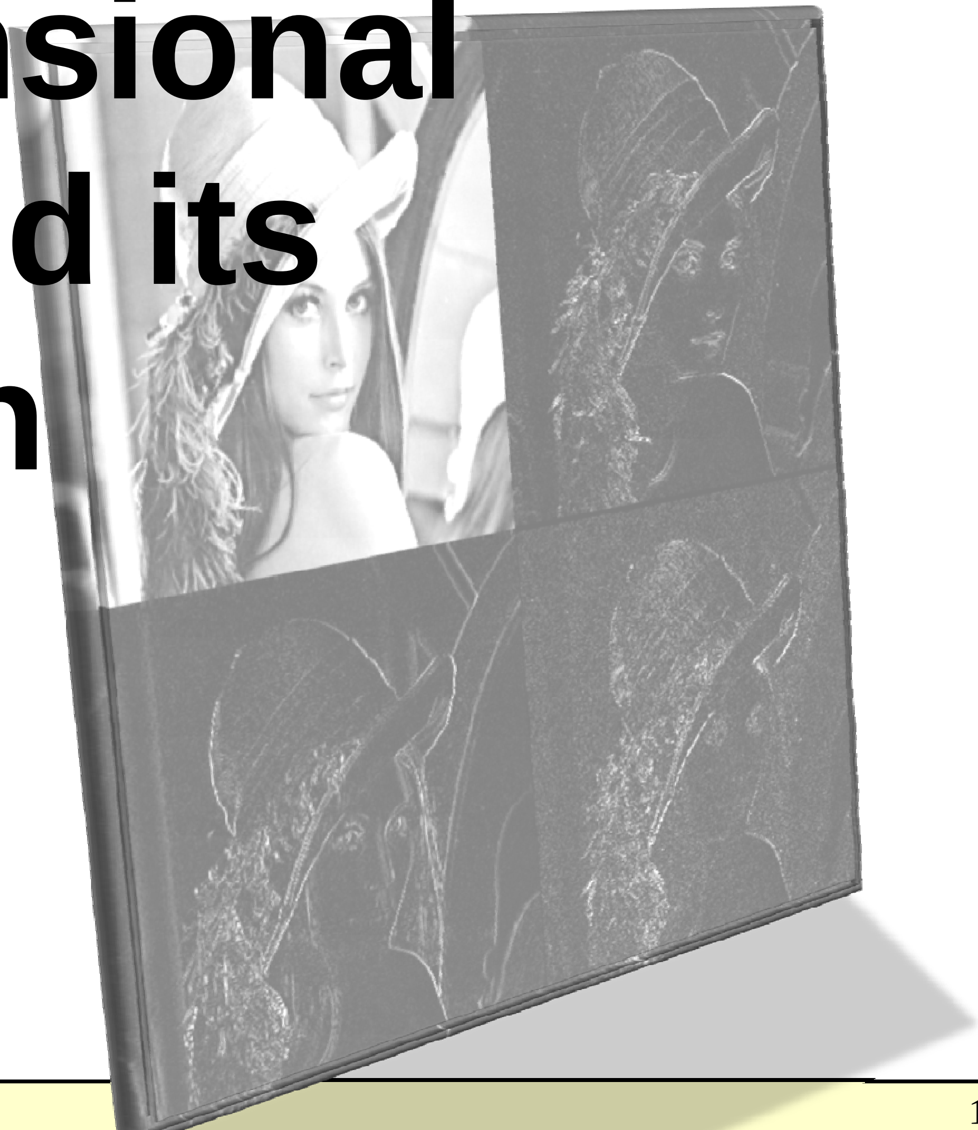
Two Dimensional Wavelet and its Application



