,8-10), with T/2-spaced FIR equalizers (16-tap, double 0.5 center spike)
- Applied stepsize of $2^{-10}$ (except DSE-CMA which used $2^{-11}$ to avoid divergence), block length $L$=20, $\alpha$=0.65
- $R_b$ for selective update method was chosen using an ad hoc approach and ranged between $d$/8 and $d$/12, where d is the distance between symbol points
- MSE calculated as instantaneous squared error over the slicer for 100-1000 iterations
- Graphical results shown for SPIB microwave channel #2

# Simulation Results for CMA-Based Algorithms

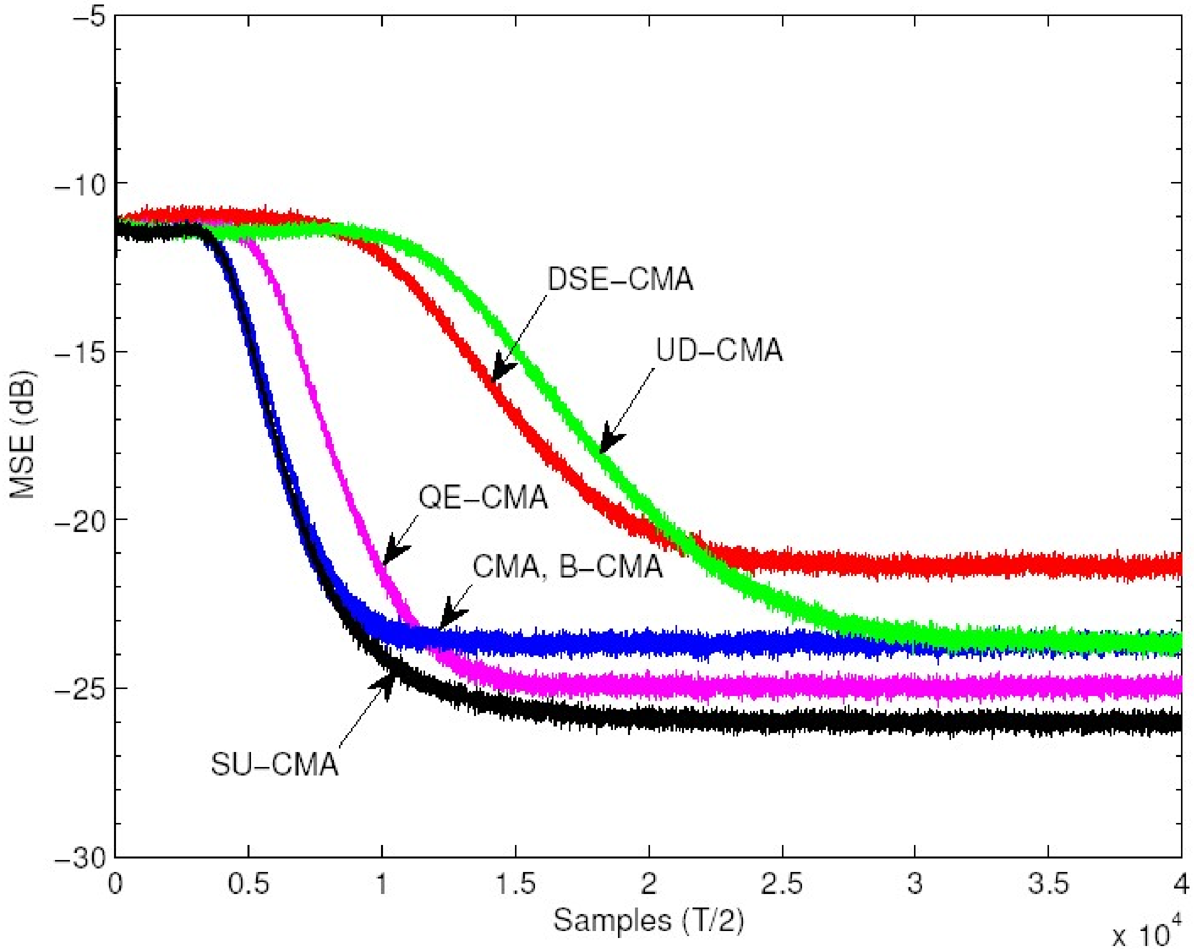


Figure 3: CMA simulation results.