Iman Makaremi

Outline

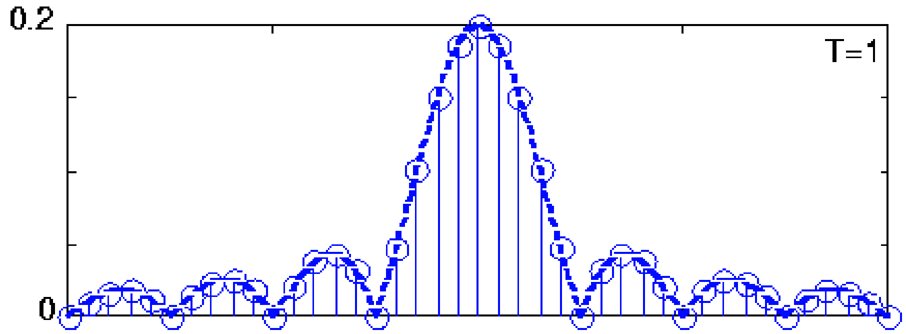
- Problem Statement
- Fourier Transform
- Short Time Fourier Transform
- One Dimensional Wavelet
- Two Dimensional Wavelet
- Two Dimensional Wavelet in Application
- Conclusion

Problem Statement

- What is wavelet?
- How and where can I use it?

Fourier Transform

- In early 1800 was introduced by Josef Fourier
- It is good for analyzing periodic functions.
- Has a sinusoidal basis.
- Just contains frequency domain information.
- Time domain information are lost.

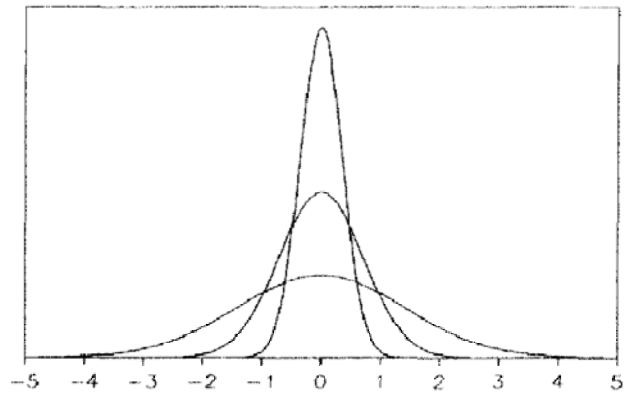


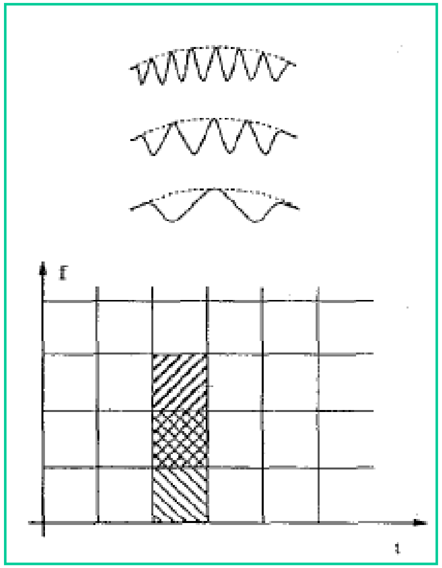
Short Time Fourier Transform

- In 1964,Gabor introduced windowed Fourier atoms.
- The support area of the basis has reduced using windows.

$$g_\alpha(t) = \frac{1}{2\sqrt{\pi\alpha}} e^{-\frac{t^2}{4\alpha}},$$

$$g_{b,\omega}(t) = e^{i\omega t} g_\alpha(t-b).$$





How does wavelet work?

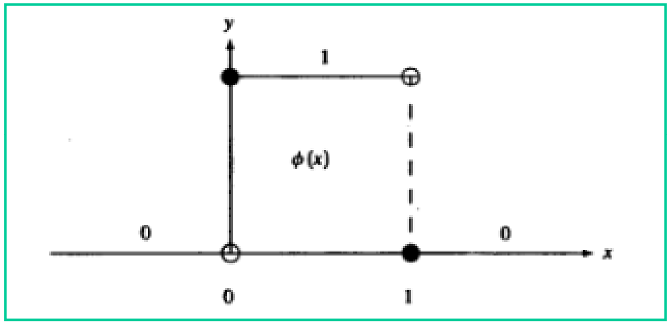
- A row of an image [8 4 1 3]
- Representing in the *Haar basis*

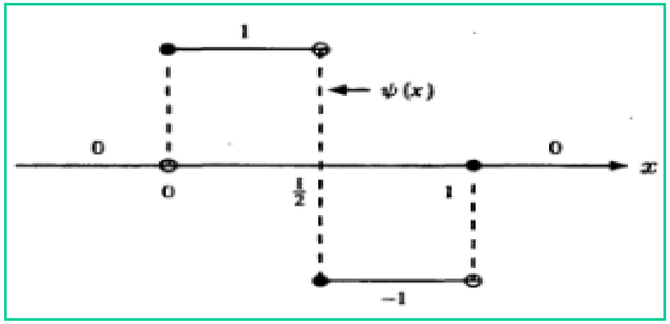
Resolution	Average	Detail Coefficients
4	[8 4 1 3]	
2	[6 2]	[2 -1]
1	[4]	[2]

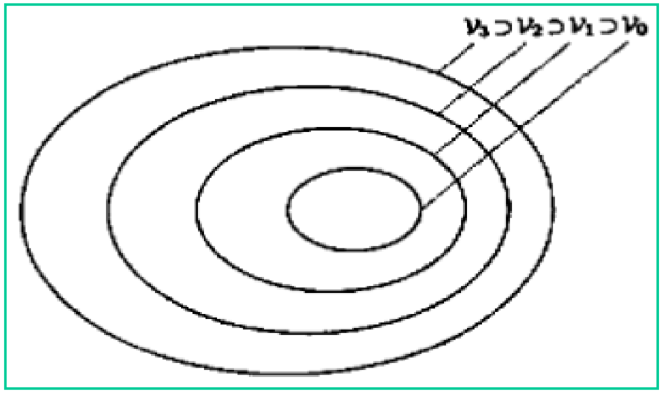
- The wavelet transform is [4 2 2 -1].
- Two Function in wavelet transform,
  - Scaling Function (Average, Lowpass Filter)
  - Wavelet Function (Detail, Highpass Filter)

1D Haar Wavelet

- Haar Scaling Function
  - V_0 the space of all function $\sum_{k \in Z} a_k \varphi(x-k)$
  - V_j the space of all function $\sum_{k \in Z} a_k \varphi(2^j x-k)$
- Haar Wavelet Function
  - W_0 the space of all function $\sum_{k \in Z} a_k \psi(x-k)$
  - W_j the space of all function $\sum_{k \in Z} a_k \psi(2^j x-k)$
- W_j is the orthogonal complement of V_j in V_{j+1} .
- Multiresolution Concept



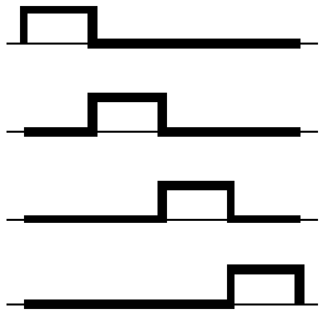




1D Haar Wavelet

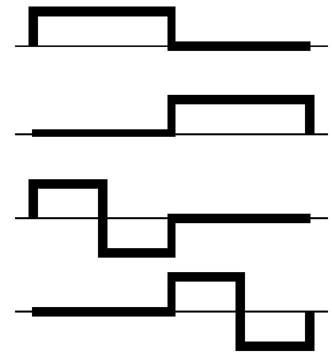
$$\mathcal{I}(x) = c_0^2\,\phi_0^2(x) + c_1^2\,\phi_1^2(x) + c_2^2\,\phi_2^2(x) + c_3^2\,\phi_3^2(x).$$