# Simulation Results for MMA-Based Algorithms

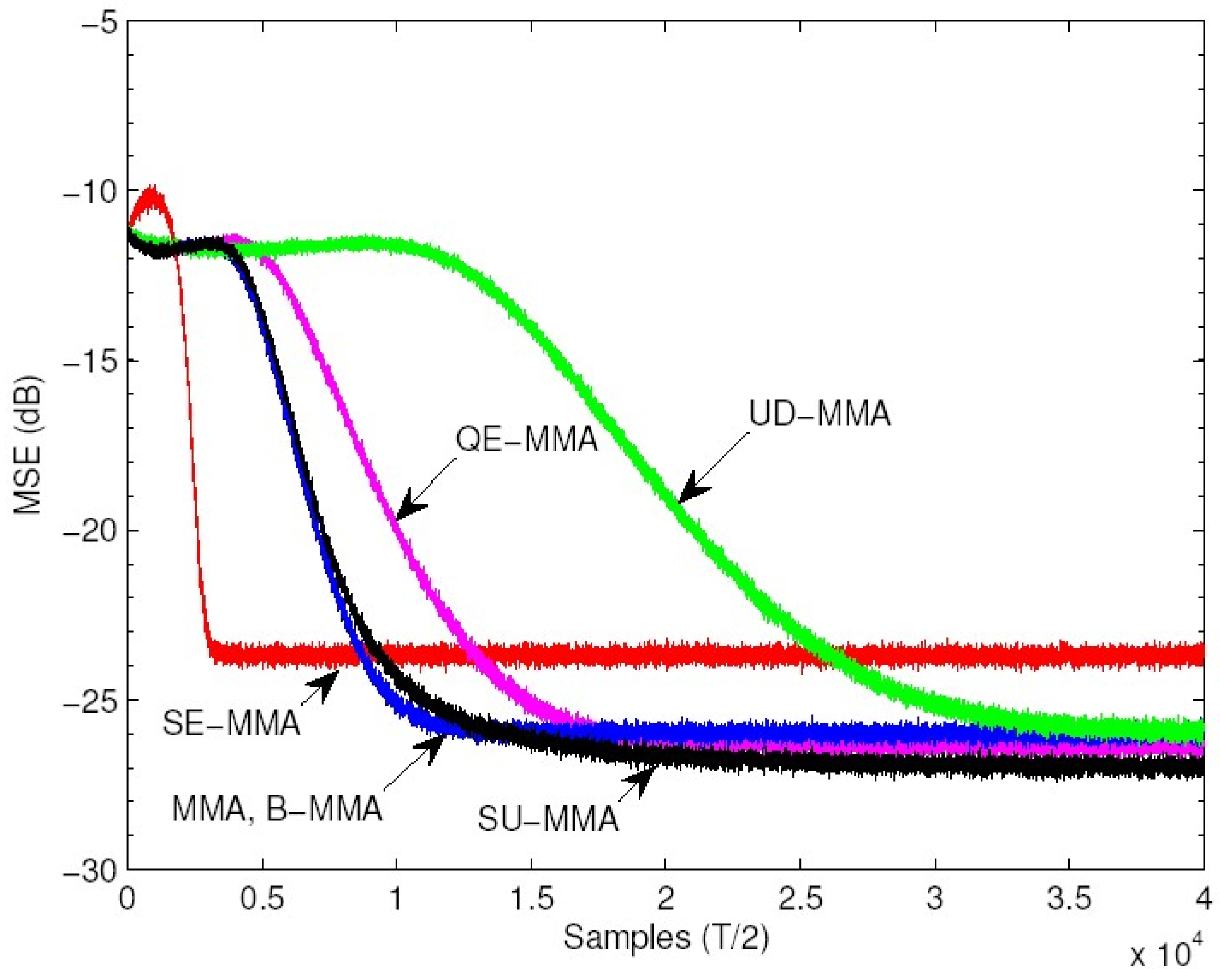


Figure 3: MMA simulation results.

# Simulation Results

- Quantitative results have been averaged over all channels
- In the table to follow, the MSE corresponds to the steady-state MSE, M is the misadjustment, and TTC is the time-to-convergence which was taken as the number of samples required to reach 90% of the steady-state MSE
- Misadjustment is the ratio of excess MSE (EMSE) to the minimum theoretical MSE (MMSE), where EMSE is the difference between the steady-state MSE and the MMSE

# Quantitative Simulation Results

Table 1: Quantitative Simulation Results.

| | Performance Measures | | | Tap Update Percentage | |
|---|---|---|---|---|---|
| Method | MSE | M | TTC | Transient | Steady-State |
| CMA | -23.8834 | 0.2055 | 8587 | 100.00 | 100.00 |
| DSE-CMA | -21.1884 | 0.2939 | 17892 | 100.00 | 100.00 |
| QE-CMA | -25.3812 | 0.1573 | 12902 | 100.00 | 100.00 |
| B-CMA | -23.8353 | 0.2072 | 8783 | 5.00 | 5.00 |
| UD-CMA | -23.7006 | 0.2115 | 24525 | 33.33 | 33.33 |
| SU-CMA | -26.1220 | 0.1324 | 10953 | 73.05 | 14.95 |
| MMA | -26.4679 | 0.1218 | 10506 | 100.00 | 100.00 |
| SE-MMA | -24.0006 | 0.2016 | 2530 | 100.00 | 100.00 |
| QE-MMA | -27.0522 | 0.1039 | 17250 | 100.00 | 100.00 |
| B-MMA | -26.4454 | 0.1226 | 10506 | 5.00 | 5.00 |
| UD-MMA | -26.0838 | 0.1346 | 29880 | 33.33 | 33.33 |
| SU-MMA | -27.5051 | 0.0888 | 11933 | 75.65 | 14.43 |

Proposed algorithms have the lowest misadjustment and same rate of convergence as original algorithms

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# Conclusions

- Simulations have confirmed that on average, the proposed selective update method achieves similar transient behavior and lower steady-state MSE and misadjustment than the original algorithm
- After convergence, the percentage of tap updates for the selective update method is considerably reduced (<15%)
- Performance gains obtained using the selective update method serve to validate this technique as being computationally-efficient as well as an effective method for blind equalization

# Thank You!
# Questions or Comments?

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