$$\mathcal{I}(x) = 8 \times \quad + 4 \times \quad + 1 \times \quad + 3 \times$$



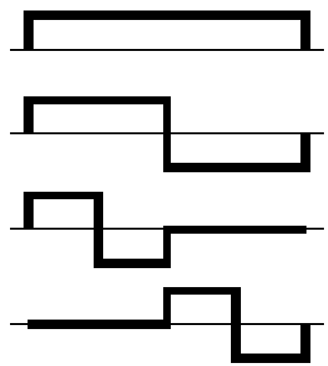
$$\mathcal{I}(x) = c_0^1\,\phi_0^1(x) + c_1^1\,\phi_1^1(x) + d_0^1\,\psi_0^1(x) + d_1^1\,\psi_1^1(x)$$

$$= 6 \times \quad + 2 \times \quad + 2 \times \quad + -1 \times$$



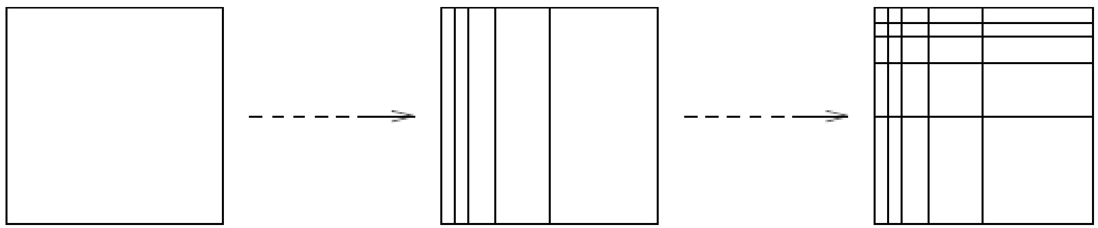
$$\mathcal{I}(x) = c_0^0\,\phi_0^0(x) + d_0^0\,\psi_0^0(x) + d_0^1\,\psi_0^1(x) + d_1^1\,\psi_1^1(x)$$

$$= 4 \times \quad + 2 \times \quad + 2 \times \quad + -1 \times$$

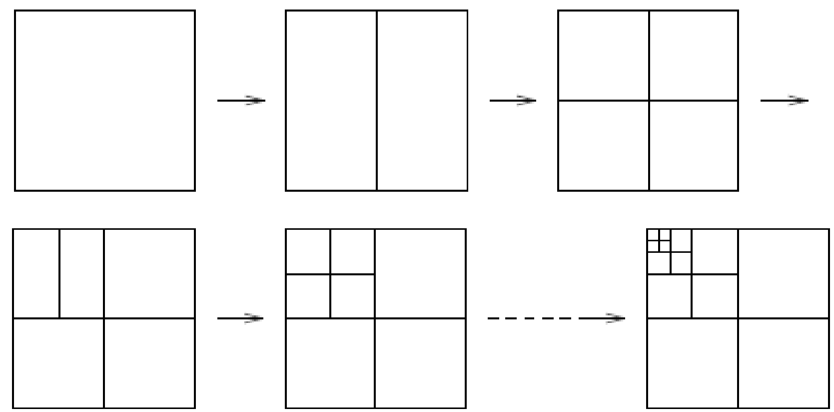


2D Haar Wavelet

- Two approaches for dealing with 2D signals.
  - Standard decomposition

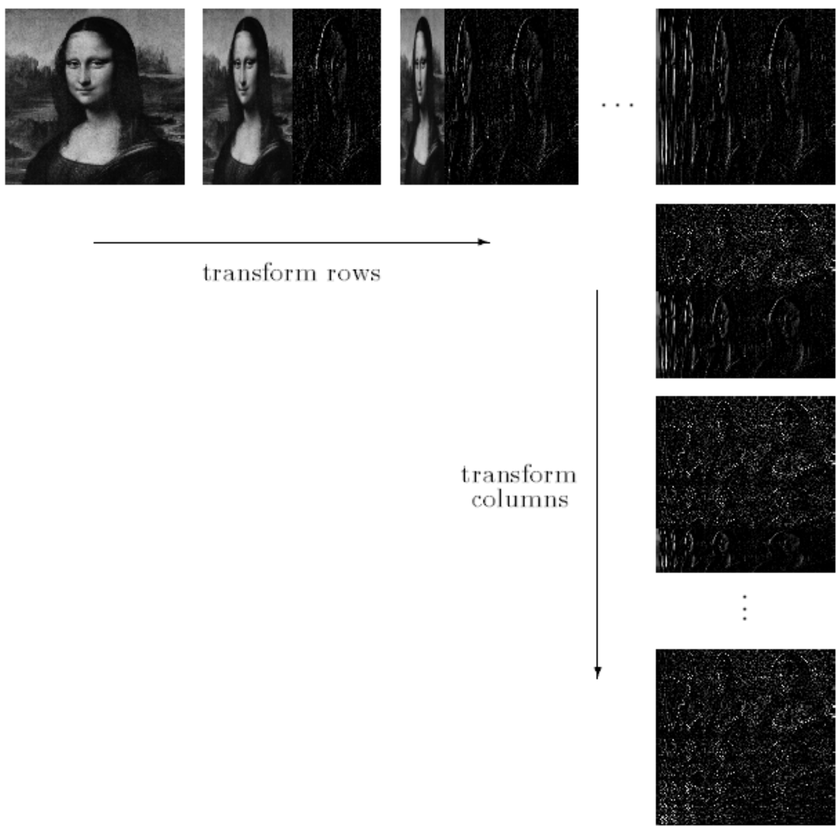


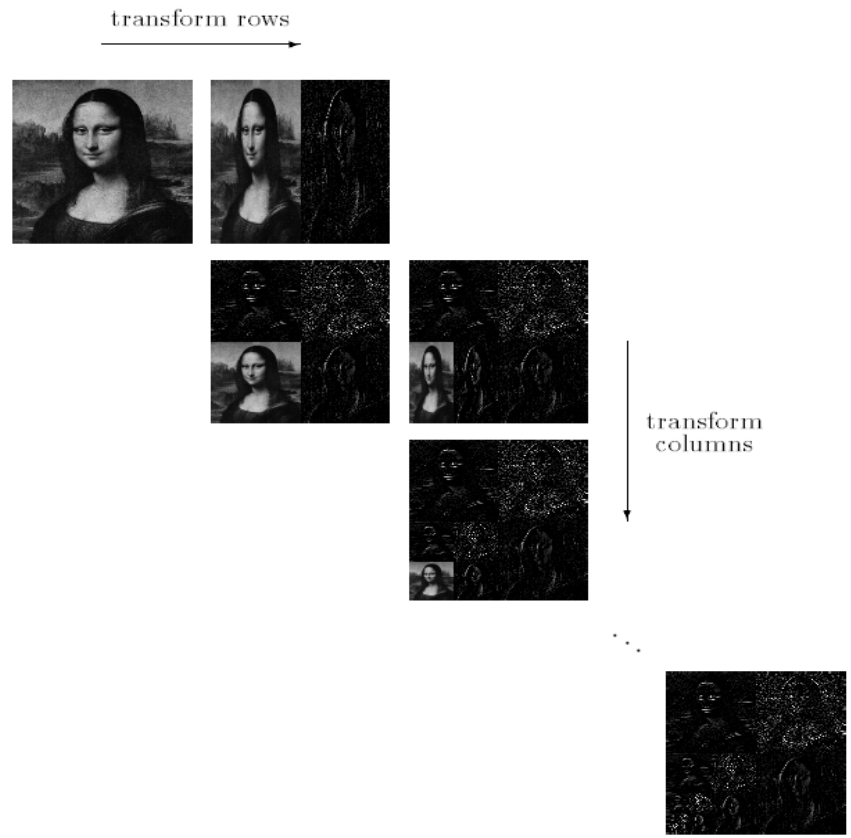
  - Non-standard decomposition



- Advantages and disadvantages
  - S.D. is simple.
  - N.S.D is more efficient in computation.

2D Haar Wavelet





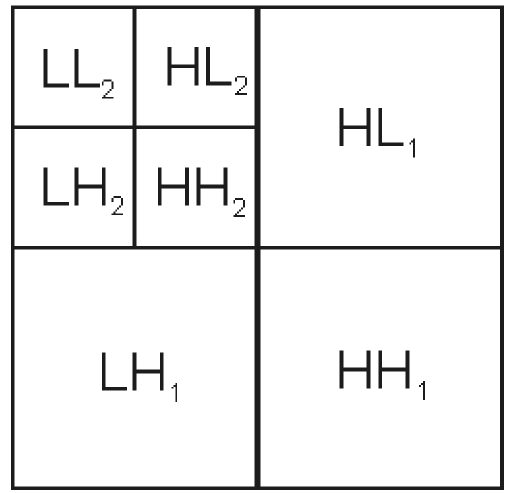
2D Haar Wavelet

- Non-standard Approach
  - Scaling Function

$$\Phi(x,y) = \varphi(x)\varphi(y),$$

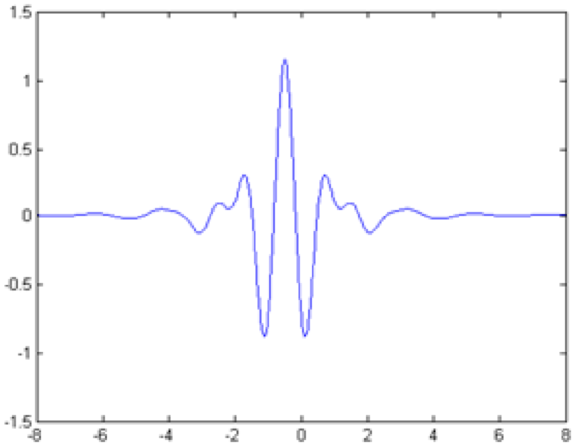
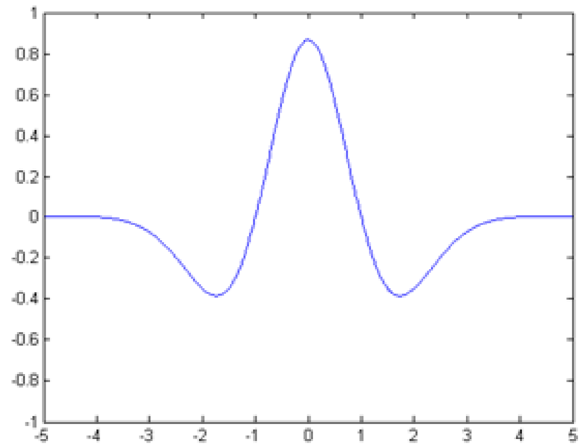
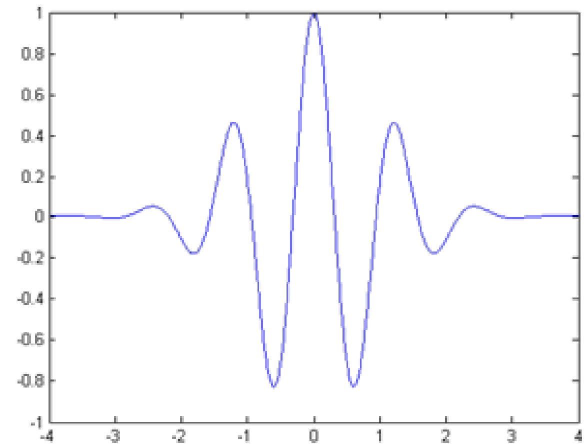
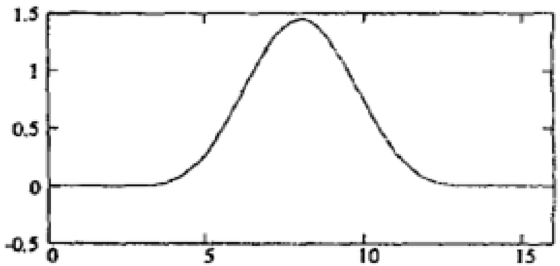
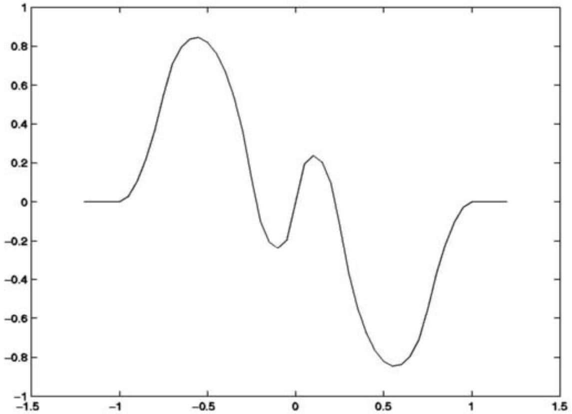
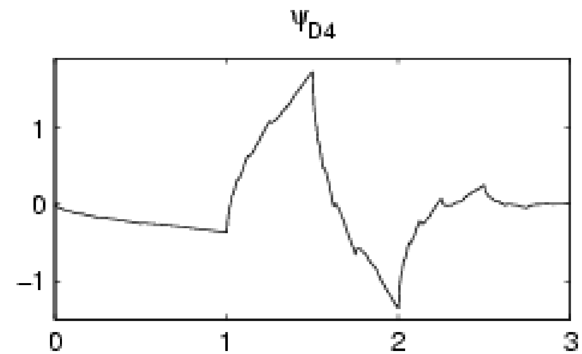
  - Wavelet Functions

$$\Psi_1(x,y) = \varphi(x)\psi(y),$$
$$\Psi_2(x,y) = \psi(x)\varphi(y),$$
$$\Psi_3(x,y) = \psi(x)\psi(y).$$



Wavelet Functions

- Different purposes, different wavelets,






,


2D Wavelet in Application

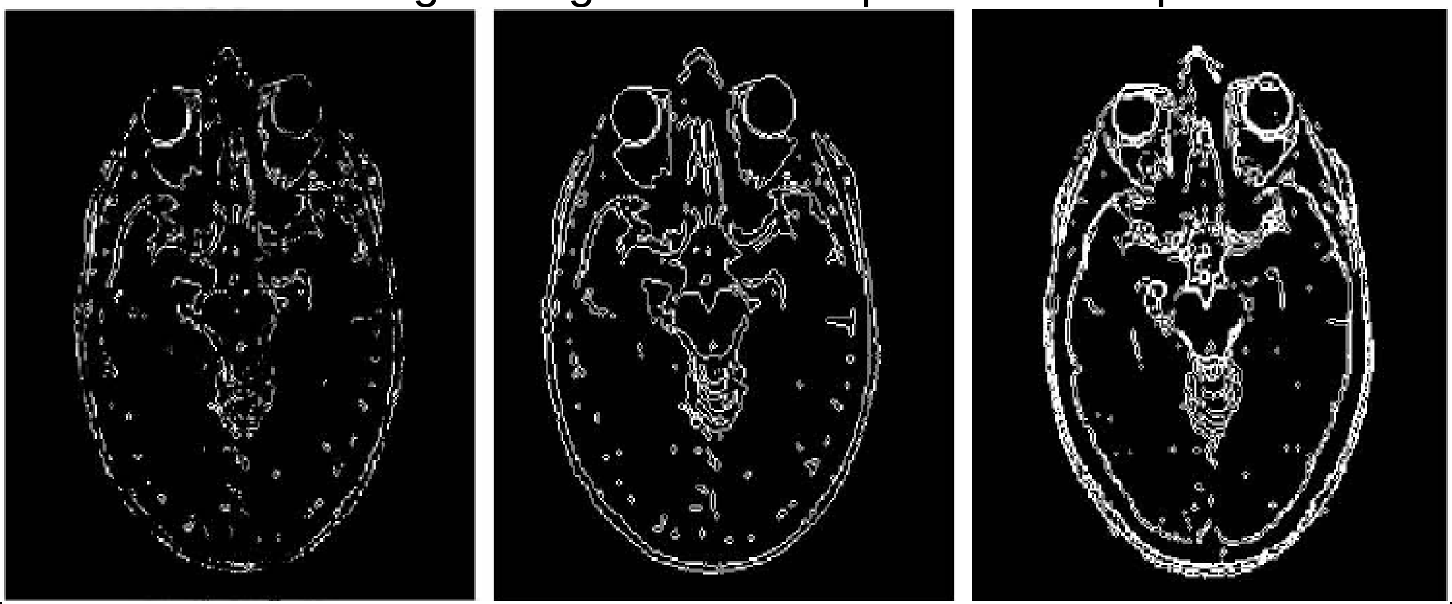
- Filtering
- Image Comparison
- Fault Prognosis
- Classification
- Document Analysis
- Hardware Implementation
- Feature Extracting(*)

Feature Extraction

- “Wavelet Segmentation for Fetal Ultrasound Images”, 2001
  - Efficient algorithm for segmentation
  - Decomposes the input image into a multiresolution space
  - Feature vector for each pixel in the wavelet domain including
    - Mean
    - Variance
  - Fuzzy C-means Clustering for segmentation
  - Less noisy segmented image

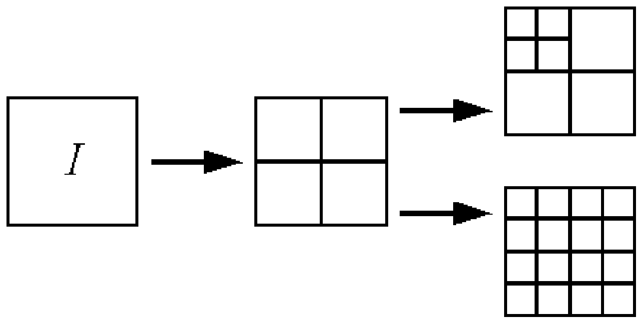
Feature Extraction

- "Efficient Segmentation in MRI Applying Discrete Wavelet Transform", 2005
  - Main goal is a better identification of abrupt changes without increasing the presence of noise.
  - A sliding window scanning the image and calculating the wavelet coefficients
  - Use of ICA to extract *Independent* wavelet descriptors
  - Kohenen's Self Organizing Feature Map as the unsupervised classifier



Feature Extracting

- "Choosing Best Basis in Wavelet Packets for Fingerprint Matching" 2004
  - Wavelet packets: An over-complete version of regular wavelet decomposition
  - The ridge-valley structure of fingerprint
  - Energy in a sub-band reaches a local MAX when the scale of wavelet packet basis best matches the spatial freq. of the ridge-valley struct.
  - Energy of different sub-bands gives information regarding both the edge spatial frequency as well as the ridge orientation.
  - A tree selection algorithm for selecting the best informative branch.



Feature Extracting

- "Advanced System for Automating Eddy-Current Nondestructive Evaluation", 2000
  - Measuring a metal's impedance using c-scanning mode, which gives a matrix format data, like an image.
  - Images divided into sub-images, wavelet decomposition is applied on them.
  - Average, standard deviation of each sub-images as the feature vector.
  - A neural network as a classifier to classify sub-surface flaws.

Conclusion

- About five books and 17 papers
- Most of the papers were published after 2000
- More focus on feature extraction application
- Different methods for feature extraction